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INITIAL SHAPE ANALYSIS OF CABLE-STAYED BRIDGES DURING CONSTRUCTION BY THE CANTILEVER METHOD

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สมชาย พงศวัต: การวิเคราะห์หารูปร่างเริ่มแรกของสะพานขึงในระหว่างการก่อสร้างด้วยวิธีคานยื่น (INITIAL SHAPE ANALYSIS OF CABLE-STAYED BRIDGES DURING CONSTRUCTION BY THE CANTILEVER METHOD)
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งานวิจัยนี้ศึกษาการวิเคราะห์หารูปร่างเริ่มแรกของสะพานขึงในระหว่างการก่อสร้างด้วยวิธีคานยื่น มีวัตถุประสงค์หลักเพื่อปรับปรุงวิธีการคำนวนหารูปร่างเริ่มแรกให้ดีขึ้นกว่างานวิจัยในอดีต การวิเคราะห์โครงสร้างสะพานในแต่ละขั้นตอนการก่อสร้างจะอาศัยวิธีไฟไนต์เอลิเมนต์ร่วมกับเทคนิคของโครงสร้างย่อย การวิเคราะห์จะพิจารณากระบวนการเดินหน้าตามลำดับการก่อสร้างจริง โดยคำนึงถึงผลของความไม่เชิงเส้นทางเรขาคณิตเนื่องจากการหย่อนของสายเคเบิล การหารูปร่างเริ่มแรกโดยวิธีทำซ้ำในที่นี้จะประยุกต์ใช้ระเบียบวิธีการผ่อนปรนเกินสืบเนื่องเพื่อเร่งอัตราการลู่เข้าของคำตอบให้ได้เร็วขึ้น

ผลจากการวิเคราะห์กรณีศึกษาของสะพานขึงจำนวนสี่ประเภทพบว่า การใช้ระเบียบวิธีการผ่อนปรนเกินสืบเนื่องจะช่วยให้คำตอบของรูปร่างเริ่มแรกมีการลู่เข้าได้เร็วขึ้น อย่างไรก็ตาม ค่าตัวคูณการผ่อนปรนเกิน ที่เหมาะสมที่สุดนั้น จะไม่สามารถหาได้โดยตรง และมีค่าแตกต่างไปในแต่ละปัญหา ทั้งนี้ จากกรณีศึกษาพบว่า ค่าตัวคูณการผ่อนปรนเกินที่เหมาะสมนั้น จะอยู่ระหว่าง 1.1 ถึง 1.9 ในกรณีที่สะพานขึงมีจำนวนสายเคเบิลน้อย อาจใช้ค่าตัวคูณการผ่อนปรนเกินระหว่าง 1.1 ถึง 1.4 แต่ถ้าสะพานขึงมีจำนวนสายเคเบิลมาก ค่าตัวคูณการผ่อนปรนเกินที่มีค่ามาก จะเหมาะสมกว่า และจะทำให้อัตราการลู่เข้าของคำตอบของรูปร่างเริ่มแรกโดยวีธีทำซ้ำเร็วขึ้นมาก โดยเฉพาะอย่างยิ่งในการวิเคราะห์แบบไร้เชิงเส้น

ภาควิชา : วิศวกรรมโยธา ลายมือชื่อนิสิต : ....
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SOMXAI PHONGSAWAT : INITIAL SHAPE ANALYSIS OF CABLE-STAYED BRIDGES DURING CONSTRUCTION BY THE CANTILEVER METHOD. THESIS PRINCIPAL ADVISOR : ASST. PROF. WATANACHAI SMITTAKORN, Ph.D., THESIS COADVISOR : KHAMPASEUTH THEPVONGSA, D.Eng., 54 pp.

Initial shape analysis of cable-stayed bridges during construction by cantilever method is under investigation in this study. The main objective is to improve the calculation procedure, given in recent studies, for finding the initial shape of such bridges. A finite element computational algorithm is formulated for the analysis of the bridges at each construction stage using substructuring technique. Forward process analysis in accordance with the actual construction sequence is performed and geometric nonlinearity due to the cable sag is taken into account. Successive over-relaxation (SOR) technique is employed to accelerate the convergence rate of the shape iteration in finding the initial shape of the bridges.

Four different types of cable-stayed bridges are examined as case studies. The results from these case studies show that the convergence rate of the shape iteration, for finding the initial shape of the bridges during construction, can be improved by using the SOR technique. However, the optimum value of the over-relaxation factor cannot exactly be determined since it varies from problem to problem and is often determined empirically. Nevertheless, appropriate over-relaxation factor found in the case studies ranges between 1.1 and 1.9. For the bridges with a small number of cables, the value of the over-relaxation factor from 1.1 to 1.4 may be used. A large value of the over-relaxation factor tends to be suitable for the bridges with a large number of cables and significant improvement of the convergence of the shape iteration can be achieved especially for nonlinear analysis.

Department: Civil Engineering Student's signature ....

Field of study: Infrastructure in Civil Engineering Thesis principal advisor's signature: ....

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LIST OF SYMBOLS

A	element cross-sectional area
E	modulus of elasticity
E_{eq}	equivalent modulus of elasticity
F	element axial forces
$\{F\}$	vector of global nodal forces
i	construction stage
I	moment of inertia of the cross-sectional area
k	shape iteration step
k_e	element stiffness matrix in local coordinate system
$[K]$	structure global stiffness matrix
l	horizontal projected length of the cable
L	element length
L_R	reference length selected for the shape iteration
T	tension force in the cable
T_0	estimated initial cable force
$\{U\}$	vector of known and unknown structure nodal displacements
u_j	local degrees of freedom
w	cable weight per unit length
W	dead load of girder segment
W_{eq}	weight of machine equipments
α	inclined angle of the cable
δ_i	displacement at control points
ε	allowable tolerance for the shape iteration
$\lambda_{\max}$	maximum eigenvalue of the structure global stiffness matrix
ω	over-relaxation factor
ω_{opt}	optimum over-relaxation factor

CHAPTER 1

INTRODUCTION

1.1 Problem Statement

Due to advantages such as economy, ease of construction, and their aesthetic appearance, cable-stayed bridges have rapidly gained popularity in the last five decades. However, this kind of bridge is a very complicated structure with large degrees of indeterminacy. Thus, advanced technology is required in both design and construction. The complete design of a cable-stayed bridge consists of preliminary design and detailed design. The purpose of the preliminary design is to determine tentative dimensions of structural members by analyzing of the whole bridge structure. The results from the preliminary design are then used for the detailed design which includes initial shape analysis, static deflection analysis, and dynamic analysis, etc. First, the initial shape analysis has to be carried out. An initial shape of the bridge is the target configuration of the bridge under the action of dead load of girders, towers and tension force in cables. It provides geometric configuration as well as prestressing distribution of the bridge. The purpose of the initial shape analysis is to find such tension force in the cables under the action of dead load of girders and towers which yield the initial shape of the bridge. Based on the determined initial shape, the static deflection analysis under live load and dynamic analysis can then be performed for checking the deflection and dynamic response of the bridge, respectively.

The importance of the analysis and design of such bridges during construction, however, cannot be overemphasized. History shows that failure of the bridges happened much more often during construction than under service loads since erection procedure affects the internal force distribution in the bridge structures. The structural behavior of the bridge changes significantly as the construction progresses. As a result, the structural system might become unsafe or unstable. Therefore, the geometric configuration and prestressing distribution of the bridge structure during erection stage under the action of the dead load of girders and towers, and tension force in the cables have to be determined and examined in details. Hence, the initial shape of the bridge during construction is important and has to be determined for the purpose of checking and controlling the erection procedure so that the target configuration of the bridge is achieved after construction. This research involves developing a finite element computational algorithm and the improvement of the solution computation for the initial shape analysis of cable-stayed bridges under the action of the dead load of girders and towers and tension force in the cables during construction by the cantilever method.

