การประยุกต์ใช้การปรับให้สอดคล้องและค้นหาความผิดพลาดอย่างเห็นได้ชัดของข้อมูลในถังผสม สำหรับกระบวนการผลิตแบตเตอรี

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APPLICATION OF DATA RECONCILIATION AND GROSS ERROR DETECTION IN MIXING TANK FOR BATTERY PRODUCTION PROCESS

Miss Jidapa Somsupun

A Thesis Submitted in Partial Fulfillment of the Requirements
for the Degree of Master of Engineering Program in Chemical Engineering
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Field of Study	Chemical Engineering
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การควบคุมและการศึกษาพฤติกรรมของกระบวนการในโรงงานอุตสาหกรรมเคมีนั้น ต้องอาศัยเครื่องมือวัดเพื่อให้ได้มาซึ่งข้อมูลของระบบที่ต้องการ ซึ่งข้อมูลที่ได้จากการวัดจะมีความผิดพลาดของข้อมูลเนื่องจากเครื่องมือวัดมีความผิดพลาดในการวัดรวมอยู่ด้วย ค่าความผิดพลาดของข้อมูลที่ได้จากการวัดนั้นสามารถทำให้ประสิทธิภาพของกระบวนการลดต่ำลง ดังนั้นเทคนิคการปรับให้สอดคล้องและค้นหาความผิดพลาดอย่างเห็นได้ชัดของข้อมูล ได้ถูกนำมาประยุกต์ใช้อย่างแพร่หลาย เพื่อลดผลกระทบจากความผิดพลาดอย่างเห็นได้ชัดของข้อมูลที่ได้จากการวัด ดังนั้นงานวิจัยนี้ จึงสนใจที่จะศึกษาการประยุกต์ใช้เทคนิคดังกล่าว กับถังผสมสำหรับกระบวนการผลิตแบตเตอรี

งานวิจัยนี้แบ่งกรณีศึกษาออกเป็น 2 กรณีด้วยกัน กรณีศึกษาที่ 1 มีวัตถุประสงค์เพื่อหาข้อมูลที่ได้จากการปรับให้สอดคล้อง โดยใช้หลักการกระจายตัว 3 แบบคือ Contaminated normal distribution, Lorentzian distribution, และ Hampel's redescending M-estimator กรณีศึกษาที่ 2 เป็นกรณีทดสอบ มีวัตถุประสงค์เพื่อทดสอบประสิทธิภาพของการกระจายตัว 3 แบบดังกล่าวเมื่อมีการเปลี่ยนระดับความรุนแรงของความผิดพลาดอย่างเห็นได้ชัดของข้อมูลที่ได้จากการวัดในรูปแบบของการเปลี่ยนขนาดและการเปลี่ยนปริมาณของความผิดพลาดอย่างเห็นได้ชัด ซึ่งผลจากการใช้วิธีดังกล่าวพบว่า กรณีที่ข้อมูลวัดมีเฉพาะความผิดพลาดแบบสุ่ม (Random Error) ทั้งสามวิธีให้ผลออกมาเป็นที่น่าพอใจ สำหรับกรณีที่มีทั้งความผิดพลาดแบบสุ่ม (Random Error) และความผิดพลาดอย่างเห็นได้ชัด (Gross Error) พบว่า วิธีของ Hampel's redescending M-estimator ให้ผลดีที่สุดสำหรับถังผสมของกระบวนการผลิตแบตเตอรี

ภาควิชา วิศวกรรมเคมี	ลายมือชื่อนิสิต.....จิดาภา โสมสุพรรณ์.....
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Process measurements are taken in chemical plant for the purpose of evaluating process control or process performance. In general, measured data inherently contain inaccurate information because measurements are obtained with imperfect instruments. The error in measured data can lead to significant deterioration in process performance. Data reconciliation and gross errors detection techniques are the techniques widely applied in various production processes for reducing the effect of measurement error. Therefore, the application of these techniques in mixing tank for battery production process is very attractive.

This work is divided into two case studies. The first case study is to determine the reconciled data from the data reconciliation problem formulated by using three different methods: Contaminated normal distribution, Lorentzian distribution, and Hampel's redescending M-estimator. The other case study is to test the performance of these three methods under the condition of measurement with different degree of error. The results of these methods for nominal case which contain only random errors the three different methods used provide good reconciled value and in the test case which contain both gross errors and random errors, the ρ function of Hampel's redescending M-estimator is the most appropriated function for data reconciliation in mixing tank of battery production process.

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NOMENCLATURES

NOMENCLATURE

A	Process matrix
Lb	Lower bound
Q	Variance-Covariance matrix
Ub	Upper bound
a_H , b_H , c_H	Tuning constants of the ρ function of Hampel's redescending M-estimator
b_{CN} , p_{CN}	Tuning constants of the ρ function of contaminated normal distribution
c_L	Tuning constants of the ρ function of Lorentzian distribution
f	Equality process model constraint
g	Inequality process model constraint
n	Total number of variable
x	True value of measured variable
$x_{nominal}$	Reconciled data in a nominal case
x_{test}	Reconciled data in a test case
x^*	Reconciled variable
y	Measured variable

NOMENCLATURE

u	Unmeasured variable

SUBSCRIPTS

CN	Method of contaminated normal distribution
H	Method of Hampel's redescending M-estimator
L	Method of Lorentzian distribution
i	Variable number $(1,\ n)$
j	Steady state number $(1,\ J)$
k	Regressor variable number $(0,\ K)$
l	Data set number $(1,\ l_i)$
$nominal$	Nominal case
$test$	Test case
u	Unmeasured variable
x	Reconciled variable

GREEK LETTER

ε	Standard error
σ	Standard deviation
ρ	Monotone function

CHAPTER I

INTRODUCTION

This chapter intro to the research which consists of the importance and reasons for the research, the objectives, the scopes, the contributions, the activity plan, and the overview of the research presented in the following.

1.1 Importance and Reasons for Research

In any chemical process, many of variables such as flow rates, temperature, pressures, level, compositions, etc. are routinely measured and recorded from the process instrumentation and laboratory analyzer for the purpose of process control, process optimization and process economic evaluation. In general, measured data inherently contains inaccurate information since measurements are obtained with imperfect instruments.

The total error in measurement can be represented as the sum of the contributions from two types of errors: *random errors* and *gross errors*. The first one, it can arise from different sources some of which may be beyond the control of the design engineer, they cannot be completely eliminated and are always present in any measurement. The other one, gross errors is generated from nonrandom events such as instrument malfunctioning (due to improper installation of measuring devices), miscalibration, wear or corrosion of sensors, solid deposits, etc.

The random and gross errors in measured data can lead to significant deterioration in process performance. Small errors lead to deterioration in the performance of control system, whereas lager errors can nullify gains achievable through process optimization. In some cases, erroneous data can also drive the process into an uneconomic, or even worse, an unsafe operating regime. Therefore, it is important to reduce the effect of random and gross errors. Several data processing techniques can be used to reduce the effect of random and gross errors. One of these techniques widely applied in various chemical processes is *data reconciliation* and *gross errors detection* techniques.

As mentioned previously, data reconciliation and gross errors detection are the interested techniques for reducing the effect of random and gross errors in measurement. Therefore, it is very challenging to apply theses techniques for the actual process such as battery production process which is the studied process of this work.

1.2 Research Objective

The objective of the research is as follows:

1. To take advantage of information redundancy on a process to make a cross- check of real time process measurements.
2. To develop valid model of mixing tank for battery production process by combining data reconciliation with gross error detection.

1.3 Scope of Research

The scope of the research is as follows:

1. The mixing tank for battery production process is considered in this study.
2. An application of data reconciliation with gross error detection in mixing tank for battery production process is considered.
3. The data reconciliation problem is formulated as the optimization problem by robust method using robust distribution functions. The distribution functions that we studied are as follows:
    - Contaminated normal distribution.
    - Lorentzian distribution
    - Hampel’s redescending M-estimator.

1.4 Contribution of Research

The contribution of the research is as follows:

1. A mathematical model for mixing tank of battery production process is developed.
2. A current status of process variables and unmeasured variables of mixing tank for battery production process are estimated.

1.5 Activity Plan of Research

The activity plan of the research is as follows:

1. Review relevant information regarding mixing tank for battery production process and data reconciliation reviewed.
2. A mathematical model for mixing tank of battery production process is developed.
3. Industrial process operation data are collected.
4. Various different data reconciliation and gross error detection techniques are perform in order to estimate current status of the process.
5. Performances of the process calculated from reconcile and real process performances are compared.
6. Make a document.

1.6 Overview of Thesis

The organization of the research is as follows:

Chapter I is an introduction to the research which consists of the objective, the scope, the contribution, activity plan, and the overview.

Chapter II reviews the past work related to the battery production process, and the application of data reconciliation in chemical process.

Chapter III shows the theory of battery production process and data reconciliation.

Chapter IV presents the application of data reconciliation for battery production process.

Chapter V presents the conclusions of this work and the recommendations for the future work

CHAPTER II

LITERATURE REVIEW

This chapter reviews the work and the application of data reconciliation in any chemical process.

2.1 Data Reconciliation

For more than twenty years, reconciliation problem has received consideration in the literature. Kuehn and Davidson (1961) used Lagrange multipliers to solve for optimal adjustments to measurements for the case when either all or none of component flow rates are measured. Much more work has subsequently been done by Mah et al. (1976), Romagnoli and Stephanopoulos (1981) and Mah and Tamhane (1982). Britt and Leucke (1973) and Knepper and Gorman (1980) provided an algorithm that can be used to adjust plant data to meet the constraints. In the last ten years, Hlavacek (1977) and Mah (1981) developed procedures to handle very large flowsheets.

In order to reduce the number of balance equations to a minimum number, Vaclavek et al. (1976) proposed a two-step reduction. Later, Stanley and Mah (1981a) developed the concepts of global and local observability of state variables, given a set of measurements and constraints, for the nonlinear problem. They also (1981b) applied graph theory to mass-energy flow networks to classify unmeasured variables as globally (or locally) observable or unobservable. Crowe et al. (1983) used a matrix projection method to decompose the problem so that the measured and unmeasured variables can be evaluated sequentially. The essence is to construct a matrix, which is orthogonal to the matrix in balance equations, which corresponds to unmeasured quantities. The problem can then be divided into a minimization problem to reconcile redundant measurements and then equation solution for the unmeasured variables. In 1986, Crowe extended this method for problems with bilinear constraints. The unknown component flow rates and extents of reaction are deleted, as in linear case, by the constant projection matrix. Then, the unknown total flow rates are deleted via a second projection matrix, which is stochastic because of the definition in terms of measured concentrations. The adjustments to component flow rates are iterative determined, starting with guessed values of unmeasured total flow rates.

Gertler and Almasy (1973) treated the linear dynamic data reconciliation. They showed that the dynamic material balance model could be represented by continuous-state space equations or after discretization by a sampled input-output representation.

Narasimhan and Mah (1988) have extended the formulation of the hypothesis of Generalized Likelihood Ratio (GLR) method proposed by Willsky and Jones (1974) for gross error identification in closed-loop dynamic processes described by a stochastic linear discrete model. For estimating the time of occurrence of the gross error, a simple chi-square test on the innovations (measurement residuals) is used, which is computationally more efficient than the method used by Willsky and Jones. Through simulation studies of a level control process the appropriate selection of parameters of the GLR method is investigated. A new method for incorporating of data reconciliation and gross error detection was proposed by Narasimhan and Harikumar (1991). In Part I, the reconciliation problem that includes bounds on the process variables has been solved using a Quadratic Programming (QP) algorithm. More importantly, a method to obtain the statistical distributions of measurement residuals and constraint residuals has been developed which is useful for gross error detection. Gross error detection methods based on this approach are described in Par II. Simulation results show that compared to currently available methods, the proposed methods give better gross error detection performance and more accurate estimates which always satisfy the bounds especially when tight bounds are specified.

Almasy (1990) has presented a method for dynamic data reconciliation in state space model form, in which the environmental effects (EE) are described by a random walk process. The method is based upon using linear conservation equations to reconcile measured states. In this approach only balance equations are utilized. Other modeling equations are neglected due to claims that dynamic filtering can not be performed sufficiently quickly unless the model is linear. The data reconciliation in this case is reduced to a discrete Kalman Filter as in the quasi-steady state problem. After that, Darouach and Zasadzinski present a new on-line estimation algorithm for the systems of dynamic material balance equations in 1991. In this work, the generalized linear dynamic model or singular model, for which the standard state space representation and the Kalman filtering can not be applied, is used to develop a new algorithm to solve the linear dynamic material balance problem. This algorithm is based on the method developed in the steady-state case and leads to a recursivescheme, which is very useful in real-time processing. It reduces the computational problem such as singularities and round-off errors that may occur in complex systems. Convergence conditions are given and verified for the dynamic material balance case.

The data reconciliation procedures can be extended to analyze unit operations to obtain performance parameter estimates, for example, tray efficiencies for distillation, heat transfer coefficients (Stephenson and Shewchuk, 1986) and reaction rate constants. Hlavacek (1977) suggested that parameter estimation could be done sequentially after reconciliation or simultaneously with it. MacDonald and Howat (1988) combined data reconciliation with process parameter estimation in an application involving a single stage flash and flash efficiency. The data reconciliation techniques are successfully extended to estimate flash efficiency. Two developments are presented. The first is a sequential, decoupled procedure that reconciles the data to satisfy the material and energy balances, and then estimates the process parameters using maximum-likelihood estimation. The second is a coupled procedure that simultaneously reconciles the data to satisfy the constraints and estimate the process parameters. The former is computationally faster and is more easily adapted to the existing reconciliation algorithms, but is not statistically rigorous. The later is statistically rigorous.

Weiss et al. (1996) successfully applied data reconciliation to an industrial pyrolysis reactor. Both linear and non-linear methods were used to solve the data reconciliation problem. The linear methods, which included successive linearization, yielded results very similar to those from the non-linear method. The large computational time required by the non-linear method could not be justified, and the majority of the study used only the successive linearization method. The approach was tested using plant data collected at regular intervals over a full operational cycle of the reactor. The overall heat transfer coefficient, one of the operating parameters of the pyrolysis reactor, calculated using reconciled data showed a trend consistent with plant experience and could be used to determine better regeneration cycle time of the reactor.

Tjoa and Biegler (1991) introduced a method which simultaneously reconciles the data and detects the gross errors by combining the treatment of small measurement errors and gross errors into a so-called contaminated Gaussian objective function instead minimize an objective function that is constructed using maximum likelihood principle to construct a new distribution function, which takes into account both contributions from random and gross errors. The advantages of minimizing this objective function are that it gives unbiased estimates in the presence of gross errors and that simultaneously a gross-error detection test can be constructed based on their distribution functions without the assumption on the linearity of the constraints. Furthermore, the structure of this objective function can be exploited under certain conditions. Thus, efficient nonlinear programming strategies, similar to the hybrid SQP method introduced by Tjoa and Biegler in 1991 for least squares objective functions, can also be developed. The effectiveness of this strategy is demonstrated on nonlinear example problems.

Chen, Pike, Hertwig and Hoppe (1998) studied optimal implementation of on-line optimization for Monsanto sulfuric acid contact plant. In data validation step, simultaneous gross error detection and data reconciliation algorithms are used to detect and rectify the gross errors in measurements. These algorithms are measurement test method using a normal distribution, Tjoa-Biegler’s method using a contaminated Gaussian distribution, and robust method using robust distribution functions (Lorentzian distribution, Fair distribution). In summary, the evaluation of influence functions for the probability distributions shows that the contaminated Gaussian and Lorentzian distributions have influence functions that are relatively insensitive to gross errors. Methods based on the contaminated Gaussian distribution should have the best performance for reconciling measurements when moderate size gross errors are present (rang $3\sigma - 30\sigma$), and methods using the Lorentzian distribution should be more effective for very large gross errors.

Özyurt and Pike (2004) compared different objective functions with the contaminated Gaussian function regarding their effectiveness in detecting gross errors of simultaneous procedures for data reconciliation and gross error detection is established. These procedures depending on the results from robust statistics reduce the effect of the gross errors. They provide comparable results to those from methods such as modified iterative measurement test method (MIMT) without requiring an iterative procedure. In addition to deriving new robust methods, novel gross error detection criteria are described and their performance is tested. The comparative results of the introduced methods are given for five literatures and more importantly, two industrial cases. Methods based on the Cauchy distribution and Hampel's redescending M-estimator give promising results for data reconciliation and gross error detection with less computation (OP and AVTI).

Mansour and Ellis (2008) presented a methodology of on-line optimization that consists of steady state detection, data reconciliation, gross error detection, parameter estimation and optimization, and applied under simulation. The methodology is implemented and tested on a two continuous stirred tank reactors (CSTR) system. The results shown that the application of this methodology can be used to achieve the optimum operating point of the system and also reduces the time wasted for the system to settle down.

Faber et al. (2006) studied the implementation of online optimization with model updating and data reconciliation for the ammonia hydrogen sulfide circulation scrubbing, a common industrial coke-oven-gas purification process. The process model was improved by using the data reconciliation technique to reconcile the measurement data before being used for model updating. The objective function of the reconciliation problem was formulated with gross error using the fair function. The results of this work shown that the obtained model which using the reconciled data can be used to build an online optimization problem for that process and achieved a satisfactory agreement.

CHAPTER III

THEORY

This chapter presents the theory of battery production process and data reconciliation. The organization of this chapter is as follows: the brief background of battery production process, and the basic concept of data reconciliation.

3.1 Battery Production Process

3.1.1 What is a Battery

A battery consists of one or more electro-chemical cells. Although the terms battery and cell are often used interchangeably cells are the building blocks of which batteries are constructed. Batteries consist of one or more cells that are electrically connected.

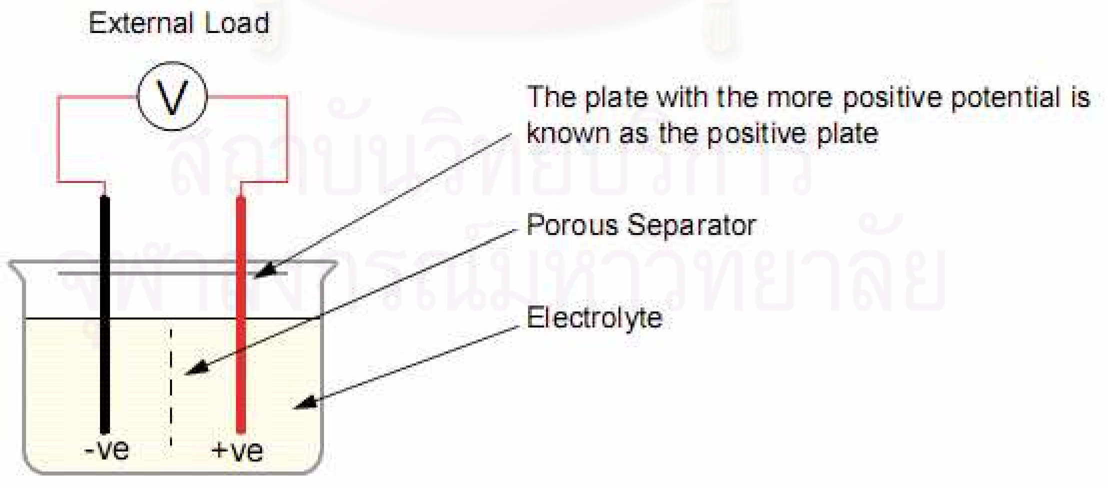


Figure 3.1 A single battery electro-chemical cell

A cell normally consists of the four principal components shown in Figure 3.1. These are:

- A positive electrode that receives electrons from the external circuit when the cell is discharged,
- A negative electrode that donates electrons to the external circuit as the cell discharges,
- Electrolyte which provides a mechanism for charge to flow between positive and negative electrodes,
- A separator which electrically isolates the positive and negative electrodes.

In some designs, physical distance between the electrodes provides the electrical isolation and the separator is not needed. In addition to the critical elements listed above, cells intended for commercial batteries normally require a variety of packaging and current collection apparatus to be complete.

When a battery or cell is inserted into a circuit, it completes a loop which allows charge to flow uniformly around the circuit. In the external part of the circuit, the charge flow is electrons resulting in electrical current. Within the cell, the charge flows in the form of ions that are transported from one electrode to the other. The positive electrode receives electrons from the external circuit on discharge. These electrons then react with the active materials of the positive electrode in “reduction” reactions that continue the flow of charge through the electrolyte to the negative electrode. At the negative electrode, “oxidation” reaction between the active materials of the negative electrode and the charge flowing through the electrolyte results in surplus electrons that can be donated to the external circuit.

It is important to remember that the system is closed. For every electron generated in an oxidation reaction at the negative electrode, there is an electron consumed in a reduction reaction at the positive.

As the process continues, the active materials become depleted and the reactions slow down until the battery is no longer capable of supplying electrons. At this point the battery is discharged.

The world of batteries divides into two major classes: primary and secondary batteries. Primary batteries such as the common torch battery are used once and replaced. The chemical reactions that supply current in them are irreversible. Secondary batteries can be recharged and reused. They use reversible chemical reactions. By reversing the flow of electricity i.e. putting current in rather than taking it out, the chemical reactions are reversed to restore active material that had been depleted.

Secondary batteries are also known as rechargeable batteries, storage batteries or accumulators.

3.1.2 Basic Battery Concepts

There are two parameters that measure battery performance: voltage and capacity. In very simple terms, the voltage is the force propelling each of the electrons coming out of a battery and the capacity is the number of electrons that can be obtained from a battery. How these parameters relate to batteries is explained below.

- *Voltage*

All lead batteries work on the same set of reactions and use the same active materials. At the positive electrode, lead dioxide (PbO_2) is converted to lead sulphate ($PbSO_4$) and at the negative electrode, sponge metallic lead (Pb) is also converted to lead sulphate ($PbSO_4$). The electrolyte is a dilute mixture of sulphuric acid that provides the sulphation for the discharge reactions. The reduction and oxidation reactions each produce a fixed potential. The sum of the reduction and oxidation potential is the voltage of the cell.

For example, the discharge reaction at the positive electrode for a lead-acid cell is

$$PbO_2 + SO_4^{-2} + 4H^+ + 2e^- \rightarrow PbSO_4 + 2H_2O$$

which has a potential of 1.685 volts. The reaction at the negative electrode is

$$Pb + SO_4^{-2} \rightarrow PbSO_4 + 2e^-$$

which has a potential of 0.356 volts. This means that the overall voltage of a lead-acid cell is 2.04 volts. This value is known as the standard electrode potential. Other factors, such as the acid concentration can also affect the voltage of a lead-acid cell. The typical open circuit voltage of commercial lead-acid cells is around 2.15 volts. Thus, the voltage of any battery cell is established depending on the cell chemistry. Nickel-cadmium cells are about 1.2 volts, lead-acid cells are about 2.0 volts, and lithium cells may be as high as nearly 4 volts. Cells can be connected together so that their voltages accumulate. This means lead-acid batteries with nominal voltages of 2v, 4v, 6v, etc. are available.

- *Capacity*

While the voltage of a cell is fixed by its chemistry, cell capacity is variable depending on the quantity of active materials it contains. Individual cells may range in capacity from fractions of an ampere-hour to many thousands of ampere-hours. The capacity of a cell is essentially the number of electrons that can be obtained from it. Since current is the number of electrons per unit time, cell capacity is the current supplied by a cell over time and is normally measured in ampere-hours (Ah).

- *Voltage and Capacity*

Batteries normally consist of multiple cells that are electrically connected. The way that the electrical connections are made determines the voltage and capacity of the battery. If the positive terminal of one cell is connected to the negative terminal of the next and so on through the battery the result, as illustrated in Figure 3.2, is called a series connected battery. The voltage of this type of battery is the sum of the individual cell voltages. For example, a 12-volt battery consists of 6 x 2-volt lead-acid cells connected in series. Although the voltages add, the cell capacity is fixed at the value for the individual cell.

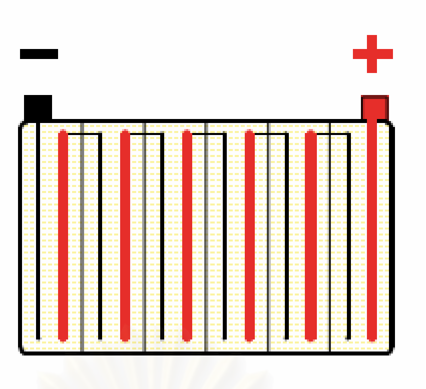


Figure 3.2 12V battery block (6 cells in series)

The other way to connect cells within a battery is to connect the negative terminal from one cell to the negative of the next cell and to connect the positive terminal to the positive terminal. When this is done throughout the battery, the result is the parallel-connected battery. The capacities of the individual cells add to make the battery capacity but the battery voltage remains as the volt-age of the individual cell.

Series-connected batteries are far more common than parallel-connected. Usually it is easier to get added capacity by just using a larger cell rather than a parallel-connected battery.

All of the battery connections may be made internally so that it is difficult to determine the number of cells by external examination. However, knowing the voltage of the basic cell, it is easy to determine the number of cells by dividing the cell voltage into the battery voltage.

Cells used for batteries should always be identical. Mixing cells of different chemistry or different size may be hazardous and should be avoided.

3.1.3 Battery Construction

While there are various choices for a rechargeable system, lead-acid batteries are still the most commonly used for UPS and Stand-by systems. Lead-acid batteries are usually more economical and have a high tolerance for abuse. The fact that all of the batteries used for starting, lighting, and ignition (SLI) service on automobiles and trucks are lead-acid indicates their ability to withstand varied forms of maltreatment.

Lead-acid batteries also provide motive power for everything from fork-lift to submarines. Lead-acid batteries are also mainstays of the backup systems that provide power when the electrical supply fails. Now, development of the sealed-lead battery has allowed lead technology to be used in applications such as electronics that need a clean power source.

Purely for convenience, batteries are made in 12 volt blocks using six cells but are also available in 6 volt (three cell), 4 volt (two cell) and even 2 volt (single cell) blocks. There are two main types of lead-acid batteries which may be used in back up power applications (as shown in Table 3.1):

Table 3.1 Two main types of lead-acid batteries which may be used in back up power applications

OPEN-VENTED	SEALED/VRLA
Older technology	Environmentally friendly
Separate battery room	Suitable for office environments
Regular routine maintenance	Low regular routine maintenance
Separate safety requirements	Self-contained. Safe
Store/use in vertical position	Store/use in any orientation

- *Open Vented*

Secondary batteries are made so that their chemical reactions can be reversed. This feature enables them to be recharged efficiently after they have delivered their electric energy. The most common types of secondary batteries are lead-acid and nickel-cadmium storage batteries.

Lead-acid batteries consist of a plastic or hard-rubber container that holds three or six cells. Each cell has two sets of lattice like electrodes or plates. The frames of these structures, called grids, are made of a lead-antimony alloy. The meshes (open spaces) of the negative electrode are filled with a mass of pure lead in spongy form.

The meshes of the positive electrode contain lead dioxide, a compound of lead and oxygen. An electrolyte of sulphuric acid and water surrounds the electrodes.

Most lead-acid storage batteries have six cells. Each cell contains two sets of lead electrodes called plates. The plates are separated by plastic or rubber sheets. A solution of sulphuric acid, called the electrolyte, surrounds the plates. Each terminal post on the outside of the battery is connected to one set of plates. Vent holes in the case allow water to be added to the electrolyte and also permit gases produced in the cell to escape.

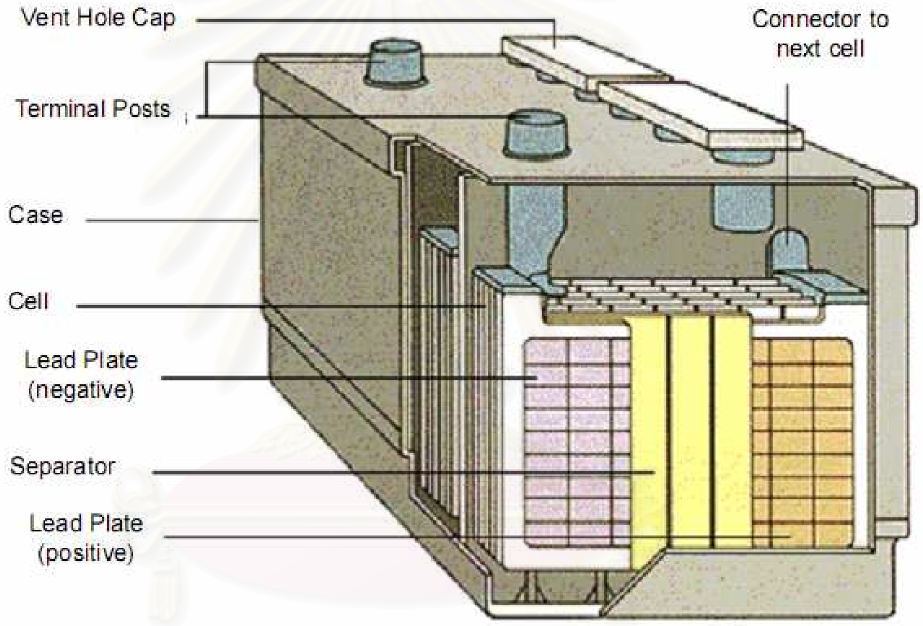


Figure 3.3 Typical lead acid battery construction.

During the discharge process, chemical reactions take place between the electrode materials and the electrolyte. At the negative electrode, atoms of pure lead react with negative sulphate ions of the electrolyte. The negative sulphate ions, along with positive hydrogen ions, form when sulphuric acid dissolves in water. As the lead atoms combine with the sulphate ions, each lead atom loses two electrons and becomes a molecule of lead sulphate.

The electrons lost by the lead atoms flow from the negative electrode to the positive electrode through a device using the electric current. At the positive electrode, they are captured by molecules of lead dioxide, which in turn combine with the hydrogen and sulphate ions of the electrolyte. This reaction produces lead sulphate and water.

Adding together the positive and negative electrode reactions yields a combined discharge reaction. Thus, sulphuric acid is consumed and water is produced during battery use. Eventually the sulphuric acid becomes so diluted that the necessary chemical reactions can no longer occur.

After a lead-acid battery loses its ability to supply electricity, it can be recharged by means of a battery charger. The battery charger forces electrons through the battery in a direction opposite to that of the discharge process. This action reverses the chemical reactions that occur when a bat-tery discharges.

The reversed reactions of the charging process restore the electrode materials to their original form. They also increase the amount of sulphuric acid in the electrolyte to a satisfactory level. After a battery has been charged, it can again produce current.

Nickel-cadmium storage batteries operate on the same general principles as lead-acid batteries but use different chemical substances. In a nickel-cadmium battery, the negative electrode is made of cadmium and the positive electrode of nickel oxide. A solution of potassium hydroxide serves as the electrolyte.

The chemical composition of a nickel-cadmium battery allows the battery container to be sealed airtight, which prevents the corrosive electrolyte from leaking. Because of this advantage, nickel-cadmium batteries are used in drills, garden tools, and other portable equipment. Most space satellites use these batteries.

- *Sealed Rechargeable Batteries*

Development work on the sealed-lead battery was begun in 1967 by subsidiaries of The Gates

Corporation with first commercial uses occurring in the early Seventies. It has become an accepted high-performance power source for clean applications including computer power and power backup, telecommunications, emergency lighting, security alarms, and consumer products. It is also becoming popular in cordless tools and appliances, electric vehicles, and other applications which require frequent discharges.

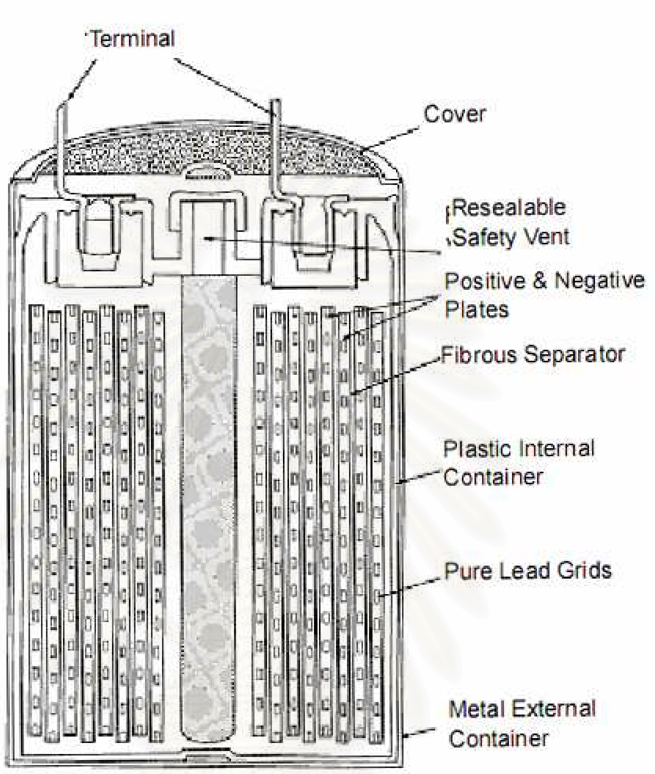


Figure 3.4 Sealed lead acid battery construction.

The sealed lead cell shown in Figure 3.4 consists of positive and negative electrodes and their accompanying separators that are wound in a spiral pattern. The electrodes consist of pure lead grids pasted with mixtures of lead oxides. These oxides are converted to the proper active materials when the cell receives its first charge in a process called formation. The pure lead supporting grids allow the flexibility needed for winding the plate and also give excellent corrosion resistance to prolong cell life. The separator consists of a fibrous glass mat. The cell works as a starved electrolyte system where the quantity of electrolyte is limited to the amount that is either absorbed in the plates or wets the fibers in the separator. The result is open gas paths between the plates that allow gases evolved during overcharge to diffuse from one plate to the other where they are recombined. This recombination provides a closed system reducing venting of gases under normal overcharge conditions. A resealing safety vent is provided to handle pressure buildup during abusive overcharges. Since the electrolyte is recycled, the water loss that requires routine maintenance or limits life is minimized. The sealed-lead system has proven to provide high performance and long life in a clean, compact package.

3.1.4 Battery Process Background

The principal of modern lead acid batteries was discovered in the laboratory by the French physicist, Mr. Gaston Plante in 1860. Using pure lead plates and dilute sulfuric acid as an electrolyte he was able to charge the plates and then measure a current when discharging them through a load. Twenty years later Mr. Henri Tudor, a Luxembourg Engineer, started manufacturing lead acid batteries using pure lead Plante plates for the positive plate and a pasted negative plate.

The lead acid battery has been changed and improved many times over the years but the electrode reaction in all lead acid batteries is basically the same regardless of design or construction. As the battery is discharged the lead dioxide positive active material and the sponge lead negative material both react with the dilute sulfuric acid electrolyte to form lead sulphate and water, during charging this process is reversed.

The efficiency of the charge / discharge process is less than 100% because during the charge process the voltage has to be increased (over the discharge voltage) by about 7 to 10% to overcome chemical inefficiency and the cells internal resistance.

Over the years many different alloys of lead have been used to reduce the high cost of manufacturing the pure lead Plante plates.

Antimony lead alloy was first used, with up to 9% alloyed with pure lead; this caused the cell to gas freely and increased the current needed to charge the batteries. In later designs, the antimony was reduced to less then 2% to reduce the maintenance interval, in example, the intervals between the need to add distilled or de-ionized water.

In the 1960s calcium was alloyed with lead, this resulted in longer cell life, reduced load current requirements and reduced the need to add water to the cell, i.e. reduce the maintenance interval. In the early 1980's sealed valve regulated VRLA batteries were developed for use in telecommunications applications.

It has been shown that in vented / flooded electrolyte cells the positive electrode in a lead acid battery accepts charging less efficiently then the negative plate. Under typical charging conditions oxygen evolution at the positive plate peaks at a charge ampere hour / discharge ampere hour ratio of approximately 0.94, whereas the rate of hydrogen evolution at the negative plate reaches a maximum at a charge amp hour / discharge amp hour ration of 1:0.

This feature is used in the design of sealed valve regulated cells which oversize the negative plate to design a 'positive limited' cell, thus reducing the overall gas evolution, i.e. limiting the hydrogen evolution. Gas evolution was further reduced in sealed valve regulated cells by using antimony free lead alloys with reduced gassing rates and immobilized electrolyte either gelled or absorbed.

Thus today there are three main types of vented / flooded batteries:

(1) the pure Plante lead plate,

(2) the flat pasted plate and

(3) the tubular positive plate and flat pasted negative plate batteries.

Battery types (2) and (3) are commercially available in either antimony or calcium lead alloys.

The Plante positive plate batteries can provide long life but at a relatively high expense, the flat pasted plate and tubular plate cells are therefore commercially the most popular today.

With the vented / flooded batteries there is the presence of hydrogen and oxygen thus batteries must be housed in well vented, preferably air conditioned, and battery rooms. Maintenance / cleaning or corroded terminals and the frequent addition of water to the cells add to the battery total maintenance costs. This maintenance can be greatly reduced by the use of gas recombining caps which reduce the vented gas and thus maintenance by up to 75%. However, the now well established sealed valve regulated VRLA batteries are becoming the industry norm due to their considerable cost saving features:

1. They do not need to be put in special battery rooms as they do not gas appreciably in normal service, thus they can be placed in battery racks or cabinets or be placed free standing in the equipment room.

2. They are spill-proof and leak-proof therefore the maintenance to terminals and intercell interconnects is minimal

3. They generate less hydrogen gas, when properly charged and used, and are a safer product.

4. They are much more compact in design, saving up to 75% of the present floor space requirements.

5. Due to high production levels the cost of sealed valve regulated batteries is about the same as the installed cost of vented/flooded batteries.

3.2 Data Reconciliation

3.2.1 Introduction to Data Reconciliation and Gross Error Diagnosis

In any modern chemical plant, petrochemical process or refinery, hundreds or even thousands of variables such as flow rates, temperatures, pressures, levels, compositions, etc. are routinely measured and automatically recorded for the purpose of process control, online optimization or process economic evaluation. Modern computers and data acquisition systems facilitate collection and processing of a great volume of data, often sampled with a frequency of the order of minutes or even seconds. The use of computers not only allows data to be obtained at a greater frequency, but has also resulted in the elimination of errors present in manual recording. This in itself has greatly improved the accuracy and validity of process data. However, the increased amount of information can be exploited for further improving the accuracy and consistency of process data through a systematic data checking and treatment.

Process measurements are inevitably corrupted by errors during the measurement, processing and transmission of the measured signal. The total error in a measurement, which is the difference between the measured value and the true value of a variable, can be conveniently represented as the sum of the contributions from two types of errors - random errors and gross errors. The term random error implies that neither the magnitude nor the sign of the error can be predicted with certainty. In other words, if the measurement is repeated with the same instrument under identical process conditions, a different value may be obtained depending on the outcome of the random error. The only possible way these errors can be characterized is by use of probability distributions. These errors can be caused by a number of different sources such as power supply fluctuations, network transmission and signal conversion noise, analog input filtering, changes in ambient conditions, etc. Since, these errors can arise from different sources some of which may be beyond the control of the design engineer, they cannot be completely eliminated and are always present in any measurement. They usually correspond to the high frequency components of a measured signal, and are usually small in magnitude except for some occasional spikes.

On the other hand, gross errors are caused by nonrandom events such as instrument malfunctioning (due to improper installation of measuring devices), miscalibration, wear or corrosion of sensors, solid deposits, etc. The non-random nature of these errors implies that at any given time they have a certain magnitude and sign, which may be unknown. Thus, if the measurement is repeated with the same instrument under identical conditions, the contribution of a gross error to the measured value will be the same. By following good installation and maintenance procedures, it is possible to ensure that gross errors are not present in the measurements at least for some time. Gross errors caused by sensor miscalibration may occur suddenly at a particular time and thereafter remain at a constant level or magnitude. Other gross error causes such as wear or fouling of sensors can occur gradually over a period of time and thus the magnitude of the gross error increases slowly over a relatively long time period. Thus, gross errors occur less frequently but their magnitudes are typically larger than those of random errors.

Errors in measured data can lead to significant deterioration in plant performance. Small random and gross errors can lead to deterioration in the performance of control systems, whereas larger gross errors can nullify gains achievable through process optimization. In some cases, erroneous data can also drive the process into an uneconomic, or even worse, an unsafe operating regime. It is therefore important to reduce, if not completely eliminate, the effect of both random and gross errors. Several data processing techniques can be used together to achieve this objective. In this text we describe methods which can play an important role as part of an integrated data processing strategy to reduce errors in measurements made in continuous process industries.