1.2 Literature Review

There are several studies regarding the initial shape analysis of cable-stayed bridges. Most studies published in the literature, however, dealt with finding the initial shape at the completed state of the whole bridge structure. Only a few of them involved the initial shape analysis of the bridges during construction.

For the initial shape analysis of the whole bridge structure, several approaches to determine the tension force in the cables for cable-stayed bridges have been proposed in the literature: the load balancing method, the zero displacement method, the force equilibrium method, the approach using the Newton-Raphson method, and the optimization method. Lazar, Troitsky, and Douglass (1972) used load balancing analysis to determine the tension force of stayed cables of the bridges. The influence matrices describing various sectional forces on the girders and towers, nodal displacements, reactions, and cable forces, etc., due to a unit force applied successively along each cable were first set up. A system of equations can then be formulated for the cable tension forces of each cable. Among those, one equation was written to express that the sum of the displacements due to the unknown cable tension forces should be opposite in sign to the displacements due to dead load and equal in absolute value. Thus, the unknown tension forces in stay cables can be determined by solving such system of equations.

Wang, Tseng, and Yang (1993) presented the iterative approach called 'the zero displacement method' to find the initial shape of cable-stayed bridges. By applying certain initial cable forces for starting the computation, cable forces required to produce the initial shape of the bridge can be determined by an iterative approach called 'shape iteration'. In their model, the stiffness matrix based on structural nonlinear behavior, including cable sag, beam-column, and large displacement effects, was generated to determine the structural deformation. The obtained displacements of some predominant points were then investigated in order to find out whether they had met with the target configuration or not. As soon as the poor value of the displacement was found, the shape iteration was carried out by taking the obtained cable forces as the initial cable forces for starting the next iteration. The shape iteration was repeated until the values of the displacement of the bridge converge to an allowable tolerance. The computation was then stopped and the initial shape of the bridge was found.

On the other hand, Chen et al. (2000) proposed a method called 'the force equilibrium method' to determine cable tension forces under the action of the dead loads and prestress which achieve a target moment distribution rather than the displacement of the decks. In the proposed method, three calculation stages are carried out. First of all the target moment distribution is determined by considering only the bridge deck, whereas all cables and towers are replaced by rigid simple supports. Secondly, all cables are replaced by the internal forces. The purpose of this stage is to evaluate an initial estimate of the cable forces. Finally the interaction of the towers, cables and decks is then taken into consideration to find a new moment distribution under the action of the dead loads, prestress and the initial estimated cable forces. Such moment distribution is normally different from the target moment distribution. Hence, adjustment of cable forces at some control sections must be introduced which is done by iteration.

Kim and Lee (2001)' presented an approach based on the Newton-Raphson method to determine the target configurations of cable-supported structures under dead loads. In this approach, a linearized equilibrium equation of a cable element, which includes the nodal coordinates and the unstrained element length as unknowns, is formulated using the analytical solution of an elastic catenary cable. An incremental equilibrium equation for a single cable is formed with the proposed equilibrium matrices of cable elements. The geometry of the target configuration of a cable-supported structure under dead loads is utilized to solve the incremental equilibrium equation to find the initial tensions in the cables that yield the target configuration of the cable-supported structures under the actions of dead loads.

In the optimization method, Tori, Ikeda, and Nagasaki (1986) proposed a non-iterative optimum analysis to find the cable tension forces. They expressed the cross sectional area of each member as a function of the sectional force and the designed allowable stresses by their derivation. Therefore, the objective function indicating the total cost of the bridge, which is related to the cross sectional area of each member, can easily be expressed by a linear form of the sectional forces. By taking the sectional forces as residuals and the reciprocal of the designed allowable stresses as weights, they transformed the original objective function into a new form using the weighted least-squares method. The transformed objective function is a quadratic form of the sectional forces. Sequentially, the relationships between the sectional forces of all members and the tension forces in cables, i.e., the design variables, were established using the influence matrix. As a result, the transformed objective function can be obtained as a quadratic function of the cable tension forces. By zeroing the partial derivatives of the transformed objective function with each design variable respectively, the tension forces in cables can be determined by solving the system of equations. In other words, unconstrained minimization was considered in their optimal analysis approach.

Furukawa et al. (1987) proposed an optimization procedure for cable forces in cable-stayed PC bridges based on minimum strain energy criteria. A formulation combining the two stress adjustment systems, one based on the cable forces and the other on the internal prestresses in the girders, was presented by way of optimization of the cable forces. The change in cable forces due to creep of concrete was taken into consideration in the optimization process.

Negrao and Simoes (1997), Simoes and Negrao (2000) proposed a multiobjective optimization with goals of minimum cost of materials, stresses and displacements to determine the optimum cable forces that yield the initial equilibrium configuration of the cable-stayed bridges. The constraints for the actual optimization procedure must be imposed very carefully for the resulting schemes to remain within practical limits.

For the suspension bridge, Kim, Lee, and Chang (2002) carried out nonlinear shape finding analysis for a self-anchored suspension bridge of which the main cable adopts a three-dimensional profile. The unstrained lengths of main cables and hangers are calculated using the elastic catenary cable element. The proposed procedure consists of two successive steps of nonlinear analysis. The first step focuses on the cable-only system and the second on the total bridge system. For the cable-only system, the preliminary configuration of the main cable is calculated based on the conventional method utilizing simplified force equilibrium at each node of the main cable. Then, the iterative nonlinear analysis is carried out and repeated to obtain the target configuration of the main cable. For the total bridge system, the deformations at several check positions can be successfully suppressed by introducing the proper initial forces in the cables.

All these solutions, however, are based on the configuration of the final structure and do not take into account the actual construction process. This is very problematic, because as stated by Cruz, Mari, and Roca (1998), the construction sequence influences considerably the distribution of internal forces in the completed structure. This has also been recognized by Behin and Murray (1992) who described the so-called 'backward solution' whereby a desired geometry and stress distribution are defined for the finalized structure. The structure is then virtually disassembled based on the assumption that the sequence of events during disassembly is the opposite of that which occurs during assembly, including the tensioning of stay-cables. Behin and Murray (1992) included various nonlinearities into a computer code that followed this concept.

Janjic, Pircher, and Pircher (2003) outlined a method called 'the unit load method' that allows the definition of a desired-moment distribution in the final structure under dead load. It then computes the tensioning strategy that will achieve exactly the distribution. This is done by analyzing some structural models for a unit load case of bending moments at the points for which the desired-moment distribution is given and the results are stored. A system of linear equations can then be set up with one equation for each point. This system of equations can be directly solved for the unknown multiplication factor for the unit load cases which give the exact values for the bending moment at each selected point achieving the desired-moment distribution. In the proposed method, construction methods, changes in the structural system (for example due to the individual construction stages), time-dependent effects, such as creep and shrinkage or relaxation of prestressing tendons, and geometrically nonlinear behavior are taken into account in the analysis.

Wang, Tang, and Zheng (2004) applied their zero displacement method for finding the initial shape of cable-stayed bridges during each stage of erection. Two computational processes are established: the forward process analysis (FPA) and the backward process analysis (BPA). The FPA is performed in accordance with the actual construction sequence in the bridge construction. Meanwhile the BPA follows the reverse direction of the construction sequence of the bridge. At each erection stage, the finite element analysis model is rebuilt. Then, the system of equations is set up and solved anew under the action of dead load and member forces determined in the previous stage for finding the corresponding new initial shape. Their approach, however, suffers from a slow convergence rate since when the poor value of the displacement was found, the shape iteration was carried out. The systems of linear equations must be solved repeatedly to update the cable forces until the values of the displacements of the bridge converge to be within an allowable tolerance. As a result, significant amount of computational effort is required.

The purpose of this research is to improve the convergence rate of the shape iteration in finding the initial shape of the bridge at each stage of construction by the cantilever method using the successive over-relaxation (SOR) technique.

1.3 Objectives

Based on the problems stated earlier, the objectives of this research are as follows:

1. To study the procedure of initial shape analysis of cable-stayed bridges during construction by the cantilever method.
2. To present a finite element computational algorithm for the analysis using the substructuring technique.
3. To improve the convergence rate of the shape iteration using the successive over-relaxation (SOR) technique.
4. To compare the results of cable forces, displacements, and computational effort (in term of number of shape iteration) with previous studies.

1.4 Scopes of the Research

The scopes of this research are summarized as follows:

1. Consider only static analysis subject to dead load.
2. Consider only the forward process analysis of cable-stayed bridges in two dimensions.
3. The material properties are linear and elastic.
4. Consider geometric nonlinearity due to the cable sag effect in the analysis, but geometric nonlinearities due to the beam-column and large displacement effects are ignored.
5. Consider only straight horizontal bridges.

CHAPTER 2

THEORIES

This chapter presents necessary theories required for the initial shape analysis of cable-stayed bridges during construction by the cantilever method. First, the finite element model used to represent the bridge structure is illustrated. Then, the procedure for the initial shape analysis of the bridges during construction by the cantilever method is described. Model for the forward process analysis employing the substructuring technique and shape iteration technique by applying the successive over-relaxation (SOR) technique are also presented.

2.1 Finite Element Model

The finite element model of a cable-stayed bridge structure as shown in Figure 2.1 is developed using the beam element to model the tower and girder segments and the cable element to model the cables as shown in Figure 2.2 and 2.3, respectively.

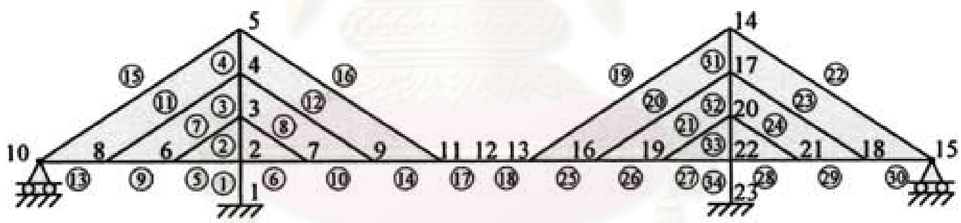


Figure 2.1 Finite element model of a double-tower symmetric harp cable-stayed bridge

2.1.1 Beam Element

Elementary beam theory which neglects the shear deformation is employed in this study. The beam element as shown in Figure 2.2 is a straight line member with a constant cross-sectional area and two nodes with six degrees of freedom. The element stiffness matrix in local coordinate system is given by

$$k_e = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \tag{2.1}$$

where E is the modulus of elasticity; A is the cross-sectional area; I is the moment of inertia of the cross-sectional area; and L is the element length. This beam element is used to model the girders and towers of the bridge.

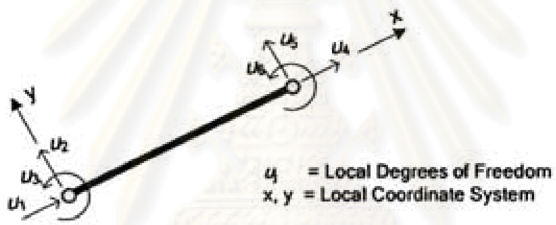


Figure 2.2 Beam element

2.1.2 Cable Element

A cable is a line element which is capable of transmitting axial tension force only. When a cable is subjected to its own weight and applied tension force, it will sag into a catenary shape as shown in Figure 2.3. The axial stiffness of the cable depends on the amount of the sag. In order to obtain a more accurate result, cable sag effect has to be taken into account when the cable element is used in the analysis. To include the sag effect in the cable stays, it is convenient to use a straight line element with an equivalent modulus of elasticity proposed by Ernst (1965). The equivalent modulus of elasticity depends on the magnitude of the cable tension force and is given by

$$E_{eq} = \frac{E}{1 + \frac{(wl)^2 AE}{12T^3}} \tag{2.2}$$

where E_{eq} is the equivalent modulus of elasticity of the cable, E is modulus of elasticity, A is the cross-sectional area, w is the cable weight per unit length, l is the horizontal projected length of the cable, and T is the tension force in the cable. The stiffness matrix in local coordinate system of the cable element with one degree of freedom (relative axial deformation) as shown in Figure 2.3 is given by

$$k_e = [k_e]_{1x1} = \begin{cases} \left[\dfrac{E_{eq}A}{L}\right] & \text{for } u_1 > 0 \\ [0] & \text{for } u_1 < 0 \end{cases} \tag{2.3}$$

where L is the chord length of the cable element. The cable stiffness becomes zero and no axial force in the cable for $u_1 < 0$, i.e., when shortening occurs. However, if the cable sag effect is ignored, the cable element becomes a straight line element with E_{eq} equal to E, hence the stiffness matrix becomes $k_e = [EA/L]$.

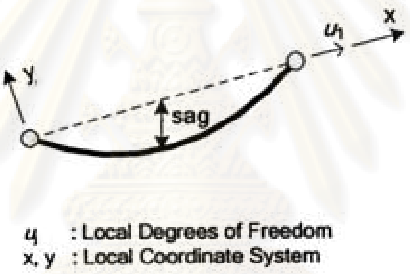


Figure 2.3 Cable element with sag

2.1.3 System Stiffness Equation

The element stiffness matrix in local coordinate system of each element must be transformed into the global coordinate system, and then assembled into the structure global stiffness. The system stiffness equation has the linear form as follows:

$$\{F\} = [K]\{U\} \tag{2.4}$$

where $\{F\}$ is the vector of global nodal forces, $[K]$ is the structure global stiffness matrix, and $\{U\}$ is the vector of known and unknown structure nodal displacements. After the imposition of the boundary conditions, equation (2.4) is solved for unknown structure nodal displacements. The member forces of each element can then be determined.

2.3 Initial Shape Analysis of Cable-Stayed Bridges during Construction by the Cantilever method

The initial shape analysis of cable-stayed bridges during construction by the cantilever method is under investigation in this study. According to Wang et al. (2004), the initial shape of a cable-stayed bridge is the target configuration of the bridge under the influence of the dead load of towers and girders and the pretension in the cables. It provides geometric configuration as well as prestress distribution of the bridge under such loads. The purpose of the initial shape analysis is to find such tension force in the cables under the action of dead load of girders and towers which yield the initial shape of the bridge. The analysis is started with an estimated initial cable forces. The new equilibrium position of the bridge under the action of dead load and such initial cable forces is then determined. Since the bridge span is long, large deflection and bending moment might exist in the towers and girders. The shape iteration has to be performed in order to reduce such deflection and bending moment. The shape iteration is carried out by taking the obtained axial forces in the towers and girders and tension forces in the cables from the present iteration step as the initial element forces for the next iteration. The equilibrium position of the bridge under the action of dead load and such initial element forces will be determined again.