Research and development in the area of signal conditioning have led to the design of analog and digital filters which can be used to attenuate the effect of high frequency noise in the measurements. Large gross errors can be initially detected by using various data validation checks. These include checking whether the measured data and the rate at which it is changing is within predefined operational limits. Nowadays smart sensors are available which can perform diagnostic checks to determine whether there is any hardware problem in measurement and whether the measured data is acceptable. More sophisticated techniques include statistical quality control tests (SQC) which can be used to detect significant errors (outliers) in process data.. These techniques are usually applied to each measured variable separately. Thus, although these methods improve the accuracy of the measurements, they do not make use of a process model and hence do not ensure consistency of the data with respect to the inter-relationships between different process variables. Nevertheless, these techniques must be used as a first step to reduce the effect of random errors in the data and to eliminate obvious gross errors. It is possible to further reduce the effect of random error and also eliminate systematic gross errors in the data by exploiting the relationships that are known to exist between different variables of a process. The techniques of data reconciliation and gross error detection that have been developed in the field of chemical engineering during the past 35 years for this purpose is the principal focus of this book.

Data reconciliation is a technique that has been developed to improve the accuracy of measurements by reducing the effect of random errors in the data. The principal difference between data reconciliation and other filtering techniques is that data reconciliation explicitly makes use of process model constraints and obtains estimates of process variables by adjusting process measurements so that the estimates satisfy the constraints. The reconciled estimates are expected to be more accurate than the measurements and, more importantly, are also consistent with the known relationships between process variables as defined by the constraints. In order for data reconciliation to be effective, there should be no gross errors either in the measurements or in the process model constraints. Gross error detection is a companion technique to data reconciliation that has been developed to identify and eliminate gross errors. Thus data reconciliation and gross error detection are applied together to improve accuracy of measured data.

Data reconciliation and gross error detection both achieve error reduction only by exploiting the redundancy property of measurements. Typically, in any process the variables are related to each other through physical constraints such as material or energy conservation laws. Given a set of such system constraints, a minimum number of error free measurements is required in order to calculate all of the system parameters and variables. If there are more measurements than this minimum, then redundancy exists in the measurements which can be exploited. This type of redundancy is usually called spatial redundancy and the system of equations is said to be overdetermined. Data reconciliation cannot be performed without spatial redundancy. With no extra measured information, the system is just determined and no correction to erroneous measurements is possible. Further, if fewer variables than necessary to determine the system are measured, the system is underdetermined and the values of some variables can be estimated only through other means or if additional measurements are provided.

A second type of redundancy that exists in measurements is temporal redundancy. This arises due to the fact that measurements of process variables are made continually in time at a high sampling rate, producing more data than necessary to determine a steady state process. If the process is assumed to be in a steady state, then temporal redundancy can be exploited by simply averaging the measurements, and applying steady state data reconciliation to the averaged values. However, if the process state is dynamic then the evolution of the process state is described by differential equations corresponding to mass and energy balances, which inherently capture both the temporal and spatial redundancy of measured variables. For such a process, dynamic data reconciliation and gross error detection techniques have been developed to obtain accurate estimates consistent with the differential model equations of the process.

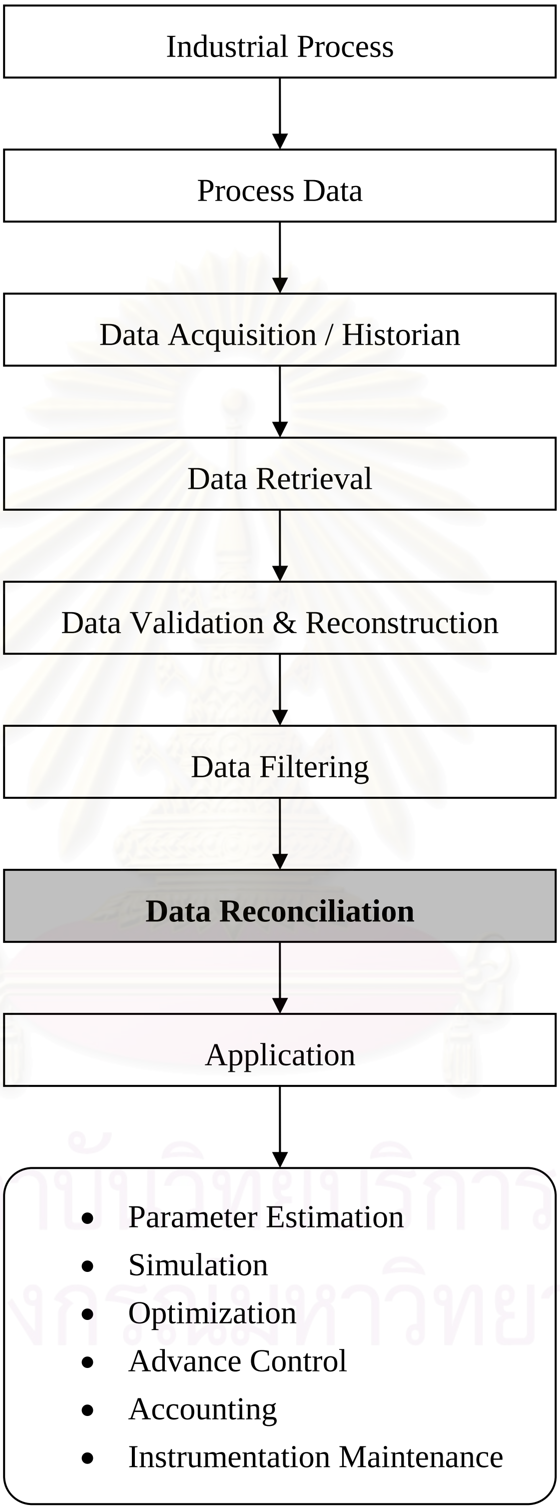


Figure 3.5 Online Data Collection and Conditioning System.

Signal processing and data reconciliation techniques for error reduction can be applied to industrial processes as part of an integrated strategy referred to as data conditioning or data rectification. Figure 3.1 illustrates the various operations and the position occupied by data reconciliation in data conditioning for online industrial applications

3.2.2 Definition Different Objective Functions for Formulate Data Reconciliation Problem is as the Optimization Problem

Maximum Likelihood Estimation (MLE).

If the measurement error distribution follows a normal distribution, the Data Reconciliation problem can be posed as a Maximum Likelihood Estimation (MLE) problem, where the probability of the estimated (reconciled) process variables (*y*) is maximized given the measurement set (*y*) as shown in Equation 3.1

$$\max \ P\{x/y\} \tag{3.1}$$

According to Bayes' theorem, the probability of the process variables given the measurements can be written in terms of the probability of the measurements given the reconciled process variables, the probability density function of the process variables $P\{x\}$ and the probability density function of the measurements $P\{y\}$.

$$\max \ P\{x/y\} = \max \ \frac{P\{y/x\}\, P\{x\}}{P\{y\}} \tag{3.2}$$

The denominator term (independent of x) acts as normalizing constant and does not need to be further considered for optimization. The first term in the numerator represents the probability density of the measurements given the reconciled process variables, x, which is the distribution of the measurements errors $P(y-x)$. Finally, $P\{x\}$is a binary assumption, that is equal to 1 if the constraints are satisfied (under this assumption the $P\{x\}$ term converted to a set of constraints and the original problem is converted to a constrained optimization) and equal to 0 otherwise.

$$P(y-x) = P(\varepsilon) \tag{3.3}$$

If sensor errors are independents the product of this probability over all sensors yields to:

$$P\{x/y\}=\prod_i \exp\left\{-\frac{1}{2}\left(\frac{y_i-x_i}{\sigma_i}\right)^2\right\}=\exp\left\{-\frac{1}{2}\sum_i\left(\frac{y_i-x_i}{\sigma_i}\right)^2\right\} \qquad (3.4)$$

Taking the negative logarithm of the maximization of the objective function represented in Equation 3.4 results in the minimization of the conventional WLS formulation. The symmetric and positive defined matrix Q contains the variance-covariance elements of the measurement errors and thus quantifies the uncertainty in each measured value. Then the success of data reconciliation technique relies on the hypothesis that the error is normally distributed and on the evaluation of matrix Q.

3.2.3 Gross Error Detection (GED)

The technique of DR crucially depend on the assumption that d values, only random error are present in the data and systematic errors either in the measurement or the model equation are not present. If this assumption is valid, reconciliation can lead to large adjustments being made to the measured values, and the resulting estimates can be very inaccurate and even infeasible. Thus it is important to identify such systematic or gross error before the final reconciled estimated are obtained.

There are two major types of gross errors. One is related to the instrument performance and includes measurement bias, drifting, miscalibration, and total instrument failure. The other is constraint model-related and includes unaccounted loss of material and energy resulting from leaks from process equipment or model inaccuracies due to inaccuracies parameters.

Usually gross errors are associated with sensor faults. In Figure 3.1, illustrates graphically the most common types of instrument; bias, complete failure, drifting, and precision degradation.

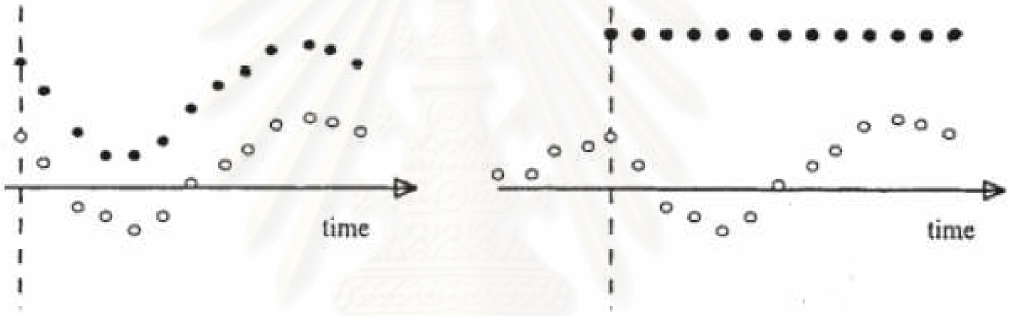
Various techniques have been designed for the detection and elimination of these two types of gross errors: statistical tests approaches. Any comprehensive gross error detection strategy should preferably processes the following capabilities:

- Ability to detect the presence of one or more gross error in the data (the detection problem)

- Ability to identify the type and location of the gross error (the identification problem)

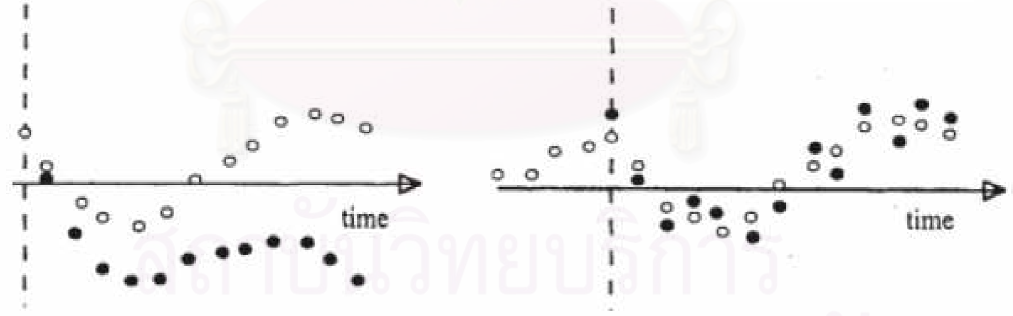
- Ability to locate and identify multiple gross error which may be present simultaneous in the data (the multiple gross error identification problem)

- Ability to estimate the magnitude of the gross error (the estimation problem)



(a) Bias

(a) Complete Failure



(c) Drifting

(a) Precision Degradation

○ ○ ○ Corrected Data

● ● ● Corrupted Data

Figure 3.6 Types of gross errors (Narasimhan & Jordache, 2000)

A number of statistical tests are derived from this basic statistical principle and are able to detect gross errors. But not all statistical tests are able to identify different types and location of gross errors. Some basic statistical tests are able to detect only measurement error (biases). Other statistic test can only detect process model error or leaks. On the other hand, the generalized likelihood ratio test, which is derived from maximum likelihood estimation principle in statistics, can be used to detect both instrument problems and process leaks.

3.2.4 General Formulation

A data reconciliation problem begins with the acquisition of the process data measurements. To assess the performance of the process evaluation or control,

$$x^T = [x_1, x_2, x_3, ..., x_n] \tag{3.5}$$

is a set of system variables for which sensors are available to measure their state.

The result of a measurement session (data from the DCS) can be collected in a set of measurement vectors as follows

$$y_i^T = [\, y_{i,1}, y_{i,2}, ..., y_{i,li} \,] \; ; \qquad \text{for } i = 1,2,3,\ldots,n \tag{3.6}$$

where l_i is the number of sets of measurements taken during steady-state plant operation to estimate the system variable (x_i). l_i is equal to one if we are interested in the snapshot of the process and is greater than one if our concern is a smoothed average within a time window of interest.

If there were no gross errors in the system, the difference of the measured values and the system state would have a distribution around the mean zero, i.e.

$$y_{i,1} - x_i \,, y_{i,2} - x_i \,, y_{i,3} - x_i \,, ..., y_{i,li} - x_i \tag{3.7}$$

is a sample from a distribution with mean zero. Also, the unknown variance of this distribution can be estimated by using plant’s historical data.

The states of the system variables are determined by using the constraints that describe the process. Therefore, using a proper objective function in an NLP, estimates of the x_i's can be obtained which are expected to minimize these differences.

The formulation for the data reconciliation problem with the generalized least squares method has its root in the general regression model. Let us define a single measurement of the i^{th} measured variable at the j^{th} steady state as $y_{i,j}$. The k^{th} fixed regressor (explanatory or independent) variable that we believe to explain the variation between each steady state is called $z_{k,j}$. Then a linear regression problem with fixed regressors using generalized least squares estimation is posed as:

$$\min \sum_{j=1}^{J} \frac{(y_{i,j} - \theta_0 - \theta_{1z_{1,j}} \ldots - \theta_{kz_{k,j}})^2}{\sigma_j^2} \tag{3.8}$$

for which the regression model is stated as:

$$y_{i,j} = \theta_0 + \theta_{1z_{1,j}} \ldots + \theta_{kz_{k,j}} + \varepsilon_j , \tag{3.9}$$

$$E(\varepsilon_j) = 0 , \quad Var(\varepsilon_j) = \sigma_j^2 \quad \forall j$$

An estimate for the location of the steady state can be calculated using a special case of the linear regression problem described above, where k = 0 and the sum is over the steady-state points l.

$$\min \sum_{l=1}^{l_i} \frac{(y_{i,l} - \theta_0)^2}{\sigma_{i,l}^2} \tag{3.10}$$

The corresponding regression model is

$$y_{i,l} = \theta_0 + \varepsilon_{i,l} , \tag{3.11}$$

$$E(\varepsilon_{i.l}) = 0 , \quad Var(\varepsilon_{i,l}) = \sigma_{i,l}^2 \quad \forall l$$

The minimization problem Equation 3.6 can also be written as

$$\min \sum_{l=1}^{l_i} \frac{(y_{i,l} - x_{i,l})^2}{\sigma_{i,l}^{\;2}} \tag{3.12}$$

such that

$$x_{i,1} = x_{i,2} = \ldots = x_{i,l_i} = \theta_0$$

Formulation Equation 3.12 is equivalent to Equation 3.13 shown in the following.

$$\min \sum_{l=1}^{l_i} \frac{(y_{i,l} - x_{i,l})^2}{\sigma_{i,l}^{\;2}} \tag{3.13}$$

such that

$$Ax_i = 0, \qquad A((l_i - 1) \times l_i) = \begin{bmatrix} 1 & -1 & 0 & . & 0 \\ 0 & . & . & . & 0 \\ 0 & . & . & . & 0 \\ 0 & . & 1 & -1 & 0 \\ 0 & . & . & 1 & -1 \end{bmatrix}$$

If the measured values are standardized with their true values and standard deviations, they are pragmatically assumed to be random variables from the same distribution (univariate) with zero mean and unit deviation. Then similar to Equation 3.13, but with a general matrix A for the linear case, additional constraints, and l_i = 1, data reconciliation problem can be stated as:

$$\min \sum_{l=1}^{n} \frac{(y_{i,1} - x_{i,1})^2}{\sigma_{i,1}^{\;2}} \tag{3.14}$$

such that

$Ax = 0$,

A is the process matrix

$Lb \le x \le Ub$

Formulation Equation 3.14 can be further generalized to include the unmeasured variables (u) and nonlinear process model constraints (f, g), which is frequently used in the data reconciliation literature.

$$\min\ (y-x)\, Q^{-1} (y-x) \tag{3.15}$$

such that

$g(x,u) \geq 0$

$f(x,u) = 0$

$Lb_x \leq x \leq Ub_x$

$Lb_u \leq u \leq Ub_u$

where

$Q = diag\, [\sigma^2{}_{1,1}, \sigma^2{}_{2,1}, \ldots, \sigma^2{}_{n,1}]$

An optimum x_i called reconciled variables (x_i^*) to the problem Equation 3.15 is expected to result in the differences Equation 3.16

$$y_1 - x_1^*, y_2 - x_2^*, y_3 - x_3^*, \ldots, y_n - x_n^* \tag{3.16}$$

from a distribution with zero mean.

A fundamental method to determine whether the measurements are from a distribution with zero mean is applying hypothesis testing with H_0 being "μ is 0" and H_1 being "μ is not equal to 0", where μ denotes the mean. The test statistic for this procedure is

$$t = \frac{\hat{E}(y_i - x_i^*) - 0}{\sqrt{\hat{V}(y_i - x_i^*)}} \tag{3.17}$$

where $\hat{E}$ is an estimate for the expected value of $(y_i - x_i{}^*)$ and $\hat{V}$ is an estimate for the variance. This test statistic in Equation 3.17 is the basis of classical gross error detection procedures. If a particular probability distribution function can be assumed for t, larger t values will describe less likely instances and provide proof for the truth of the hypothesis H_1, i.e. the existence of a gross error (outlier).

CHAPTER IV

APPLICATION OF DATA RECONCILIATION

The application of data reconciliation for the paste mixing tank of the battery production process is presented in this chapter. The organization of this chapter is divided into three sections; the first section is the battery production process, next is the formulation and the performance evaluation of data reconciliation, and the final section is the case studies of the data reconciliation application for mixing tank of battery production process.

4.1 Battery Production Process

The battery production process concerned in this work is presented in the following. This section is divided into two subsections; the first one is the process description presented the details of battery production process, and the other one is the paste mixing tank concerned in this work.

4.1.1 Process Description

The block diagram of the battery production process is shown in Figure 4.1 which the description of the process presented in the following.

Lead Supply

- Supply lead in got.

Lead Oxide

- Melt lead to react with oxygen to get lead oxide.

- Store for paste mixing.

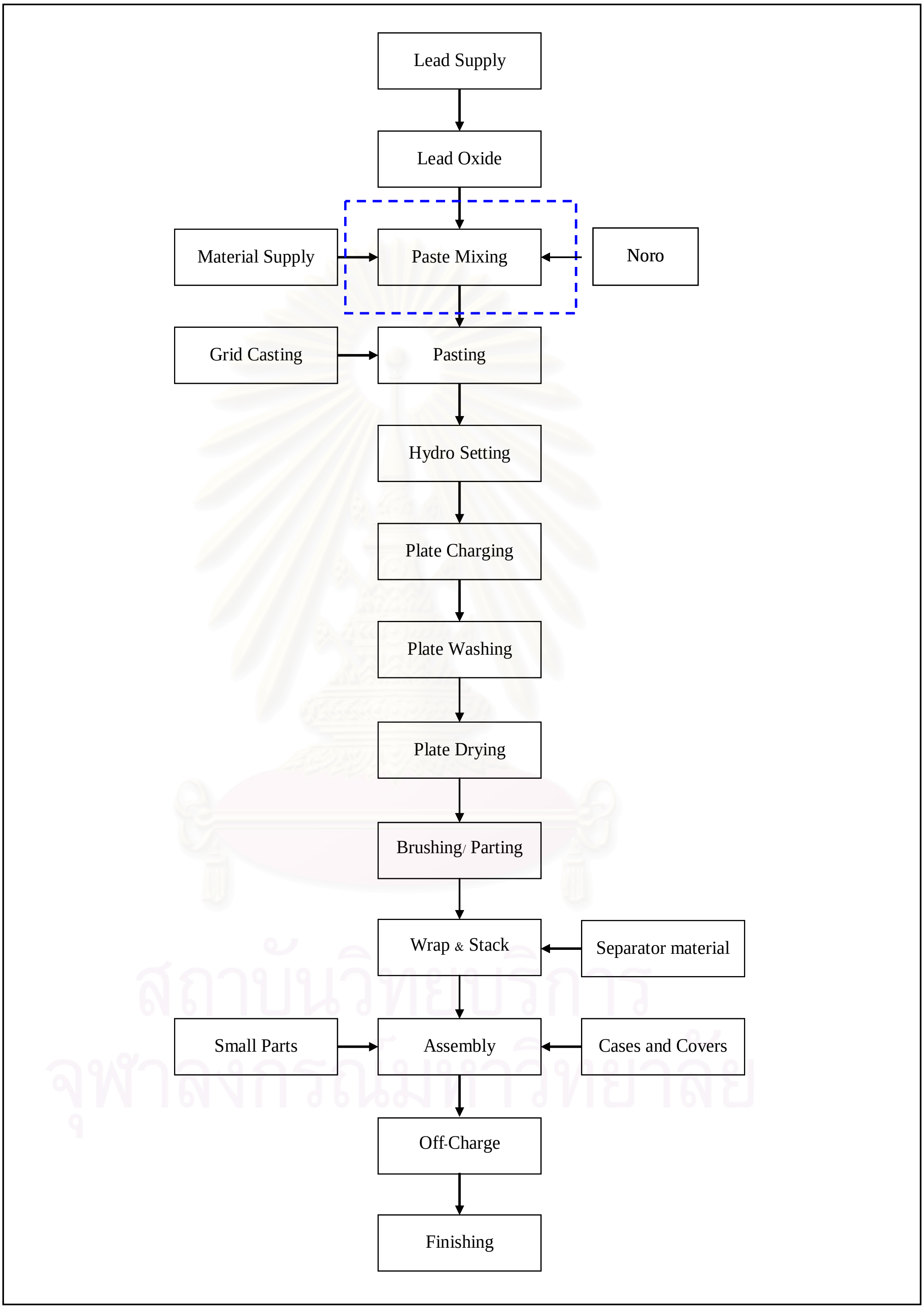


Figure 4.1 Battery production process flow

Paste Mixing

- Mix lead oxide, acid and water with additives to get positive mixes and negative mixes.

During the paste mixing process, lead oxide, water, acid, and other chemicals are blended in a mixer to form a thick paste. The major source of lead exposure in this process is from lead oxide that escapes from the paste mixing machine, dries, and becomes airborne.

Acid Mixing

- Mix acid with water to required concentrations (Specific gravities).
- Store acid.

Grid Casting

- Melt lead.
- Cast supporting frame.

Grid production and parts casting involves book casting, continuous casting, and strip casting. In all of these processes, lead pigs are melted down and the molten lead is poured into molds or continuously cast into grids, strips, or parts. Expanded metal grid production involves mechanical operations on the cast strip and is not a source of airborne lead dust or oxide. The major source of lead exposure in this process is from lead fumes and lead oxide which can become easily airborne.

Pasting

- Apply paste to grids.
- Flash dry pasted plates.

During the pasting process, lead oxide paste is applied to the grid panels in a pasting machine to fill the spaces of the grid. The major source of lead exposure in the pasting process comes from lead oxide in the paste which can become easily airborne once it dries.

Hydro Setting

- Cure/ dry pasted plates.

Hydrosetting methods vary between battery plants, ranging from placing the pasted plate racks in the workroom to placing the racks within a temperature- and humidity-controlled room or chamber. The major source of lead exposure in the hydrosetting process comes from lead oxide when the grids are handled incorrectly.

Plate Charging

- Charge positive & negative plates in tanks.

Plate Washing

- Wash positive & negative charged plates.

Plate Drying

- Dry positive & negative charged plates.

Brushing/ Parting

- Brush plates.
- Part plates (If required).

Wrap & Stack

- Envelope positive plates & stack with negative pates, to get stacked elements.

Enveloping involves placing a plate (usually positive), either automatically or manually, within porous membranes. The major sources of exposure in this process result from lead oxide being released when the plates are handled incorrectly, such as resting them against the body or handling them in unventilated areas.

Assembly

- Cast on stab or burn positive plates into common strap & burn negative plates into common strap, to get element.
- Place elements into case.
- Seal cover into case
- Seal terminals.
- Test for leaks.

Small Parts

- Cast terminal posts.
- Cast straps.

Off-Charge

- Wash the battery.
- High rate test the battery.

Finishing

- Label batteries.
- Palletize.

4.1.2 Paste Mixing Tank

This work is studied the battery production process in the unit of the paste mixing tank which the lead oxide, water, acid, and other chemicals are blended. This unit has the flow diagram of ten variables: lead oxide (x0), water (x1), sulfuric acid (x2), additive F (x3), noro (x4), additive L (x5), additive B (x6), additive C (x7), oil (x8), and paste (x9) as shown in Figure 4.2.

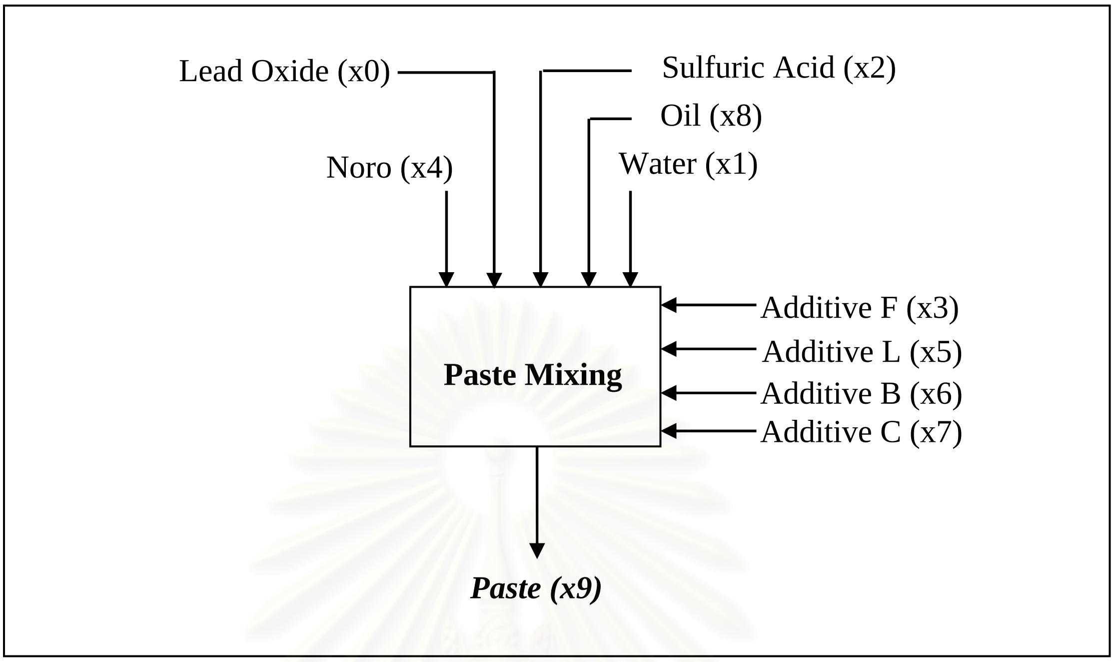


Figure 4.2 Paste mixing tank

The mathematical model of the paste mixing tank is concerned only the mass flow which the flows of all ten variables can be measured. The flow balances around the paste mixing tank can be written as:

$$0 = x0 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 - x9 \tag{4.1}$$

In addition, the measured values of all ten variables collected 50 sets from the actual process are shown in Appendix A. Theses data will be used for studying the application of data reconciliation in the Section 4.3. Note that these values do not satisfy the above equation, since they contain unknown errors.

4.2 Data Reconciliation

As mentioned in chapter III, the general data reconciliation problem deals with a weighted least-squares minimization of the measurement adjustments subject to the model constraints. The least-squares objective function is formulated under the assumption that all the measurements are normally distributed, without taking into account gross errors. However, the influence of these gross errors on the estimates can be minimized by defining robust objective functions which this work is concerned only three methods used for formulating the data reconciliation problem.

4.2.1 Optimization Formulation

The data reconciliation problem of this work is formulated with gross errors which use the general robust objective function proposed by Huber (1981) as shown in the following.

$$\min_{x_i} \sum_i \rho\left(\frac{y_i - x_i}{\sigma_i}\right) \tag{4.2}$$

where ρ is any reasonable monotone function of $\left(\frac{y_i - x_i}{\sigma_i}\right)$ which is called the standard error (ε_i), y_i is the measured value, x_i is the system value, and σ_i is the deviation value. Note that the ρ function shown in Equation 4.2 can be different function when the problem is formulated by using different method.

In addition, three different ρ functions formulated by using three different methods shown in the following are studied in this work.

<u>Contaminated normal distribution</u>

$$-ln\left\{(1-p_{CN})exp\left(-\frac{{\varepsilon_i}^2}{2}\right) + \frac{p_{CN}}{b_{CN}}exp\left(-\frac{{\varepsilon_i}^2}{2{b_{CN}}^2}\right)\right\} \tag{4.3}$$

where b_{CN}, and p_{CN} are tuning constants.

<u>Lorentzian distribution</u>

$$-\frac{1}{1+\left(\frac{{\varepsilon_i}^2}{2{c_L}^2}\right)} \tag{4.4}$$

where c_L is tuning constant.

<u>Hampel’s redescending M-estimator</u>

$$
\begin{cases}
\frac{1}{2}\varepsilon_i^{\,2} & ; 0 \le |\varepsilon_i| \le a_H \\[2ex]
a_H|\varepsilon_i| - \frac{a_H^{\,2}}{2} & ; a_H < |\varepsilon_i| \le b_H \\[2ex]
a_H b_H - \frac{a_H^{\,2}}{2} + (c_H - b_H)\frac{a_H^{\,2}}{2}\left[1 - \left(\frac{c_H - |\varepsilon_i|}{c_H - b_H}\right)^2\right] & ; b_H < |\varepsilon_i| \le c_H \\[2ex]
a_H b_H - \frac{a_H^{\,2}}{2} + (c_H - b_H)\frac{a_H^{\,2}}{2} & ; c_H < |\varepsilon_i|
\end{cases}
\tag{4.5}
$$

where a_H, b_H, and c_H are tuning constants.

4.2.2 Performance Evaluation

In order to test the performance of three different methods, the relative absolute error (RAE) is used, which defined in the following:

$$\%RAE = \left|\frac{x_{base} - x_{study}}{x_{study}}\right| \times 100 \tag{4.6}$$

where x_{base} is the data in a base case, and x_{study} is the data in a case study.

4.3 Case Studies Simulation

This work is concerned in two case studies. The first case study is a nominal case which has the objective to determine the reconciled data from the measured data in the actual process by using three different methods for formulating the data reconciliation problem. The other case study is a test case which has the objective to test the performance of three different methods under two conditions, the first one is the size of measurement error changed, and the other one is the amount of data set of measurement error changed.

4.3.1 Nominal Case

This case study is to determine the reconciled data from the data reconciliation problem in Equation 4.2 formulated in term of ρ function. In this work, the data reconciliation problem is concerned the formulation based on only three different methods: Contaminated normal distribution (Equation 4.3), Lorentzian distribution (Equation 4.4), and Hampel’s redescending M-estimator (Equation 4.5).

As mentioned in the section 4.2, the data reconciliation problem has tuning constants in ρ function. If the ρ function of the objective function has properly tuning constants, the good reconciled data is achieved. The tuning constants of each ρ function are different which the tuning constants of three ρ functions are as follows: b_{CN} and p_{CN} for Contaminated normal distribution, c_L for Lorentzian distribution, and a_H, b_H, and c_H for Hampel’s redescending M-estimator. The tuning constants of three ρ functions used in this work are shown in Table 4.1 (efficiency values are approximately 95.5 %) which Özyurt and Pike (2004) are proposed.

Table 4.1 The proper tuning constants of three different ρ functions.

Three ρ functions	**Tuning constants**
Contaminated normal distribution	$b_{CN} = 0.1$ $p_{CN} = 0.235$
Lorentzian distribution	$c_L = 0.3$
Hampel’s redescending M-estimator	$a_H = 0.5$ $b_H = 1.0$ $c_H = 2.0$

In addition, the measured data which will be reconciled in this work consist of ten variables defined as *x*0, *x*1, *x*2, *x*3, *x*4, *x*5, *x*6, *x*7, *x*8, and *x*9 as seen in the section of the paste mixing tank. Due to the actual values of all ten variables contain unknown errors which may be caused by the accuracy of all measurement, the variable defined as *x*10 is added in the simulation for detecting theses errors. Therefore the results of reconciled data in the following have eleven variables.

4.3.1.1 Simulation Results of Nominal Case

The simulation results of the data reconciliation problem formulated by using three different methods are shown in Table 4.2 and Figure 4.3. The RAE values of reconciled data used to evaluate the performance of three different methods in the reconciliation of measured data are shown in Table 4.3 and Figure 4.4. Note that the standard data is used as the base case for the RAE calculation shown in Equation 4.6 and the average RAE value is chosen to compare the performance of three different methods. Moreover, the results of reconciled data and the measured data are depicted in Figure 4.5 to Figure 4.14.

As seen in the results of the data reconciliation problem in a nominal case, three different methods provide good reconciled values compared with the standard values which the RAE values of all ten variables are below 7%, especially the variables as $x0$, $x1$, and $x2$ which have the effect on product quality directly to provide RAE values below 4% which the ρ function of Lorentzian distribution provide the best RAE value.

However, this work is evaluated the performance of three different methods by using the average RAE value of all measured variables as mentioned earlier. As seen in the average RAE value of three different methods, the ρ function of Contaminated normal distribution, Lorentzian distribution, and Hampel's redescending M-estimator provide the average RAE value as 1.83%, 2.08%, and 1.91%, respectively. This result can be concluded that the ρ function of Contaminated normal distribution provides the best performance with lowest RAE value.

4.3.1.2 Discussions of Nominal Case

a) The tuning constants in the ρ function of each robust method are tuned for the same efficiency values, which is required for a reasonable comparison between different methods.

b) The comparison of the data reconciliation performance shows that the ρ function of the Contaminated normal distribution is the most effective with 1.83 % of RAE.

Table 4.2 The data reconciliation results of three different ρ functions in a nominal case.

xi	Standard	Three ρ functions		
		Contaminated	**Lorentzian**	**Hampel**
x0	800.00	799.53	799.62	799.52
x1	50.00	51.66	51.35	51.31
x2	60.00	61.36	61.15	61.30
x3	0.50	0.50	0.51	0.51
x4	240.00	240.92	241.51	241.53
x5	1.00	1.05	1.06	1.05
x6	2.50	2.51	2.51	2.51
x7	1.00	1.00	1.01	1.00
x8	1.50	1.50	1.50	1.50
x9	1156.50	1084.28	1085.09	1085.00
x10	-	-75.74	-75.22	-75.24

Table 4.3 The RAE percentage of three different ρ functions in a nominal case

xi	Three ρ function functions		
	Contaminated	**Lorentzian**	**Hampel**
x0	0.06	0.05	0.06
x1	3.32	2.71	2.62
x2	2.27	1.92	2.17
x3	0.00	2.22	2.00
x4	0.38	0.63	0.64
x5	5.00	5.69	5.00
x6	0.40	0.50	0.40
x7	0.30	0.89	0.00
x8	0.31	0.02	0.00
x9	6.24	6.17	6.18
Average	**1.83**	**2.08**	**1.91**

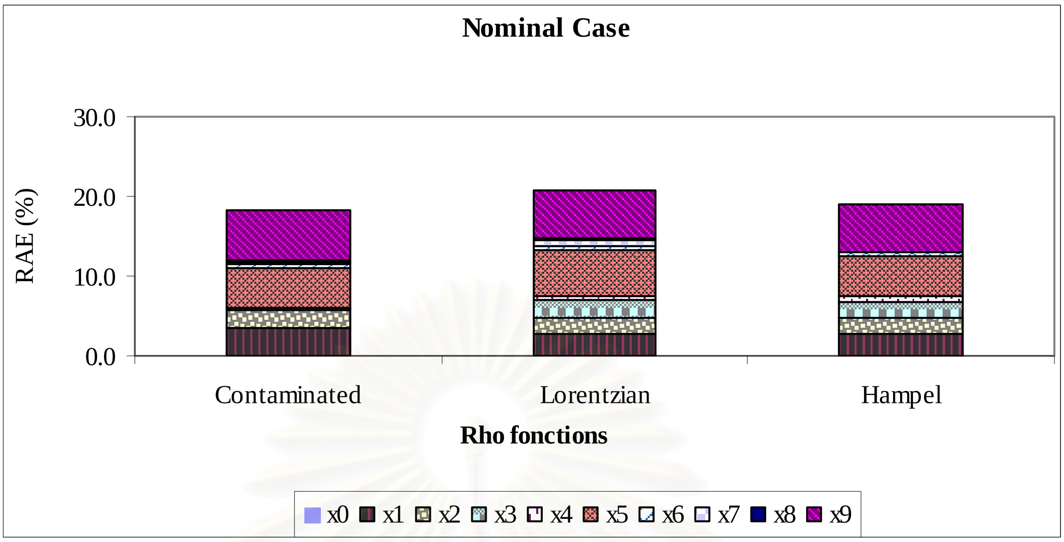


Figure 4.3 The RAE percentage of data reconciliation result of all ten variables (*x*0 to *x*9) and total in a nominal case

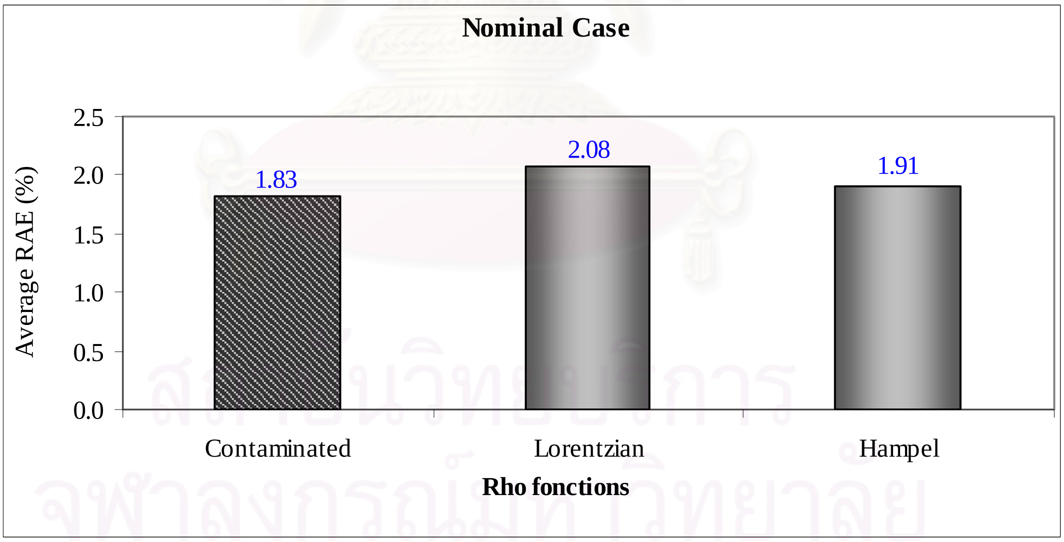


Figure 4.4 The average RAE percentage of data reconciliation result in a nominal case

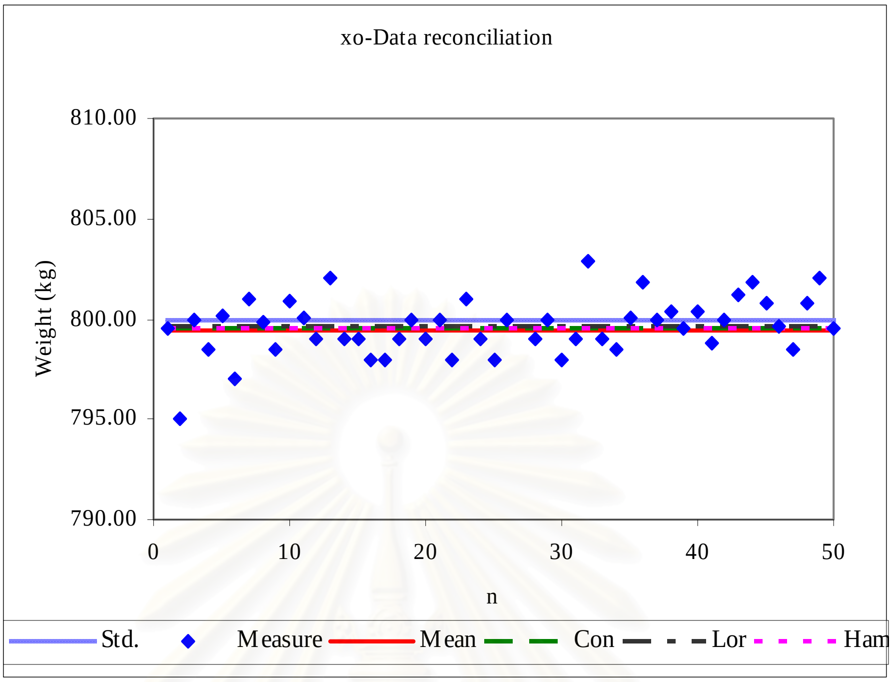


Figure 4.5 The data reconciliation result of *x*0 in a nominal case.