For the shape iteration, control points (usually at the connection between girders and cables e.g. nodes 7, 9, and 11 in Figure 2.1) are chosen for checking the convergence tolerance. In each shape iteration, the ratio of the vertical displacement at control points to the bridge main span length will be checked whether the allowable tolerance is achieved or not. The convergence check can also be carried out in a similar manner for the towers where control points are the tip of the tower and the connection between towers and cables e.g. nodes 3, 4, and 5 in Figure 2.1. The ratio of horizontal displacement at such control points to the tower height will then be checked. The convergence tolerance for the shape iteration can be expressed as follows:

$$\left|\frac{\delta_i}{L_R}\right| \leq \varepsilon \tag{2.5}$$

where δ_i is the displacement at control points, L_R is the reference length selected for the shape iteration. The main span length and tower height are selected as the reference lengths for checking the deflection of the girder and tower, respectively, $\varepsilon = 10^{-4}$ is used as the allowable tolerance. The shape iteration will be repeated until the allowable tolerance is satisfied, then the computation will be stopped and the initial shape of the bridge is obtained.

Initial shape analysis of cable-stayed bridges during construction incorporates the initial shape analysis procedure into the forward process analysis. For the initial shape analysis in this study, only dead load of girders is taken into account while dead load of towers and cables are ignored. However, the cable sag effect due to the cable dead load is included. In order to perform the initial shape analysis of a cable-stayed bridge during construction, construction sequence of the bridge should be clearly defined to consider the assembly of towers, girder segments, cables, boundary conditions, applied loads, etc. Each stage must be defined to represent a meaningful structural system, which changes during construction. As an example, the construction sequence of a double-tower symmetric harp cable-stayed bridge by the cantilever method is illustrated in Figure 2.4. There are 8 construction stages. Starting from construction of towers T at stage 1, the girder segments G1 are sequentially added to each side of each tower. In order to achieve a required architectural geometry, at stage 3 cable stays are attached and post-tensioned to prevent excessive deflection and overstress when additional segments are added. This procedure is repeated until the spans are closed at the abutments and closure is achieved at the center of midspan.

After the construction sequence has been defined, the forward process analysis is carried out accordingly. The analysis is started from the towers at stage 1, and then continued stage by stage until stage 8, where the bridge is closed at the center of main span and the analysis is completed. At construction stages of even number (2, 4, 6) the girder segments are erected without cables. Due to their own weight, large deflection and bending moment might exist in the girders. At construction stages of odd number (3, 5, 7) the cables are installed and post-tensioned to lift up the girders to certain position. The bending moment is then reduced. The tension force in the cables required to lift up the girders to the desired elevation is computed by the shape iteration described earlier. In this study, the estimated initial cable forces for starting

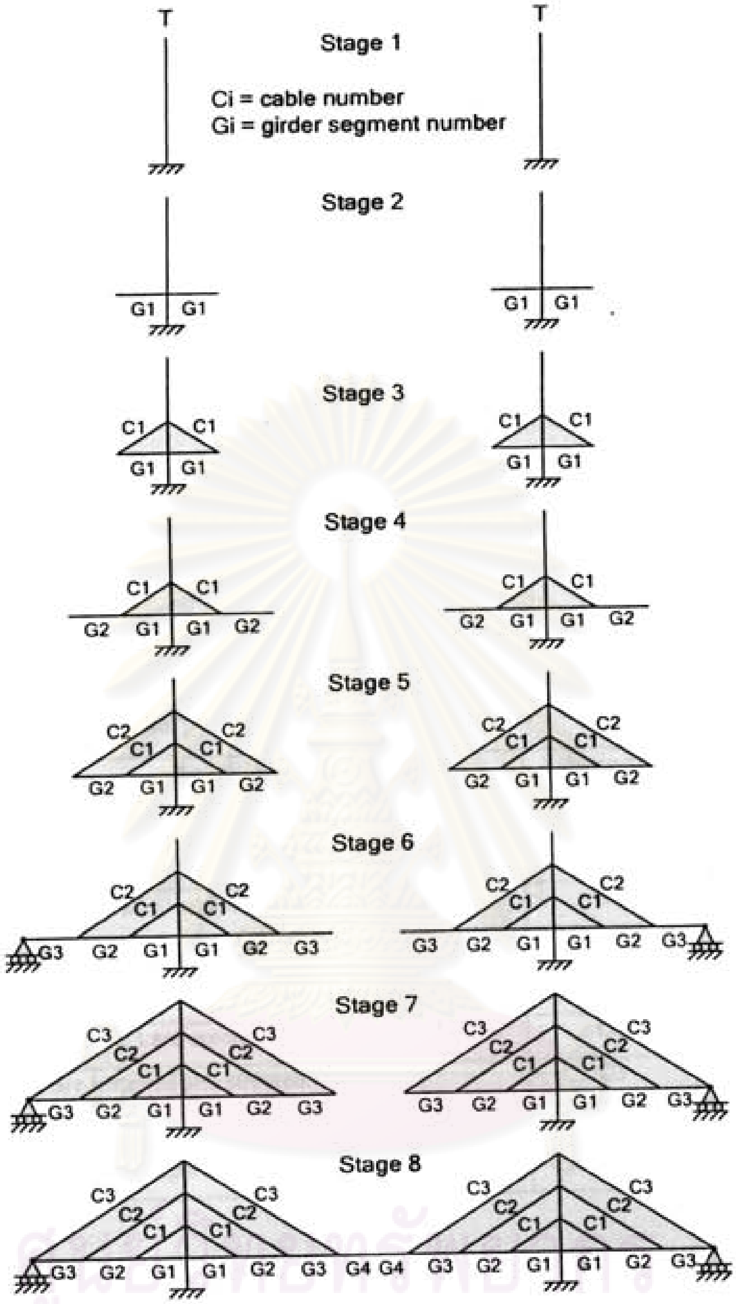


Figure 2.4 Construction sequence of the double-tower symmetric harp cable-stayed bridge by the cantilever method

the shape iteration is calculated by the formula proposed by Wang et al. (2004) as follows:

$$T_0 = \frac{3WL + 8W_{eq}}{8\sin\alpha} \tag{2.6}$$

where T_0 is the estimated initial cable force, W is the dead load of a girder segment, W_{eq} is the weight of machine equipments, L is the length of the girder segment, α is the inclined angle of the cable.

2.4 Model for the Forward Process Analysis

To perform the forward process analysis of cable-stayed bridges, construction sequence of the double-tower symmetrical harp cable-stayed bridge as shown in Figure 2.4 is taken as an example of the analysis. By employing the concept of the substructuring technique (see McGuire et al. (2000)), structural members and loads associated with each stage of construction are grouped as a 'substructure' as shown in Table 2.1 and Figure 2.5. Then these substructures are assembled and analyzed stage by stage according to the construction sequence.

Table 2.1 Substructures and loads associated with each construction stage of the double-tower symmetric harp cable-stayed bridge

Stage	Activities	Substructure	Loads
1	Towers T are constructed.	T	-
2	Girder segments G1 are erected.	G1	Self-weight
3	Cables C1 are installed and stressed.	C1	Cable pretension
4	Girder segments G2 are erected.	G2	Self-weight
5	Cables C2 are installed and stressed.	C2	Cable pretension
6	Girder segments G3 are erected.	G3	Self-weight
7	Cables C3 are installed and stressed.	C3	Cable pretension
8	Girder segments G4 are erected and the bridge is closed at the center of main span.	G4	Self-weight

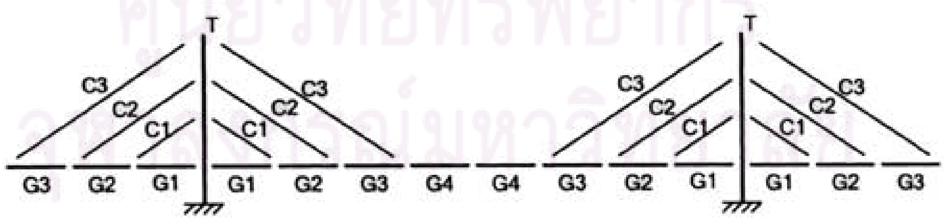


Figure 2.5 Identification of substructures associated with each construction stage of the double-tower symmetric harp cable-stayed bridge

2.5 Shape Iteration Technique

In this study, the successive over-relaxation (SOR) technique (Chapra and Canale, 2006) is employed to improve the convergence rate of the shape iteration. In the shape iteration in the previous study, the element axial forces determined from the present iteration, including compression in the tower and girder segments and tension in the cables, are used as the initial element forces for starting the next iteration. Here modification is made to element axial forces determined from the present iteration using the SOR technique before taking them as the initial element forces for starting the next iteration as follows:

$$F_i^{k+1} = \omega F_i^{k+1^*} + (1-\omega) F_i^k \tag{2.7}$$

where k is shape iteration step, F_i^k and F_i^{k+1} are the initial element forces for starting the present and next shape iteration, respectively, $F_i^{k+1^*}$ are the element axial forces determined from the present iteration, and ω is relaxation factor ranging from 0 to 2.

If $\omega = 1$, the results are unmodified. But if $1 < \omega \le 2$, the results of the present iteration are weighted to accelerate the convergence as shown in Figure 2.6. This modification is called 'over-relaxation'. On the other hand, if $0 \le \omega < 1$, which is called 'under-relaxation', this modification is used to make the results which diverge from the true solution converge. In general practice, the value of ω is usually between 1 and 2, which is called 'over-relaxation factor', for improving the convergence.

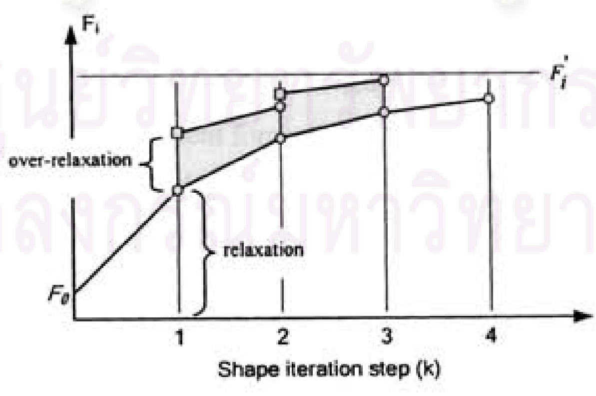


Figure 2.6 Successive over-relaxation technique

CHAPTER 3

COMPUTER PROGRAM

This chapter presents a computer program which has been developed in this study for the initial shape analysis of cable-stayed bridges during construction by the cantilever method using the substructuring technique. Here flowchart of the program is shown and the computer program is briefly described.

3.1 Flowchart of Program

Based on the initial shape analysis of cable-stayed bridges during construction by the cantilever method described in Chapter 2, the procedure of such analysis is briefly summarized as follows:

1) Input the geometric and physical data of the bridge.
2) Input dead load of girders.
3) Input the estimated initial cable forces to start the computation.
4) Assemble (or reassemble) the analysis model according to each construction stage. Then, set up and solve system of equations to find the equilibrium position and element forces.
5) For the construction stages of odd numbers (3, 5, 7, etc.) where the cables are installed and stressed, the displacement at control points are investigated whether they meet the allowable tolerance or not. If yes, the equilibrium configuration is the desired initial shape of the bridge. Otherwise, the shape iteration using SOR technique is carried out by taking newly obtained element forces as the initial element forces for next iteration, and repeat steps 4 and 5.
6) Output of the initial shape includes geometric configuration and element forces.

The construction stages 5 – 7 as shown in Figure 3.1 are chosen as examples for explaining the analysis procedure which is summarized as follows:

1) At stage 5, after finding the initial shape of the bridge, the geometric configuration and the initial force in members are known.
2) At stage 6, based on the determined (constructed) structural shape at stage 5,

the girder elements number 13, 14, 25, and 30 are installed and the analysis model is rebuilt. The system of equations is then formulated and solve for displacement and element forces. No shape iteration is performed here since the cables are not yet installed.

3) At stage 7, the cables number 15, 16, 19, and 22 are installed and stressed. The estimated initial cable forces and dead load of the girders are applied. The displacement and element forces are then determined anew after solving the system of equations. The elevation of the girders and the deflection of the towers are checked whether the allowable tolerance of the shape iteration achieved or not. If not, the shape iteration is carried out. Otherwise, the initial shape of the bridge at stage 7 is found and the analysis proceeds to the next construction stage.
4) Computation is continued for the next stage until the final stage is reached, i.e., the bridge is completely constructed.

The flowchart of the program for the initial shape analysis of cable-stayed bridges during construction by the cantilever method is shown in Figure 3.2.

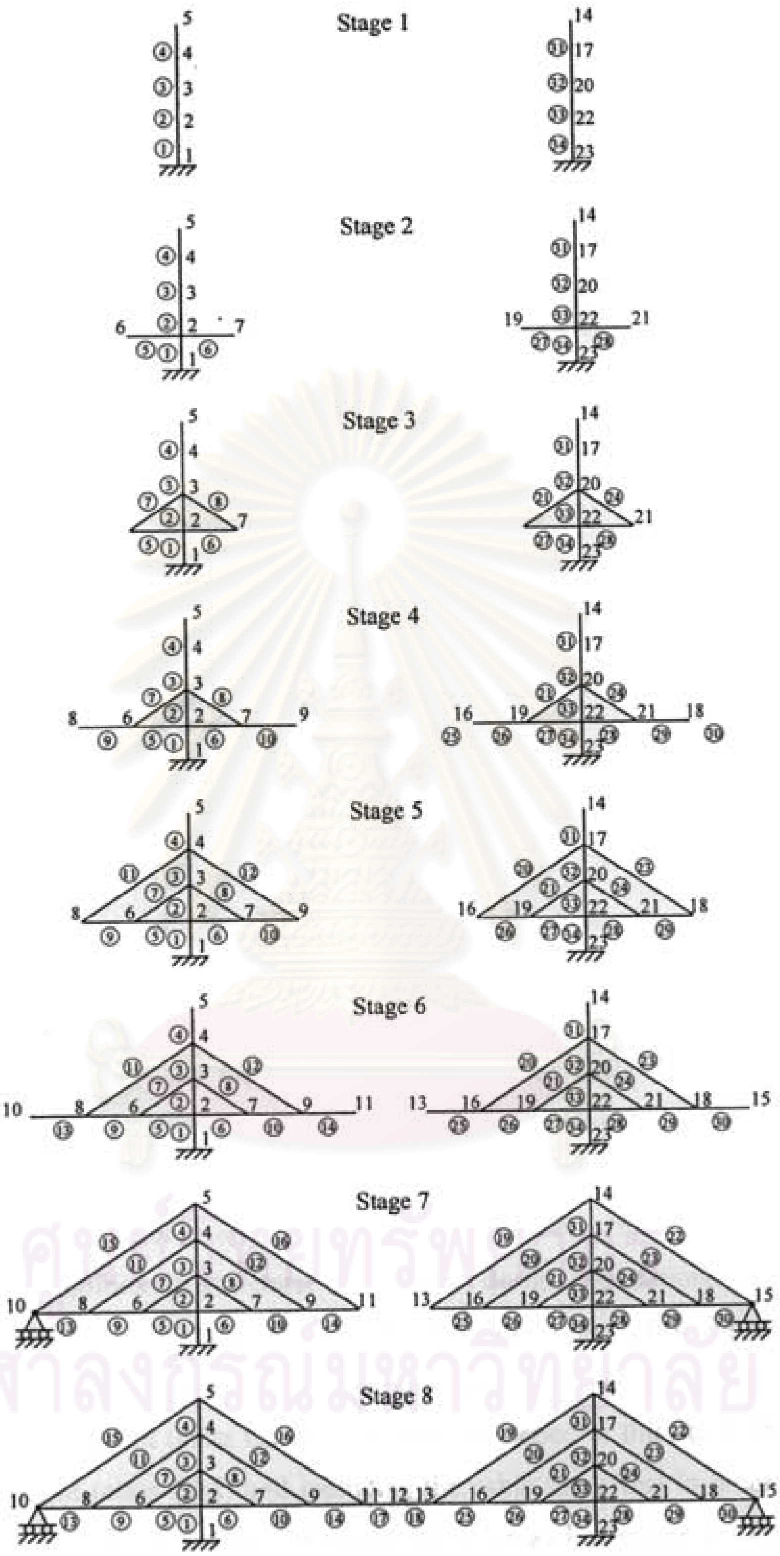


Figure 3.1 Finite element model of the double-tower symmetric harp cable-stayed bridge at each construction stage