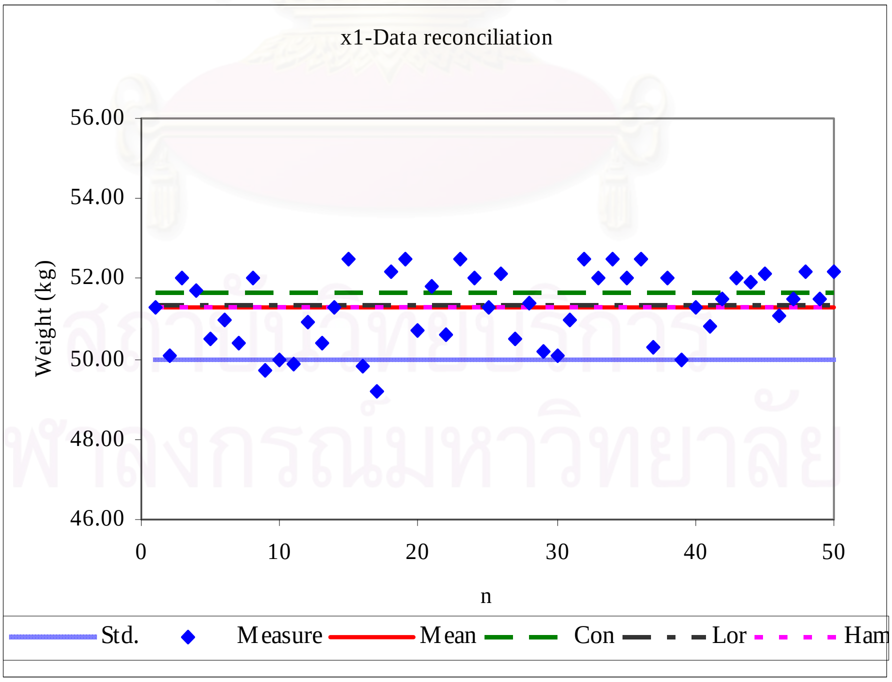


Figure 4.6 The data reconciliation result of *x*1 in a nominal case.

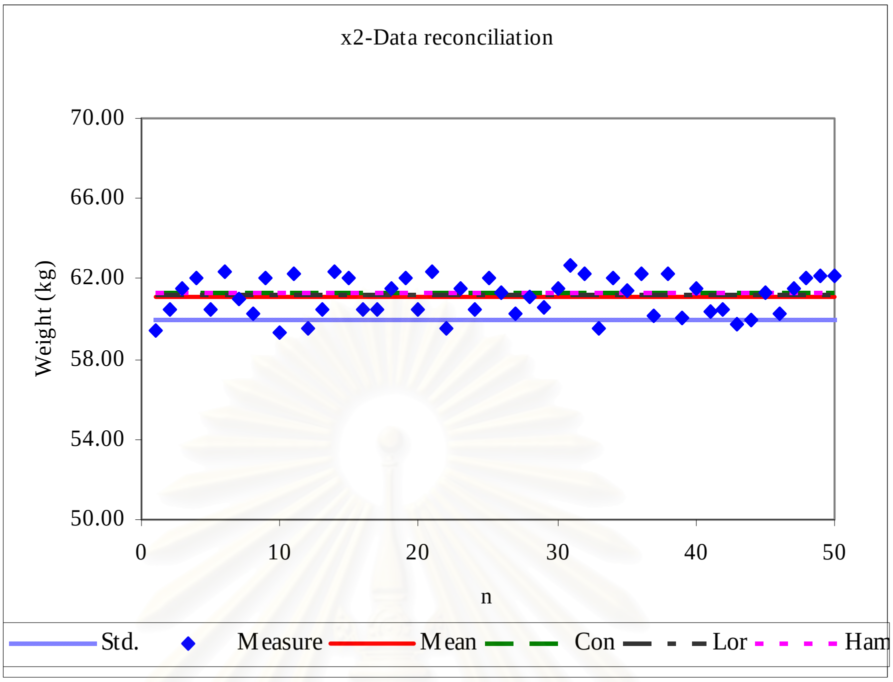


Figure 4.7 The data reconciliation result of *x*2 in a nominal case.

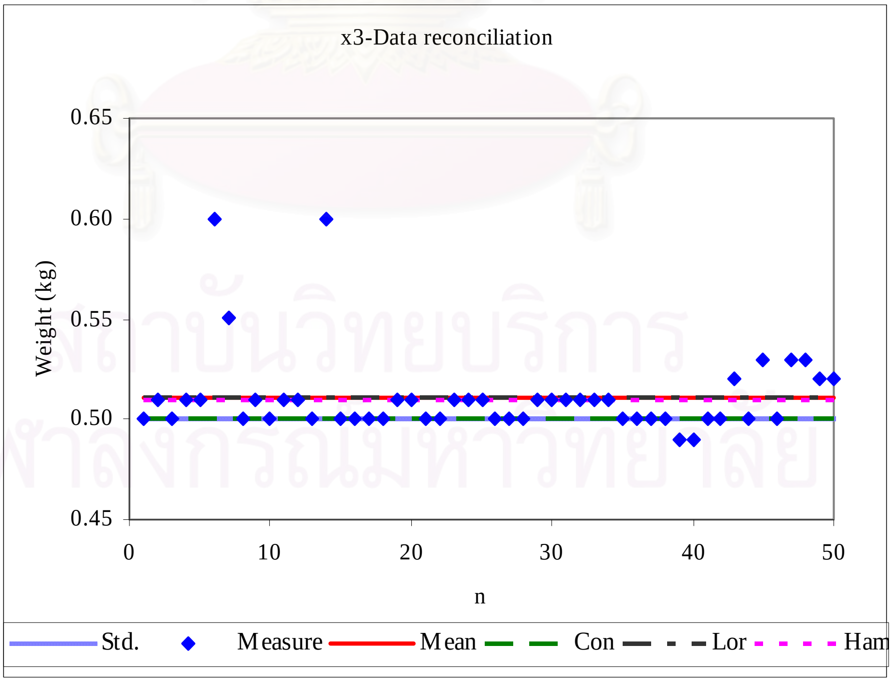


Figure 4.8 The data reconciliation result of *x*3 in a nominal case.

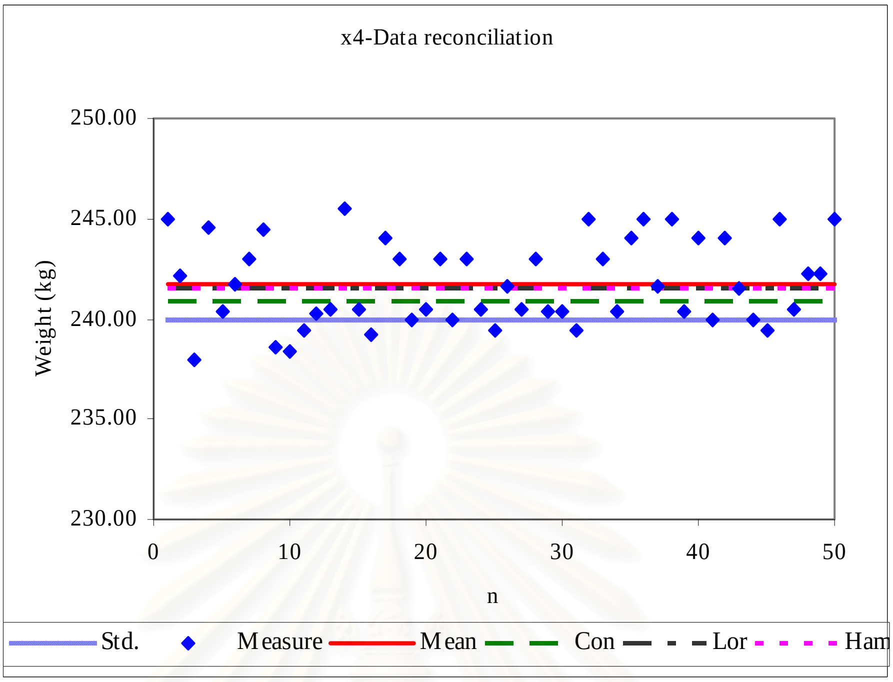


Figure 4.9 The data reconciliation result of *x*4 in a nominal case.

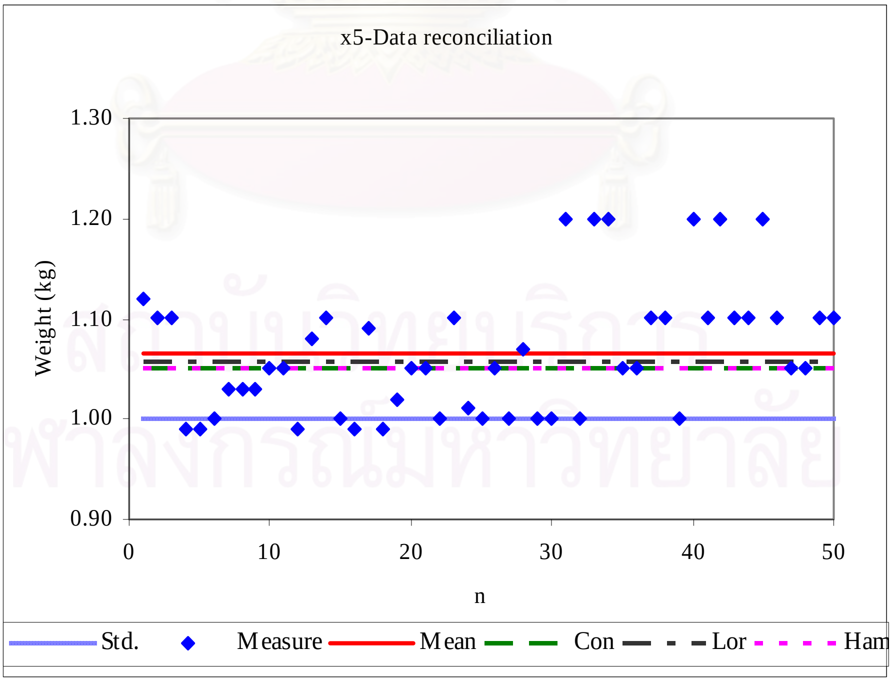


Figure 4.10 The data reconciliation result of *x*5 in a nominal case.

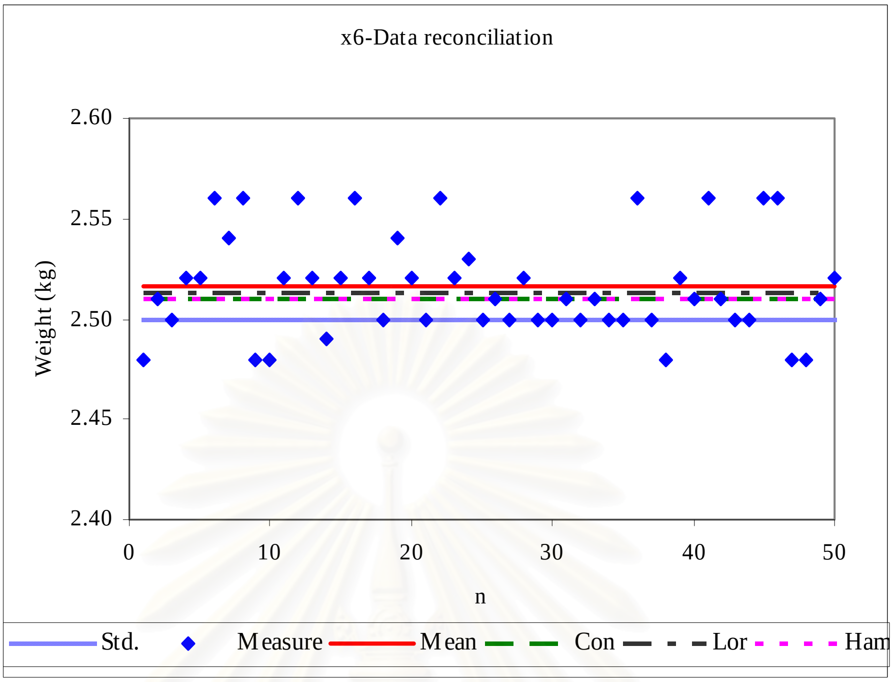


Figure 4.11 The data reconciliation result of *x*6 in a nominal case.

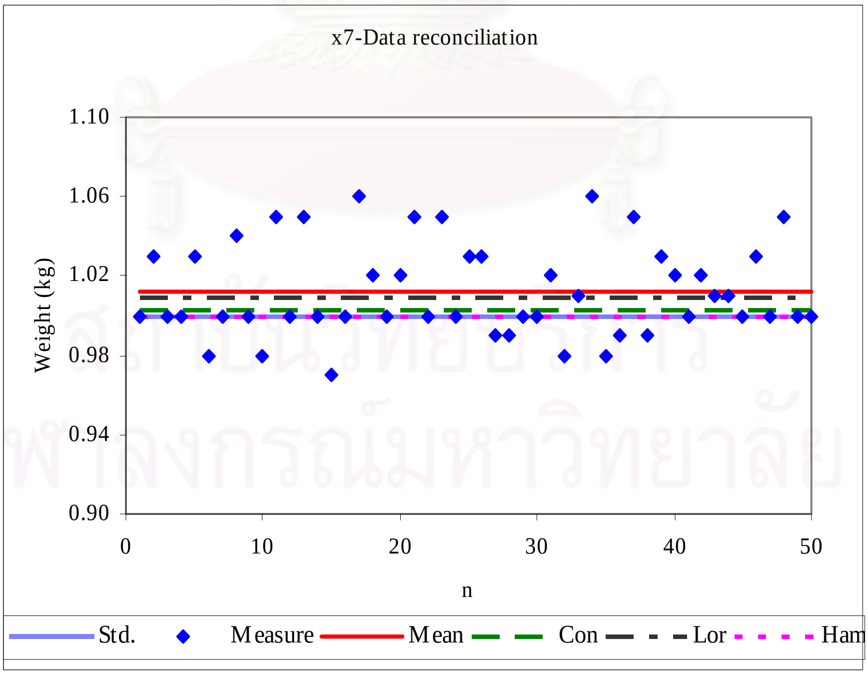


Figure 4.12 The data reconciliation result of *x*7 in a nominal case.

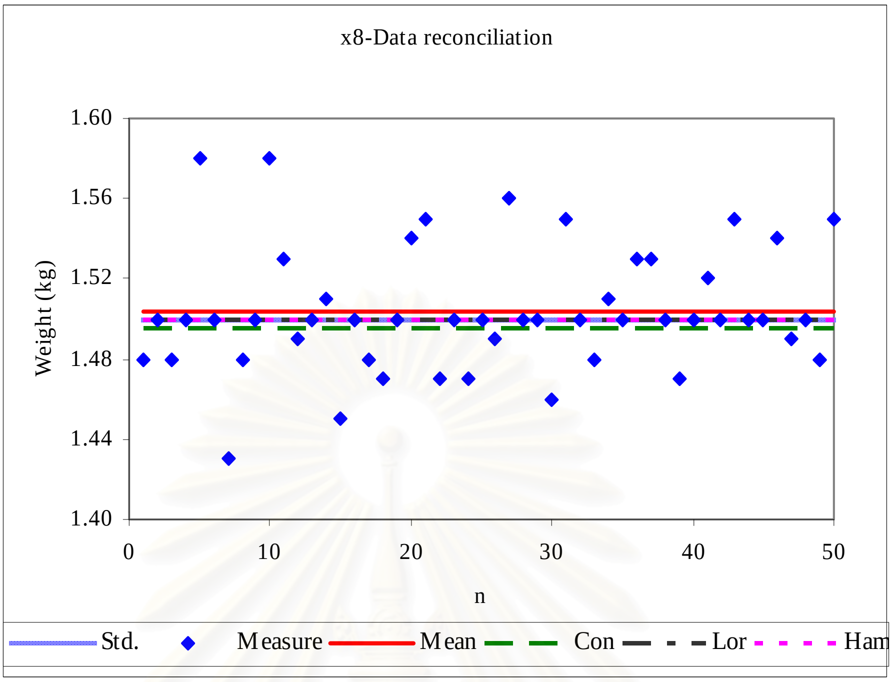


Figure 4.13 The data reconciliation result of *x*8 in a nominal case.

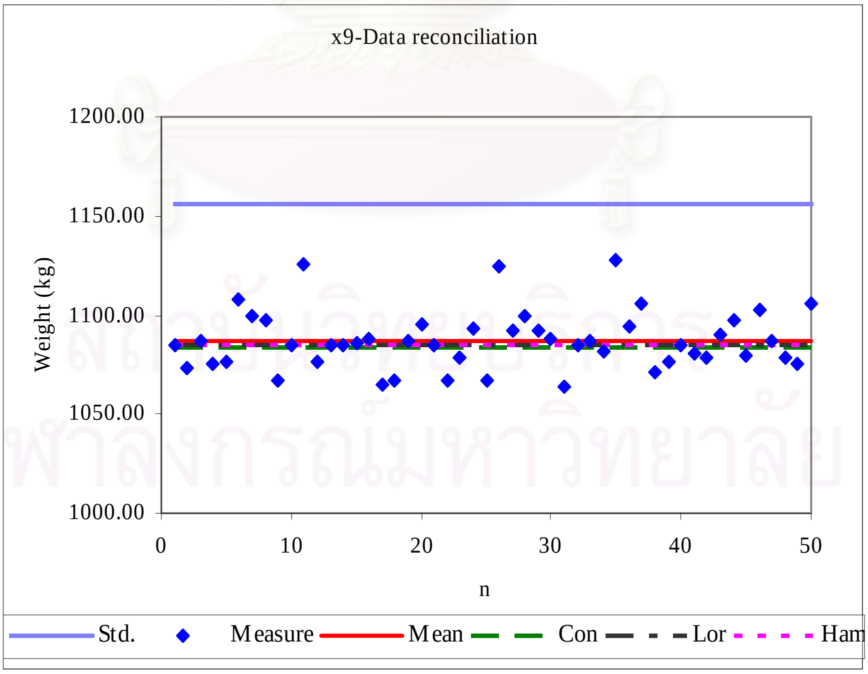


Figure 4.14 The data reconciliation result of *x*9 in a nominal case.

4.3.2 Test Case

As mentioned previously, the objective of this case is to test the performance of three different methods under two conditions: the size of measurement error changed and the amount of data set of measurement error changed. Therefore, the test cases are designed to divide the case study into three cases as the following:

- *± 10% error size and 10%, 20%, and 30% error amount*
- *± 20% error size and 10%, 20%, and 30% error amount*
- *± 30% error size and 10%, 20%, 30% and 100% error amount*

The data reconciliation problems in test cases are formulated as a nominal case. The problem in Equation 4.2 is formulated by using three different ρ functions: Contaminated normal distribution (Equation 4.3), Lorentzian distribution (Equation 4.4), and Hampel's redescending M-estimator (Equation 4.5) with the tuning constants in the ρ function as shown in Table 4.1.

Moreover, three different methods formulated the data reconciliation problem are evaluated the performance by comparing the average RAE values which use the reconciled data in a nominal case as the base case for calculating the RAE value as shown in Equation 4.6.

4.3.2.1 Simulation Results of Test Case

The simulation results of the data reconciliation problem and the RAE values of reconciled data in the test case divided into three case studies as mentioned earlier are presented in the following:

- ± ***10% error size and 10%, 20% and 30% error amount***

This case study is subdivided in to + 10% error size and – 10% error size cases. The results of the data reconciliation problem and the RAE values of reconciled data are shown in Table 4.4 and Table 4.5, respectively, for the case of + 10% error size and Table 4.6 and Table 4.7, respectively, for the case of – 10% error size. Moreover, the results of the variables as $x0$, $x1$, and $x2$ which have the effect on product quality directly are depicted in Figure 4.15 to Figure 4.20.

The case of + 10% error size

As seen in the results of the data reconciliation problem in the case of + 10% error size, three different methods still provide good reconciled values compared with the standard values even though the amount of error of measurement data changed is increased which the average RAE values of all three methods are below 4%. To insist the performance of methods provided the best RAE values, the case of 30% error amount is concerned. As this result, it can be concluded that the ρ function of Contaminated normal distribution provide 1.54% of the average RAE value. Notice that the ρ function of Lorentzian distribution provides the worst average RAE values, especially the case of 30% error amount to provide 3.46% of the average RAE value.

The case of – 10% error size

As seen in the results of the data reconciliation problem in the case of – 10% error size, three different methods still provide good reconciled values as in the case of + 10% error size which the average RAE values of all three methods in this case are below 5%. Moreover, it can be concluded that the ρ function of Hampel’s redescending M-estimator provide the best average RAE values with 0.48% and the ρ function of Lorentzian distribution still provides the worst average RAE values with 4.59% as seen in the case of 30% error amount.

Table 4.4 The data reconciliation results of three different ρ functions in the case of +10% error sizes.

xi	Contaminated			Lorent			Hampel		
	10%	20%	30%	10%	20%	30%	10%	20%	30%
x0	799.59	799.69	800.34	799.93	804.38	813.86	800.00	800.10	800.20
x1	51.48	51.32	51.54	51.49	51.67	52.32	51.54	51.62	52.00
x2	61.18	61.28	61.22	61.24	61.64	62.29	61.45	61.88	62.00
x3	0.51	0.51	0.51	0.51	0.51	0.52	0.51	0.51	0.51
x4	241.94	241.80	242.17	242.09	243.30	246.20	242.51	242.28	243.56
x5	1.05	1.05	1.06	1.07	1.08	1.08	1.06	1.08	1.10
x6	2.51	2.51	2.52	2.52	2.53	2.56	2.51	2.52	2.54
x7	1.01	1.01	1.01	1.01	1.02	1.03	1.01	1.02	1.02
x8	1.50	1.50	1.51	1.51	1.52	1.53	1.50	1.50	1.51
x9	1084.27	1084.27	1085.45	1085.10	1085.09	1086.16	1085.00	1085.00	1085.00
x10	-76.51	-76.39	-76.44	-76.25	-82.54	-95.25	-76.75	-66.42	-79.43

Table 4.5 The RAE percentage of three different ρ functions in the case of +10% error sizes.

xi	Contaminated			Lorent			Hampel		
	10%	20%	30%	10%	20%	30%	10%	20%	30%
x0	0.01	0.00	0.08	0.02	0.58	1.76	0.03	0.04	0.06
x1	0.35	0.66	0.24	0.27	0.62	1.88	0.29	0.45	1.19
x2	0.29	0.13	0.23	0.14	0.80	1.86	0.24	0.95	1.14
x3	11.46	11.46	11.46	0.22	0.22	1.74	0.26	0.26	0.26
x4	0.42	0.36	0.52	0.20	0.70	1.90	0.40	0.31	0.84
x5	0.30	0.30	1.25	1.24	2.19	2.19	0.95	2.86	4.76
x6	0.10	0.10	0.50	0.29	0.69	1.89	0.00	0.40	1.20
x7	0.70	0.70	0.70	0.11	1.10	2.09	1.00	2.00	2.00
x8	0.31	0.31	0.98	0.69	1.35	2.02	0.00	0.00	0.67
x9	0.09	0.09	0.20	0.36	0.36	0.45	0.01	0.01	0.01
x10	0.67	0.83	0.76	3.75	4.19	20.24	1.45	12.20	5.00
Average	**1.34**	**1.36**	**1.54**	**0.66**	**1.16**	**3.46**	**0.42**	**1.77**	**1.56**

Table 4.6 The data reconciliation results of three different ρ functions in the case of –10% error sizes.

xi	Contaminated			Lorent			Hampel		
	10%	20%	30%	10%	20%	30%	10%	20%	30%
x0	799.36	799.14	798.29	798.99	794.44	784.88	799.36	799.00	798.29
x1	51.40	51.25	51.24	51.26	50.86	50.30	51.40	51.00	51.24
x2	61.15	61.18	60.95	61.04	60.72	59.88	61.15	60.52	60.95
x3	0.50	0.51	0.51	0.50	0.50	0.50	0.50	0.50	0.51
x4	241.86	241.62	241.39	241.73	240.15	237.32	241.86	240.48	241.39
x5	1.04	1.03	1.03	1.05	1.04	1.03	1.04	1.04	1.03
x6	2.51	2.51	2.51	2.51	2.50	2.47	2.51	2.50	2.51
x7	1.01	1.00	1.00	1.01	1.00	0.99	1.01	1.00	1.00
x8	1.50	1.50	1.50	1.50	1.49	1.47	1.50	1.50	1.50
x9	1084.28	1084.28	1084.27	1085.09	1085.10	1085.09	1084.28	1085.01	1084.27
x10	-76.05	-75.48	-74.15.	-74.51	-67.60	-53.75	-76.05	-72.53	-74.15

Table 4.7 The RAE percentage of three different ρ functions in the case of –10% error sizes.

xi	Contaminated			Lorent			Hampel		
	10%	20%	30%	10%	20%	30%	10%	20%	30%
x0	0.04	0.07	0.18	0.10	0.67	1.86	0.05	0.09	0.18
x1	0.51	0.80	0.82	0.18	0.96	2.05	0.02	0.76	0.29
x2	0.34	0.29	0.67	0.19	0.71	2.08	0.25	1.27	0.57
x3	13.19	11.46	11.46	2.17	2.17	2.17	1.71	1.71	0.26
x4	0.39	0.29	0.19	0.05	0.61	1.78	0.14	0.44	0.06
x5	0.66	1.61	1.61	0.65	1.60	2.55	0.95	0.95	1.90
x6	0.10	0.10	0.10	0.10	0.50	1.70	0.00	0.40	0.00
x7	0.70	0.30	0.30	0.11	0.88	1.87	1.00	0.00	0.00
x8	0.31	0.31	0.31	0.02	0.65	1.98	0.00	0.00	0.00
x9	0.10	0.10	0.09	0.36	0.36	0.36	0.06	0.01	0.06
x10	1.27	2.01	3.73	5.94	14.67	32.15	0.53	4.13	1.98
Average	**1.60**	**1.58**	**1.77**	**0.90**	**2.16**	**4.59**	**0.43**	**0.89**	**0.48**

X0

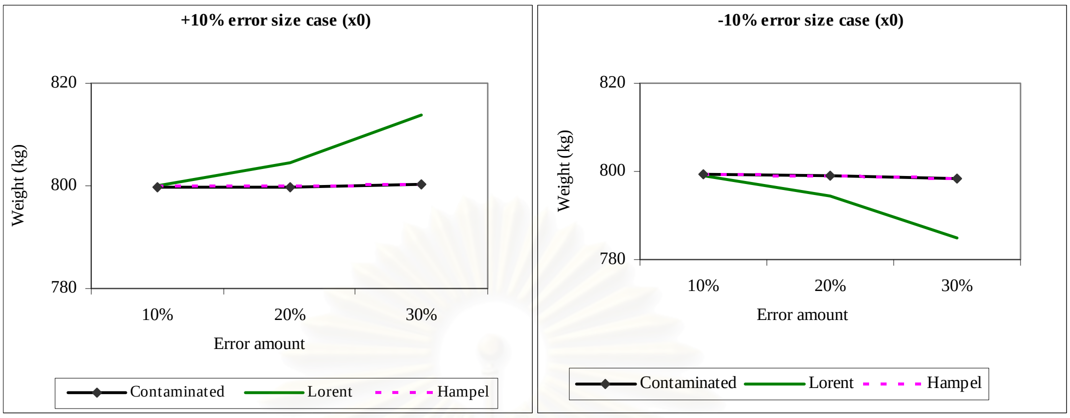


Figure 4.15 The data reconciliation result of *x*0 in the case of ±10% error sizes.

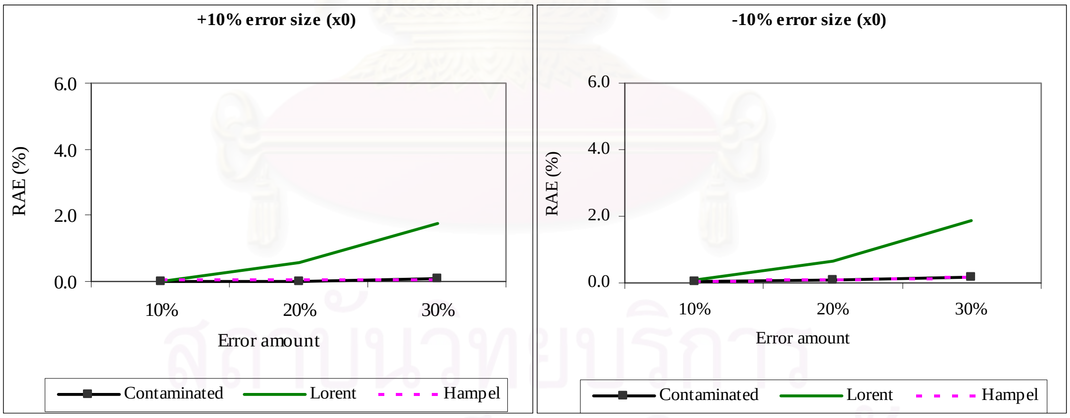


Figure 4.16 The RAE percentage of *x*0 in the case of ±10% error sizes.

X1

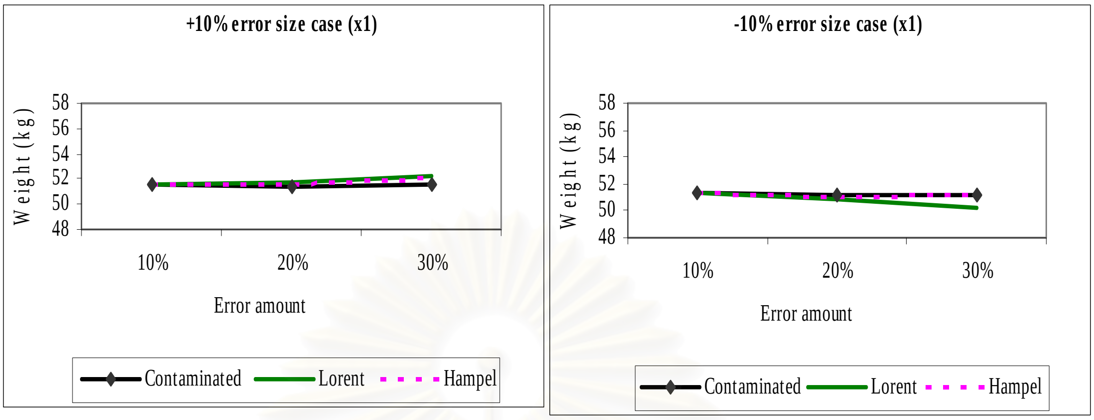


Figure 4.17 The data reconciliation result of *x*1 in the case of ±10% error sizes.

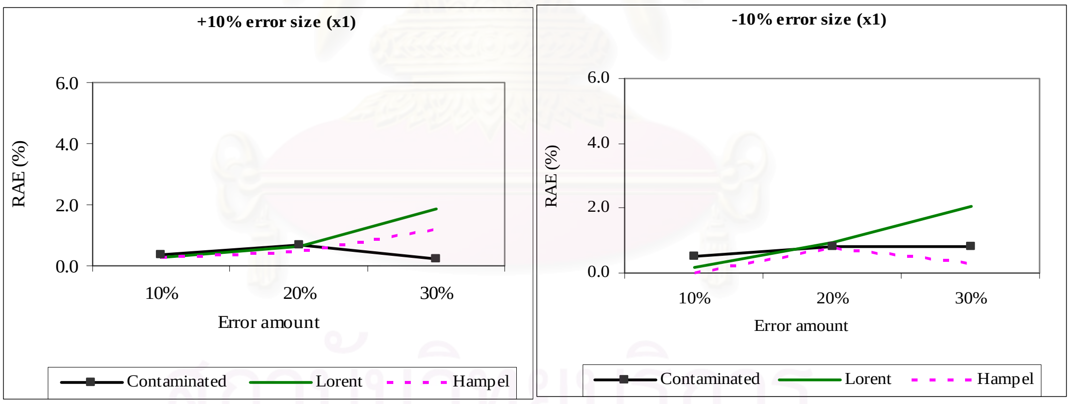


Figure 4.18 The RAE percentage of *x*1 in the case of ±10% error sizes.

<u>X2</u>

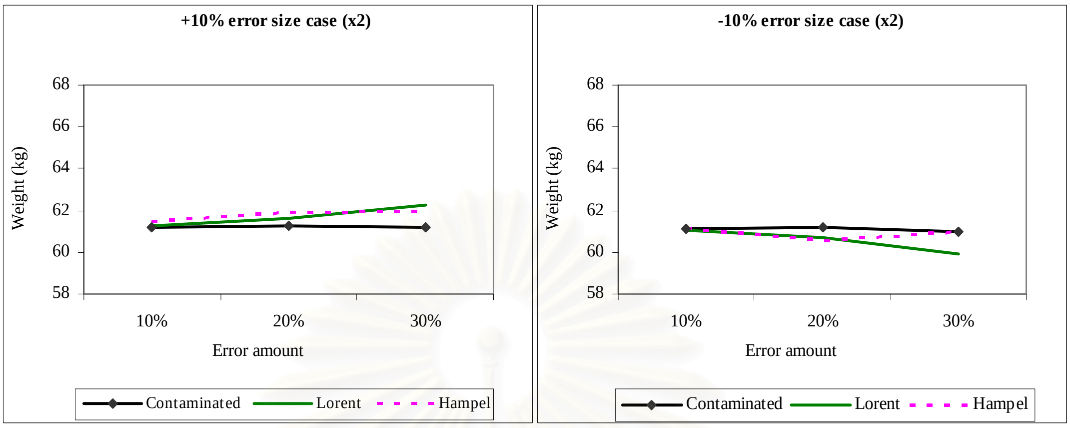


Figure 4.19 The data reconciliation result of *x*2 in the case of ±10% error sizes.

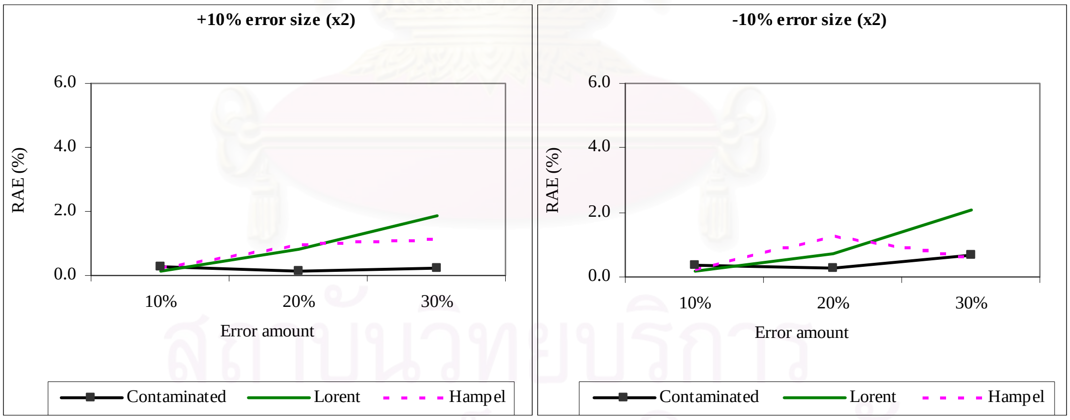


Figure 4.20 The RAE percentage of *x*2 in the case of ±10% error sizes.

- ***± 20% error size and 10%, 20%, and 30% error amount***

This case study is subdivided in to + 20% error size and – 20% error size cases. The results of the data reconciliation problem and the RAE values of reconciled data are shown in Table 4.8 and Table 4.9, respectively, for the case of + 20% error size and Table 4.10 and Table 4.11, respectively, for the case of – 20% error size. Moreover, the results of the variables as $x0$, $x1$, and $x2$ which have the effect on product quality directly are depicted in Figure 4.21 to Figure 4.26.

The case of + 20% error size

As seen in the results of the data reconciliation problem in the case of + 20% error size, the ρ functions of Contaminated normal distribution and Hampel's redescending M-estimator still provide good reconciled values with the average RAE values are below 2%. Moreover, it can be concluded that the ρ function of Hampel's redescending M-estimator provide the best average RAE values with 1.41% when the case of 30% error amount is concerned. For the ρ function of Lorentzian distribution, it provides bad results as seen from the average RAE values, especially the case of 30% error amount to provide 6.71% of the average RAE value.

The case of – 20% error size

As seen in the results of the data reconciliation problem in the case of – 20% error size, the ρ functions of Contaminated normal distribution and Hampel's redescending M-estimator still provide good reconciled values as in the case of + 20% error size which the average RAE values of all three methods in this case are below 3%. Moreover, it can be concluded that the ρ function of Hampel's redescending M-estimator provide the best average RAE values with 0.81% and the ρ function of Lorentzian distribution still provides the worst average RAE values with 8.41% as seen in the case of 30% error amount.

Table 4.8 The data reconciliation results of three different ρ functions in the case of +20% error sizes.

xi	Contaminated			Lorent			Hampel		
	10%	20%	30%	10%	20%	30%	10%	20%	30%
x0	799.60	800.18	801.32	800.28	809.46	824.78	799.81	799.99	800.17
x1	51.35	51.35	51.29	51.39	51.97	53.12	51.50	51.98	52.00
x2	61.12	61.10	61.27	61.18	61.87	63.31	61.47	61.50	62.00
x3	0.51	0.51	0.51	0.51	0.52	0.53	0.51	0.51	0.51
x4	241.87	242.01	242.00	242.09	244.85	250.07	241.60	242.99	242.54
x5	1.05	1.05	1.08	1.07	1.08	1.11	1.05	1.08	1.10
x6	2.52	2.52	2.52	2.52	2.55	2.61	2.52	2.52	2.52
x7	1.01	1.01	1.02	1.02	1.02	1.05	1.01	1.00	1.03
x8	1.50	1.50	1.51	1.51	1.52	1.56	1.50	1.50	1.51
x9	1084.28	1084.28	1086.70	1085.09	1085.09	1088.14	1085.00	1085.00	1085.02
x10	-76.26	-76.96	-75.82	-76.47	-89.75	-110.00	-75.97	-78.08	-78.36

Table 4.9 The RAE percentage of three different ρ functions in the case of +20% error sizes.

xi	Contaminated			Lorent			Hampel		
	10%	20%	30%	10%	20%	30%	10%	20%	30%
x0	0.01	0.06	0.20	0.07	1.21	3.13	0.01	0.03	0.05
x1	0.60	0.60	0.72	0.07	1.20	3.44	0.21	1.15	1.19
x2	0.39	0.42	0.14	0.04	1.17	3.53	0.28	0.33	1.14
x3	11.46	11.46	11.46	0.22	1.74	3.70	0.26	0.26	0.26
x4	0.39	0.45	0.44	0.20	1.34	3.50	0.03	0.60	0.42
x5	0.30	0.30	3.16	1.24	2.19	5.02	0.00	2.86	4.76
x6	0.50	0.50	0.50	0.29	1.49	3.88	0.40	0.40	0.40
x7	0.70	0.70	1.69	1.10	1.10	4.07	1.00	0.00	3.00
x8	0.31	0.31	0.98	0.69	1.35	4.02	0.00	0.00	0.67
x9	0.10	0.10	0.32	0.36	0.36	0.64	0.01	0.01	0.01
x10	0.99	0.09	1.57	3.47	13.29	38.86	0.42	3.21	3.58
Average	**1.43**	**1.36**	**1.93**	**0.70**	**2.40**	**6.71**	**0.24**	**0.80**	**1.41**

Table 4.10 The data reconciliation results of three different ρ functions in the case of –20% error sizes.

xi	Contaminated			Lorent			Hampel		
	10%	**20%**	**30%**	**10%**	**20%**	**30%**	**10%**	**20%**	**30%**
x0	799.12	799.11	797.26	798.43	789.69	770.41	799.00	799.00	797.26
x1	51.31	51.27	51.00	51.24	50.63	49.28	51.30	51.04	51.00
x2	61.08	61.02	60.89	61.01	60.28	58.84	60.68	60.50	60.89
x3	0.51	0.51	0.51	0.51	0.50	0.49	0.50	0.50	0.51
x4	241.72	241.67	240.72	241.51	238.79	232.77	240.51	240.47	240.72
x5	1.05	1.04	1.04	1.06	1.03	1.02	1.05	1.03	1.04
x6	2.51	2.52	2.51	2.51	2.49	2.43	2.51	2.50	2.51
x7	1.01	1.01	1.01	1.01	1.00	0.97	1.00	1.00	1.01
x8	1.50	1.52	1.50	1.50	1.48	1.45	1.50	1.49	1.50
x9	1084.27	1084.28	1084.27	1085.10	1085.10	1085.09	1085.03	1085.01	1084.27
x10	-75.55	-75.36	-72.17	-73.66	-60.79	-32.56	-73.02	-72.51	-72.17

Table 4.11 The RAE percentage of three different ρ functions in the case of –20% error sizes.

xi	Contaminated			Lorent			Hampel		
	10%	**20%**	**30%**	**10%**	**20%**	**30%**	**10%**	**20%**	**30%**
x0	0.07	0.07	0.30	0.17	1.26	3.67	0.09	0.09	0.31
x1	0.68	0.76	1.28	0.22	1.41	4.04	0.18	0.68	0.76
x2	0.45	0.55	0.76	0.23	1.43	3.78	1.01	1.31	0.67
x3	11.46	11.46	11.46	0.22	2.17	4.13	1.71	1.71	0.26
x4	0.33	0.31	0.09	0.04	1.17	3.66	0.42	0.44	0.34
x5	0.30	0.66	0.66	0.29	2.55	3.49	0.00	1.90	0.95
x6	0.10	0.50	0.10	0.10	0.90	3.29	0.00	0.40	0.00
x7	0.70	0.70	0.70	0.11	0.88	3.86	0.00	0.00	1.00
x8	0.31	1.65	0.31	0.02	1.31	3.31	0.00	0.67	0.00
x9	0.09	0.10	0.09	0.36	0.36	0.36	0.01	0.01	0.06
x10	1.92	2.16	6.30	7.02	23.26	58.90	3.48	4.15	4.60
Average	**1.49**	**1.72**	**2.01**	**0.80**	**3.34**	**8.41**	**0.63**	**1.03**	**0.81**

X0

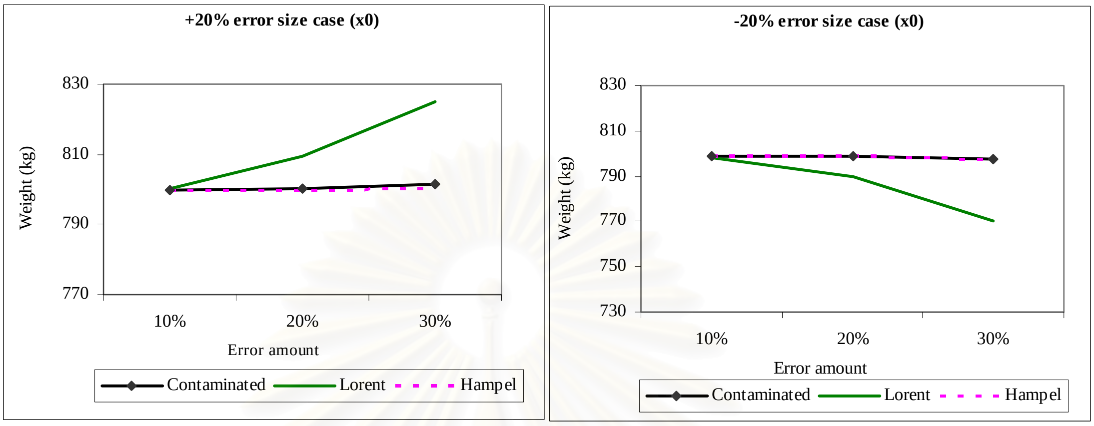


Figure 4.21 The data reconciliation result of *x*0 in the case of ±20% error sizes.