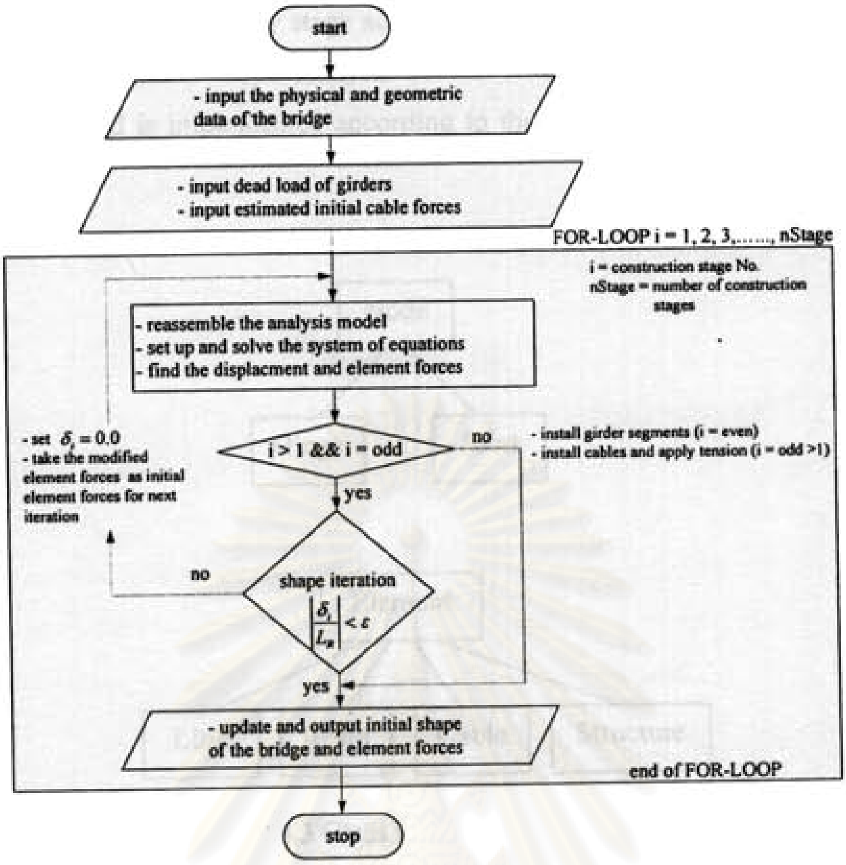


Figure 3.2 Flowchart of the program

3.2 Description of Computer Program

In order to carry out the initial shape analysis of cable-stayed bridges during construction by the cantilever method, a finite element computer program employing the concept of substructuring has been developed. The program is an extension of JSM (Smittakorn, 2008) which is an object-oriented finite element software developed as a toolbox for structural analysis and design applications, written in Java language. Classes available in JSM, such as *Hinge, Joint, Link, Beam, Structure, Material, Section, PointLoad, UniformLoad, Show, PrintOut*, have been utilized. In addition, cable element (class Cable) has been developed in this study by modifying the truss element (class *Link*) and including the cable sag effect in order to use it in case of nonlinear analysis. Class hierarchy of nodes and elements are established as shown in Figure 3.3. Class *Structure,* which is capable to deal with the problem of substructuring analysis, has been used to group the structural members and loads associated with each construction stage as substructures. Then these substructures are assembled and solved stage by stage according to the construction sequence. Finally, the part for the initial shape analysis of the bridges during construction by the cantilever method is implemented according to the flowchart shown in the previous section.

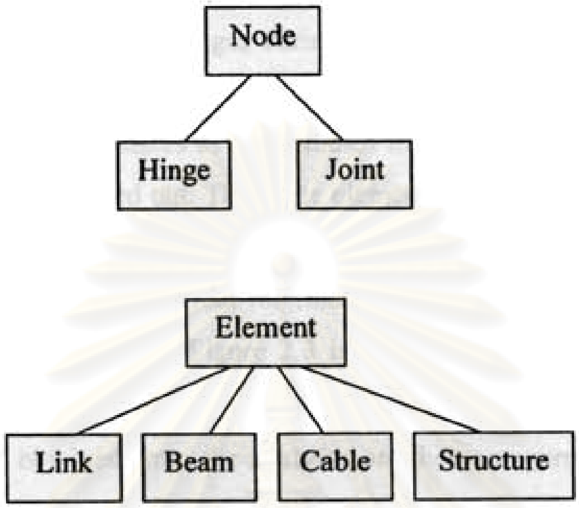


Figure 3.3 Class hierarchy of the program

CHAPTER 4

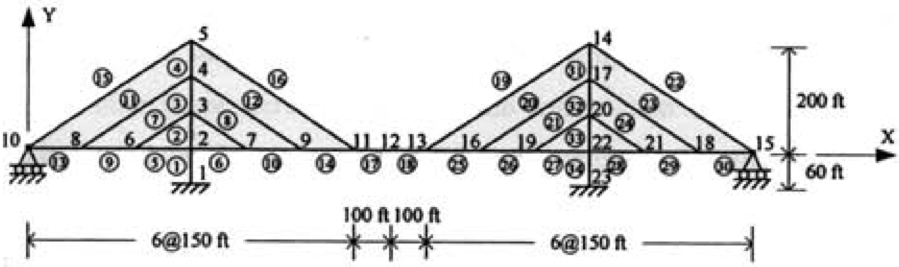
CASE STUDIES

This study presents a finite element computational algorithm for the initial shape analysis of cable-stayed bridges during construction by the cantilever method using the substructuring technique. In this chapter, four different types of cable-stayed bridges are taken from literatures as case studies for the analysis and both linear and nonlinear analyses are carried out. The finite element model of the bridge is idealized using the beam element as shown in Figure 2.2 for girder and tower segments and cable element without the sag effect for linear analysis, whereas the cable element with the sag effect as shown in Figure 2.3 is used for nonlinear analysis. Here the allowable tolerance $\varepsilon = 10^{-4}$ is used to terminate the shape iteration.

The results obtained from the analyses are then compared. For case study 1, comparison between the results determined from the developed program in this study and those from Wang et al. (2004) for linear analysis is made in order to verify the accuracy and efficiency of the developed program. The results which are compared include number of shape iteration (NSI), displacement at control points, and cable forces. For case studies 2, 3, and 4, only the results from the developed program in this study with the variation of the over-relaxation factor of the successive over-relaxation (SOR) technique are compared to demonstrate the improvement of the convergence rate of the shape iteration in finding the initial shape of cable-stayed bridges during construction.