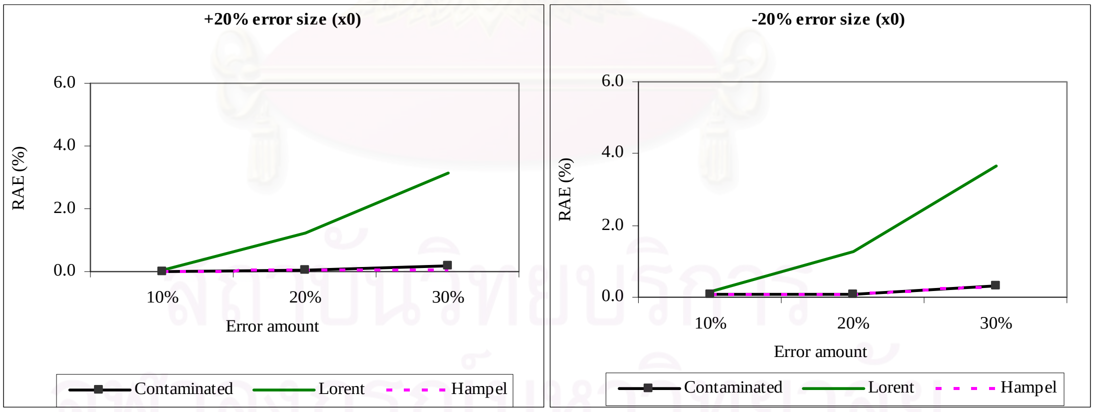


Figure 4.22 The RAE percentage of *x*0 in the case of ±20% error sizes.

X1

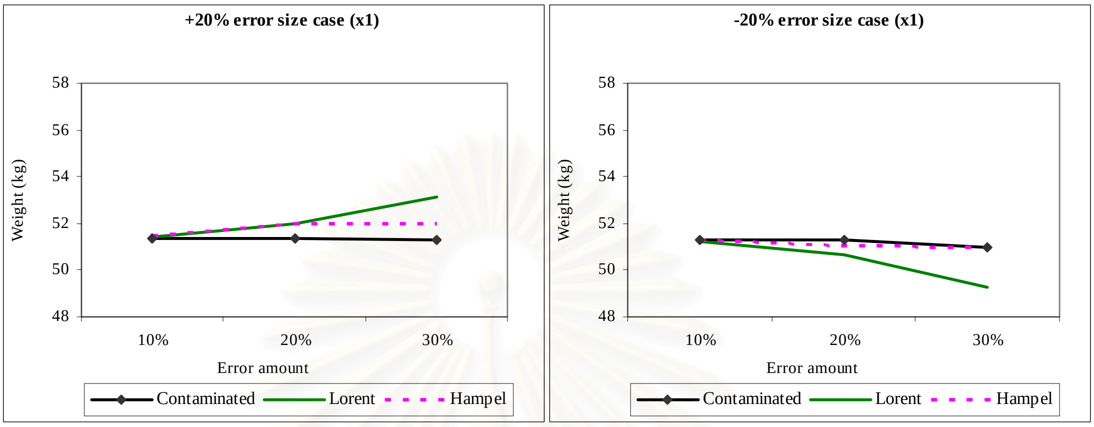


Figure 4.23 The data reconciliation result of *x*1 in the case of ±20% error sizes.

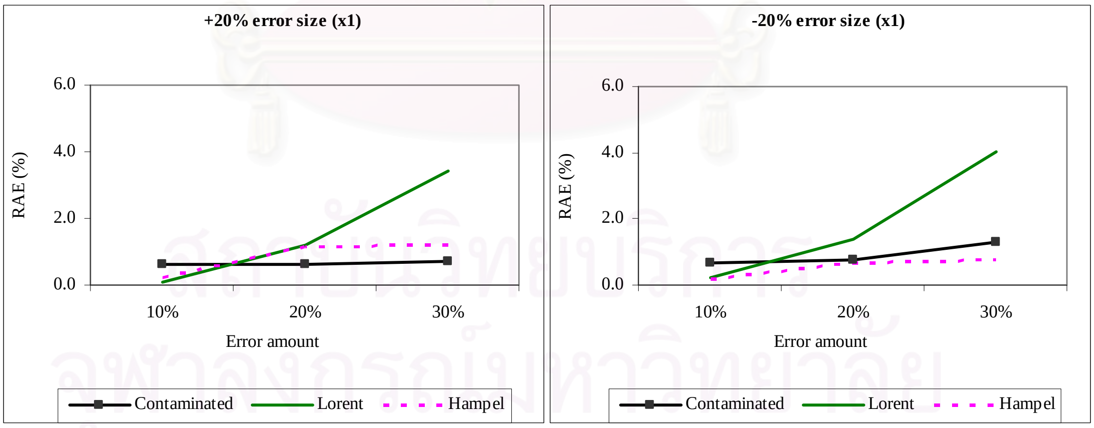


Figure 4.24 The RAE percentage of *x*1 in the case of ±20% error sizes.

X2

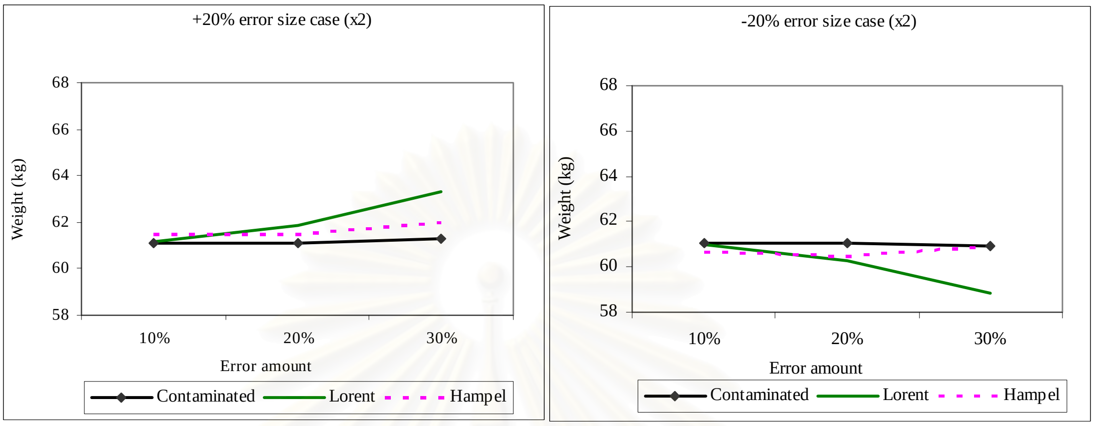


Figure 4.25 The data reconciliation result of *x*2 in the case of ±20% error sizes.

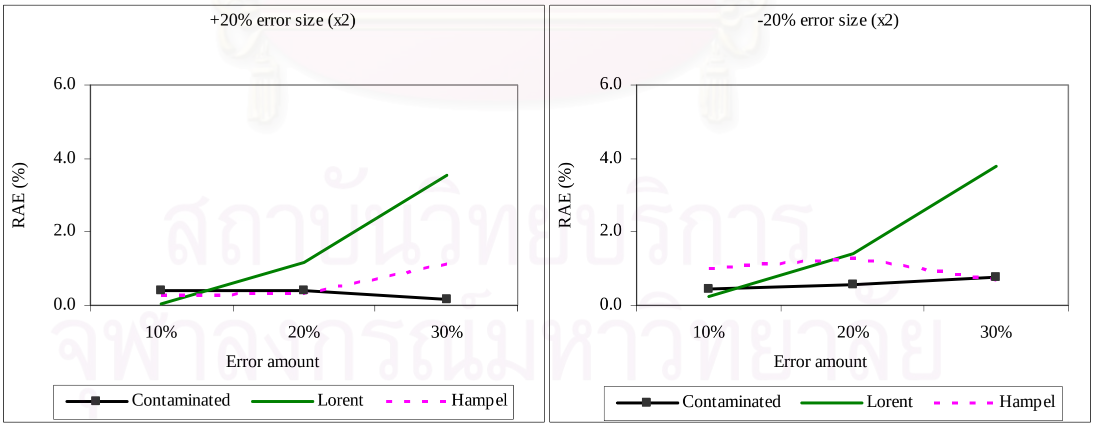


Figure 4.26 The RAE percentage of *x*2 in the case of ±20% error sizes.

- ***± 30% error size and 10%, 20%, 30% and 100% error amount***

This case study is subdivided as follow: case I ± 30% error size and 10%, 20%, and 30%error amount, case II ±30% error size and 10%, 20%, 30%, and 100% error amount. The results of the reconciled values for case I are shown in Figure 4.27 to Figure 4.46 and the variables as $x0$, $x1$, and $x2$ which have the effect on product quality directly are depicted in Figure 4.47 to Figure 4.52. And The results of the reconciled values for case II are shown in Figure 4.53 to Figure 4.58**.**

The case of + 30% error size

As seen in the results of the data reconciliation problem in the case of + 30% error size, three different methods provide satisfy reconciled values even though the size and the amount of measurement data error are increased to 30%. The ρ function of Hampel's redescending M-estimator still provides the best average RAE values with 1.57% and the ρ function of Lorentzian distribution still provides the worst average RAE values with 7.80% as seen in the case of 30% error amount.

The case of – 30% error size

As seen in the results of the data reconciliation problem in the case of – 30% error size, the ρ functions of Hampel's redescending M-estimator provides better reconciled values than the others, especially in the case of 30% error amount to provide the best average RAE value with 0.79%. Moreover, it can be conclude that the ρ function of Lorentzian distribution provides the worst average RAE values, especially the case of 30% error amount to provide 4.48% of the average RAE value.

The case of x0 ±30% error size

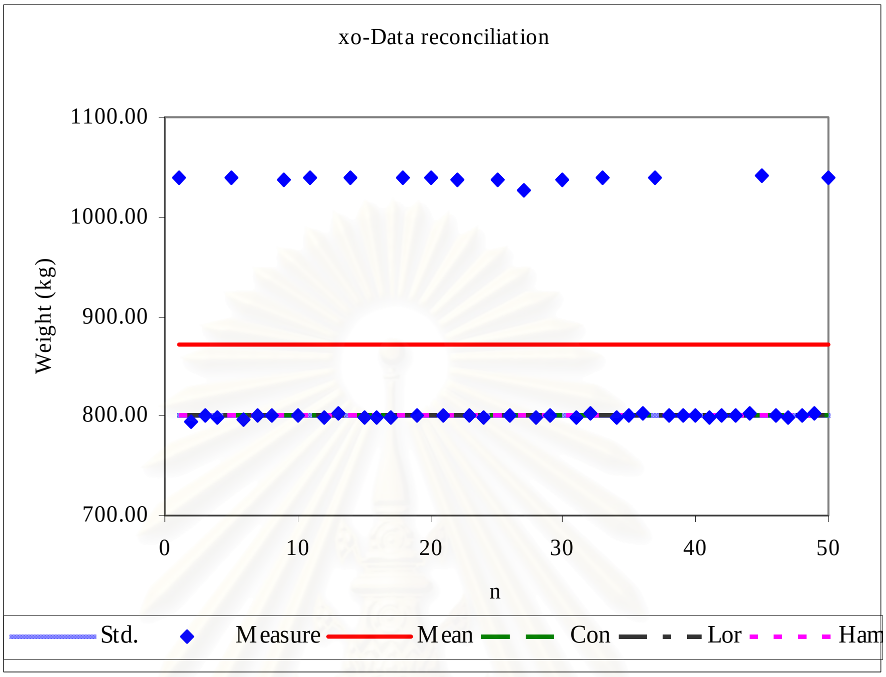


Figure 4.27 The data reconciliation result of *x*0 in the test case +30% error size.

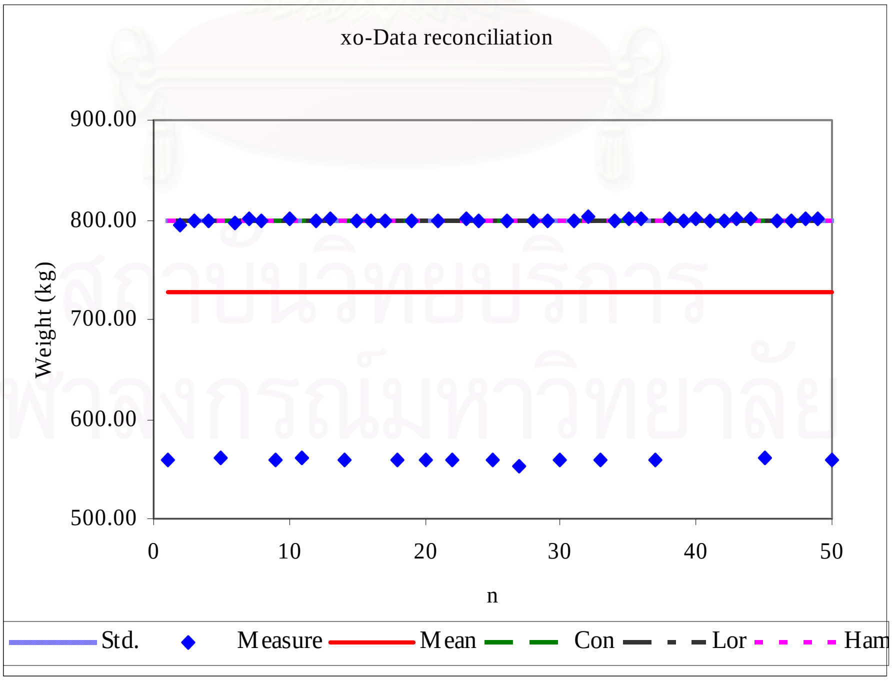


Figure 4.28 The data reconciliation result of *x*0 in the test case -30% error size.

The case of x1 ±30% error size

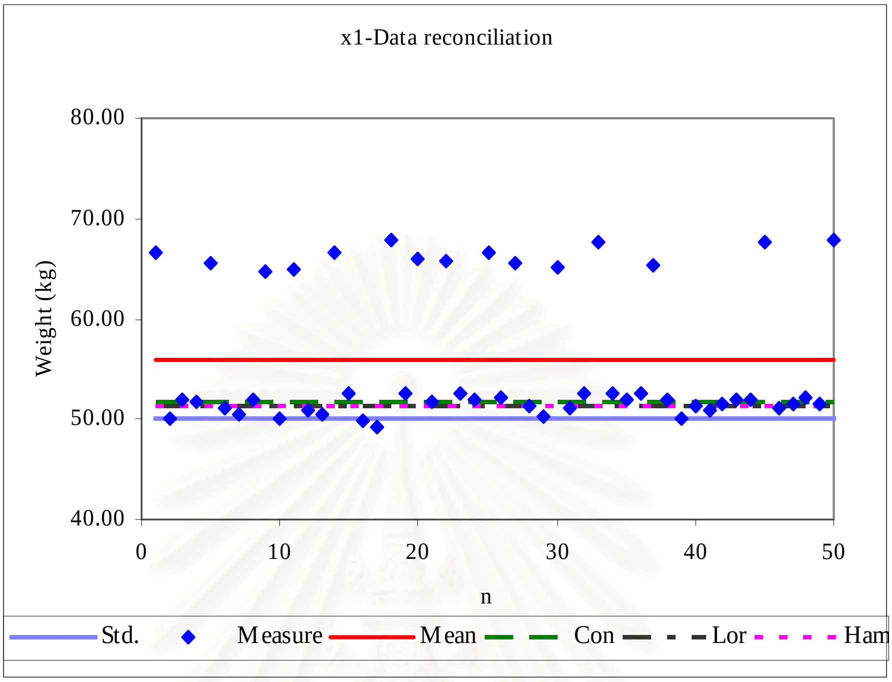


Figure 4.29 The data reconciliation result of *x*1 in the test case +30% error size.

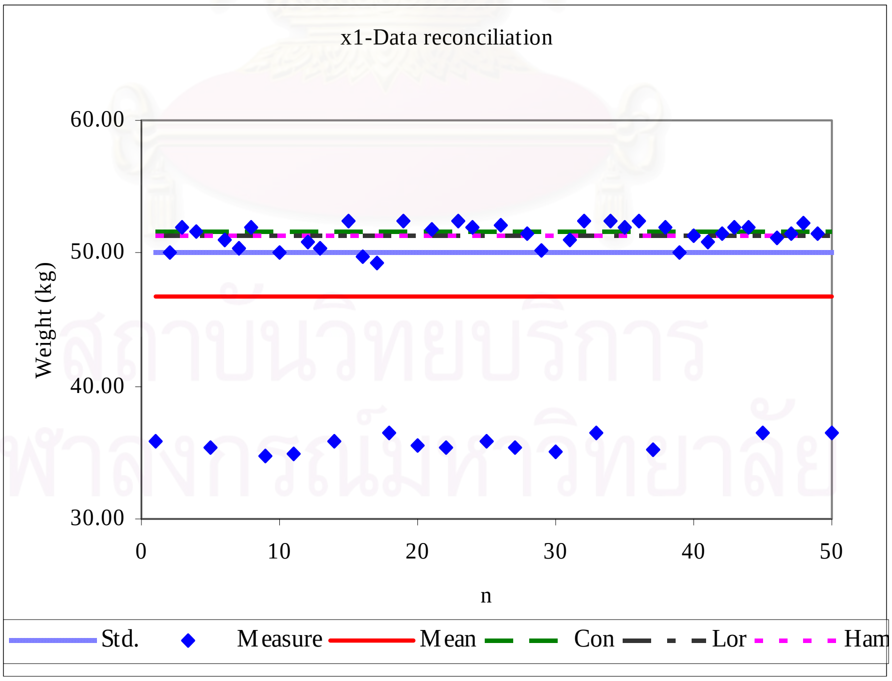


Figure 4.30 The data reconciliation result of *x*1 in the test case -30% error size.

The case of x2 +30% error size

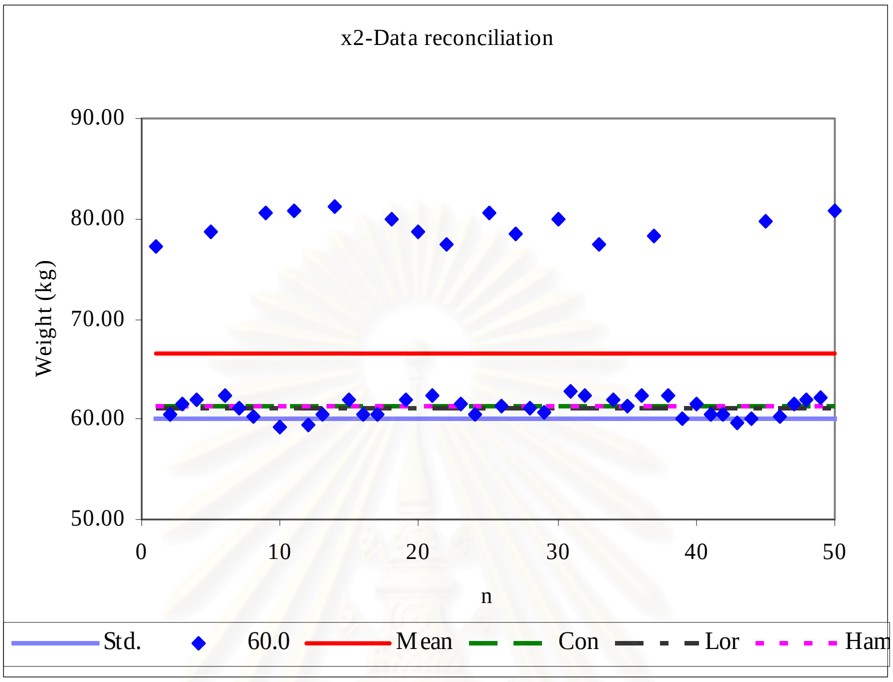


Figure 4.31 The data reconciliation result of *x*2 in the test case +30% error size.

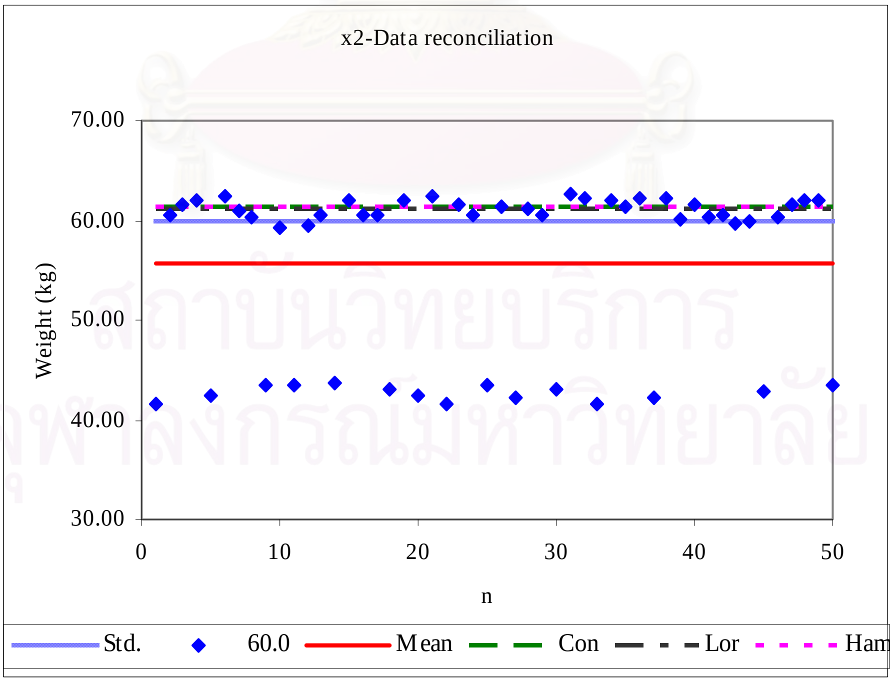


Figure 4.32 The data reconciliation result of *x*2 in the test case -30% error size.

The case of x3 ±30% error size

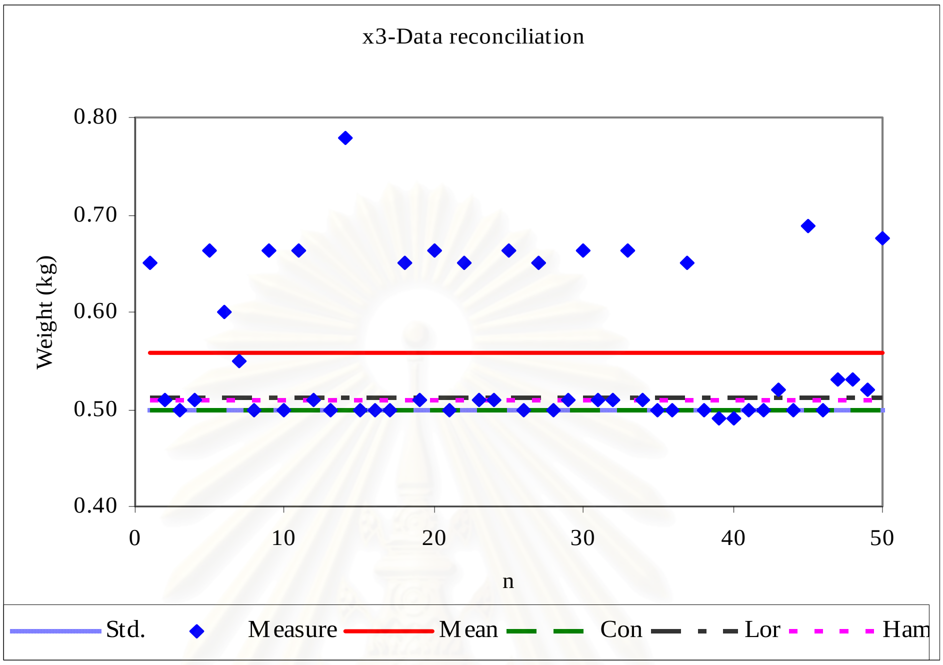


Figure 4.33 The data reconciliation result of *x*3 in the test case +30% error size.

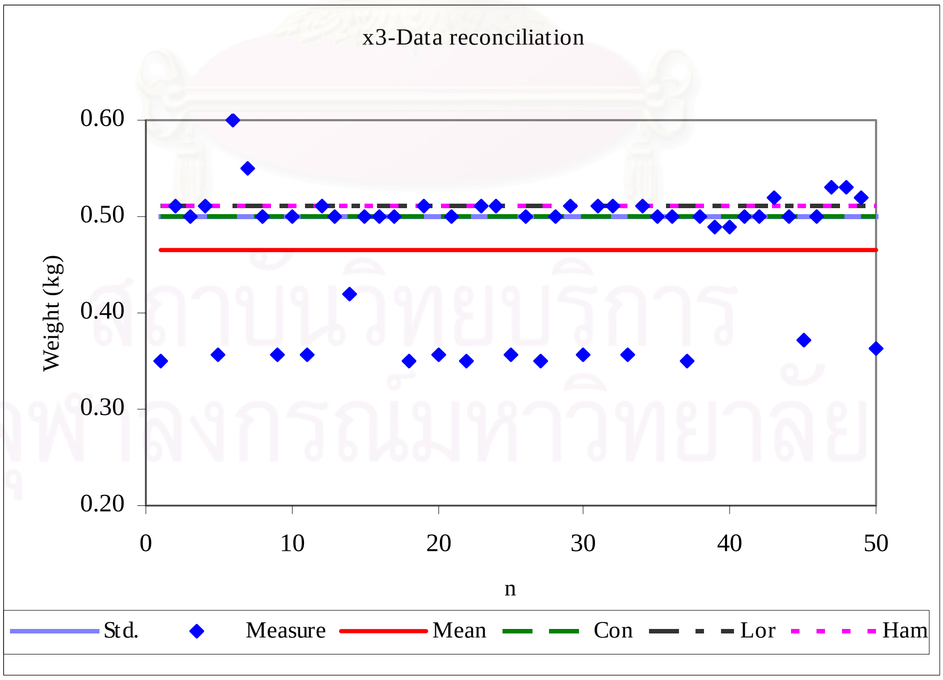


Figure 4.34 The data reconciliation result of *x*3 in the test case -30% error size.

The case of x4 ±30% error size

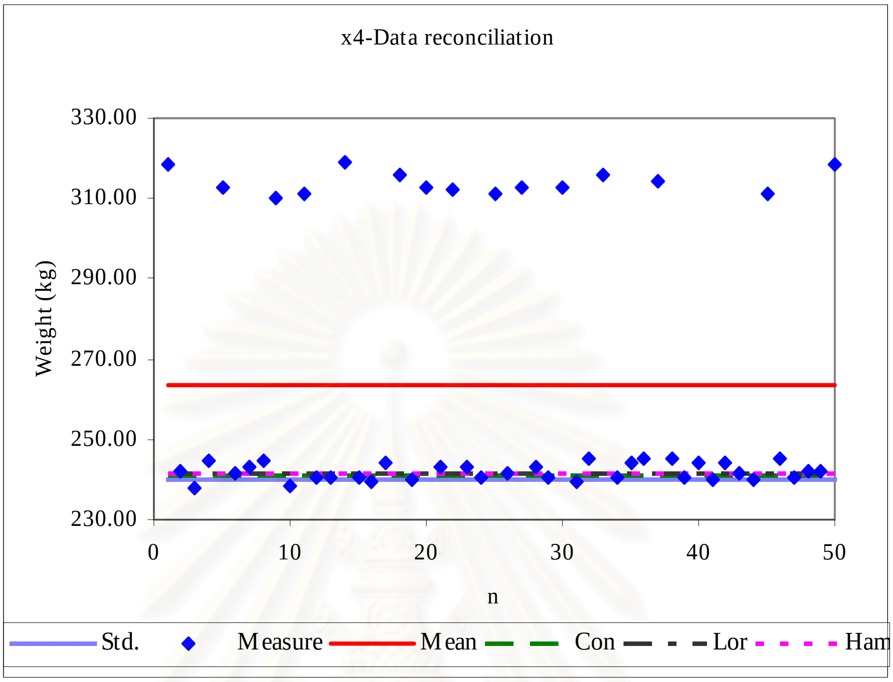


Figure 4.35 The data reconciliation result of *x*4 in the test case +30% error size.

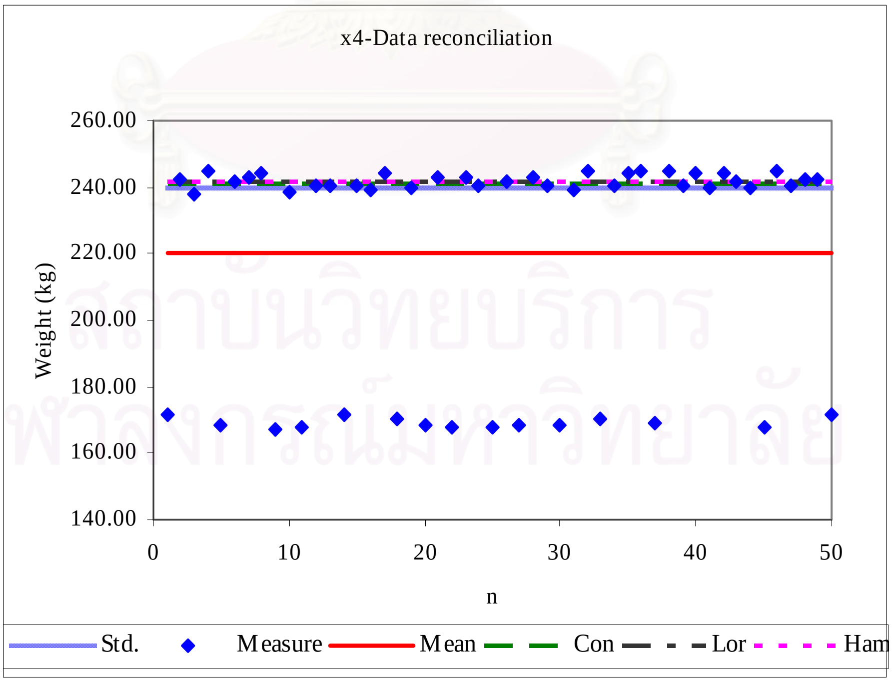


Figure 4.36 The data reconciliation result of *x*4 in the test case -30% error size.

The case of x5 ±30% error size

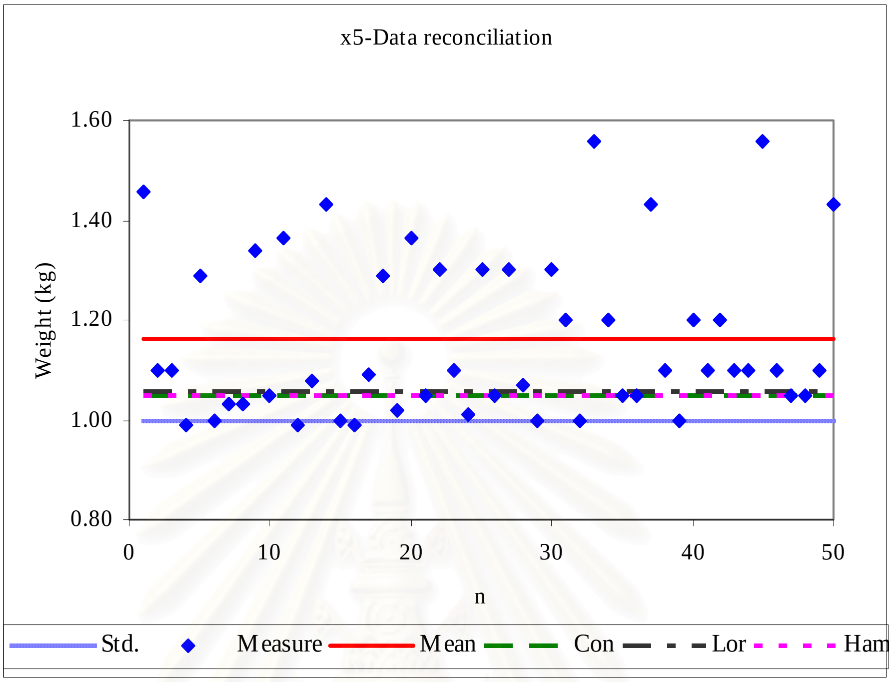


Figure 4.37 The data reconciliation result of *x*5 in the test case +30% error size.

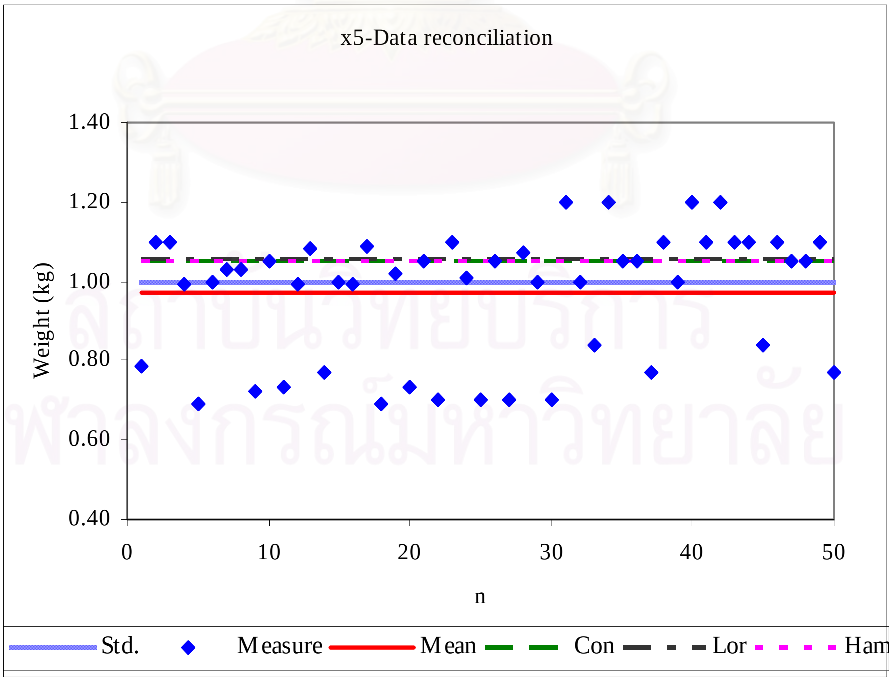


Figure 4.38 The data reconciliation result of *x*5 in the test case -30% error size.

The case of x6 +30% error size

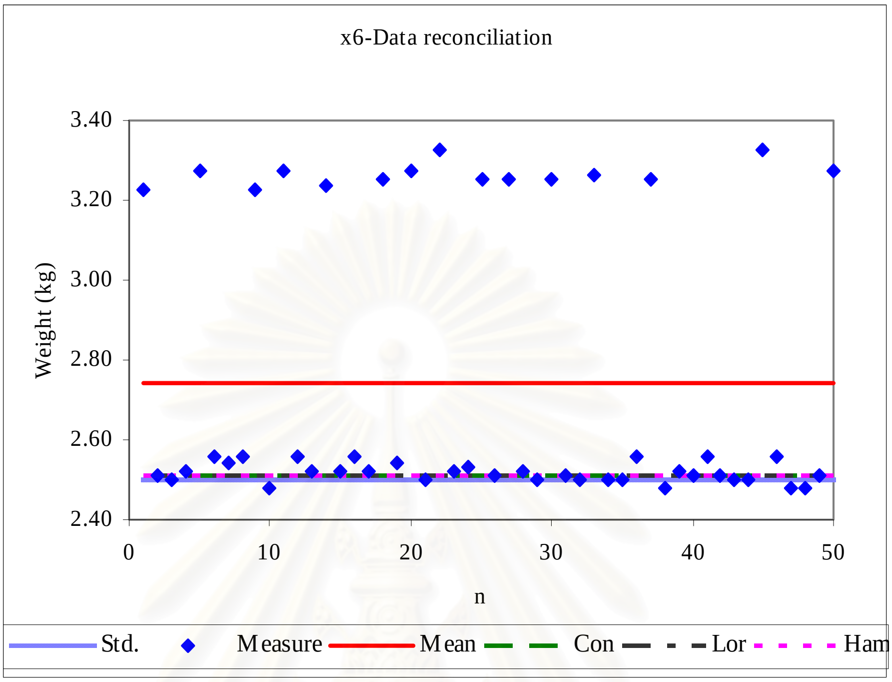


Figure 4.39 The data reconciliation result of *x*6 in the test case +30% error size.

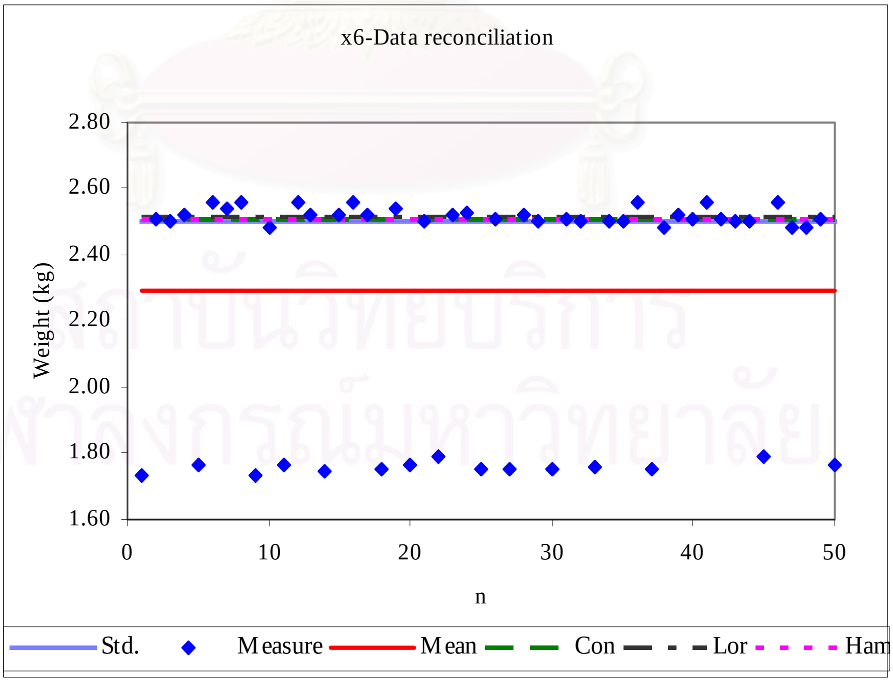


Figure 4.40 The data reconciliation result of *x*6 in the test case -30% error size.