4.1 Case Study 1: A Double-Tower Symmetric Harp Cable-Stayed Bridge

For case study 1, a double-tower symmetric harp cable-stayed bridge taken from Wang et al. (2004) as shown in Figure 4.1 is analyzed. The tower is 260 ft high. The main and side span lengths of the bridge are 1,100 ft and 450 ft, respectively. The length of each girder segment is 150 ft except the mid main spans where the length of each segment is 100 ft. There are 12 stayed cables arranged in harp pattern. The bridge is constructed by the cantilever method and there are 8 construction stages as shown in Figure 2.4. The physical properties and weight of the bridge structure are shown in Figure 4.1.



girder, tower $E = 4{,}320{,}000$ ksf

cable $E = 4{,}320{,}000$ ksf

girder $I = 131.$ ft^4, $A = 3.44$ ft^2

tower $I = 24.4, 40, 50, 60.0$ ft^4 (from top to bottom)

cable exterior: $A = 0.452$ ft^2

interior : $A = 0.174$ ft^2

dead load girder: $W = 6.0$ kips/ft,

cable : exterior $w = 0.221$ kips/ft

interior $w = 0.085$ kips/ft

Figure 4.1 The double-tower symmetric harp cable-stayed bridge

The forward process analysis is carried out following the order of the construction sequence as shown in Figure 2.4. The analysis is started from the towers at stage 1 and continued stage by stage until stage 8 where the bridge is completely constructed. The control points for the shape iteration are chosen at node 7, 9, and 11 on the girders. The vertical displacements at these nodes are investigated during the shape iteration whether they meet the allowable tolerance or not.

The comparison of number of shape iteration (NSI) at construction stages 3, 5, 7, and 8 between the results from the previous study and this study with the variation of the over-relaxation factor from 1 to 2 for linear and nonlinear analysis is shown in Table 4.1 and Table 4.2, respectively. For linear analysis, NSI used in this study with $\omega = 1.1$ and 1.2 which are the optimum values are 1, 3, 4, and 3 while those from the previous study are 1, 3, 5, and 3, respectively. It can be seen at construction stage 7 that 3 shape iterations are required to meet the allowable tolerance in this study, whereas 4 shape iterations were performed in the previous study. In the case of nonlinear analysis, NSI used with the optimum value of $\omega = 1.4$ are 1, 3, 3, and 5, whereas NSI used with $\omega = 1.0$ are 1, 3, 6, and 6 which are equivalent to those used in the shape iteration procedure of the previous study. It is seen that, for example, NSI used at construction stage 7 with $\omega = 1.4$ is only 3 while that used with $\omega = 1.0$ is 6. The total number shape iteration used, for the construction stages where the shape iteration is carried out, in this study with the optimum over-relaxation factor for linear analysis ($\omega = 1.1$ and 1.2) and nonlinear analysis ($\omega = 1.4$) are less than those from the previous study ($\omega = 1.0$) about 8.33% and 25%, respectively.

Table 4.1 Number of shape iteration (NSI) used between the results from Wang et al. (2004) and this study with the variation of the over-relaxation factor from 1 to 2 for linear analysis

	Wang et al. 2004	Over-relaxation factor ω (This Study)										
		1.0*	*1.1***	*1.2*	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
Stage3	1	1	*1*	*1*	1	1	1	1	1	1	1	1
Stage5	3	3	*3*	*3*	3	3	3	3	4	5	5	7
Stage7	5	5	*4*	*4*	4	4	5	5	7	9	13	24
Stage8	3	3	*3*	*3*	4	4	4	6	6	8	10	16
$\sum$	12	12	*11*	*11*	12	12	13	15	18	23	29	48

Remarks: *For w = 1.0 the results are equivalent to those used in the shape iteration procedure of Wang et al. 2004. **Italicized data means the optimum over-relaxation factor.

Table 4.2 Number of shape iteration (NSI) used with the variation of the over-relaxation factor from 1 to 2 for nonlinear analysis

	Over-relaxation factor ω										
	1.0	1.1	1.2	1.3	*1.4*	1.5	1.6	1.7	1.8	1.9	2.0
Stage3	1	1	1	1	*1*	1	1	1	1	1	1
Stage5	3	3	3	3	*3*	3	3	4	5	5	7
Stage7	6	6	5	4	*3*	5	5	6	7	8	15
Stage8	6	6	6	5	*5*	4	6	6	8	10	20
$\sum$	16	16	15	13	*12*	13	15	17	21	24	43

Remarks: Italicized data means the optimum over-relaxation factor.

Figure 4.2 and 4.3 show the comparison of convergence of the vertical displacement at control points (Node7, Node9, and Node11) and tension forces in cable element (CE) 8, 12, and 16, respectively, at construction stage 7 during the shape iteration for linear analysis between the results from this study with ω = 1.2 and Wang et al. (2004). It should be noted in Figure 4.2 that the results of the vertical displacement at control points from this study and the previous study are quite different since the supports exist at both ends of the bridge at construction stage 6 and 7 in this study (see Figure 2.4). However, this causes only small difference after the shape iteration is completed. Figure 4.4 and 4.5 show the comparison of convergence of the vertical displacement at the same control points and tension forces in the same cable elements as in linear analysis, respectively, at construction stage 7 during the shape iteration for nonlinear analysis between the results with ω = 1.0 and ω = 1.4. It can be seen obviously from both linear and nonlinear analysis that the convergence rate of the shape iteration can be improved by using the SOR technique.

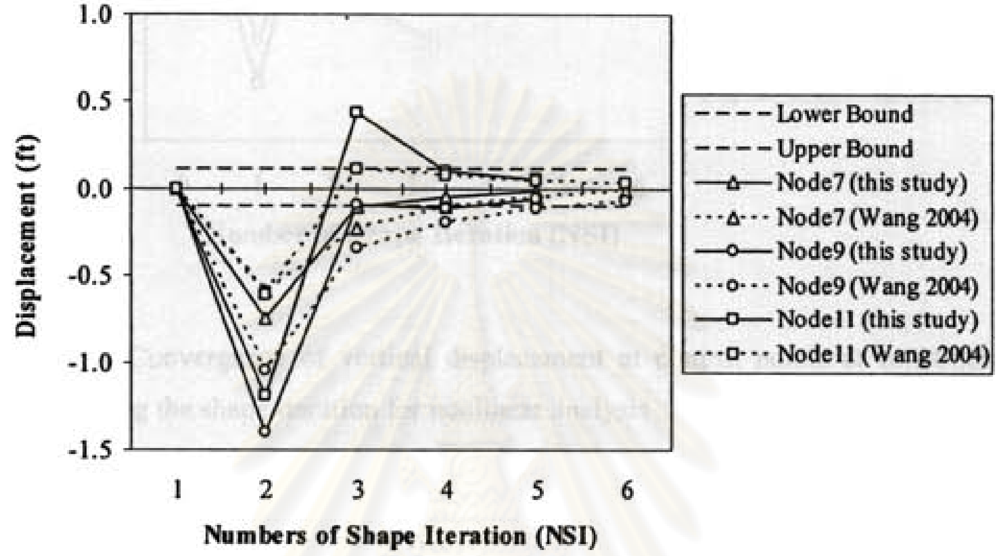


Figure 4.2 Convergence of vertical displacement at control points at construction stage 7 during the shape iteration for linear analysis