The case of x7 ±30% error size

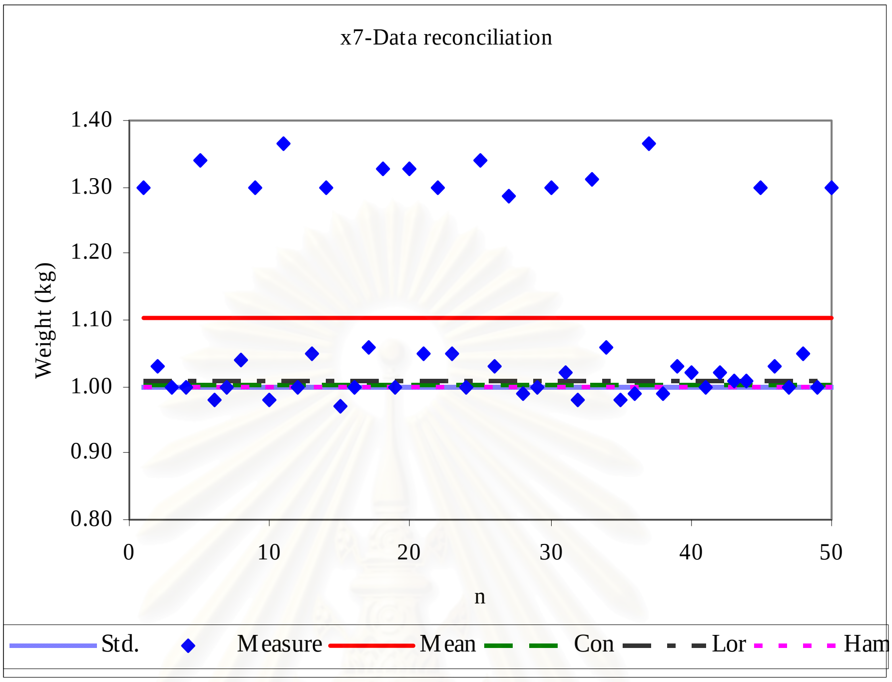


Figure 4.41 The data reconciliation result of *x*7 in the test case +30% error size.

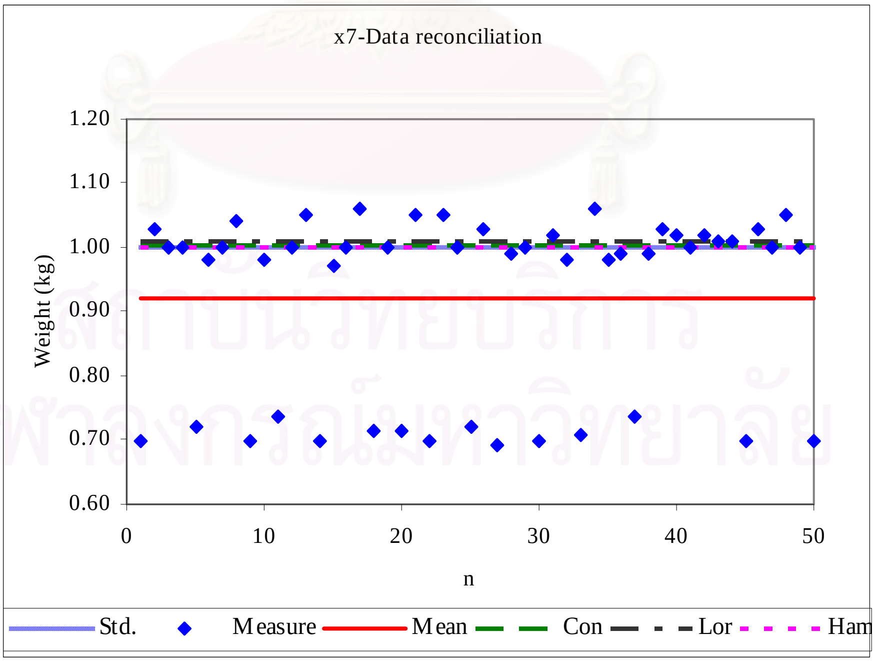


Figure 4.42 The data reconciliation result of *x*7 in the test case -30% error size.

The case of x8±30% error size

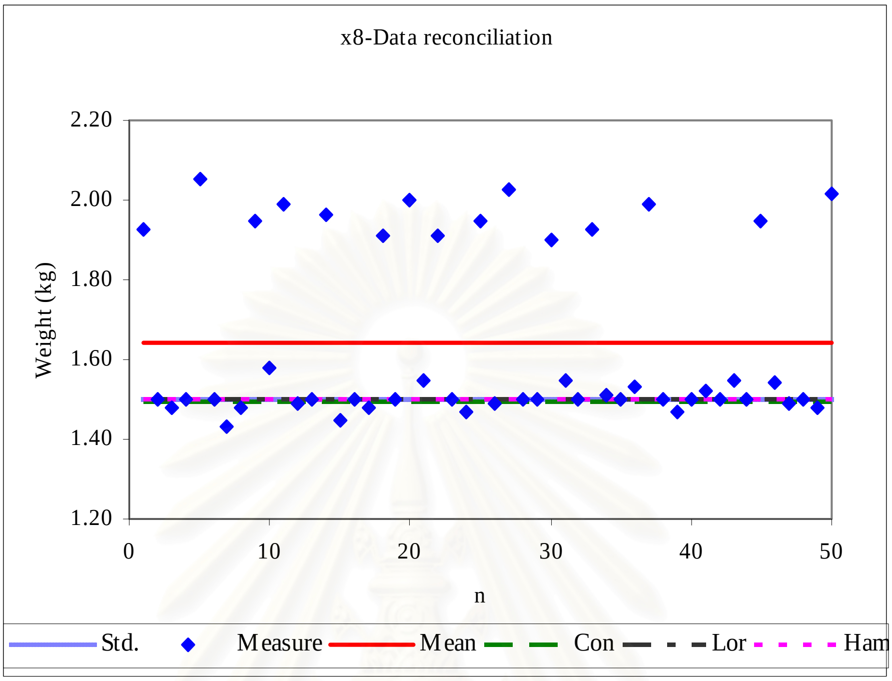


Figure 4.43 The data reconciliation result of *x*8 in the test case +30% error size.

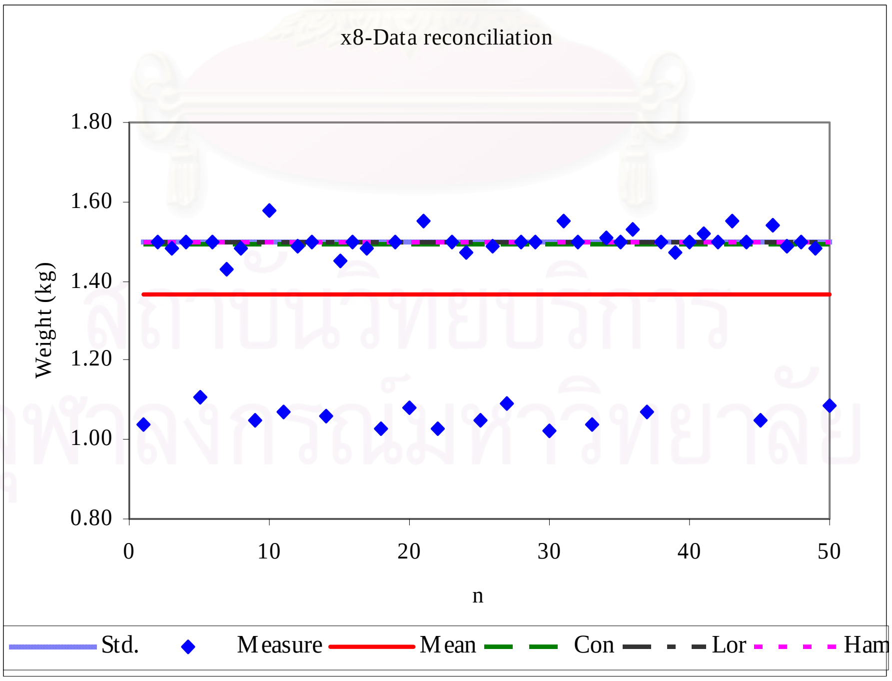


Figure 4.44 The data reconciliation result of *x*8 in the test case -30% error size.

The case of x9 ±30% error size

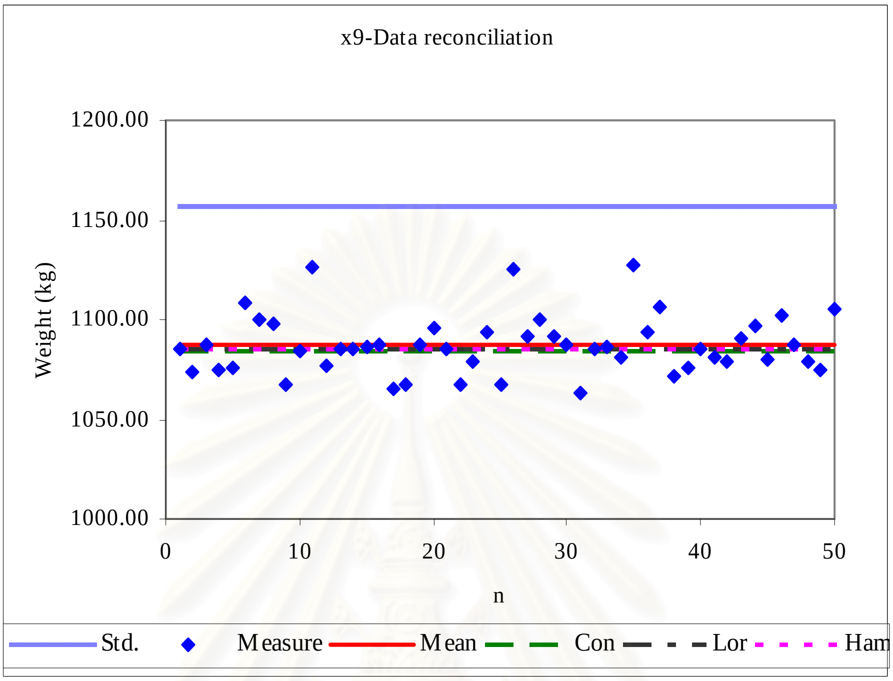


Figure 4.45 The data reconciliation result of *x*9 in the test case +30% error size.

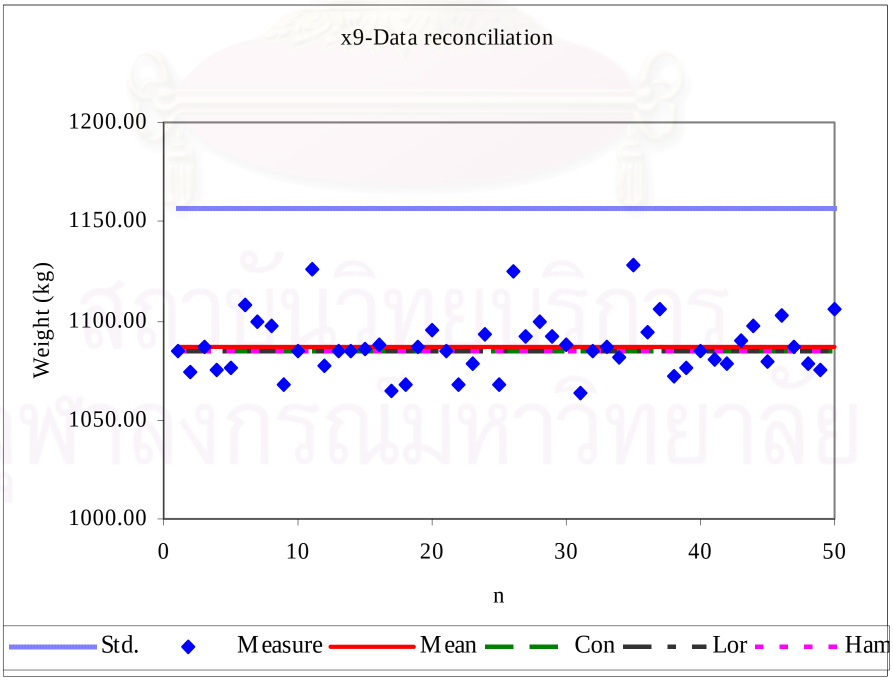


Figure 4.46 The data reconciliation result of *x*9 in the test case -30% error size

Table 4.12 The data reconciliation results of three different ρ functions in the case of +30% error sizes..

xi	Contaminated			Lorent			Hampel		
	10%	**20%**	**30%**	**10%**	**20%**	**30%**	**10%**	**20%**	**30%**
x0	799.62	800.10	802.82	800.64	814.19	817.58	799.81	799.97	800.71
x1	51.23	51.38	51.61	51.30	52.28	54.01	51.50	51.83	52.00
x2	61.11	61.32	61.42	61.19	62.36	64.29	61.47	61.71	62.00
x3	0.51	0.51	0.52	0.51	0.52	0.54	0.51	0.51	0.51
x4	241.68	242.24	242.86	242.01	246.43	252.57	241.60	243.00	243.49
x5	1.05	1.06	1.08	1.06	1.09	1.13	1.05	1.09	1.10
x6	2.52	2.52	2.53	2.52	2.56	2.65	2.52	2.52	2.52
x7	1.01	1.01	1.02	1.01	1.03	1.07	1.01	1.02	1.03
x8	1.50	1.50	1.51	1.51	1.53	1.58	1.50	1.50	1.50
x9	1084.28	1084.28	1084.28	1085.10	1085.09	1085.43	1085.00	1085.00	1085.01
x10	-75.96	-77.38	-81.09	-76.66	-96.90	-110.00	-75.97	-78.15	-79.85

Table 4.13 The RAE percentage of three different ρ functions in the case of +30% error sizes.

xi	Contaminated			Lorent			Hampel		
	10%	**20%**	**30%**	**10%**	**20%**	**30%**	**10%**	**20%**	**30%**
x0	0.01	0.05	0.39	0.11	1.80	2.23	0.01	0.03	0.12
x1	0.84	0.55	0.10	0.10	1.80	5.17	0.21	0.85	1.19
x2	0.40	0.06	0.10	0.06	1.97	5.13	0.28	0.67	1.14
x3	11.46	11.46	9.72	0.22	1.74	5.65	0.26	0.26	0.26
x4	0.31	0.54	0.80	0.16	1.99	4.53	0.03	0.61	0.81
x5	0.30	1.25	3.16	0.29	3.13	6.92	0.00	3.81	4.76
x6	0.50	0.50	0.90	0.29	1.89	5.47	0.40	0.40	0.40
x7	0.70	0.70	1.69	0.11	2.09	6.06	1.00	2.00	3.00
x8	0.31	0.31	0.98	0.69	2.02	5.35	0.00	0.00	0.00
x9	0.10	0.10	0.10	0.36	0.36	0.39	0.01	0.01	0.01
x10	1.38	0.46	5.28	3.23	22.32	38.86	0.42	3.30	5.55
Ave.	**1.48**	**1.45**	**2.11**	**0.51**	**3.74**	**7.80**	**0.24**	**1.08**	**1.57**

Table 4.14 The data reconciliation results of three different ρ functions in the case of –30% error sizes.

xi	Contaminated			Lorent			Hampel		
	10%	**20%**	**30%**	**10%**	**20%**	**30%**	**10%**	**20%**	**30%**
x0	798.92	798.51	796.73	797.89	784.49	756.27	798.92	798.99	798.98
x1	51.18	51.28	51.19	51.11	50.35	48.53	51.18	51.15	50.96
x2	61.05	61.19	60.91	60.97	60.07	57.80	61.05	60.81	60.50
x3	0.51	0.51	0.50	0.51	0.50	0.48	0.51	0.50	0.50
x4	241.46	241.76	240.97	241.16	237.43	228.72	241.46	240.50	240.42
x5	1.04	1.06	1.05	1.05	1.04	1.00	1.04	1.05	1.02
x6	2.52	2.51	2.51	2.51	2.47	2.38	2.52	2.50	2.50
x7	1.01	1.01	1.00	1.01	0.99	0.96	1.01	1.00	1.00
x8	1.50	1.50	1.50	1.50	1.47	1.42	1.50	1.50	1.49
x9	1084.28	1084.27	1084.28	1085.10	1085.09	1085.10	1084.28	1085.01	1085.01
x10	-74.92	-75.06	-72.10	-72.61	-53.72	-12.46	-74.92	-72.99	-72.35

Table 4.15 The RAE percentage of three different ρ functions in the case of –30% error sizes.

xi	Contaminated			Lorent			Hampel		
	10%	**20%**	**30%**	**10%**	**20%**	**30%**	**10%**	**20%**	**30%**
x0	0.10	0.15	0.37	0.23	1.91	5.44	0.10	0.10	0.10
x1	0.93	0.74	0.91	0.47	1.95	5.50	0.41	0.47	0.84
x2	0.50	0.27	0.73	0.30	1.77	5.48	0.41	0.80	1.31
x3	11.46	11.46	13.19	0.22	2.17	6.08	0.26	1.71	1.71
x4	0.22	0.35	0.02	0.19	1.73	5.34	0.03	0.43	0.46
x5	0.66	1.25	0.30	0.65	1.60	5.38	0.95	0.00	2.86
x6	0.50	0.10	0.10	0.10	1.70	5.28	0.40	0.40	0.40
x7	0.70	0.70	0.30	0.11	1.87	4.85	1.00	0.00	0.00
x8	0.31	0.31	0.31	0.02	1.98	5.31	0.00	0.00	0.67
x9	0.10	0.09	0.10	0.36	0.36	0.36	0.06	0.01	0.01
x10	2.73	2.55	6.39	8.34	32.14	84.37	0.97	3.52	4.36
Ave.	**1.42**	**1.42**	**1.51**	**0.25**	**1.57**	**4.48**	**0.34**	**0.37**	**0.79**

Case I ± 30% error size and 10%, 20%, and 30% error amount

X0

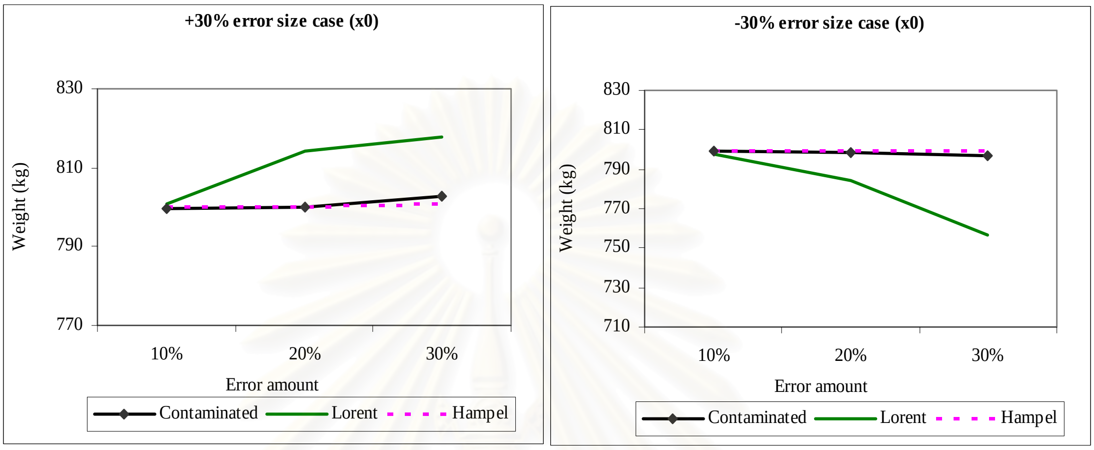


Figure 4.47 The data reconciliation result of *x*0 in the case of ±30% error sizes and 10%, 20%, and 30% error amount.

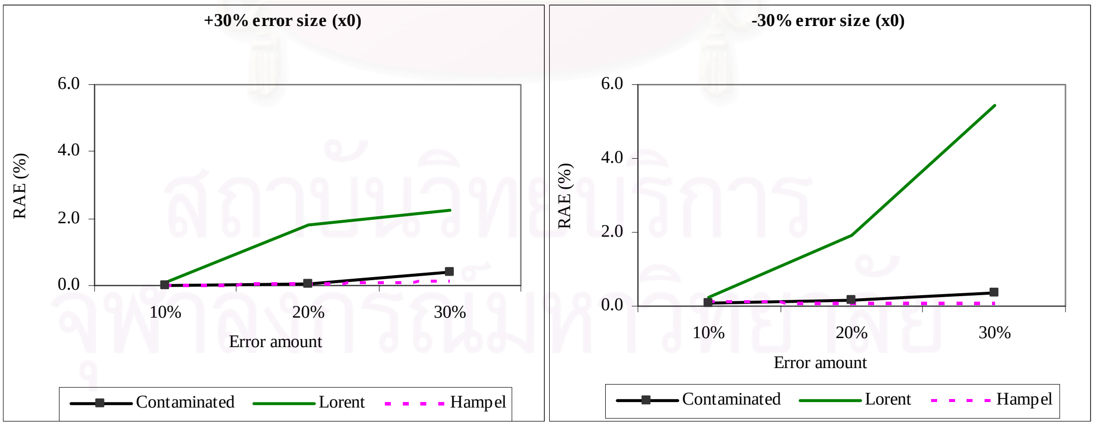


Figure 4.48 The RAE percentage of *x*0 in the case of ±30% error sizes and 10%, 20%, and 30% error amount.

X1

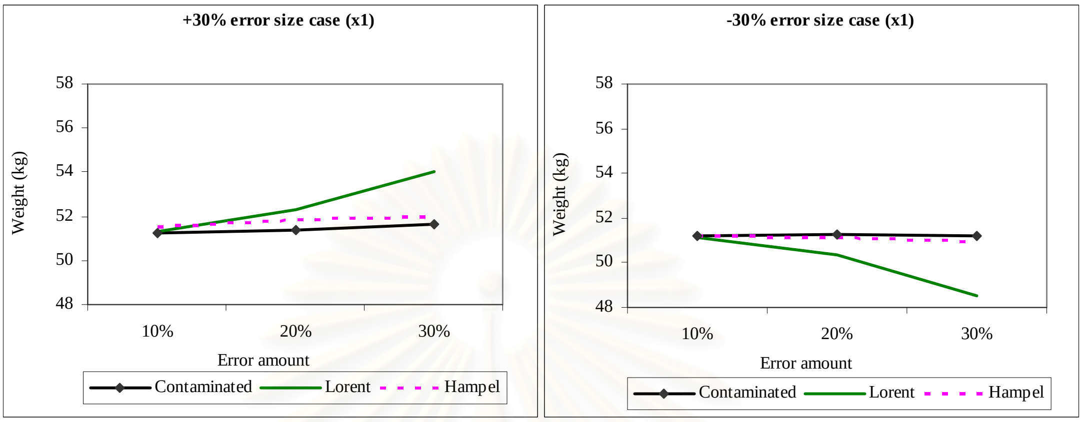


Figure 4.49 The data reconciliation result of $x1$ in the case of ±30% error sizes and 10%, 20%, and 30% error amount.

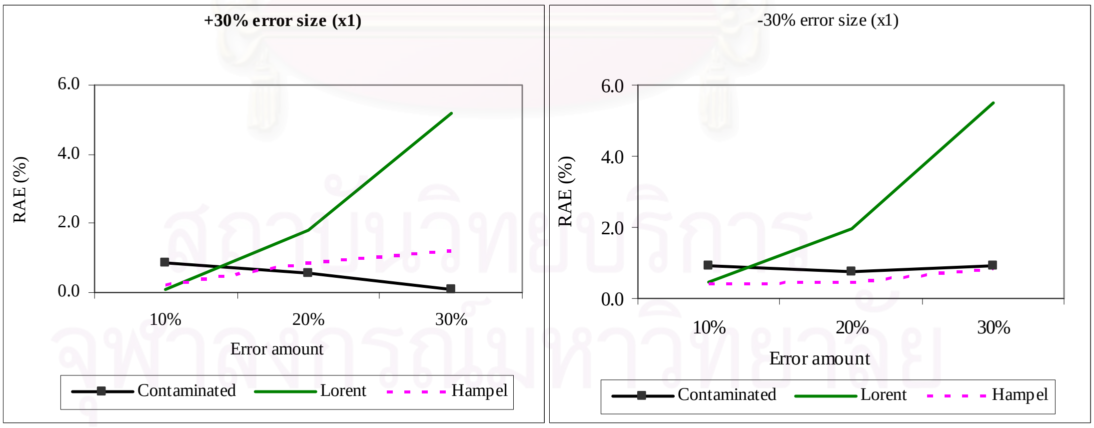


Figure 4.50 The RAE percentage of $x1$ in the case of ±30% error sizes and 10%, 20%, and 30% error amount.

X2

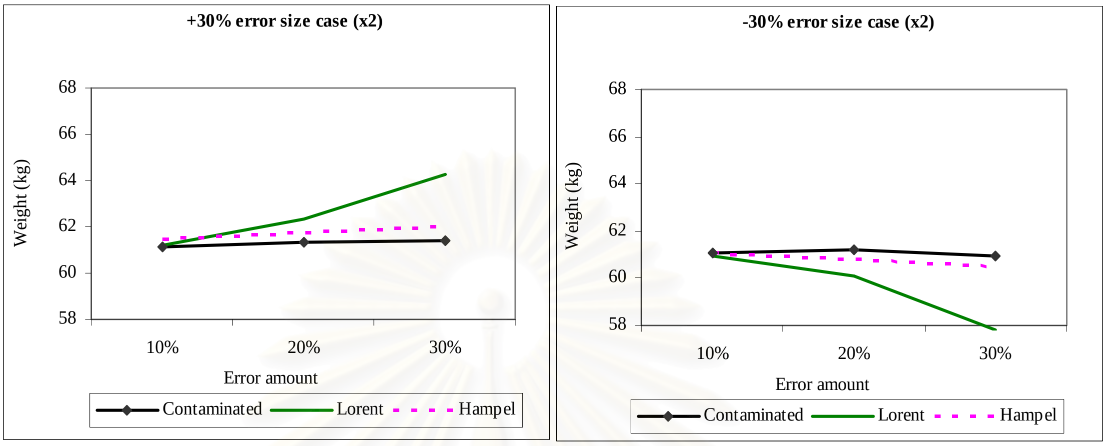


Figure 4.51 The data reconciliation result of *x*2 in the case of ±30% error sizes and 10%, 20%, and 30% error amount.

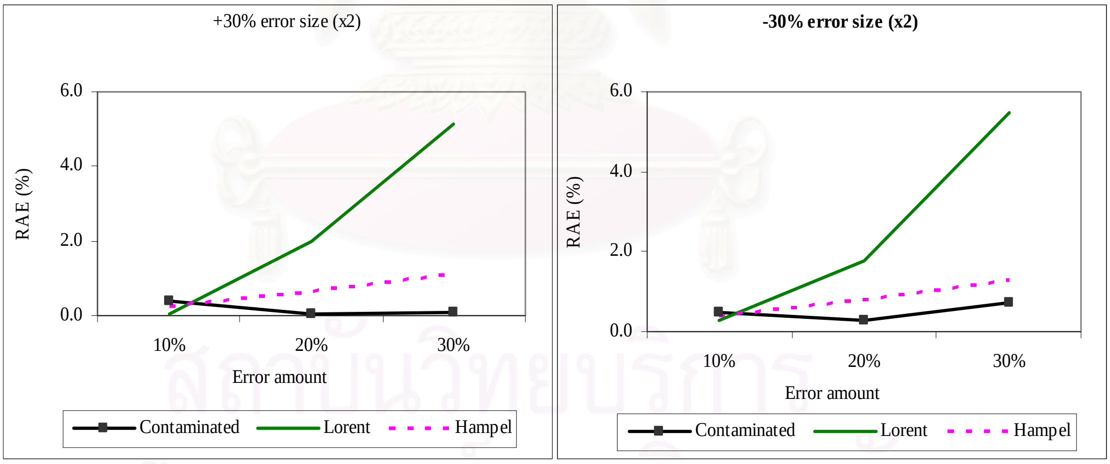


Figure 4.52 The RAE percentage of *x*2 in the case of ±30% error sizes and 10%, 20%, and 30% error amount.

Case II ± 30% error size and 10%, 20%, 30%, and 100% error amount

X0

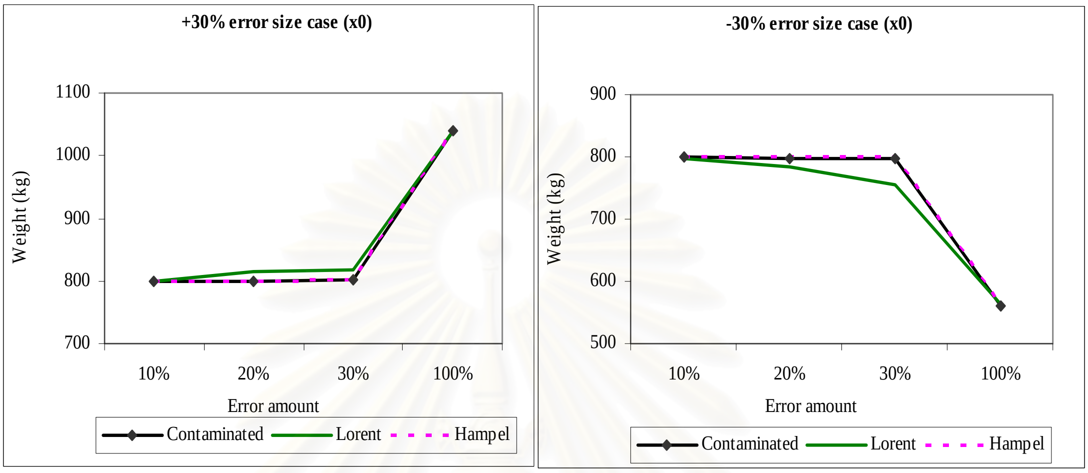


Figure 4.53 The data reconciliation result of x0 in the case of ±30% error sizes and 10%, 20%, 30%, and 100% error amount.

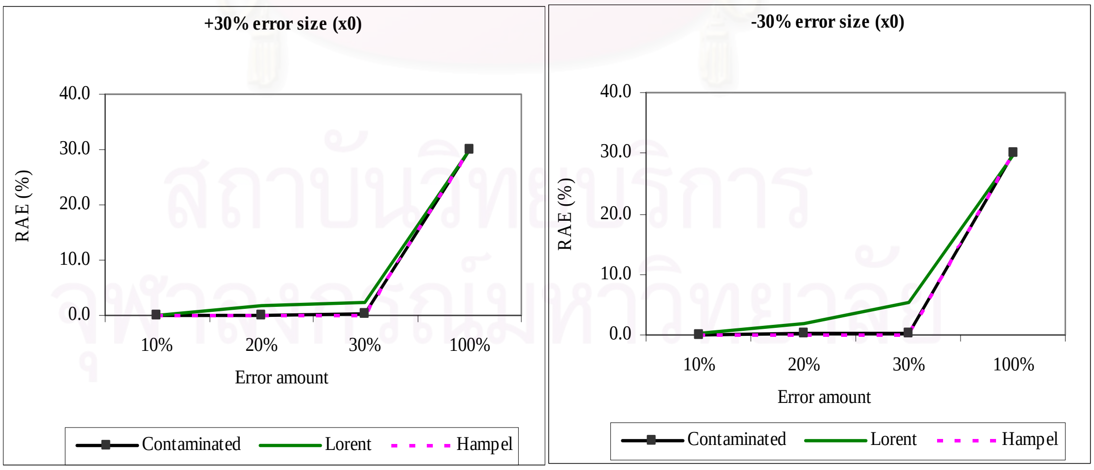


Figure 4.54 The RAE percentage of x0 in the case of ±30% error sizes and 10%, 20%, 30%, and 100% error amount.

X1

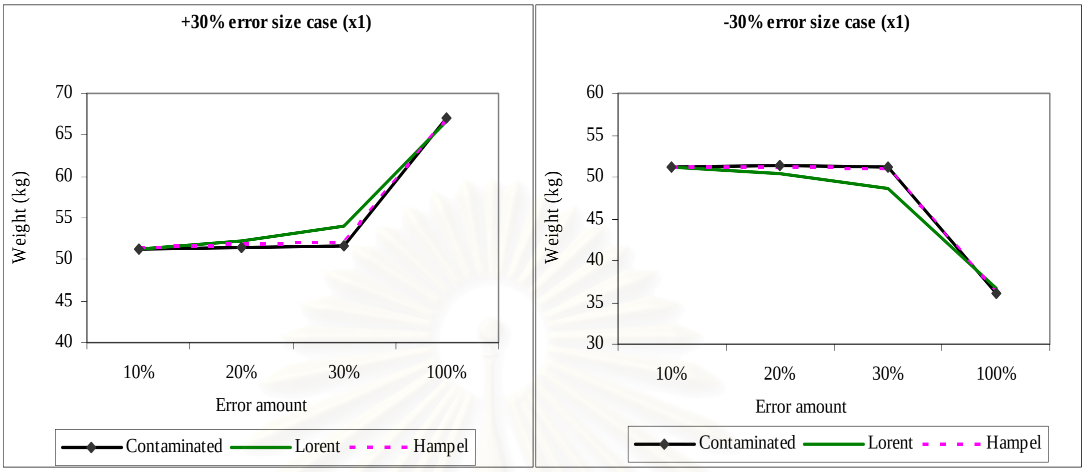


Figure 4.55 The data reconciliation result of x1 in the case of ±30% error sizes and 10%, 20%, 30%, and 100% error amount.

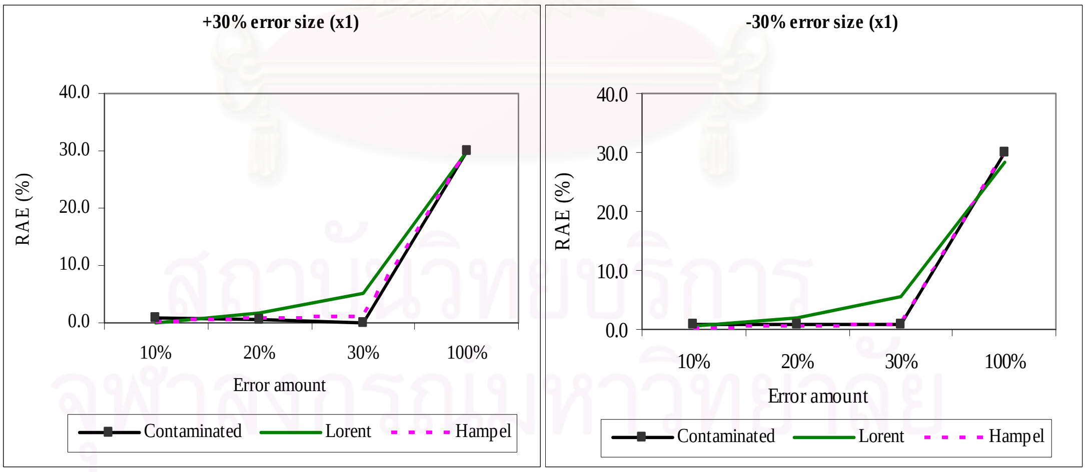


Figure 4.56 The RAE percentage of x1 in the case of ±30% error sizes and 10%, 20%, 30%, and 100% error amount.

X2

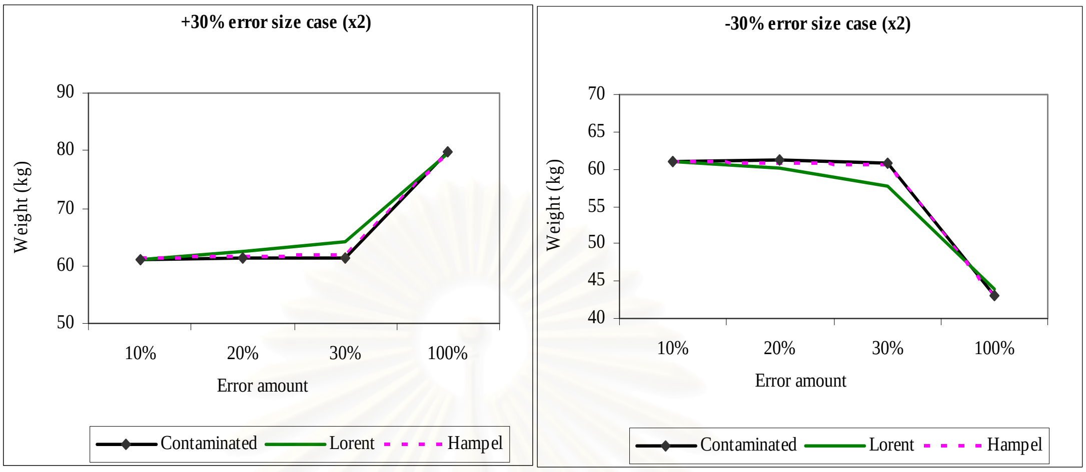


Figure 4.57 The data reconciliation result of x2 in the case of ±30% error sizes and 10%, 20%, 30%, and 100% error amount.

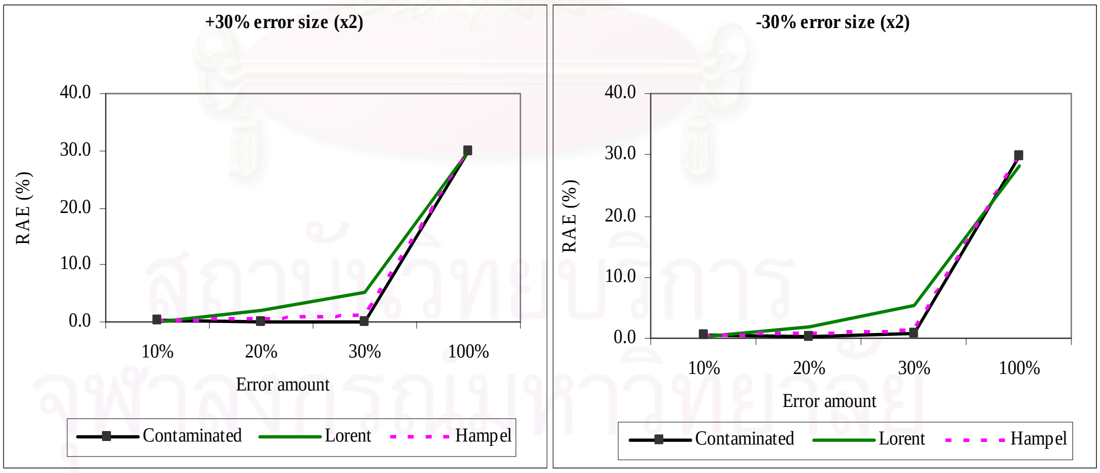


Figure 4.58 The RAE percentage of x2 in the case of ±30% error sizes and 10%, 20%, 30%, and 100% error amount.

Furthermore, the results of data reconciliation in the worst case are presented in the following which the case of ±30% error size and 10%, 20%, and 30% error amount (case I) is shown in Table 4.16 and depicted in Figure 4.59 and the case of ±30% error size and 10%, 20%, 30%, and 100% error amount (case II) is depicted in Figure 4.60 and the case of 30% error amount and ±10%, ±20%, and ±30% error size is shown in Table 4.17 and depicted in Figure 4.61.

As seen in these results, it can be concluded that three different methods provide satisfy reconciled values even though the size and the amount of measurement data error is changed provided the average RAE values of all three method below 9%. Moreover, it can be concluded that the ρ functions of Hampel's redescending M-estimator is the most appropriated function used for formulating the data reconciliation as seen from the average RAE values of all cases provided the values below 2%.

4.3.2.2 Discussions of Test Case

a) For the testing of changing error amount, detailed statistical results were obtained from 1000 runs, and the details of data reconciliation results shown in Table 4.4, 4.6, 4.8, 4.10, 4.12, and 4.14. The results of relative absolute error and average relative absolute error shown in Table 4.5, 4.7, 4.9, 4.11, 4.13, and 4.15.

b) The average relative absolute error results of changing error amount from 10% to 30% depicted in Table 4.16. The function of Hampel's redescending M-estimator shows the best performance with lowest average relative absolute error.

c) The average relative absolute error results of changing error amount from 10% to 100% depicted in Figure 60. The three difference ρ functions show the nearly values for average relative absolute error.

d) The average relative absolute error results of changing error size for the most of error amount (30% error amount) depicted in Table

4.17. The function of Hampel's redescending M-estimator shows the best performance with lowest average relative absolute error.

e) The relative absolute error results of changing error size and error amount in the most case for the function of Hampel's redescending M-estimator shows x10 variable is the most relative absolute error variable.

<u>Case I</u>

Table 4.16 The comparison of average RAE for the case of ±30% error sizes and 10%, 20%, and 30% error amount.

Error amount	**Three rho functions**		
	Contaminated	**Lorentzian**	**Hampel**
+30% error size case			
10%	1.48	0.51	0.24
20%	1.45	3.74	1.08
30%	2.11	7.8	1.57
-30% error size case			
10%	1.42	0.25	0.34
20%	1.42	1.57	0.37
30%	1.51	4.48	0.79

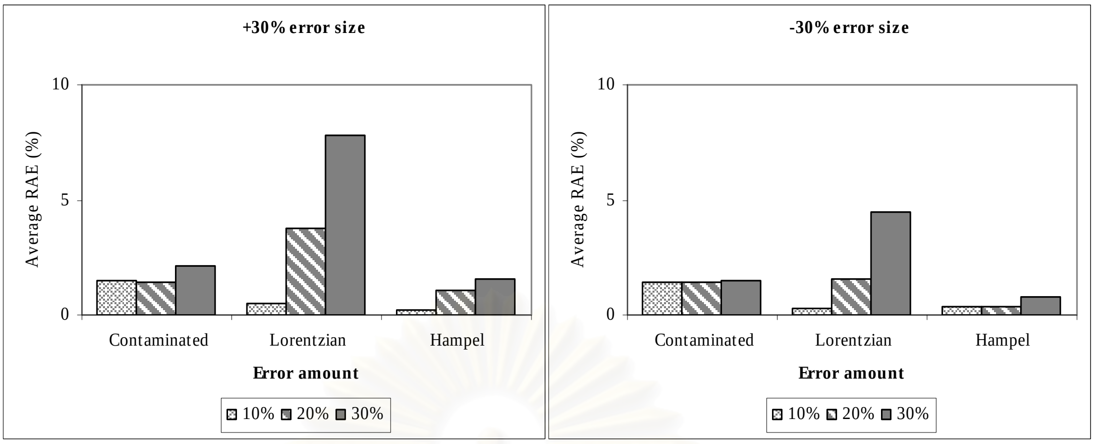


Figure 4.59 The RAE percentage for the case of ±30% error sizes and 10%, 20%, and 30% error amount.

<u>Case II</u>

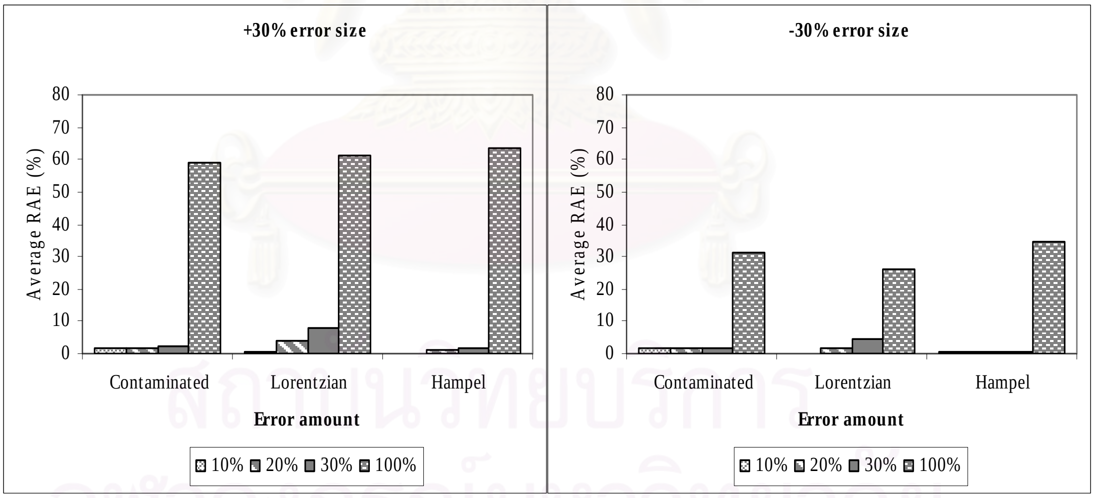


Figure 4.60 The RAE percentage for the case of ±30% error sizes and 10%, 20%, and 30% error amount.

Table 4.17 The comparison of average RAE for the 30% error amount and ±10%, ±20%, and ±30% error size.

Error size	Three rho functions		
	Contaminated	**Lorentzian**	**Hampel**
+10%	1.54	3.46	1.56
+20%	1.93	6.71	1.41
+30%	2.11	7.8	1.57
-10%	1.77	4.59	0.48
-20%	2.01	8.41	0.81
-30%	1.51	4.48	0.79

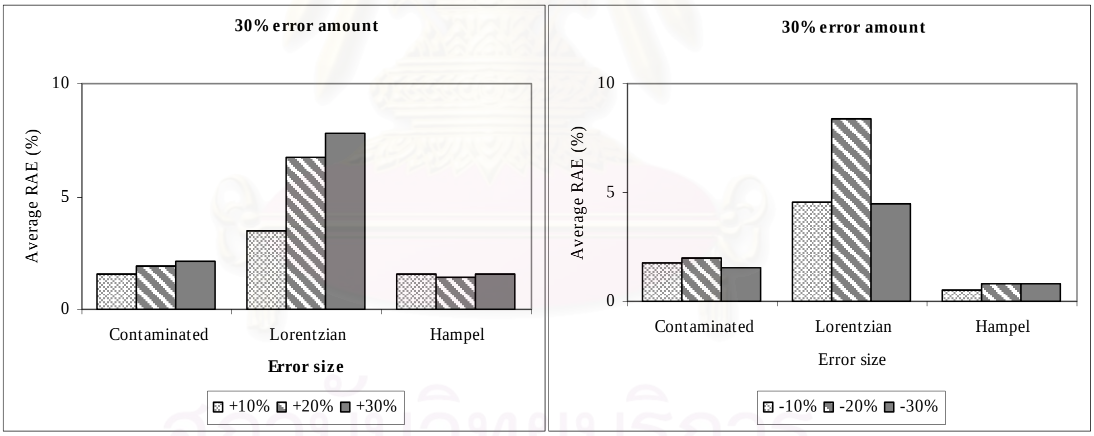


Figure 4.61 The RAE percentage for the 30% error amount and ±10%, ±20%, and ±30% error size.

CHAPTER V

CONCLUSION AND RECOMMENDATIONS

This chapter concludes this thesis and provides a summary of work done during the course of this research. Some recommendations for future work are also given at the final section.

5.1 Conclusions

In general, measured data inherently contains inaccurate information since measurements are obtained with imperfect instruments. The errors in measured data can lead to significant deterioration in process performance. Data reconciliation and gross errors detection techniques are the techniques widely applied in various chemical processes to adjust the process data to satisfy the constrains of the system model and provides estimates for unmeasured variables and process parameters, which are used in the consecutive economic optimization step.

In this work, the simultaneous data reconciliation and gross error detection techniques are applied in the mixing tank of battery production process at steady state condition for cross-check of real time process measurements. Furthermore, three different robust methods as follow: Contaminated normal distribution, Lorentzian distribution, and Hampel's redescending M-estimator used for formulating the data reconciliation problem are studied and evaluated the performance of each method by using the RAE value to determined the appropriated method for mixing tank of battery production process.

This work is divided into two case studies. The first case study has the objective to determine the reconciled data from the data reconciliation problem formulated by using three different methods. The other case study is a test case which has the objective to test the performance of these three methods under two conditions, the first one is the size of measurement error changed, and the other one is the amount of data set of measurement error changed.

The two case studies is to determine the reconciled data solved from the data reconciliation problem formulated in term of the ρ function which this work is studied only three different ρ functions: Contaminated normal distribution, Lorentzian distribution, and Hampel's redescending M-estimator.

The results of the data reconciliation problem formulated by using three different methods can be seen that in the case of nominal, which contain only random errors, the reconciled data obtained from three different ρ functions are closed to true value because measurement data of each variables in process have normal distribution therefore all functions perform well in this case.

For comparison, in the test case, which contain both gross errors and random errors, it can be seen that three different ρ functions provide good reconciled values same as nominal case. For the testing of error size changed when we increase size of error from 10%, 20% to 30%, the results of increasing effect to Lorentzian distribution function but it does not effect to Contaminated normal distribution function and Hampel's redescending M-estimator function. For the testing of error amount changed when we increase amount of error from ±10%, ±20% to ±30%, the results of increasing effect to Lorentzian distribution function but it does not effect to Contaminated normal distribution function and Hampel's redescending M-estimator function. Therefore, the increasing of the size and the amount of error does not effect to Contaminated normal distribution function and Hampel's redescending M-estimator function for mixing tank of battery production process. For the testing of error amount changed when we increase amount of error from 10%, 20%, 30% to 100%, the results of relative absolute error for 100% error amount effect to three difference ρ functions.

As seen in the results, it can be concluded that three different methods provide satisfy reconciled values even though the size and the amount of measurement data error is changed, especially the Contaminated normal distribution function and

Hampel’s redescending M-estimator function to provide better reconciled values than Lorentzian distribution function provided not good reconciled values in the test case.

Moreover, it can also be concluded that the ρ functions of Hampel’s redescending M-estimator is the most appropriated function used for data reconciliation in mixing tank of battery production process.

5.2 Recommendations for Future Work

For the future direction, data reconciliation and gross error detection techniques studied in this work should be applied to the other process. Furthermore, the process control or optimization problem with the application of data reconciliation and gross error detection techniques should also study in the future work.

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APPENDICES

APPENDIX A

MEASURED DATA

A.1 Measured Data in a Nominal Case

No.	Reconciled variable									
	x0	**x1**	**x2**	**x3**	**x4**	**x5**	**x6**	**x7**	**x8**	**x9**
1	799.50	51.30	59.40	0.50	245.00	1.12	2.48	1.00	1.48	1085.00
2	795.00	50.10	60.50	0.51	242.10	1.10	2.51	1.03	1.50	1073.60
3	800.00	52.00	61.50	0.50	238.00	1.10	2.50	1.00	1.48	1087.28
4	798.50	51.70	62.00	0.51	244.60	0.99	2.52	1.00	1.50	1075.20
5	800.20	50.50	60.50	0.51	240.40	0.99	2.52	1.03	1.58	1076.25
6	797.00	51.00	62.40	0.60	241.70	1.00	2.56	0.98	1.50	1108.01
7	801.00	50.40	61.00	0.55	243.00	1.03	2.54	1.00	1.43	1099.80
8	799.80	52.00	60.30	0.50	244.40	1.03	2.56	1.04	1.48	1097.40
9	798.50	49.70	62.00	0.51	238.60	1.03	2.48	1.00	1.50	1067.50
10	800.90	50.00	59.30	0.50	238.40	1.05	2.48	0.98	1.58	1084.65
11	800.10	49.90	62.20	0.51	239.40	1.05	2.52	1.05	1.53	1125.91
12	799.00	50.90	59.50	0.51	240.30	0.99	2.56	1.00	1.49	1076.77
13	802.00	50.40	60.50	0.50	240.50	1.08	2.52	1.05	1.50	1084.83
14	799.00	51.30	62.40	0.60	245.50	1.10	2.49	1.00	1.51	1085.00
15	799.00	52.50	62.00	0.50	240.50	1.00	2.52	0.97	1.45	1085.80
16	798.00	49.80	60.50	0.50	239.20	0.99	2.56	1.00	1.50	1087.45
17	798.00	49.20	60.50	0.50	244.00	1.09	2.52	1.06	1.48	1064.88
18	799.00	52.20	61.50	0.50	243.00	0.99	2.50	1.02	1.47	1067.50
19	800.00	52.50	62.00	0.51	240.00	1.02	2.54	1.00	1.50	1087.28
20	799.00	50.70	60.50	0.51	240.50	1.05	2.52	1.02	1.54	1095.48
21	800.00	51.80	62.40	0.50	243.00	1.05	2.50	1.05	1.55	1085.00
22	798.00	50.60	59.50	0.50	240.00	1.00	2.56	1.00	1.47	1067.50
23	801.00	52.50	61.50	0.51	243.00	1.10	2.52	1.05	1.50	1078.80
24	799.00	52.00	60.50	0.51	240.50	1.01	2.53	1.00	1.47	1093.49
25	798.00	51.30	62.00	0.51	239.40	1.00	2.50	1.03	1.50	1067.50
26	800.00	52.10	61.30	0.50	241.60	1.05	2.51	1.03	1.49	1124.92

No.	Reconciled variable									
	x0	**x1**	**x2**	**x3**	**x4**	**x5**	**x6**	**x7**	**x8**	**x9**
27	789.00	50.50	60.30	0.50	240.50	1.00	2.50	0.99	1.56	1092.00
28	799.00	51.40	61.10	0.50	243.00	1.07	2.52	0.99	1.50	1099.62
29	800.00	50.20	60.60	0.51	240.40	1.00	2.50	1.00	1.50	1091.72
30	798.00	50.10	61.50	0.51	240.40	1.00	2.50	1.00	1.46	1087.45
31	799.00	51.00	62.70	0.51	239.40	1.20	2.51	1.02	1.55	1063.66
32	802.90	52.50	62.30	0.51	245.00	1.00	2.50	0.98	1.50	1085.00
33	799.00	52.00	59.50	0.51	243.00	1.20	2.51	1.01	1.48	1086.75
34	798.50	52.50	62.00	0.51	240.40	1.20	2.50	1.06	1.51	1081.47
35	800.10	52.00	61.40	0.50	244.00	1.05	2.50	0.98	1.50	1127.70
36	801.80	52.50	62.30	0.50	245.00	1.05	2.56	0.99	1.53	1093.93
37	800.00	50.30	60.20	0.50	241.60	1.10	2.50	1.05	1.53	1106.25
38	800.40	52.00	62.30	0.50	245.00	1.10	2.48	0.99	1.50	1071.53
39	799.50	50.00	60.10	0.49	240.40	1.00	2.52	1.03	1.47	1076.25
40	800.40	51.30	61.50	0.49	244.00	1.20	2.51	1.02	1.50	1085.00
41	798.80	50.80	60.40	0.50	240.00	1.10	2.56	1.00	1.52	1080.71
42	800.00	51.50	60.50	0.50	244.00	1.20	2.51	1.02	1.50	1078.63
43	801.20	52.00	59.70	0.52	241.50	1.10	2.50	1.01	1.55	1090.25
44	801.80	51.90	60.00	0.50	240.00	1.10	2.50	1.01	1.50	1097.05
45	800.80	52.10	61.30	0.53	239.40	1.20	2.56	1.00	1.50	1079.70
46	799.60	51.10	60.30	0.50	245.00	1.10	2.56	1.03	1.54	1102.50
47	798.50	51.50	61.50	0.53	240.50	1.05	2.48	1.00	1.49	1086.93
48	800.80	52.20	62.00	0.53	242.30	1.05	2.48	1.05	1.50	1078.80
49	802.00	51.50	62.10	0.52	242.30	1.10	2.51	1.00	1.48	1075.03
50	799.50	52.20	62.10	0.52	245.00	1.10	2.52	1.00	1.55	1105.37
Mean	799.40	51.27	61.11	0.51	241.77	1.07	2.52	1.01	1.50	1087.12
Lower	789.00	49.20	59.30	0.49	238.00	0.99	2.48	0.97	1.43	1063.66
Upper	802.90	52.50	62.70	0.60	245.50	1.20	2.56	1.06	1.58	1127.70

A.2 Measured Data in Test Cases

- ***The Case of + 10% Error Size and 10% Error Amount***

No.	Reconciled variable									
	x0	x1	x2	x3	x4	x5	x6	x7	x8	x9
1	799.50	51.30	59.40	0.50	245.00	1.12	2.48	1.00	1.48	1085.00
2	795.00	50.10	60.50	0.51	242.10	1.10	2.51	1.03	1.50	1073.60
3	800.00	52.00	61.50	0.50	238.00	1.10	2.50	1.00	1.48	1087.28
4	798.50	51.70	62.00	0.51	244.60	0.99	2.52	1.00	1.50	1075.20
5	800.20	50.50	60.50	0.51	240.40	0.99	2.52	1.03	1.58	1076.25
6	797.00	51.00	62.40	0.60	241.70	1.00	2.56	0.98	1.50	1108.01
7	801.00	50.40	61.00	0.55	243.00	1.03	2.54	1.00	1.43	1099.80
8	799.80	52.00	60.30	0.50	244.40	1.03	2.56	1.04	1.48	1097.40
9	878.35	54.67	68.20	0.56	262.46	1.13	2.73	1.10	1.65	1067.50
10	800.90	50.00	59.30	0.50	238.40	1.05	2.48	0.98	1.58	1084.65
11	800.10	49.90	62.20	0.51	239.40	1.05	2.52	1.05	1.53	1125.91
12	878.90	55.99	65.45	0.56	264.33	1.09	2.82	1.10	1.64	1076.77
13	802.00	50.40	60.50	0.50	240.50	1.08	2.52	1.05	1.50	1084.83
14	799.00	51.30	62.40	0.60	245.50	1.10	2.49	1.00	1.51	1085.00
15	799.00	52.50	62.00	0.50	240.50	1.00	2.52	0.97	1.45	1085.80
16	798.00	49.80	60.50	0.50	239.20	0.99	2.56	1.00	1.50	1087.45
17	798.00	49.20	60.50	0.50	244.00	1.09	2.52	1.06	1.48	1064.88
18	799.00	52.20	61.50	0.50	243.00	0.99	2.50	1.02	1.47	1067.50
19	800.00	52.50	62.00	0.51	240.00	1.02	2.54	1.00	1.50	1087.28
20	799.00	50.70	60.50	0.51	240.50	1.05	2.52	1.02	1.54	1095.48
21	800.00	51.80	62.40	0.50	243.00	1.05	2.50	1.05	1.55	1085.00
22	798.00	50.60	59.50	0.50	240.00	1.00	2.56	1.00	1.47	1067.50
23	881.10	57.75	67.65	0.56	267.30	1.21	2.77	1.16	1.65	1078.80
24	799.00	52.00	60.50	0.51	240.50	1.01	2.53	1.00	1.47	1093.49
25	798.00	51.30	62.00	0.51	239.40	1.00	2.50	1.03	1.50	1067.50
26	800.00	52.10	61.30	0.50	241.60	1.05	2.51	1.03	1.49	1124.92
27	789.00	50.50	60.30	0.50	240.50	1.00	2.50	0.99	1.56	1092.00
28	799.00	51.40	61.10	0.50	243.00	1.07	2.52	0.99	1.50	1099.62
29	800.00	50.20	60.60	0.51	240.40	1.00	2.50	1.00	1.50	1091.72
30	877.80	55.11	67.65	0.56	264.44	1.10	2.75	1.10	1.61	1087.45

No.	Reconciled variable									
	x0	**x1**	**x2**	**x3**	**x4**	**x5**	**x6**	**x7**	**x8**	**x9**
31	799.00	51.00	62.70	0.51	239.40	1.20	2.51	1.02	1.55	1063.66
32	802.90	52.50	62.30	0.51	245.00	1.00	2.50	0.98	1.50	1085.00
33	799.00	52.00	59.50	0.51	243.00	1.20	2.51	1.01	1.48	1086.75
34	798.50	52.50	62.00	0.51	240.40	1.20	2.50	1.06	1.51	1081.47
35	800.10	52.00	61.40	0.50	244.00	1.05	2.50	0.98	1.50	1127.70
36	801.80	52.50	62.30	0.50	245.00	1.05	2.56	0.99	1.53	1093.93
37	800.00	50.30	60.20	0.50	241.60	1.10	2.50	1.05	1.53	1106.25
38	800.40	52.00	62.30	0.50	245.00	1.10	2.48	0.99	1.50	1071.53
39	799.50	50.00	60.10	0.49	240.40	1.00	2.52	1.03	1.47	1076.25
40	800.40	51.30	61.50	0.49	244.00	1.20	2.51	1.02	1.50	1085.00
41	878.68	55.88	66.44	0.55	264.00	1.21	2.82	1.10	1.67	1080.71
42	800.00	51.50	60.50	0.50	244.00	1.20	2.51	1.02	1.50	1078.63
43	801.20	52.00	59.70	0.52	241.50	1.10	2.50	1.01	1.55	1090.25
44	801.80	51.90	60.00	0.50	240.00	1.10	2.50	1.01	1.50	1097.05
45	800.80	52.10	61.30	0.53	239.40	1.20	2.56	1.00	1.50	1079.70
46	799.60	51.10	60.30	0.50	245.00	1.10	2.56	1.03	1.54	1102.50
47	798.50	51.50	61.50	0.53	240.50	1.05	2.48	1.00	1.49	1086.93
48	800.80	52.20	62.00	0.53	242.30	1.05	2.48	1.05	1.50	1078.80
49	802.00	51.50	62.10	0.52	242.30	1.10	2.51	1.00	1.48	1075.03
50	799.50	52.20	62.10	0.52	245.00	1.10	2.52	1.00	1.55	1105.37
Mean	807.39	51.78	61.72	0.52	244.18	1.08	2.54	1.02	1.52	1087.12
Lower	789.00	49.20	59.30	0.49	238.00	0.99	2.48	0.97	1.43	1063.66
Upper	881.10	57.75	68.20	0.60	267.30	1.21	2.82	1.16	1.67	1127.70

- ***The Case of - 10% Error Size and 10% Error Amount***

No.	Reconciled variable									
	x0	**x1**	**x2**	**x3**	**x4**	**x5**	**x6**	**x7**	**x8**	**x9**
1	799.50	51.30	59.40	0.50	245.00	1.12	2.48	1.00	1.48	1085.00
2	795.00	50.10	60.50	0.51	242.10	1.10	2.51	1.03	1.50	1073.60
3	800.00	52.00	61.50	0.50	238.00	1.10	2.50	1.00	1.48	1087.28
4	798.50	51.70	62.00	0.51	244.60	0.99	2.52	1.00	1.50	1075.20
5	800.20	50.50	60.50	0.51	240.40	0.99	2.52	1.03	1.58	1076.25
6	797.00	51.00	62.40	0.60	241.70	1.00	2.56	0.98	1.50	1108.01

No.	Reconciled variable									
	x0	**x1**	**x2**	**x3**	**x4**	**x5**	**x6**	**x7**	**x8**	**x9**
7	801.00	50.40	61.00	0.55	243.00	1.03	2.54	1.00	1.43	1099.80
8	799.80	52.00	60.30	0.50	244.40	1.03	2.56	1.04	1.48	1097.40
9	718.65	44.73	55.80	0.46	214.74	0.93	2.23	0.90	1.35	1067.50
10	800.90	50.00	59.30	0.50	238.40	1.05	2.48	0.98	1.58	1084.65
11	800.10	49.90	62.20	0.51	239.40	1.05	2.52	1.05	1.53	1125.91
12	719.10	45.81	53.55	0.46	216.27	0.89	2.30	0.90	1.34	1076.77
13	802.00	50.40	60.50	0.50	240.50	1.08	2.52	1.05	1.50	1084.83
14	799.00	51.30	62.40	0.60	245.50	1.10	2.49	1.00	1.51	1085.00
15	799.00	52.50	62.00	0.50	240.50	1.00	2.52	0.97	1.45	1085.80
16	798.00	49.80	60.50	0.50	239.20	0.99	2.56	1.00	1.50	1087.45
17	798.00	49.20	60.50	0.50	244.00	1.09	2.52	1.06	1.48	1064.88
18	799.00	52.20	61.50	0.50	243.00	0.99	2.50	1.02	1.47	1067.50
19	800.00	52.50	62.00	0.51	240.00	1.02	2.54	1.00	1.50	1087.28
20	799.00	50.70	60.50	0.51	240.50	1.05	2.52	1.02	1.54	1095.48
21	800.00	51.80	62.40	0.50	243.00	1.05	2.50	1.05	1.55	1085.00
22	798.00	50.60	59.50	0.50	240.00	1.00	2.56	1.00	1.47	1067.50
23	720.90	47.25	55.35	0.46	218.70	0.99	2.27	0.95	1.35	1078.80
24	799.00	52.00	60.50	0.51	240.50	1.01	2.53	1.00	1.47	1093.49
25	798.00	51.30	62.00	0.51	239.40	1.00	2.50	1.03	1.50	1067.50
26	800.00	52.10	61.30	0.50	241.60	1.05	2.51	1.03	1.49	1124.92
27	789.00	50.50	60.30	0.50	240.50	1.00	2.50	0.99	1.56	1092.00
28	799.00	51.40	61.10	0.50	243.00	1.07	2.52	0.99	1.50	1099.62
29	800.00	50.20	60.60	0.51	240.40	1.00	2.50	1.00	1.50	1091.72
30	718.20	45.09	55.35	0.46	216.36	0.90	2.25	0.90	1.31	1087.45
31	799.00	51.00	62.70	0.51	239.40	1.20	2.51	1.02	1.55	1063.66
32	802.90	52.50	62.30	0.51	245.00	1.00	2.50	0.98	1.50	1085.00
33	799.00	52.00	59.50	0.51	243.00	1.20	2.51	1.01	1.48	1086.75
34	798.50	52.50	62.00	0.51	240.40	1.20	2.50	1.06	1.51	1081.47
35	800.10	52.00	61.40	0.50	244.00	1.05	2.50	0.98	1.50	1127.70
36	801.80	52.50	62.30	0.50	245.00	1.05	2.56	0.99	1.53	1093.93
37	800.00	50.30	60.20	0.50	241.60	1.10	2.50	1.05	1.53	1106.25
38	800.40	52.00	62.30	0.50	245.00	1.10	2.48	0.99	1.50	1071.53
39	799.50	50.00	60.10	0.49	240.40	1.00	2.52	1.03	1.47	1076.25
40	800.40	51.30	61.50	0.49	244.00	1.20	2.51	1.02	1.50	1085.00
41	718.92	45.72	54.36	0.45	216.00	0.99	2.30	0.90	1.37	1080.71
42	800.00	51.50	60.50	0.50	244.00	1.20	2.51	1.02	1.50	1078.63

No.	Reconciled variable									
	x0	**x1**	**x2**	**x3**	**x4**	**x5**	**x6**	**x7**	**x8**	**x9**
43	801.20	52.00	59.70	0.52	241.50	1.10	2.50	1.01	1.55	1090.25
44	801.80	51.90	60.00	0.50	240.00	1.10	2.50	1.01	1.50	1097.05
45	800.80	52.10	61.30	0.53	239.40	1.20	2.56	1.00	1.50	1079.70
46	799.60	51.10	60.30	0.50	245.00	1.10	2.56	1.03	1.54	1102.50
47	798.50	51.50	61.50	0.53	240.50	1.05	2.48	1.00	1.49	1086.93
48	800.80	52.20	62.00	0.53	242.30	1.05	2.48	1.05	1.50	1078.80
49	802.00	51.50	62.10	0.52	242.30	1.10	2.51	1.00	1.48	1075.03
50	799.50	52.20	62.10	0.52	245.00	1.10	2.52	1.00	1.55	1105.37
Mean	791.41	50.76	60.50	0.51	239.37	1.06	2.49	1.00	1.49	1087.12
Lower	718.20	44.73	53.55	0.45	214.74	0.89	2.23	0.90	1.31	1063.66
Upper	802.90	52.50	62.70	0.60	245.50	1.20	2.56	1.06	1.58	1127.70

- ***The Case of + 10% Error Size and 20% Error Amount***

No.	Reconciled variable									
	x0	**x1**	**x2**	**x3**	**x4**	**x5**	**x6**	**x7**	**x8**	**x9**
1	799.50	51.30	59.40	0.50	245.00	1.12	2.48	1.00	1.48	1085.00
2	795.00	50.10	60.50	0.51	242.10	1.10	2.51	1.03	1.50	1073.60
3	800.00	52.00	61.50	0.50	238.00	1.10	2.50	1.00	1.48	1087.28
4	798.50	51.70	62.00	0.51	244.60	0.99	2.52	1.00	1.50	1075.20
5	800.20	50.50	60.50	0.51	240.40	0.99	2.52	1.03	1.58	1076.25
6	797.00	51.00	62.40	0.60	241.70	1.00	2.56	0.98	1.50	1108.01
7	961.20	60.48	73.20	0.66	291.60	1.24	3.05	1.20	1.72	1099.80
8	799.80	52.00	60.30	0.50	244.40	1.03	2.56	1.04	1.48	1097.40
9	798.50	49.70	62.00	0.51	238.60	1.03	2.48	1.00	1.50	1067.50
10	800.90	50.00	59.30	0.50	238.40	1.05	2.48	0.98	1.58	1084.65
11	800.10	49.90	62.20	0.51	239.40	1.05	2.52	1.05	1.53	1125.91
12	799.00	50.90	59.50	0.51	240.30	0.99	2.56	1.00	1.49	1076.77
13	802.00	50.40	60.50	0.50	240.50	1.08	2.52	1.05	1.50	1084.83
14	799.00	51.30	62.40	0.60	245.50	1.10	2.49	1.00	1.51	1085.00
15	799.00	52.50	62.00	0.50	240.50	1.00	2.52	0.97	1.45	1085.80
16	957.60	59.76	72.60	0.60	287.04	1.19	3.07	1.20	1.80	1087.45
17	798.00	49.20	60.50	0.50	244.00	1.09	2.52	1.06	1.48	1064.88

No.	Reconciled variable									
	x0	**x1**	**x2**	**x3**	**x4**	**x5**	**x6**	**x7**	**x8**	**x9**
18	799.00	52.20	61.50	0.50	243.00	0.99	2.50	1.02	1.47	1067.50
19	960.00	63.00	74.40	0.61	288.00	1.22	3.05	1.20	1.80	1087.28
20	799.00	50.70	60.50	0.51	240.50	1.05	2.52	1.02	1.54	1095.48
21	800.00	51.80	62.40	0.50	243.00	1.05	2.50	1.05	1.55	1085.00
22	798.00	50.60	59.50	0.50	240.00	1.00	2.56	1.00	1.47	1067.50
23	801.00	52.50	61.50	0.51	243.00	1.10	2.52	1.05	1.50	1078.80
24	799.00	52.00	60.50	0.51	240.50	1.01	2.53	1.00	1.47	1093.49
25	798.00	51.30	62.00	0.51	239.40	1.00	2.50	1.03	1.50	1067.50
26	800.00	52.10	61.30	0.50	241.60	1.05	2.51	1.03	1.49	1124.92
27	789.00	50.50	60.30	0.50	240.50	1.00	2.50	0.99	1.56	1092.00
28	799.00	51.40	61.10	0.50	243.00	1.07	2.52	0.99	1.50	1099.62
29	960.00	60.24	72.72	0.61	288.48	1.20	3.00	1.20	1.80	1091.72
30	798.00	50.10	61.50	0.51	240.40	1.00	2.50	1.00	1.46	1087.45
31	799.00	51.00	62.70	0.51	239.40	1.20	2.51	1.02	1.55	1063.66
32	802.90	52.50	62.30	0.51	245.00	1.00	2.50	0.98	1.50	1085.00
33	799.00	52.00	59.50	0.51	243.00	1.20	2.51	1.01	1.48	1086.75
34	798.50	52.50	62.00	0.51	240.40	1.20	2.50	1.06	1.51	1081.47
35	800.10	52.00	61.40	0.50	244.00	1.05	2.50	0.98	1.50	1127.70
36	801.80	52.50	62.30	0.50	245.00	1.05	2.56	0.99	1.53	1093.93
37	800.00	50.30	60.20	0.50	241.60	1.10	2.50	1.05	1.53	1106.25
38	960.48	62.40	74.76	0.60	294.00	1.32	2.98	1.19	1.80	1071.53
39	799.50	50.00	60.10	0.49	240.40	1.00	2.52	1.03	1.47	1076.25
40	800.40	51.30	61.50	0.49	244.00	1.20	2.51	1.02	1.50	1085.00
41	798.80	50.80	60.40	0.50	240.00	1.10	2.56	1.00	1.52	1080.71
42	800.00	51.50	60.50	0.50	244.00	1.20	2.51	1.02	1.50	1078.63
43	801.20	52.00	59.70	0.52	241.50	1.10	2.50	1.01	1.55	1090.25
44	801.80	51.90	60.00	0.50	240.00	1.10	2.50	1.01	1.50	1097.05
45	800.80	52.10	61.30	0.53	239.40	1.20	2.56	1.00	1.50	1079.70
46	799.60	51.10	60.30	0.50	245.00	1.10	2.56	1.03	1.54	1102.50
47	798.50	51.50	61.50	0.53	240.50	1.05	2.48	1.00	1.49	1086.93
48	800.80	52.20	62.00	0.53	242.30	1.05	2.48	1.05	1.50	1078.80
49	802.00	51.50	62.10	0.52	242.30	1.10	2.51	1.00	1.48	1075.03
50	799.50	52.20	62.10	0.52	245.00	1.10	2.52	1.00	1.55	1105.37
Mean	815.40	52.29	62.33	0.52	246.60	1.09	2.57	1.03	1.53	1087.12
Lower	789.00	49.20	59.30	0.49	238.00	0.99	2.48	0.97	1.45	1063.66
Upper	961.20	63.00	74.76	0.66	294.00	1.32	3.07	1.20	1.80	1127.70

- ***The Case of - 10% Error Size and 20% Error Amount***

No.	**Reconciled variable**									
	x0	**x1**	**x2**	**x3**	**x4**	**x5**	**x6**	**x7**	**x8**	**x9**
1	799.50	51.30	59.40	0.50	245.00	1.12	2.48	1.00	1.48	1085.00
2	795.00	50.10	60.50	0.51	242.10	1.10	2.51	1.03	1.50	1073.60
3	800.00	52.00	61.50	0.50	238.00	1.10	2.50	1.00	1.48	1087.28
4	798.50	51.70	62.00	0.51	244.60	0.99	2.52	1.00	1.50	1075.20
5	800.20	50.50	60.50	0.51	240.40	0.99	2.52	1.03	1.58	1076.25
6	797.00	51.00	62.40	0.60	241.70	1.00	2.56	0.98	1.50	1108.01
7	640.80	40.32	48.80	0.44	194.40	0.82	2.03	0.80	1.14	1099.80
8	799.80	52.00	60.30	0.50	244.40	1.03	2.56	1.04	1.48	1097.40
9	798.50	49.70	62.00	0.51	238.60	1.03	2.48	1.00	1.50	1067.50
10	800.90	50.00	59.30	0.50	238.40	1.05	2.48	0.98	1.58	1084.65
11	800.10	49.90	62.20	0.51	239.40	1.05	2.52	1.05	1.53	1125.91
12	799.00	50.90	59.50	0.51	240.30	0.99	2.56	1.00	1.49	1076.77
13	802.00	50.40	60.50	0.50	240.50	1.08	2.52	1.05	1.50	1084.83
14	799.00	51.30	62.40	0.60	245.50	1.10	2.49	1.00	1.51	1085.00
15	799.00	52.50	62.00	0.50	240.50	1.00	2.52	0.97	1.45	1085.80
16	638.40	39.84	48.40	0.40	191.36	0.79	2.05	0.80	1.20	1087.45
17	798.00	49.20	60.50	0.50	244.00	1.09	2.52	1.06	1.48	1064.88
18	799.00	52.20	61.50	0.50	243.00	0.99	2.50	1.02	1.47	1067.50
19	640.00	42.00	49.60	0.41	192.00	0.82	2.03	0.80	1.20	1087.28
20	799.00	50.70	60.50	0.51	240.50	1.05	2.52	1.02	1.54	1095.48
21	800.00	51.80	62.40	0.50	243.00	1.05	2.50	1.05	1.55	1085.00
22	798.00	50.60	59.50	0.50	240.00	1.00	2.56	1.00	1.47	1067.50
23	801.00	52.50	61.50	0.51	243.00	1.10	2.52	1.05	1.50	1078.80
24	799.00	52.00	60.50	0.51	240.50	1.01	2.53	1.00	1.47	1093.49
25	798.00	51.30	62.00	0.51	239.40	1.00	2.50	1.03	1.50	1067.50
26	800.00	52.10	61.30	0.50	241.60	1.05	2.51	1.03	1.49	1124.92
27	789.00	50.50	60.30	0.50	240.50	1.00	2.50	0.99	1.56	1092.00
28	799.00	51.40	61.10	0.50	243.00	1.07	2.52	0.99	1.50	1099.62
29	640.00	40.16	48.48	0.41	192.32	0.80	2.00	0.80	1.20	1091.72
30	798.00	50.10	61.50	0.51	240.40	1.00	2.50	1.00	1.46	1087.45
31	799.00	51.00	62.70	0.51	239.40	1.20	2.51	1.02	1.55	1063.66
32	802.90	52.50	62.30	0.51	245.00	1.00	2.50	0.98	1.50	1085.00
33	799.00	52.00	59.50	0.51	243.00	1.20	2.51	1.01	1.48	1086.75
34	798.50	52.50	62.00	0.51	240.40	1.20	2.50	1.06	1.51	1081.47

No.	Reconciled variable									
	x0	**x1**	**x2**	**x3**	**x4**	**x5**	**x6**	**x7**	**x8**	**x9**
35	800.10	52.00	61.40	0.50	244.00	1.05	2.50	0.98	1.50	1127.70
36	801.80	52.50	62.30	0.50	245.00	1.05	2.56	0.99	1.53	1093.93
37	800.00	50.30	60.20	0.50	241.60	1.10	2.50	1.05	1.53	1106.25
38	640.32	41.60	49.84	0.40	196.00	0.88	1.98	0.79	1.20	1071.53
39	799.50	50.00	60.10	0.49	240.40	1.00	2.52	1.03	1.47	1076.25
40	800.40	51.30	61.50	0.49	244.00	1.20	2.51	1.02	1.50	1085.00
41	798.80	50.80	60.40	0.50	240.00	1.10	2.56	1.00	1.52	1080.71
42	800.00	51.50	60.50	0.50	244.00	1.20	2.51	1.02	1.50	1078.63
43	801.20	52.00	59.70	0.52	241.50	1.10	2.50	1.01	1.55	1090.25
44	801.80	51.90	60.00	0.50	240.00	1.10	2.50	1.01	1.50	1097.05
45	800.80	52.10	61.30	0.53	239.40	1.20	2.56	1.00	1.50	1079.70
46	799.60	51.10	60.30	0.50	245.00	1.10	2.56	1.03	1.54	1102.50
47	798.50	51.50	61.50	0.53	240.50	1.05	2.48	1.00	1.49	1086.93
48	800.80	52.20	62.00	0.53	242.30	1.05	2.48	1.05	1.50	1078.80
49	802.00	51.50	62.10	0.52	242.30	1.10	2.51	1.00	1.48	1075.03
50	799.50	52.20	62.10	0.52	245.00	1.10	2.52	1.00	1.55	1105.37
Mean	783.40	50.25	59.88	0.50	236.94	1.05	2.47	0.99	1.47	1087.12
Lower	638.40	39.84	48.40	0.40	191.36	0.79	1.98	0.79	1.14	1063.66
Upper	802.90	52.50	62.70	0.60	245.50	1.20	2.56	1.06	1.58	1127.70

- ***The Case of + 10% Error Size and 30% Error Amount***

No.	Reconciled variable									
	x0	**x1**	**x2**	**x3**	**x4**	**x5**	**x6**	**x7**	**x8**	**x9**
1	1039.35	66.69	77.22	0.65	318.50	1.46	3.22	1.30	1.92	1085.00
2	795.00	50.10	60.50	0.51	242.10	1.10	2.51	1.03	1.50	1073.60
3	800.00	52.00	61.50	0.50	238.00	1.10	2.50	1.00	1.48	1087.28
4	798.50	51.70	62.00	0.51	244.60	0.99	2.52	1.00	1.50	1075.20
5	800.20	50.50	60.50	0.51	240.40	0.99	2.52	1.03	1.58	1076.25
6	797.00	51.00	62.40	0.60	241.70	1.00	2.56	0.98	1.50	1108.01
7	801.00	50.40	61.00	0.55	243.00	1.03	2.54	1.00	1.43	1099.80
8	799.80	52.00	60.30	0.50	244.40	1.03	2.56	1.04	1.48	1097.40
9	798.50	49.70	62.00	0.51	238.60	1.03	2.48	1.00	1.50	1067.50

No.	Reconciled variable									
	x0	x1	x2	x3	x4	x5	x6	x7	x8	x9
10	800.90	50.00	59.30	0.50	238.40	1.05	2.48	0.98	1.58	1084.65
11	800.10	49.90	62.20	0.51	239.40	1.05	2.52	1.05	1.53	1125.91
12	799.00	50.90	59.50	0.51	240.30	0.99	2.56	1.00	1.49	1076.77
13	802.00	50.40	60.50	0.50	240.50	1.08	2.52	1.05	1.50	1084.83
14	1038.70	66.69	81.12	0.78	319.15	1.43	3.24	1.30	1.96	1085.00
15	799.00	52.50	62.00	0.50	240.50	1.00	2.52	0.97	1.45	1085.80
16	798.00	49.80	60.50	0.50	239.20	0.99	2.56	1.00	1.50	1087.45
17	798.00	49.20	60.50	0.50	244.00	1.09	2.52	1.06	1.48	1064.88
18	799.00	52.20	61.50	0.50	243.00	0.99	2.50	1.02	1.47	1067.50
19	800.00	52.50	62.00	0.51	240.00	1.02	2.54	1.00	1.50	1087.28
20	799.00	50.70	60.50	0.51	240.50	1.05	2.52	1.02	1.54	1095.48
21	800.00	51.80	62.40	0.50	243.00	1.05	2.50	1.05	1.55	1085.00
22	798.00	50.60	59.50	0.50	240.00	1.00	2.56	1.00	1.47	1067.50
23	1041.30	68.25	79.95	0.66	315.90	1.43	3.28	1.37	1.95	1078.80
24	799.00	52.00	60.50	0.51	240.50	1.01	2.53	1.00	1.47	1093.49
25	798.00	51.30	62.00	0.51	239.40	1.00	2.50	1.03	1.50	1067.50
26	800.00	52.10	61.30	0.50	241.60	1.05	2.51	1.03	1.49	1124.92
27	789.00	50.50	60.30	0.50	240.50	1.00	2.50	0.99	1.56	1092.00
28	799.00	51.40	61.10	0.50	243.00	1.07	2.52	0.99	1.50	1099.62
29	800.00	50.20	60.60	0.51	240.40	1.00	2.50	1.00	1.50	1091.72
30	798.00	50.10	61.50	0.51	240.40	1.00	2.50	1.00	1.46	1087.45
31	799.00	51.00	62.70	0.51	239.40	1.20	2.51	1.02	1.55	1063.66
32	1043.77	68.25	80.99	0.66	318.50	1.30	3.25	1.27	1.95	1085.00
33	799.00	52.00	59.50	0.51	243.00	1.20	2.51	1.01	1.48	1086.75
34	798.50	52.50	62.00	0.51	240.40	1.20	2.50	1.06	1.51	1081.47
35	800.10	52.00	61.40	0.50	244.00	1.05	2.50	0.98	1.50	1127.70
36	801.80	52.50	62.30	0.50	245.00	1.05	2.56	0.99	1.53	1093.93
37	800.00	50.30	60.20	0.50	241.60	1.10	2.50	1.05	1.53	1106.25
38	800.40	52.00	62.30	0.50	245.00	1.10	2.48	0.99	1.50	1071.53
39	799.50	50.00	60.10	0.49	240.40	1.00	2.52	1.03	1.47	1076.25
40	800.40	51.30	61.50	0.49	244.00	1.20	2.51	1.02	1.50	1085.00
41	798.80	50.80	60.40	0.50	240.00	1.10	2.56	1.00	1.52	1080.71
42	800.00	51.50	60.50	0.50	244.00	1.20	2.51	1.02	1.50	1078.63
43	801.20	52.00	59.70	0.52	241.50	1.10	2.50	1.01	1.55	1090.25
44	801.80	51.90	60.00	0.50	240.00	1.10	2.50	1.01	1.50	1097.05
45	1041.04	67.73	79.69	0.69	311.22	1.56	3.33	1.30	1.95	1079.70

No.	Reconciled variable									
	x0	x1	x2	x3	x4	x5	x6	x7	x8	x9
46	799.60	51.10	60.30	0.50	245.00	1.10	2.56	1.03	1.54	1102.50
47	798.50	51.50	61.50	0.53	240.50	1.05	2.48	1.00	1.49	1086.93
48	800.80	52.20	62.00	0.53	242.30	1.05	2.48	1.05	1.50	1078.80
49	802.00	51.50	62.10	0.52	242.30	1.10	2.51	1.00	1.48	1075.03
50	799.50	52.20	62.10	0.52	245.00	1.10	2.52	1.00	1.55	1105.37
Mean	823.42	52.83	62.95	0.53	249.08	1.10	2.59	1.04	1.55	1087.12
Lower	789.00	49.20	59.30	0.49	238.00	0.99	2.48	0.97	1.43	1063.66
Upper	1043.77	68.25	81.12	0.78	319.15	1.56	3.33	1.37	1.96	1127.70

- ***The Case of - 10% Error Size and 30% Error Amount***

No.	Reconciled variable									
	x0	x1	x2	x3	x4	x5	x6	x7	x8	x9
1	559.65	35.91	41.58	0.35	171.50	0.78	1.74	0.70	1.04	1085.00
2	795.00	50.10	60.50	0.51	242.10	1.10	2.51	1.03	1.50	1073.60
3	800.00	52.00	61.50	0.50	238.00	1.10	2.50	1.00	1.48	1087.28
4	798.50	51.70	62.00	0.51	244.60	0.99	2.52	1.00	1.50	1075.20
5	800.20	50.50	60.50	0.51	240.40	0.99	2.52	1.03	1.58	1076.25
6	797.00	51.00	62.40	0.60	241.70	1.00	2.56	0.98	1.50	1108.01
7	801.00	50.40	61.00	0.55	243.00	1.03	2.54	1.00	1.43	1099.80
8	799.80	52.00	60.30	0.50	244.40	1.03	2.56	1.04	1.48	1097.40
9	798.50	49.70	62.00	0.51	238.60	1.03	2.48	1.00	1.50	1067.50
10	800.90	50.00	59.30	0.50	238.40	1.05	2.48	0.98	1.58	1084.65
11	800.10	49.90	62.20	0.51	239.40	1.05	2.52	1.05	1.53	1125.91
12	799.00	50.90	59.50	0.51	240.30	0.99	2.56	1.00	1.49	1076.77
13	802.00	50.40	60.50	0.50	240.50	1.08	2.52	1.05	1.50	1084.83
14	559.30	35.91	43.68	0.42	171.85	0.77	1.74	0.70	1.06	1085.00
15	799.00	52.50	62.00	0.50	240.50	1.00	2.52	0.97	1.45	1085.80
16	798.00	49.80	60.50	0.50	239.20	0.99	2.56	1.00	1.50	1087.45
17	798.00	49.20	60.50	0.50	244.00	1.09	2.52	1.06	1.48	1064.88
18	799.00	52.20	61.50	0.50	243.00	0.99	2.50	1.02	1.47	1067.50
19	800.00	52.50	62.00	0.51	240.00	1.02	2.54	1.00	1.50	1087.28
20	799.00	50.70	60.50	0.51	240.50	1.05	2.52	1.02	1.54	1095.48

No.	Reconciled variable									
	x0	x1	x2	x3	x4	x5	x6	x7	x8	x9
21	800.00	51.80	62.40	0.50	243.00	1.05	2.50	1.05	1.55	1085.00
22	798.00	50.60	59.50	0.50	240.00	1.00	2.56	1.00	1.47	1067.50
23	560.70	36.75	43.05	0.36	170.10	0.77	1.76	0.74	1.05	1078.80
24	799.00	52.00	60.50	0.51	240.50	1.01	2.53	1.00	1.47	1093.49
25	798.00	51.30	62.00	0.51	239.40	1.00	2.50	1.03	1.50	1067.50
26	800.00	52.10	61.30	0.50	241.60	1.05	2.51	1.03	1.49	1124.92
27	789.00	50.50	60.30	0.50	240.50	1.00	2.50	0.99	1.56	1092.00
28	799.00	51.40	61.10	0.50	243.00	1.07	2.52	0.99	1.50	1099.62
29	800.00	50.20	60.60	0.51	240.40	1.00	2.50	1.00	1.50	1091.72
30	798.00	50.10	61.50	0.51	240.40	1.00	2.50	1.00	1.46	1087.45
31	799.00	51.00	62.70	0.51	239.40	1.20	2.51	1.02	1.55	1063.66
32	562.03	36.75	43.61	0.36	171.50	0.70	1.75	0.69	1.05	1085.00
33	799.00	52.00	59.50	0.51	243.00	1.20	2.51	1.01	1.48	1086.75
34	798.50	52.50	62.00	0.51	240.40	1.20	2.50	1.06	1.51	1081.47
35	800.10	52.00	61.40	0.50	244.00	1.05	2.50	0.98	1.50	1127.70
36	801.80	52.50	62.30	0.50	245.00	1.05	2.56	0.99	1.53	1093.93
37	800.00	50.30	60.20	0.50	241.60	1.10	2.50	1.05	1.53	1106.25
38	800.40	52.00	62.30	0.50	245.00	1.10	2.48	0.99	1.50	1071.53
39	799.50	50.00	60.10	0.49	240.40	1.00	2.52	1.03	1.47	1076.25
40	800.40	51.30	61.50	0.49	244.00	1.20	2.51	1.02	1.50	1085.00
41	798.80	50.80	60.40	0.50	240.00	1.10	2.56	1.00	1.52	1080.71
42	800.00	51.50	60.50	0.50	244.00	1.20	2.51	1.02	1.50	1078.63
43	801.20	52.00	59.70	0.52	241.50	1.10	2.50	1.01	1.55	1090.25
44	801.80	51.90	60.00	0.50	240.00	1.10	2.50	1.01	1.50	1097.05
45	560.56	36.47	42.91	0.37	167.58	0.84	1.79	0.70	1.05	1079.70
46	799.60	51.10	60.30	0.50	245.00	1.10	2.56	1.03	1.54	1102.50
47	798.50	51.50	61.50	0.53	240.50	1.05	2.48	1.00	1.49	1086.93
48	800.80	52.20	62.00	0.53	242.30	1.05	2.48	1.05	1.50	1078.80
49	802.00	51.50	62.10	0.52	242.30	1.10	2.51	1.00	1.48	1075.03
50	799.50	52.20	62.10	0.52	245.00	1.10	2.52	1.00	1.55	1105.37
Mean	775.38	49.71	59.27	0.50	234.47	1.03	2.44	0.98	1.46	1087.12
Lower	559.30	35.91	41.58	0.35	167.58	0.70	1.74	0.69	1.04	1063.66
Upper	802.00	52.50	62.70	0.60	245.00	1.20	2.56	1.06	1.58	1127.70

- ***The Case of + 20% Error Size and 10% Error Amount***

No.	**Reconciled variable**									
	x0	**x1**	**x2**	**x3**	**x4**	**x5**	**x6**	**x7**	**x8**	**x9**
1	799.50	51.30	59.40	0.50	245.00	1.12	2.48	1.00	1.48	1085.00
2	795.00	50.10	60.50	0.51	242.10	1.10	2.51	1.03	1.50	1073.60
3	880.00	57.20	67.65	0.55	261.80	1.21	2.75	1.10	1.63	1087.28
4	798.50	51.70	62.00	0.51	244.60	0.99	2.52	1.00	1.50	1075.20
5	800.20	50.50	60.50	0.51	240.40	0.99	2.52	1.03	1.58	1076.25
6	797.00	51.00	62.40	0.60	241.70	1.00	2.56	0.98	1.50	1108.01
7	801.00	50.40	61.00	0.55	243.00	1.03	2.54	1.00	1.43	1099.80
8	879.78	57.20	66.33	0.55	268.84	1.13	2.82	1.14	1.63	1097.40
9	798.50	49.70	62.00	0.51	238.60	1.03	2.48	1.00	1.50	1067.50
10	800.90	50.00	59.30	0.50	238.40	1.05	2.48	0.98	1.58	1084.65
11	800.10	49.90	62.20	0.51	239.40	1.05	2.52	1.05	1.53	1125.91
12	878.90	55.99	65.45	0.56	264.33	1.09	2.82	1.10	1.64	1076.77
13	802.00	50.40	60.50	0.50	240.50	1.08	2.52	1.05	1.50	1084.83
14	799.00	51.30	62.40	0.60	245.50	1.10	2.49	1.00	1.51	1085.00
15	799.00	52.50	62.00	0.50	240.50	1.00	2.52	0.97	1.45	1085.80
16	798.00	49.80	60.50	0.50	239.20	0.99	2.56	1.00	1.50	1087.45
17	877.80	54.12	66.55	0.55	268.40	1.20	2.77	1.17	1.63	1064.88
18	799.00	52.20	61.50	0.50	243.00	0.99	2.50	1.02	1.47	1067.50
19	800.00	52.50	62.00	0.51	240.00	1.02	2.54	1.00	1.50	1087.28
20	799.00	50.70	60.50	0.51	240.50	1.05	2.52	1.02	1.54	1095.48
21	880.00	56.98	68.64	0.55	267.30	1.16	2.75	1.16	1.71	1085.00
22	798.00	50.60	59.50	0.50	240.00	1.00	2.56	1.00	1.47	1067.50
23	801.00	52.50	61.50	0.51	243.00	1.10	2.52	1.05	1.50	1078.80
24	799.00	52.00	60.50	0.51	240.50	1.01	2.53	1.00	1.47	1093.49
25	798.00	51.30	62.00	0.51	239.40	1.00	2.50	1.03	1.50	1067.50
26	880.00	57.31	67.43	0.55	265.76	1.16	2.76	1.13	1.64	1124.92
27	789.00	50.50	60.30	0.50	240.50	1.00	2.50	0.99	1.56	1092.00
28	799.00	51.40	61.10	0.50	243.00	1.07	2.52	0.99	1.50	1099.62
29	800.00	50.20	60.60	0.51	240.40	1.00	2.50	1.00	1.50	1091.72
30	798.00	50.10	61.50	0.51	240.40	1.00	2.50	1.00	1.46	1087.45
31	799.00	51.00	62.70	0.51	239.40	1.20	2.51	1.02	1.55	1063.66
32	802.90	52.50	62.30	0.51	245.00	1.00	2.50	0.98	1.50	1085.00
33	878.90	57.20	65.45	0.56	267.30	1.32	2.76	1.11	1.63	1086.75

No.	Reconciled variable									
	x0	**x1**	**x2**	**x3**	**x4**	**x5**	**x6**	**x7**	**x8**	**x9**
34	798.50	52.50	62.00	0.51	240.40	1.20	2.50	1.06	1.51	1081.47
35	800.10	52.00	61.40	0.50	244.00	1.05	2.50	0.98	1.50	1127.70
36	801.80	52.50	62.30	0.50	245.00	1.05	2.56	0.99	1.53	1093.93
37	800.00	50.30	60.20	0.50	241.60	1.10	2.50	1.05	1.53	1106.25
38	800.40	52.00	62.30	0.50	245.00	1.10	2.48	0.99	1.50	1071.53
39	879.45	55.00	66.11	0.54	264.44	1.10	2.77	1.13	1.62	1076.25
40	800.40	51.30	61.50	0.49	244.00	1.20	2.51	1.02	1.50	1085.00
41	878.68	55.88	66.44	0.55	264.00	1.21	2.82	1.10	1.67	1080.71
42	800.00	51.50	60.50	0.50	244.00	1.20	2.51	1.02	1.50	1078.63
43	801.20	52.00	59.70	0.52	241.50	1.10	2.50	1.01	1.55	1090.25
44	801.80	51.90	60.00	0.50	240.00	1.10	2.50	1.01	1.50	1097.05
45	800.80	52.10	61.30	0.53	239.40	1.20	2.56	1.00	1.50	1079.70
46	799.60	51.10	60.30	0.50	245.00	1.10	2.56	1.03	1.54	1102.50
47	798.50	51.50	61.50	0.53	240.50	1.05	2.48	1.00	1.49	1086.93
48	800.80	52.20	62.00	0.53	242.30	1.05	2.48	1.05	1.50	1078.80
49	802.00	51.50	62.10	0.52	242.30	1.10	2.51	1.00	1.48	1075.03
50	879.45	57.42	68.31	0.57	269.50	1.21	2.77	1.10	1.71	1105.37
Mean	815.39	52.30	62.32	0.52	246.61	1.09	2.57	1.03	1.53	1087.12
Lower	789.00	49.70	59.30	0.49	238.40	0.99	2.48	0.97	1.43	1063.66
Upper	880.00	57.42	68.64	0.60	269.50	1.32	2.82	1.17	1.71	1127.70

- ***The Case of - 20% Error Size and 10% Error Amount***

No.	Reconciled variable									
	x0	**x1**	**x2**	**x3**	**x4**	**x5**	**x6**	**x7**	**x8**	**x9**
1	799.50	51.30	59.40	0.50	245.00	1.12	2.48	1.00	1.48	1085.00
2	795.00	50.10	60.50	0.51	242.10	1.10	2.51	1.03	1.50	1073.60
3	720.00	46.80	55.35	0.45	214.20	0.99	2.25	0.90	1.33	1087.28
4	798.50	51.70	62.00	0.51	244.60	0.99	2.52	1.00	1.50	1075.20
5	800.20	50.50	60.50	0.51	240.40	0.99	2.52	1.03	1.58	1076.25
6	797.00	51.00	62.40	0.60	241.70	1.00	2.56	0.98	1.50	1108.01
7	801.00	50.40	61.00	0.55	243.00	1.03	2.54	1.00	1.43	1099.80
8	719.82	46.80	54.27	0.45	219.96	0.93	2.30	0.94	1.33	1097.40

No.	Reconciled variable									
	x0	**x1**	**x2**	**x3**	**x4**	**x5**	**x6**	**x7**	**x8**	**x9**
9	798.50	49.70	62.00	0.51	238.60	1.03	2.48	1.00	1.50	1067.50
10	800.90	50.00	59.30	0.50	238.40	1.05	2.48	0.98	1.58	1084.65
11	800.10	49.90	62.20	0.51	239.40	1.05	2.52	1.05	1.53	1125.91
12	719.10	45.81	53.55	0.46	216.27	0.89	2.30	0.90	1.34	1076.77
13	802.00	50.40	60.50	0.50	240.50	1.08	2.52	1.05	1.50	1084.83
14	799.00	51.30	62.40	0.60	245.50	1.10	2.49	1.00	1.51	1085.00
15	799.00	52.50	62.00	0.50	240.50	1.00	2.52	0.97	1.45	1085.80
16	798.00	49.80	60.50	0.50	239.20	0.99	2.56	1.00	1.50	1087.45
17	718.20	44.28	54.45	0.45	219.60	0.98	2.27	0.95	1.33	1064.88
18	799.00	52.20	61.50	0.50	243.00	0.99	2.50	1.02	1.47	1067.50
19	800.00	52.50	62.00	0.51	240.00	1.02	2.54	1.00	1.50	1087.28
20	799.00	50.70	60.50	0.51	240.50	1.05	2.52	1.02	1.54	1095.48
21	720.00	46.62	56.16	0.45	218.70	0.95	2.25	0.95	1.40	1085.00
22	798.00	50.60	59.50	0.50	240.00	1.00	2.56	1.00	1.47	1067.50
23	801.00	52.50	61.50	0.51	243.00	1.10	2.52	1.05	1.50	1078.80
24	799.00	52.00	60.50	0.51	240.50	1.01	2.53	1.00	1.47	1093.49
25	798.00	51.30	62.00	0.51	239.40	1.00	2.50	1.03	1.50	1067.50
26	720.00	46.89	55.17	0.45	217.44	0.95	2.26	0.93	1.34	1124.92
27	789.00	50.50	60.30	0.50	240.50	1.00	2.50	0.99	1.56	1092.00
28	799.00	51.40	61.10	0.50	243.00	1.07	2.52	0.99	1.50	1099.62
29	800.00	50.20	60.60	0.51	240.40	1.00	2.50	1.00	1.50	1091.72
30	798.00	50.10	61.50	0.51	240.40	1.00	2.50	1.00	1.46	1087.45
31	799.00	51.00	62.70	0.51	239.40	1.20	2.51	1.02	1.55	1063.66
32	802.90	52.50	62.30	0.51	245.00	1.00	2.50	0.98	1.50	1085.00
33	719.10	46.80	53.55	0.46	218.70	1.08	2.26	0.91	1.33	1086.75
34	798.50	52.50	62.00	0.51	240.40	1.20	2.50	1.06	1.51	1081.47
35	800.10	52.00	61.40	0.50	244.00	1.05	2.50	0.98	1.50	1127.70
36	801.80	52.50	62.30	0.50	245.00	1.05	2.56	0.99	1.53	1093.93
37	800.00	50.30	60.20	0.50	241.60	1.10	2.50	1.05	1.53	1106.25
38	800.40	52.00	62.30	0.50	245.00	1.10	2.48	0.99	1.50	1071.53
39	719.55	45.00	54.09	0.44	216.36	0.90	2.27	0.93	1.32	1076.25
40	800.40	51.30	61.50	0.49	244.00	1.20	2.51	1.02	1.50	1085.00
41	718.92	45.72	54.36	0.45	216.00	0.99	2.30	0.90	1.37	1080.71
42	800.00	51.50	60.50	0.50	244.00	1.20	2.51	1.02	1.50	1078.63
43	801.20	52.00	59.70	0.52	241.50	1.10	2.50	1.01	1.55	1090.25
44	801.80	51.90	60.00	0.50	240.00	1.10	2.50	1.01	1.50	1097.05

No.	Reconciled variable									
	x0	x1	x2	x3	x4	x5	x6	x7	x8	x9
45	800.80	52.10	61.30	0.53	239.40	1.20	2.56	1.00	1.50	1079.70
46	799.60	51.10	60.30	0.50	245.00	1.10	2.56	1.03	1.54	1102.50
47	798.50	51.50	61.50	0.53	240.50	1.05	2.48	1.00	1.49	1086.93
48	800.80	52.20	62.00	0.53	242.30	1.05	2.48	1.05	1.50	1078.80
49	802.00	51.50	62.10	0.52	242.30	1.10	2.51	1.00	1.48	1075.03
50	719.55	46.98	55.89	0.47	220.50	0.99	2.27	0.90	1.40	1105.37
Mean	783.41	50.24	59.89	0.50	236.93	1.04	2.47	0.99	1.47	1087.12
Lower	718.20	44.28	53.55	0.44	214.20	0.89	2.25	0.90	1.32	1063.66
Upper	802.90	52.50	62.70	0.60	245.50	1.20	2.56	1.06	1.58	1127.70

- ***The Case of + 10% Error Size and 20% Error Amount***

No.	Reconciled variable									
	x0	x1	x2	x3	x4	x5	x6	x7	x8	x9
1	799.50	51.30	59.40	0.50	245.00	1.12	2.48	1.00	1.48	1085.00
2	954.00	60.12	72.60	0.61	290.52	1.32	3.01	1.24	1.80	1073.60
3	800.00	52.00	61.50	0.50	238.00	1.10	2.50	1.00	1.48	1087.28
4	798.50	51.70	62.00	0.51	244.60	0.99	2.52	1.00	1.50	1075.20
5	800.20	50.50	60.50	0.51	240.40	0.99	2.52	1.03	1.58	1076.25
6	797.00	51.00	62.40	0.60	241.70	1.00	2.56	0.98	1.50	1108.01
7	801.00	50.40	61.00	0.55	243.00	1.03	2.54	1.00	1.43	1099.80
8	799.80	52.00	60.30	0.50	244.40	1.03	2.56	1.04	1.48	1097.40
9	798.50	49.70	62.00	0.51	238.60	1.03	2.48	1.00	1.50	1067.50
10	800.90	50.00	59.30	0.50	238.40	1.05	2.48	0.98	1.58	1084.65
11	960.12	59.88	74.64	0.61	287.28	1.26	3.02	1.26	1.84	1125.91
12	799.00	50.90	59.50	0.51	240.30	0.99	2.56	1.00	1.49	1076.77
13	802.00	50.40	60.50	0.50	240.50	1.08	2.52	1.05	1.50	1084.83
14	799.00	51.30	62.40	0.60	245.50	1.10	2.49	1.00	1.51	1085.00
15	799.00	52.50	62.00	0.50	240.50	1.00	2.52	0.97	1.45	1085.80
16	798.00	49.80	60.50	0.50	239.20	0.99	2.56	1.00	1.50	1087.45
17	798.00	49.20	60.50	0.50	244.00	1.09	2.52	1.06	1.48	1064.88
18	799.00	52.20	61.50	0.50	243.00	0.99	2.50	1.02	1.47	1067.50
19	960.00	63.00	74.40	0.61	288.00	1.22	3.05	1.20	1.80	1087.28

No.	Reconciled variable									
	x0	**x1**	**x2**	**x3**	**x4**	**x5**	**x6**	**x7**	**x8**	**x9**
20	799.00	50.70	60.50	0.51	240.50	1.05	2.52	1.02	1.54	1095.48
21	800.00	51.80	62.40	0.50	243.00	1.05	2.50	1.05	1.55	1085.00
22	798.00	50.60	59.50	0.50	240.00	1.00	2.56	1.00	1.47	1067.50
23	801.00	52.50	61.50	0.51	243.00	1.10	2.52	1.05	1.50	1078.80
24	799.00	52.00	60.50	0.51	240.50	1.01	2.53	1.00	1.47	1093.49
25	798.00	51.30	62.00	0.51	239.40	1.00	2.50	1.03	1.50	1067.50
26	960.00	62.52	73.56	0.60	289.92	1.26	3.01	1.24	1.79	1124.92
27	946.80	60.60	72.36	0.60	288.60	1.20	3.00	1.19	1.87	1092.00
28	799.00	51.40	61.10	0.50	243.00	1.07	2.52	0.99	1.50	1099.62
29	800.00	50.20	60.60	0.51	240.40	1.00	2.50	1.00	1.50	1091.72
30	798.00	50.10	61.50	0.51	240.40	1.00	2.50	1.00	1.46	1087.45
31	958.80	61.20	75.24	0.61	287.28	1.44	3.01	1.22	1.86	1063.66
32	802.90	52.50	62.30	0.51	245.00	1.00	2.50	0.98	1.50	1085.00
33	799.00	52.00	59.50	0.51	243.00	1.20	2.51	1.01	1.48	1086.75
34	798.50	52.50	62.00	0.51	240.40	1.20	2.50	1.06	1.51	1081.47
35	800.10	52.00	61.40	0.50	244.00	1.05	2.50	0.98	1.50	1127.70
36	801.80	52.50	62.30	0.50	245.00	1.05	2.56	0.99	1.53	1093.93
37	960.00	60.36	72.24	0.60	289.92	1.32	3.00	1.26	1.84	1106.25
38	800.40	52.00	62.30	0.50	245.00	1.10	2.48	0.99	1.50	1071.53
39	799.50	50.00	60.10	0.49	240.40	1.00	2.52	1.03	1.47	1076.25
40	960.48	61.56	73.80	0.59	292.80	1.44	3.01	1.22	1.80	1085.00
41	798.80	50.80	60.40	0.50	240.00	1.10	2.56	1.00	1.52	1080.71
42	960.00	61.80	72.60	0.60	292.80	1.44	3.01	1.22	1.80	1078.63
43	801.20	52.00	59.70	0.52	241.50	1.10	2.50	1.01	1.55	1090.25
44	801.80	51.90	60.00	0.50	240.00	1.10	2.50	1.01	1.50	1097.05
45	800.80	52.10	61.30	0.53	239.40	1.20	2.56	1.00	1.50	1079.70
46	799.60	51.10	60.30	0.50	245.00	1.10	2.56	1.03	1.54	1102.50
47	798.50	51.50	61.50	0.53	240.50	1.05	2.48	1.00	1.49	1086.93
48	960.96	62.64	74.40	0.64	290.76	1.26	2.98	1.26	1.80	1078.80
49	802.00	51.50	62.10	0.52	242.30	1.10	2.51	1.00	1.48	1075.03
50	799.50	52.20	62.10	0.52	245.00	1.10	2.52	1.00	1.55	1105.37
Mean	831.34	53.32	63.56	0.53	251.43	1.11	2.62	1.05	1.56	1087.12
Lower	797.00	49.20	59.30	0.49	238.00	0.99	2.48	0.97	1.43	1063.66
Upper	960.96	63.00	75.24	0.64	292.80	1.44	3.05	1.26	1.87	1127.70

- ***The Case of - 20% Error Size and 20% Error Amount***

No.	Reconciled variable									
	x0	**x1**	**x2**	**x3**	**x4**	**x5**	**x6**	**x7**	**x8**	**x9**
1	799.50	51.30	59.40	0.50	245.00	1.12	2.48	1.00	1.48	1085.00
2	636.00	40.08	48.40	0.41	193.68	0.88	2.01	0.82	1.20	1073.60
3	800.00	52.00	61.50	0.50	238.00	1.10	2.50	1.00	1.48	1087.28
4	798.50	51.70	62.00	0.51	244.60	0.99	2.52	1.00	1.50	1075.20
5	800.20	50.50	60.50	0.51	240.40	0.99	2.52	1.03	1.58	1076.25
6	797.00	51.00	62.40	0.60	241.70	1.00	2.56	0.98	1.50	1108.01
7	801.00	50.40	61.00	0.55	243.00	1.03	2.54	1.00	1.43	1099.80
8	799.80	52.00	60.30	0.50	244.40	1.03	2.56	1.04	1.48	1097.40
9	798.50	49.70	62.00	0.51	238.60	1.03	2.48	1.00	1.50	1067.50
10	800.90	50.00	59.30	0.50	238.40	1.05	2.48	0.98	1.58	1084.65
11	640.08	39.92	49.76	0.41	191.52	0.84	2.02	0.84	1.22	1125.91
12	799.00	50.90	59.50	0.51	240.30	0.99	2.56	1.00	1.49	1076.77
13	802.00	50.40	60.50	0.50	240.50	1.08	2.52	1.05	1.50	1084.83
14	799.00	51.30	62.40	0.60	245.50	1.10	2.49	1.00	1.51	1085.00
15	799.00	52.50	62.00	0.50	240.50	1.00	2.52	0.97	1.45	1085.80
16	798.00	49.80	60.50	0.50	239.20	0.99	2.56	1.00	1.50	1087.45
17	798.00	49.20	60.50	0.50	244.00	1.09	2.52	1.06	1.48	1064.88
18	799.00	52.20	61.50	0.50	243.00	0.99	2.50	1.02	1.47	1067.50
19	640.00	42.00	49.60	0.41	192.00	0.82	2.03	0.80	1.20	1087.28
20	799.00	50.70	60.50	0.51	240.50	1.05	2.52	1.02	1.54	1095.48
21	800.00	51.80	62.40	0.50	243.00	1.05	2.50	1.05	1.55	1085.00
22	798.00	50.60	59.50	0.50	240.00	1.00	2.56	1.00	1.47	1067.50
23	801.00	52.50	61.50	0.51	243.00	1.10	2.52	1.05	1.50	1078.80
24	799.00	52.00	60.50	0.51	240.50	1.01	2.53	1.00	1.47	1093.49
25	798.00	51.30	62.00	0.51	239.40	1.00	2.50	1.03	1.50	1067.50
26	640.00	41.68	49.04	0.40	193.28	0.84	2.01	0.82	1.19	1124.92
27	631.20	40.40	48.24	0.40	192.40	0.80	2.00	0.79	1.25	1092.00
28	799.00	51.40	61.10	0.50	243.00	1.07	2.52	0.99	1.50	1099.62
29	800.00	50.20	60.60	0.51	240.40	1.00	2.50	1.00	1.50	1091.72
30	798.00	50.10	61.50	0.51	240.40	1.00	2.50	1.00	1.46	1087.45
31	639.20	40.80	50.16	0.41	191.52	0.96	2.01	0.82	1.24	1063.66
32	802.90	52.50	62.30	0.51	245.00	1.00	2.50	0.98	1.50	1085.00
33	799.00	52.00	59.50	0.51	243.00	1.20	2.51	1.01	1.48	1086.75
34	798.50	52.50	62.00	0.51	240.40	1.20	2.50	1.06	1.51	1081.47

No.	Reconciled variable									
	x0	x1	x2	x3	x4	x5	x6	x7	x8	x9
35	800.10	52.00	61.40	0.50	244.00	1.05	2.50	0.98	1.50	1127.70
36	801.80	52.50	62.30	0.50	245.00	1.05	2.56	0.99	1.53	1093.93
37	640.00	40.24	48.16	0.40	193.28	0.88	2.00	0.84	1.22	1106.25
38	800.40	52.00	62.30	0.50	245.00	1.10	2.48	0.99	1.50	1071.53
39	799.50	50.00	60.10	0.49	240.40	1.00	2.52	1.03	1.47	1076.25
40	640.32	41.04	49.20	0.39	195.20	0.96	2.01	0.82	1.20	1085.00
41	798.80	50.80	60.40	0.50	240.00	1.10	2.56	1.00	1.52	1080.71
42	640.00	41.20	48.40	0.40	195.20	0.96	2.01	0.82	1.20	1078.63
43	801.20	52.00	59.70	0.52	241.50	1.10	2.50	1.01	1.55	1090.25
44	801.80	51.90	60.00	0.50	240.00	1.10	2.50	1.01	1.50	1097.05
45	800.80	52.10	61.30	0.53	239.40	1.20	2.56	1.00	1.50	1079.70
46	799.60	51.10	60.30	0.50	245.00	1.10	2.56	1.03	1.54	1102.50
47	798.50	51.50	61.50	0.53	240.50	1.05	2.48	1.00	1.49	1086.93
48	640.64	41.76	49.60	0.42	193.84	0.84	1.98	0.84	1.20	1078.80
49	802.00	51.50	62.10	0.52	242.30	1.10	2.51	1.00	1.48	1075.03
50	799.50	52.20	62.10	0.52	245.00	1.10	2.52	1.00	1.55	1105.37
Mean	767.46	49.22	58.66	0.49	232.11	1.02	2.42	0.97	1.44	1087.12
Lower	631.20	39.92	48.16	0.39	191.52	0.80	1.98	0.79	1.19	1063.66
Upper	802.90	52.50	62.40	0.60	245.50	1.20	2.56	1.06	1.58	1127.70

- ***The Case of + 20% Error Size and 30% Error Amount***

No.	Reconciled variable									
	x0	x1	x2	x3	x4	x5	x6	x7	x8	x9
1	799.50	51.30	59.40	0.50	245.00	1.12	2.48	1.00	1.48	1085.00
2	795.00	50.10	60.50	0.51	242.10	1.10	2.51	1.03	1.50	1073.60
3	800.00	52.00	61.50	0.50	238.00	1.10	2.50	1.00	1.48	1087.28
4	798.50	51.70	62.00	0.51	244.60	0.99	2.52	1.00	1.50	1075.20
5	1040.26	65.65	78.65	0.66	312.52	1.29	3.28	1.34	2.05	1076.25
6	797.00	51.00	62.40	0.60	241.70	1.00	2.56	0.98	1.50	1108.01
7	801.00	50.40	61.00	0.55	243.00	1.03	2.54	1.00	1.43	1099.80
8	799.80	52.00	60.30	0.50	244.40	1.03	2.56	1.04	1.48	1097.40
9	798.50	49.70	62.00	0.51	238.60	1.03	2.48	1.00	1.50	1067.50

No.	Reconciled variable									
	x0	**x1**	**x2**	**x3**	**x4**	**x5**	**x6**	**x7**	**x8**	**x9**
10	1041.17	65.00	77.09	0.65	309.92	1.37	3.22	1.27	2.05	1084.65
11	800.10	49.90	62.20	0.51	239.40	1.05	2.52	1.05	1.53	1125.91
12	799.00	50.90	59.50	0.51	240.30	0.99	2.56	1.00	1.49	1076.77
13	802.00	50.40	60.50	0.50	240.50	1.08	2.52	1.05	1.50	1084.83
14	799.00	51.30	62.40	0.60	245.50	1.10	2.49	1.00	1.51	1085.00
15	799.00	52.50	62.00	0.50	240.50	1.00	2.52	0.97	1.45	1085.80
16	798.00	49.80	60.50	0.50	239.20	0.99	2.56	1.00	1.50	1087.45
17	798.00	49.20	60.50	0.50	244.00	1.09	2.52	1.06	1.48	1064.88
18	799.00	52.20	61.50	0.50	243.00	0.99	2.50	1.02	1.47	1067.50
19	1040.00	68.25	80.60	0.66	312.00	1.33	3.30	1.30	1.95	1087.28
20	799.00	50.70	60.50	0.51	240.50	1.05	2.52	1.02	1.54	1095.48
21	800.00	51.80	62.40	0.50	243.00	1.05	2.50	1.05	1.55	1085.00
22	1037.40	65.78	77.35	0.65	312.00	1.30	3.33	1.30	1.91	1067.50
23	801.00	52.50	61.50	0.51	243.00	1.10	2.52	1.05	1.50	1078.80
24	799.00	52.00	60.50	0.51	240.50	1.01	2.53	1.00	1.47	1093.49
25	798.00	51.30	62.00	0.51	239.40	1.00	2.50	1.03	1.50	1067.50
26	800.00	52.10	61.30	0.50	241.60	1.05	2.51	1.03	1.49	1124.92
27	789.00	50.50	60.30	0.50	240.50	1.00	2.50	0.99	1.56	1092.00
28	799.00	51.40	61.10	0.50	243.00	1.07	2.52	0.99	1.50	1099.62
29	1040.00	65.26	78.78	0.66	312.52	1.30	3.25	1.30	1.95	1091.72
30	798.00	50.10	61.50	0.51	240.40	1.00	2.50	1.00	1.46	1087.45
31	799.00	51.00	62.70	0.51	239.40	1.20	2.51	1.02	1.55	1063.66
32	802.90	52.50	62.30	0.51	245.00	1.00	2.50	0.98	1.50	1085.00
33	1038.70	67.60	77.35	0.66	315.90	1.56	3.26	1.31	1.92	1086.75
34	798.50	52.50	62.00	0.51	240.40	1.20	2.50	1.06	1.51	1081.47
35	800.10	52.00	61.40	0.50	244.00	1.05	2.50	0.98	1.50	1127.70
36	801.80	52.50	62.30	0.50	245.00	1.05	2.56	0.99	1.53	1093.93
37	1040.00	65.39	78.26	0.65	314.08	1.43	3.25	1.37	1.99	1106.25
38	800.40	52.00	62.30	0.50	245.00	1.10	2.48	0.99	1.50	1071.53
39	1039.35	65.00	78.13	0.64	312.52	1.30	3.28	1.34	1.91	1076.25
40	800.40	51.30	61.50	0.49	244.00	1.20	2.51	1.02	1.50	1085.00
41	798.80	50.80	60.40	0.50	240.00	1.10	2.56	1.00	1.52	1080.71
42	800.00	51.50	60.50	0.50	244.00	1.20	2.51	1.02	1.50	1078.63
43	801.20	52.00	59.70	0.52	241.50	1.10	2.50	1.01	1.55	1090.25
44	801.80	51.90	60.00	0.50	240.00	1.10	2.50	1.01	1.50	1097.05
45	1041.04	67.73	79.69	0.69	311.22	1.56	3.33	1.30	1.95	1079.70

No.	**Reconciled variable**									
	x0	**x1**	**x2**	**x3**	**x4**	**x5**	**x6**	**x7**	**x8**	**x9**
46	799.60	51.10	60.30	0.50	245.00	1.10	2.56	1.03	1.54	1102.50
47	798.50	51.50	61.50	0.53	240.50	1.05	2.48	1.00	1.49	1086.93
48	800.80	52.20	62.00	0.53	242.30	1.05	2.48	1.05	1.50	1078.80
49	802.00	51.50	62.10	0.52	242.30	1.10	2.51	1.00	1.48	1075.03
50	1039.35	67.86	80.73	0.68	318.50	1.43	3.28	1.30	2.02	1105.37
Mean	847.39	54.33	64.74	0.54	256.23	1.13	2.67	1.07	1.59	1087.12
Lower	789.00	49.20	59.40	0.49	238.00	0.99	2.48	0.97	1.43	1063.66
Upper	1041.17	68.25	80.73	0.69	318.50	1.56	3.33	1.37	2.05	1127.70

- ***The Case of - 20% Error Size and 30% Error Amount***

No.	**Reconciled variable**									
	x0	**x1**	**x2**	**x3**	**x4**	**x5**	**x6**	**x7**	**x8**	**x9**
1	799.50	51.30	59.40	0.50	245.00	1.12	2.48	1.00	1.48	1085.00
2	795.00	50.10	60.50	0.51	242.10	1.10	2.51	1.03	1.50	1073.60
3	800.00	52.00	61.50	0.50	238.00	1.10	2.50	1.00	1.48	1087.28
4	798.50	51.70	62.00	0.51	244.60	0.99	2.52	1.00	1.50	1075.20
5	560.14	35.35	42.35	0.36	168.28	0.69	1.76	0.72	1.11	1076.25
6	797.00	51.00	62.40	0.60	241.70	1.00	2.56	0.98	1.50	1108.01
7	801.00	50.40	61.00	0.55	243.00	1.03	2.54	1.00	1.43	1099.80
8	799.80	52.00	60.30	0.50	244.40	1.03	2.56	1.04	1.48	1097.40
9	798.50	49.70	62.00	0.51	238.60	1.03	2.48	1.00	1.50	1067.50
10	560.63	35.00	41.51	0.35	166.88	0.74	1.74	0.69	1.11	1084.65
11	800.10	49.90	62.20	0.51	239.40	1.05	2.52	1.05	1.53	1125.91
12	799.00	50.90	59.50	0.51	240.30	0.99	2.56	1.00	1.49	1076.77
13	802.00	50.40	60.50	0.50	240.50	1.08	2.52	1.05	1.50	1084.83
14	799.00	51.30	62.40	0.60	245.50	1.10	2.49	1.00	1.51	1085.00
15	799.00	52.50	62.00	0.50	240.50	1.00	2.52	0.97	1.45	1085.80
16	798.00	49.80	60.50	0.50	239.20	0.99	2.56	1.00	1.50	1087.45
17	798.00	49.20	60.50	0.50	244.00	1.09	2.52	1.06	1.48	1064.88
18	799.00	52.20	61.50	0.50	243.00	0.99	2.50	1.02	1.47	1067.50
19	560.00	36.75	43.40	0.36	168.00	0.71	1.78	0.70	1.05	1087.28
20	799.00	50.70	60.50	0.51	240.50	1.05	2.52	1.02	1.54	1095.48

No.	**Reconciled variable**									
	x0	**x1**	**x2**	**x3**	**x4**	**x5**	**x6**	**x7**	**x8**	**x9**
21	800.00	51.80	62.40	0.50	243.00	1.05	2.50	1.05	1.55	1085.00
22	558.60	35.42	41.65	0.35	168.00	0.70	1.79	0.70	1.03	1067.50
23	801.00	52.50	61.50	0.51	243.00	1.10	2.52	1.05	1.50	1078.80
24	799.00	52.00	60.50	0.51	240.50	1.01	2.53	1.00	1.47	1093.49
25	798.00	51.30	62.00	0.51	239.40	1.00	2.50	1.03	1.50	1067.50
26	800.00	52.10	61.30	0.50	241.60	1.05	2.51	1.03	1.49	1124.92
27	789.00	50.50	60.30	0.50	240.50	1.00	2.50	0.99	1.56	1092.00
28	799.00	51.40	61.10	0.50	243.00	1.07	2.52	0.99	1.50	1099.62
29	560.00	35.14	42.42	0.36	168.28	0.70	1.75	0.70	1.05	1091.72
30	798.00	50.10	61.50	0.51	240.40	1.00	2.50	1.00	1.46	1087.45
31	799.00	51.00	62.70	0.51	239.40	1.20	2.51	1.02	1.55	1063.66
32	802.90	52.50	62.30	0.51	245.00	1.00	2.50	0.98	1.50	1085.00
33	559.30	36.40	41.65	0.36	170.10	0.84	1.76	0.71	1.04	1086.75
34	798.50	52.50	62.00	0.51	240.40	1.20	2.50	1.06	1.51	1081.47
35	800.10	52.00	61.40	0.50	244.00	1.05	2.50	0.98	1.50	1127.70
36	801.80	52.50	62.30	0.50	245.00	1.05	2.56	0.99	1.53	1093.93
37	560.00	35.21	42.14	0.35	169.12	0.77	1.75	0.74	1.07	1106.25
38	800.40	52.00	62.30	0.50	245.00	1.10	2.48	0.99	1.50	1071.53
39	559.65	35.00	42.07	0.34	168.28	0.70	1.76	0.72	1.03	1076.25
40	800.40	51.30	61.50	0.49	244.00	1.20	2.51	1.02	1.50	1085.00
41	798.80	50.80	60.40	0.50	240.00	1.10	2.56	1.00	1.52	1080.71
42	800.00	51.50	60.50	0.50	244.00	1.20	2.51	1.02	1.50	1078.63
43	801.20	52.00	59.70	0.52	241.50	1.10	2.50	1.01	1.55	1090.25
44	801.80	51.90	60.00	0.50	240.00	1.10	2.50	1.01	1.50	1097.05
45	560.56	36.47	42.91	0.37	167.58	0.84	1.79	0.70	1.05	1079.70
46	799.60	51.10	60.30	0.50	245.00	1.10	2.56	1.03	1.54	1102.50
47	798.50	51.50	61.50	0.53	240.50	1.05	2.48	1.00	1.49	1086.93
48	800.80	52.20	62.00	0.53	242.30	1.05	2.48	1.05	1.50	1078.80
49	802.00	51.50	62.10	0.52	242.30	1.10	2.51	1.00	1.48	1075.03
50	559.65	36.54	43.47	0.36	171.50	0.77	1.76	0.70	1.09	1105.37
Mean	751.41	48.21	57.48	0.48	227.32	1.00	2.36	0.95	1.41	1087.12
Lower	558.60	35.00	41.51	0.34	166.88	0.69	1.74	0.69	1.03	1063.66
Upper	802.90	52.50	62.70	0.60	245.50	1.20	2.56	1.06	1.56	1127.70

- ***The Case of + 30% Error Size and 10% Error Amount***

No.	Reconciled variable									
	x0	**x1**	**x2**	**x3**	**x4**	**x5**	**x6**	**x7**	**x8**	**x9**
1	799.50	51.30	59.40	0.50	245.00	1.12	2.48	1.00	1.48	1085.00
2	795.00	50.10	60.50	0.51	242.10	1.10	2.51	1.03	1.50	1073.60
3	880.00	57.20	67.65	0.55	261.80	1.21	2.75	1.10	1.63	1087.28
4	798.50	51.70	62.00	0.51	244.60	0.99	2.52	1.00	1.50	1075.20
5	880.22	55.55	66.55	0.56	264.44	1.09	2.77	1.13	1.74	1076.25
6	797.00	51.00	62.40	0.60	241.70	1.00	2.56	0.98	1.50	1108.01
7	801.00	50.40	61.00	0.55	243.00	1.03	2.54	1.00	1.43	1099.80
8	799.80	52.00	60.30	0.50	244.40	1.03	2.56	1.04	1.48	1097.40
9	878.35	54.67	68.20	0.56	262.46	1.13	2.73	1.10	1.65	1067.50
10	800.90	50.00	59.30	0.50	238.40	1.05	2.48	0.98	1.58	1084.65
11	880.11	54.89	68.42	0.56	263.34	1.16	2.77	1.16	1.68	1125.91
12	799.00	50.90	59.50	0.51	240.30	0.99	2.56	1.00	1.49	1076.77
13	802.00	50.40	60.50	0.50	240.50	1.08	2.52	1.05	1.50	1084.83
14	878.90	56.43	68.64	0.66	270.05	1.21	2.74	1.10	1.66	1085.00
15	799.00	52.50	62.00	0.50	240.50	1.00	2.52	0.97	1.45	1085.80
16	798.00	49.80	60.50	0.50	239.20	0.99	2.56	1.00	1.50	1087.45
17	798.00	49.20	60.50	0.50	244.00	1.09	2.52	1.06	1.48	1064.88
18	878.90	57.42	67.65	0.55	267.30	1.09	2.75	1.12	1.62	1067.50
19	800.00	52.50	62.00	0.51	240.00	1.02	2.54	1.00	1.50	1087.28
20	878.90	55.77	66.55	0.56	264.55	1.16	2.77	1.12	1.69	1095.48
21	800.00	51.80	62.40	0.50	243.00	1.05	2.50	1.05	1.55	1085.00
22	798.00	50.60	59.50	0.50	240.00	1.00	2.56	1.00	1.47	1067.50
23	881.10	57.75	67.65	0.56	267.30	1.21	2.77	1.16	1.65	1078.80
24	799.00	52.00	60.50	0.51	240.50	1.01	2.53	1.00	1.47	1093.49
25	798.00	51.30	62.00	0.51	239.40	1.00	2.50	1.03	1.50	1067.50
26	800.00	52.10	61.30	0.50	241.60	1.05	2.51	1.03	1.49	1124.92
27	789.00	50.50	60.30	0.50	240.50	1.00	2.50	0.99	1.56	1092.00
28	878.90	56.54	67.21	0.55	267.30	1.18	2.77	1.09	1.65	1099.62
29	800.00	50.20	60.60	0.51	240.40	1.00	2.50	1.00	1.50	1091.72
30	877.80	55.11	67.65	0.56	264.44	1.10	2.75	1.10	1.61	1087.45
31	799.00	51.00	62.70	0.51	239.40	1.20	2.51	1.02	1.55	1063.66
32	802.90	52.50	62.30	0.51	245.00	1.00	2.50	0.98	1.50	1085.00
33	878.90	57.20	65.45	0.56	267.30	1.32	2.76	1.11	1.63	1086.75
34	798.50	52.50	62.00	0.51	240.40	1.20	2.50	1.06	1.51	1081.47

No.	Reconciled variable									
	x0	x1	x2	x3	x4	x5	x6	x7	x8	x9
35	800.10	52.00	61.40	0.50	244.00	1.05	2.50	0.98	1.50	1127.70
36	801.80	52.50	62.30	0.50	245.00	1.05	2.56	0.99	1.53	1093.93
37	800.00	50.30	60.20	0.50	241.60	1.10	2.50	1.05	1.53	1106.25
38	800.40	52.00	62.30	0.50	245.00	1.10	2.48	0.99	1.50	1071.53
39	879.45	55.00	66.11	0.54	264.44	1.10	2.77	1.13	1.62	1076.25
40	880.44	56.43	67.65	0.54	268.40	1.32	2.76	1.12	1.65	1085.00
41	798.80	50.80	60.40	0.50	240.00	1.10	2.56	1.00	1.52	1080.71
42	800.00	51.50	60.50	0.50	244.00	1.20	2.51	1.02	1.50	1078.63
43	801.20	52.00	59.70	0.52	241.50	1.10	2.50	1.01	1.55	1090.25
44	801.80	51.90	60.00	0.50	240.00	1.10	2.50	1.01	1.50	1097.05
45	800.80	52.10	61.30	0.53	239.40	1.20	2.56	1.00	1.50	1079.70
46	879.56	56.21	66.33	0.55	269.50	1.21	2.82	1.13	1.69	1102.50
47	798.50	51.50	61.50	0.53	240.50	1.05	2.48	1.00	1.49	1086.93
48	800.80	52.20	62.00	0.53	242.30	1.05	2.48	1.05	1.50	1078.80
49	882.20	56.65	68.31	0.57	266.53	1.21	2.76	1.10	1.63	1075.03
50	799.50	52.20	62.10	0.52	245.00	1.10	2.52	1.00	1.55	1105.37
Mean	823.39	52.80	62.94	0.53	249.03	1.10	2.59	1.04	1.55	1087.12
Lower	789.00	49.20	59.30	0.50	238.40	0.99	2.48	0.97	1.43	1063.66
Upper	882.20	57.75	68.64	0.66	270.05	1.32	2.82	1.16	1.74	1127.70

- ***The Case of - 30% Error Size and 10% Error Amount***

No.	Reconciled variable									
	x0	x1	x2	x3	x4	x5	x6	x7	x8	x9
1	799.50	51.30	59.40	0.50	245.00	1.12	2.48	1.00	1.48	1085.00
2	795.00	50.10	60.50	0.51	242.10	1.10	2.51	1.03	1.50	1073.60
3	720.00	46.80	55.35	0.45	214.20	0.99	2.25	0.90	1.33	1087.28
4	798.50	51.70	62.00	0.51	244.60	0.99	2.52	1.00	1.50	1075.20
5	720.18	45.45	54.45	0.46	216.36	0.89	2.27	0.93	1.42	1076.25
6	797.00	51.00	62.40	0.60	241.70	1.00	2.56	0.98	1.50	1108.01
7	801.00	50.40	61.00	0.55	243.00	1.03	2.54	1.00	1.43	1099.80
8	799.80	52.00	60.30	0.50	244.40	1.03	2.56	1.04	1.48	1097.40
9	718.65	44.73	55.80	0.46	214.74	0.93	2.23	0.90	1.35	1067.50

No.	Reconciled variable									
	x0	x1	x2	x3	x4	x5	x6	x7	x8	x9
10	800.90	50.00	59.30	0.50	238.40	1.05	2.48	0.98	1.58	1084.65
11	720.09	44.91	55.98	0.46	215.46	0.95	2.27	0.95	1.38	1125.91
12	799.00	50.90	59.50	0.51	240.30	0.99	2.56	1.00	1.49	1076.77
13	802.00	50.40	60.50	0.50	240.50	1.08	2.52	1.05	1.50	1084.83
14	719.10	46.17	56.16	0.54	220.95	0.99	2.24	0.90	1.36	1085.00
15	799.00	52.50	62.00	0.50	240.50	1.00	2.52	0.97	1.45	1085.80
16	798.00	49.80	60.50	0.50	239.20	0.99	2.56	1.00	1.50	1087.45
17	798.00	49.20	60.50	0.50	244.00	1.09	2.52	1.06	1.48	1064.88
18	719.10	46.98	55.35	0.45	218.70	0.89	2.25	0.92	1.32	1067.50
19	800.00	52.50	62.00	0.51	240.00	1.02	2.54	1.00	1.50	1087.28
20	719.10	45.63	54.45	0.46	216.45	0.95	2.27	0.92	1.39	1095.48
21	800.00	51.80	62.40	0.50	243.00	1.05	2.50	1.05	1.55	1085.00
22	798.00	50.60	59.50	0.50	240.00	1.00	2.56	1.00	1.47	1067.50
23	720.90	47.25	55.35	0.46	218.70	0.99	2.27	0.95	1.35	1078.80
24	799.00	52.00	60.50	0.51	240.50	1.01	2.53	1.00	1.47	1093.49
25	798.00	51.30	62.00	0.51	239.40	1.00	2.50	1.03	1.50	1067.50
26	800.00	52.10	61.30	0.50	241.60	1.05	2.51	1.03	1.49	1124.92
27	789.00	50.50	60.30	0.50	240.50	1.00	2.50	0.99	1.56	1092.00
28	719.10	46.26	54.99	0.45	218.70	0.96	2.27	0.89	1.35	1099.62
29	800.00	50.20	60.60	0.51	240.40	1.00	2.50	1.00	1.50	1091.72
30	718.20	45.09	55.35	0.46	216.36	0.90	2.25	0.90	1.31	1087.45
31	799.00	51.00	62.70	0.51	239.40	1.20	2.51	1.02	1.55	1063.66
32	802.90	52.50	62.30	0.51	245.00	1.00	2.50	0.98	1.50	1085.00
33	719.10	46.80	53.55	0.46	218.70	1.08	2.26	0.91	1.33	1086.75
34	798.50	52.50	62.00	0.51	240.40	1.20	2.50	1.06	1.51	1081.47
35	800.10	52.00	61.40	0.50	244.00	1.05	2.50	0.98	1.50	1127.70
36	801.80	52.50	62.30	0.50	245.00	1.05	2.56	0.99	1.53	1093.93
37	800.00	50.30	60.20	0.50	241.60	1.10	2.50	1.05	1.53	1106.25
38	800.40	52.00	62.30	0.50	245.00	1.10	2.48	0.99	1.50	1071.53
39	719.55	45.00	54.09	0.44	216.36	0.90	2.27	0.93	1.32	1076.25
40	720.36	46.17	55.35	0.44	219.60	1.08	2.26	0.92	1.35	1085.00
41	798.80	50.80	60.40	0.50	240.00	1.10	2.56	1.00	1.52	1080.71
42	800.00	51.50	60.50	0.50	244.00	1.20	2.51	1.02	1.50	1078.63
43	801.20	52.00	59.70	0.52	241.50	1.10	2.50	1.01	1.55	1090.25
44	801.80	51.90	60.00	0.50	240.00	1.10	2.50	1.01	1.50	1097.05
45	800.80	52.10	61.30	0.53	239.40	1.20	2.56	1.00	1.50	1079.70

No.	Reconciled variable									
	x0	**x1**	**x2**	**x3**	**x4**	**x5**	**x6**	**x7**	**x8**	**x9**
46	719.64	45.99	54.27	0.45	220.50	0.99	2.30	0.93	1.39	1102.50
47	798.50	51.50	61.50	0.53	240.50	1.05	2.48	1.00	1.49	1086.93
48	800.80	52.20	62.00	0.53	242.30	1.05	2.48	1.05	1.50	1078.80
49	721.80	46.35	55.89	0.47	218.07	0.99	2.26	0.90	1.33	1075.03
50	799.50	52.20	62.10	0.52	245.00	1.10	2.52	1.00	1.55	1105.37
Mean	775.41	49.74	59.27	0.50	234.52	1.03	2.44	0.98	1.46	1087.12
Lower	718.20	44.73	53.55	0.44	214.20	0.89	2.23	0.89	1.31	1063.66
Upper	802.90	52.50	62.70	0.60	245.00	1.20	2.56	1.06	1.58	1127.70

- ***The Case of + 30% Error Size and 20% Error Amount***

No.	Reconciled variable									
	x0	**x1**	**x2**	**x3**	**x4**	**x5**	**x6**	**x7**	**x8**	**x9**
1	799.50	51.30	59.40	0.50	245.00	1.12	2.48	1.00	1.48	1085.00
2	795.00	50.10	60.50	0.51	242.10	1.10	2.51	1.03	1.50	1073.60
3	800.00	52.00	61.50	0.50	238.00	1.10	2.50	1.00	1.48	1087.28
4	958.20	62.04	74.40	0.61	293.52	1.19	3.02	1.20	1.80	1075.20
5	800.20	50.50	60.50	0.51	240.40	0.99	2.52	1.03	1.58	1076.25
6	956.40	61.20	74.88	0.72	290.04	1.20	3.07	1.18	1.80	1108.01
7	801.00	50.40	61.00	0.55	243.00	1.03	2.54	1.00	1.43	1099.80
8	799.80	52.00	60.30	0.50	244.40	1.03	2.56	1.04	1.48	1097.40
9	798.50	49.70	62.00	0.51	238.60	1.03	2.48	1.00	1.50	1067.50
10	800.90	50.00	59.30	0.50	238.40	1.05	2.48	0.98	1.58	1084.65
11	800.10	49.90	62.20	0.51	239.40	1.05	2.52	1.05	1.53	1125.91
12	958.80	61.08	71.40	0.61	288.36	1.19	3.07	1.20	1.79	1076.77
13	802.00	50.40	60.50	0.50	240.50	1.08	2.52	1.05	1.50	1084.83
14	799.00	51.30	62.40	0.60	245.50	1.10	2.49	1.00	1.51	1085.00
15	799.00	52.50	62.00	0.50	240.50	1.00	2.52	0.97	1.45	1085.80
16	798.00	49.80	60.50	0.50	239.20	0.99	2.56	1.00	1.50	1087.45
17	798.00	49.20	60.50	0.50	244.00	1.09	2.52	1.06	1.48	1064.88
18	958.80	62.64	73.80	0.60	291.60	1.19	3.00	1.22	1.76	0.00
19	800.00	52.50	62.00	0.51	240.00	1.02	2.54	1.00	1.50	1087.28
20	799.00	50.70	60.50	0.51	240.50	1.05	2.52	1.02	1.54	1095.48

No.	Reconciled variable									
	x0	**x1**	**x2**	**x3**	**x4**	**x5**	**x6**	**x7**	**x8**	**x9**
21	960.00	62.16	74.88	0.60	291.60	1.26	3.00	1.26	1.86	1085.00
22	798.00	50.60	59.50	0.50	240.00	1.00	2.56	1.00	1.47	1067.50
23	801.00	52.50	61.50	0.51	243.00	1.10	2.52	1.05	1.50	1078.80
24	958.80	62.40	72.60	0.61	288.60	1.21	3.04	1.20	1.76	1093.49
25	798.00	51.30	62.00	0.51	239.40	1.00	2.50	1.03	1.50	1067.50
26	800.00	52.10	61.30	0.50	241.60	1.05	2.51	1.03	1.49	1124.92
27	789.00	50.50	60.30	0.50	240.50	1.00	2.50	0.99	1.56	1092.00
28	799.00	51.40	61.10	0.50	243.00	1.07	2.52	0.99	1.50	1099.62
29	960.00	60.24	72.72	0.61	288.48	1.20	3.00	1.20	1.80	1091.72
30	798.00	50.10	61.50	0.51	240.40	1.00	2.50	1.00	1.46	1087.45
31	799.00	51.00	62.70	0.51	239.40	1.20	2.51	1.02	1.55	1063.66
32	802.90	52.50	62.30	0.51	245.00	1.00	2.50	0.98	1.50	1085.00
33	958.80	62.40	71.40	0.61	291.60	1.44	3.01	1.21	1.78	1086.75
34	798.50	52.50	62.00	0.51	240.40	1.20	2.50	1.06	1.51	1081.47
35	960.12	62.40	73.68	0.60	292.80	1.26	3.00	1.18	1.80	1127.70
36	801.80	52.50	62.30	0.50	245.00	1.05	2.56	0.99	1.53	1093.93
37	800.00	50.30	60.20	0.50	241.60	1.10	2.50	1.05	1.53	1106.25
38	960.48	62.40	74.76	0.60	294.00	1.32	2.98	1.19	1.80	1071.53
39	799.50	50.00	60.10	0.49	240.40	1.00	2.52	1.03	1.47	1076.25
40	960.48	61.56	73.80	0.59	292.80	1.44	3.01	1.22	1.80	1085.00
41	798.80	50.80	60.40	0.50	240.00	1.10	2.56	1.00	1.52	1080.71
42	800.00	51.50	60.50	0.50	244.00	1.20	2.51	1.02	1.50	1078.63
43	801.20	52.00	59.70	0.52	241.50	1.10	2.50	1.01	1.55	1090.25
44	962.16	62.28	72.00	0.60	288.00	1.32	3.00	1.21	1.80	1097.05
45	800.80	52.10	61.30	0.53	239.40	1.20	2.56	1.00	1.50	1079.70
46	959.52	61.32	72.36	0.60	294.00	1.32	3.07	1.24	1.85	1102.50
47	798.50	51.50	61.50	0.53	240.50	1.05	2.48	1.00	1.49	1086.93
48	800.80	52.20	62.00	0.53	242.30	1.05	2.48	1.05	1.50	1078.80
49	962.40	61.80	74.52	0.62	290.76	1.32	3.01	1.20	1.78	1075.03
50	959.40	62.64	74.52	0.62	294.00	1.32	3.02	1.20	1.86	1105.37
Mean	847.38	54.37	64.78	0.54	256.34	1.13	2.67	1.07	1.59	1065.77
Lower	789.00	49.20	59.30	0.49	238.00	0.99	2.48	0.97	1.43	0.00
Upper	962.40	62.64	74.88	0.72	294.00	1.44	3.07	1.26	1.86	1127.70

- ***The Case of - 30% Error Size and 20% Error Amount***

No.	Reconciled variable									
	x0	**x1**	**x2**	**x3**	**x4**	**x5**	**x6**	**x7**	**x8**	**x9**
1	799.50	51.30	59.40	0.50	245.00	1.12	2.48	1.00	1.48	1085.00
2	795.00	50.10	60.50	0.51	242.10	1.10	2.51	1.03	1.50	1073.60
3	800.00	52.00	61.50	0.50	238.00	1.10	2.50	1.00	1.48	1087.28
4	638.80	41.36	49.60	0.41	195.68	0.79	2.02	0.80	1.20	1075.20
5	800.20	50.50	60.50	0.51	240.40	0.99	2.52	1.03	1.58	1076.25
6	637.60	40.80	49.92	0.48	193.36	0.80	2.05	0.78	1.20	1108.01
7	801.00	50.40	61.00	0.55	243.00	1.03	2.54	1.00	1.43	1099.80
8	799.80	52.00	60.30	0.50	244.40	1.03	2.56	1.04	1.48	1097.40
9	798.50	49.70	62.00	0.51	238.60	1.03	2.48	1.00	1.50	1067.50
10	800.90	50.00	59.30	0.50	238.40	1.05	2.48	0.98	1.58	1084.65
11	800.10	49.90	62.20	0.51	239.40	1.05	2.52	1.05	1.53	1125.91
12	639.20	40.72	47.60	0.41	192.24	0.79	2.05	0.80	1.19	1076.77
13	802.00	50.40	60.50	0.50	240.50	1.08	2.52	1.05	1.50	1084.83
14	799.00	51.30	62.40	0.60	245.50	1.10	2.49	1.00	1.51	1085.00
15	799.00	52.50	62.00	0.50	240.50	1.00	2.52	0.97	1.45	1085.80
16	798.00	49.80	60.50	0.50	239.20	0.99	2.56	1.00	1.50	1087.45
17	798.00	49.20	60.50	0.50	244.00	1.09	2.52	1.06	1.48	1064.88
18	639.20	41.76	49.20	0.40	194.40	0.79	2.00	0.82	1.18	1067.50
19	800.00	52.50	62.00	0.51	240.00	1.02	2.54	1.00	1.50	1087.28
20	799.00	50.70	60.50	0.51	240.50	1.05	2.52	1.02	1.54	1095.48
21	640.00	41.44	49.92	0.40	194.40	0.84	2.00	0.84	1.24	1085.00
22	798.00	50.60	59.50	0.50	240.00	1.00	2.56	1.00	1.47	1067.50
23	801.00	52.50	61.50	0.51	243.00	1.10	2.52	1.05	1.50	1078.80
24	639.20	41.60	48.40	0.41	192.40	0.81	2.02	0.80	1.18	1093.49
25	798.00	51.30	62.00	0.51	239.40	1.00	2.50	1.03	1.50	1067.50
26	800.00	52.10	61.30	0.50	241.60	1.05	2.51	1.03	1.49	1124.92
27	789.00	50.50	60.30	0.50	240.50	1.00	2.50	0.99	1.56	1092.00
28	799.00	51.40	61.10	0.50	243.00	1.07	2.52	0.99	1.50	1099.62
29	640.00	40.16	48.48	0.41	192.32	0.80	2.00	0.80	1.20	1091.72
30	798.00	50.10	61.50	0.51	240.40	1.00	2.50	1.00	1.46	1087.45
31	799.00	51.00	62.70	0.51	239.40	1.20	2.51	1.02	1.55	1063.66
32	802.90	52.50	62.30	0.51	245.00	1.00	2.50	0.98	1.50	1085.00
33	639.20	41.60	47.60	0.41	194.40	0.96	2.01	0.81	1.18	1086.75
34	798.50	52.50	62.00	0.51	240.40	1.20	2.50	1.06	1.51	1081.47

No.	Reconciled variable									
	x0	x1	x2	x3	x4	x5	x6	x7	x8	x9
35	640.08	41.60	49.12	0.40	195.20	0.84	2.00	0.78	1.20	1127.70
36	801.80	52.50	62.30	0.50	245.00	1.05	2.56	0.99	1.53	1093.93
37	800.00	50.30	60.20	0.50	241.60	1.10	2.50	1.05	1.53	1106.25
38	640.32	41.60	49.84	0.40	196.00	0.88	1.98	0.79	1.20	1071.53
39	799.50	50.00	60.10	0.49	240.40	1.00	2.52	1.03	1.47	1076.25
40	640.32	41.04	49.20	0.39	195.20	0.96	2.01	0.82	1.20	1085.00
41	798.80	50.80	60.40	0.50	240.00	1.10	2.56	1.00	1.52	1080.71
42	800.00	51.50	60.50	0.50	244.00	1.20	2.51	1.02	1.50	1078.63
43	801.20	52.00	59.70	0.52	241.50	1.10	2.50	1.01	1.55	1090.25
44	641.44	41.52	48.00	0.40	192.00	0.88	2.00	0.81	1.20	1097.05
45	800.80	52.10	61.30	0.53	239.40	1.20	2.56	1.00	1.50	1079.70
46	639.68	40.88	48.24	0.40	196.00	0.88	2.05	0.82	1.23	1102.50
47	798.50	51.50	61.50	0.53	240.50	1.05	2.48	1.00	1.49	1086.93
48	800.80	52.20	62.00	0.53	242.30	1.05	2.48	1.05	1.50	1078.80
49	641.60	41.20	49.68	0.42	193.84	0.88	2.01	0.80	1.18	1075.03
50	639.60	41.76	49.68	0.42	196.00	0.88	2.02	0.80	1.24	1105.37
Mean	751.42	48.17	57.44	0.48	227.21	1.00	2.37	0.95	1.41	1087.12
Lower	637.60	40.16	47.60	0.39	192.00	0.79	1.98	0.78	1.18	1063.66
Upper	802.90	52.50	62.70	0.60	245.50	1.20	2.56	1.06	1.58	1127.70

- ***The Case of + 30% Error Size and 30% Error Amount***

No.	Reconciled variable									
	x0	x1	x2	x3	x4	x5	x6	x7	x8	x9
1	1039.35	66.69	77.22	0.65	318.50	1.46	3.22	1.30	1.92	1085.00
2	795.00	50.10	60.50	0.51	242.10	1.10	2.51	1.03	1.50	1073.60
3	800.00	52.00	61.50	0.50	238.00	1.10	2.50	1.00	1.48	1087.28
4	798.50	51.70	62.00	0.51	244.60	0.99	2.52	1.00	1.50	1075.20
5	1040.26	65.65	78.65	0.66	312.52	1.29	3.28	1.34	2.05	1076.25
6	797.00	51.00	62.40	0.60	241.70	1.00	2.56	0.98	1.50	1108.01
7	801.00	50.40	61.00	0.55	243.00	1.03	2.54	1.00	1.43	1099.80
8	799.80	52.00	60.30	0.50	244.40	1.03	2.56	1.04	1.48	1097.40
9	1038.05	64.61	80.60	0.66	310.18	1.34	3.22	1.30	1.95	1067.50

No.	Reconciled variable									
	x0	**x1**	**x2**	**x3**	**x4**	**x5**	**x6**	**x7**	**x8**	**x9**
10	800.90	50.00	59.30	0.50	238.40	1.05	2.48	0.98	1.58	1084.65
11	1040.13	64.87	80.86	0.66	311.22	1.37	3.28	1.37	1.99	1125.91
12	799.00	50.90	59.50	0.51	240.30	0.99	2.56	1.00	1.49	1076.77
13	802.00	50.40	60.50	0.50	240.50	1.08	2.52	1.05	1.50	1084.83
14	1038.70	66.69	81.12	0.78	319.15	1.43	3.24	1.30	1.96	1085.00
15	799.00	52.50	62.00	0.50	240.50	1.00	2.52	0.97	1.45	1085.80
16	798.00	49.80	60.50	0.50	239.20	0.99	2.56	1.00	1.50	1087.45
17	798.00	49.20	60.50	0.50	244.00	1.09	2.52	1.06	1.48	1064.88
18	1038.70	67.86	79.95	0.65	315.90	1.29	3.25	1.33	1.91	1067.50
19	800.00	52.50	62.00	0.51	240.00	1.02	2.54	1.00	1.50	1087.28
20	1038.70	65.91	78.65	0.66	312.65	1.37	3.28	1.33	2.00	1095.48
21	800.00	51.80	62.40	0.50	243.00	1.05	2.50	1.05	1.55	1085.00
22	1037.40	65.78	77.35	0.65	312.00	1.30	3.33	1.30	1.91	1067.50
23	801.00	52.50	61.50	0.51	243.00	1.10	2.52	1.05	1.50	1078.80
24	799.00	52.00	60.50	0.51	240.50	1.01	2.53	1.00	1.47	1093.49
25	1037.40	66.69	80.60	0.66	311.22	1.30	3.25	1.34	1.95	1067.50
26	800.00	52.10	61.30	0.50	241.60	1.05	2.51	1.03	1.49	1124.92
27	1025.70	65.65	78.39	0.65	312.65	1.30	3.25	1.29	2.03	1092.00
28	799.00	51.40	61.10	0.50	243.00	1.07	2.52	0.99	1.50	1099.62
29	800.00	50.20	60.60	0.51	240.40	1.00	2.50	1.00	1.50	1091.72
30	1037.40	65.13	79.95	0.66	312.52	1.30	3.25	1.30	1.90	1087.45
31	799.00	51.00	62.70	0.51	239.40	1.20	2.51	1.02	1.55	1063.66
32	802.90	52.50	62.30	0.51	245.00	1.00	2.50	0.98	1.50	1085.00
33	1038.70	67.60	77.35	0.66	315.90	1.56	3.26	1.31	1.92	1086.75
34	798.50	52.50	62.00	0.51	240.40	1.20	2.50	1.06	1.51	1081.47
35	800.10	52.00	61.40	0.50	244.00	1.05	2.50	0.98	1.50	1127.70
36	801.80	52.50	62.30	0.50	245.00	1.05	2.56	0.99	1.53	1093.93
37	1040.00	65.39	78.26	0.65	314.08	1.43	3.25	1.37	1.99	1106.25
38	800.40	52.00	62.30	0.50	245.00	1.10	2.48	0.99	1.50	1071.53
39	799.50	50.00	60.10	0.49	240.40	1.00	2.52	1.03	1.47	1076.25
40	800.40	51.30	61.50	0.49	244.00	1.20	2.51	1.02	1.50	1085.00
41	798.80	50.80	60.40	0.50	240.00	1.10	2.56	1.00	1.52	1080.71
42	800.00	51.50	60.50	0.50	244.00	1.20	2.51	1.02	1.50	1078.63
43	801.20	52.00	59.70	0.52	241.50	1.10	2.50	1.01	1.55	1090.25
44	801.80	51.90	60.00	0.50	240.00	1.10	2.50	1.01	1.50	1097.05
45	1041.04	67.73	79.69	0.69	311.22	1.56	3.33	1.30	1.95	1079.70

No.	Reconciled variable									
	x0	x1	x2	x3	x4	x5	x6	x7	x8	x9
46	799.60	51.10	60.30	0.50	245.00	1.10	2.56	1.03	1.54	1102.50
47	798.50	51.50	61.50	0.53	240.50	1.05	2.48	1.00	1.49	1086.93
48	800.80	52.20	62.00	0.53	242.30	1.05	2.48	1.05	1.50	1078.80
49	802.00	51.50	62.10	0.52	242.30	1.10	2.51	1.00	1.48	1075.03
50	1039.35	67.86	80.73	0.68	318.50	1.43	3.28	1.30	2.02	1105.37
Mean	871.27	55.86	66.60	0.56	263.50	1.16	2.74	1.10	1.64	1087.12
Lower	795.00	49.20	59.30	0.49	238.00	0.99	2.48	0.97	1.43	1063.66
Upper	1041.04	67.86	81.12	0.78	319.15	1.56	3.33	1.37	2.05	1127.70

- ***The Case of - 30% Error Size and 30% Error Amount***

No.	Reconciled variable									
	x0	x1	x2	x3	x4	x5	x6	x7	x8	x9
1	559.65	35.91	41.58	0.35	171.50	0.78	1.74	0.70	1.04	1085.00
2	795.00	50.10	60.50	0.51	242.10	1.10	2.51	1.03	1.50	1073.60
3	800.00	52.00	61.50	0.50	238.00	1.10	2.50	1.00	1.48	1087.28
4	798.50	51.70	62.00	0.51	244.60	0.99	2.52	1.00	1.50	1075.20
5	560.14	35.35	42.35	0.36	168.28	0.69	1.76	0.72	1.11	1076.25
6	797.00	51.00	62.40	0.60	241.70	1.00	2.56	0.98	1.50	1108.01
7	801.00	50.40	61.00	0.55	243.00	1.03	2.54	1.00	1.43	1099.80
8	799.80	52.00	60.30	0.50	244.40	1.03	2.56	1.04	1.48	1097.40
9	558.95	34.79	43.40	0.36	167.02	0.72	1.74	0.70	1.05	1067.50
10	800.90	50.00	59.30	0.50	238.40	1.05	2.48	0.98	1.58	1084.65
11	560.07	34.93	43.54	0.36	167.58	0.74	1.76	0.74	1.07	1125.91
12	799.00	50.90	59.50	0.51	240.30	0.99	2.56	1.00	1.49	1076.77
13	802.00	50.40	60.50	0.50	240.50	1.08	2.52	1.05	1.50	1084.83
14	559.30	35.91	43.68	0.42	171.85	0.77	1.74	0.70	1.06	1085.00
15	799.00	52.50	62.00	0.50	240.50	1.00	2.52	0.97	1.45	1085.80
16	798.00	49.80	60.50	0.50	239.20	0.99	2.56	1.00	1.50	1087.45
17	798.00	49.20	60.50	0.50	244.00	1.09	2.52	1.06	1.48	1064.88
18	559.30	36.54	43.05	0.35	170.10	0.69	1.75	0.71	1.03	1067.50
19	800.00	52.50	62.00	0.51	240.00	1.02	2.54	1.00	1.50	1087.28
20	559.30	35.49	42.35	0.36	168.35	0.74	1.76	0.71	1.08	1095.48

No.	Reconciled variable									
	x0	**x1**	**x2**	**x3**	**x4**	**x5**	**x6**	**x7**	**x8**	**x9**
21	800.00	51.80	62.40	0.50	243.00	1.05	2.50	1.05	1.55	1085.00
22	558.60	35.42	41.65	0.35	168.00	0.70	1.79	0.70	1.03	1067.50
23	801.00	52.50	61.50	0.51	243.00	1.10	2.52	1.05	1.50	1078.80
24	799.00	52.00	60.50	0.51	240.50	1.01	2.53	1.00	1.47	1093.49
25	558.60	35.91	43.40	0.36	167.58	0.70	1.75	0.72	1.05	1067.50
26	800.00	52.10	61.30	0.50	241.60	1.05	2.51	1.03	1.49	1124.92
27	552.30	35.35	42.21	0.35	168.35	0.70	1.75	0.69	1.09	1092.00
28	799.00	51.40	61.10	0.50	243.00	1.07	2.52	0.99	1.50	1099.62
29	800.00	50.20	60.60	0.51	240.40	1.00	2.50	1.00	1.50	1091.72
30	558.60	35.07	43.05	0.36	168.28	0.70	1.75	0.70	1.02	1087.45
31	799.00	51.00	62.70	0.51	239.40	1.20	2.51	1.02	1.55	1063.66
32	802.90	52.50	62.30	0.51	245.00	1.00	2.50	0.98	1.50	1085.00
33	559.30	36.40	41.65	0.36	170.10	0.84	1.76	0.71	1.04	1086.75
34	798.50	52.50	62.00	0.51	240.40	1.20	2.50	1.06	1.51	1081.47
35	800.10	52.00	61.40	0.50	244.00	1.05	2.50	0.98	1.50	1127.70
36	801.80	52.50	62.30	0.50	245.00	1.05	2.56	0.99	1.53	1093.93
37	560.00	35.21	42.14	0.35	169.12	0.77	1.75	0.74	1.07	1106.25
38	800.40	52.00	62.30	0.50	245.00	1.10	2.48	0.99	1.50	1071.53
39	799.50	50.00	60.10	0.49	240.40	1.00	2.52	1.03	1.47	1076.25
40	800.40	51.30	61.50	0.49	244.00	1.20	2.51	1.02	1.50	1085.00
41	798.80	50.80	60.40	0.50	240.00	1.10	2.56	1.00	1.52	1080.71
42	800.00	51.50	60.50	0.50	244.00	1.20	2.51	1.02	1.50	1078.63
43	801.20	52.00	59.70	0.52	241.50	1.10	2.50	1.01	1.55	1090.25
44	801.80	51.90	60.00	0.50	240.00	1.10	2.50	1.01	1.50	1097.05
45	560.56	36.47	42.91	0.37	167.58	0.84	1.79	0.70	1.05	1079.70
46	799.60	51.10	60.30	0.50	245.00	1.10	2.56	1.03	1.54	1102.50
47	798.50	51.50	61.50	0.53	240.50	1.05	2.48	1.00	1.49	1086.93
48	800.80	52.20	62.00	0.53	242.30	1.05	2.48	1.05	1.50	1078.80
49	802.00	51.50	62.10	0.52	242.30	1.10	2.51	1.00	1.48	1075.03
50	559.65	36.54	43.47	0.36	171.50	0.77	1.76	0.70	1.09	1105.37
Mean	727.54	46.68	55.62	0.46	220.04	0.97	2.29	0.92	1.37	1087.12
Lower	552.30	34.79	41.58	0.35	167.02	0.69	1.74	0.69	1.02	1063.66
Upper	802.90	52.50	62.70	0.60	245.00	1.20	2.56	1.06	1.58	1127.70

VITA

Jidapa Somsupun was born in Ubonratchatani, Thailand, on February 3, 1980. In 1999, she attended Kasetsart University, Bangkok, Thailand where she received the degree of Bachelor of Engineering in Chemical Engineering in 2003. In 2005, she entered the Graduate School of Chulalongkorn University to purpose a Master of Engineering in Chemical Engineering and completed in 2007.