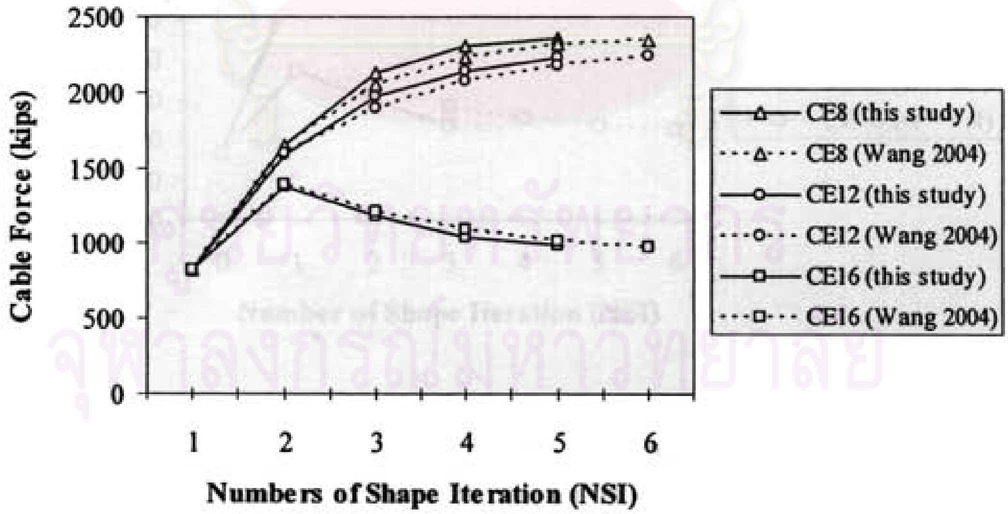


Figure 4.3 Convergence of cable forces at construction stage 7 during the shape iteration for linear analysis

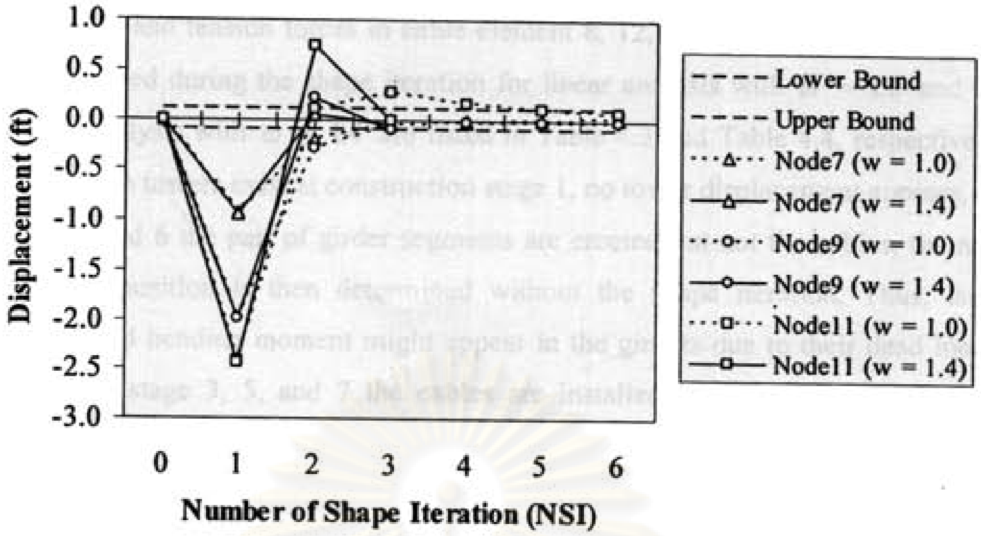


Figure 4.4 Convergence of vertical displacement at control points at construction stage 7 during the shape iteration for nonlinear analysis

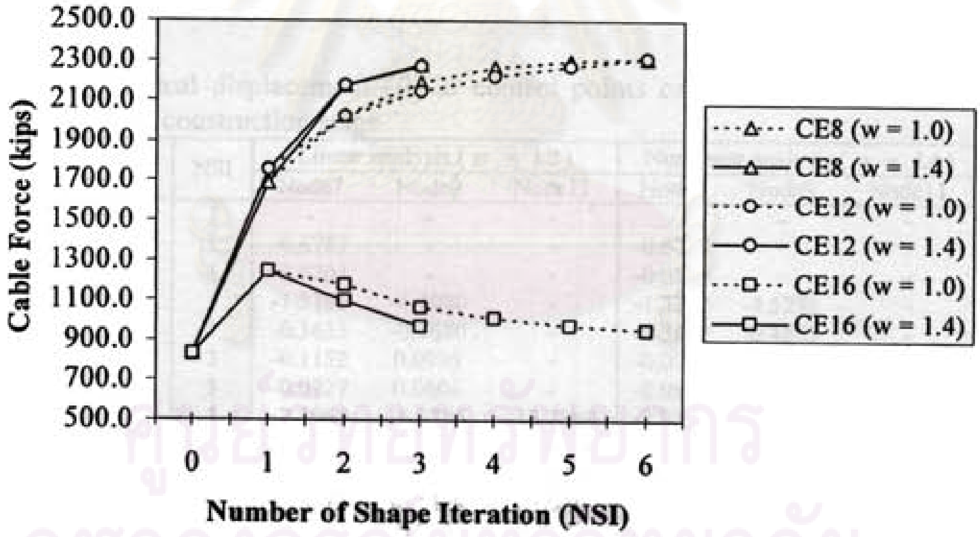


Figure 4.5 Convergence of cable forces at construction stage 7 during the shape iteration for nonlinear analysis

The variation of the vertical displacement at the control points (Node7, Node9, and Node11) and tension forces in cable element 8, 12, and 16 at each construction stage calculated during the shape iteration for linear analysis with $\omega = 1.2$ and for nonlinear analysis with $\omega = 1.4$ are listed in Table 4.3 and Table 4.4, respectively. Since only the towers exist at construction stage 1, no tower displacement appears. At stage 2, 4, and 6 the pair of girder segments are erected but not the cables, the new equilibrium position is then determined without the shape iteration. Thus, large deflection and bending moment might appear in the girders due to their dead loads. However, at stage 3, 5, and 7 the cables are installed and the shape iteration is performed to find the tension forces in the cables required to lift up the girders to the desired position for these stages. Finally, the shape iteration is done once more for the whole bridge structure at the last stage (8) in order to find the target configuration after construction. From the results in Table 4.4 the maximum difference (%) of the cable forces calculated from linear analysis with $\omega = 1.2$ and nonlinear analysis with $\omega = 1.4$ is 9.64%. Although the results from nonlinear analysis are considered more accurate, more computational effort is needed particularly in the case of the bridges with a large number of cables. This will be demonstrated clearly in the last case study.

Table 4.3 Vertical displacement (ft) at control points calculated during the shape iteration at each construction stage

Construction stage	NSI	Linear analysis ($\omega = 1.2$)			Nonlinear analysis ($\omega = 1.4$)		
		Node7	Node9	Node11	Node7	Node9	Node11
1	1	-	-	-	-	-	-
2	1	-0.6787	-	-	-0.6787	-	-
3	1	-0.0201	-	-	-0.0199	-	-
4	1	-1.3166	-4.4980	-	-1.3292	-4.5296	-
5	1	-0.3635	-0.4680	-	-0.3697	-0.4889	-
	2	-0.1152	0.0995	-	-0.0739	0.1924	-
	3	-0.0927	0.0604	-	-0.0911	0.0312	-
6	1	-2.1964	-6.3795	-12.3380	-2.2359	-6.5156	-12.5855
7	1	-0.7417	-1.3956	-1.1836	-0.9347	-1.9911	-2.4330
	2	-0.1028	-0.0923	0.4324	0.0500	0.2231	0.7438
	3	-0.0532	-0.1129	0.0946	-0.0377	-0.1014	-0.0151
	4	-0.0118	-0.0633	0.0479	-	-	-
8	1	-0.5798	-1.4047	-2.2838	-0.8669	-2.2393	-3.7453
	2	0.0801	0.1552	0.0060	0.1852	0.3327	0.0479
	3	0.0397	0.0817	-0.0566	0.0255	0.0751	-0.1565
	4	-	-	-	0.0110	0.0650	-0.1099
	5	-	-	-	0.0068	0.0624	-0.0830

Remarks: NSI – number of shape iteration.

Table 4.4 Cable force (kips) calculated during the shape iteration at each construction stage

stage	NSI	Linear analysis (ω = 1.2)			Nonlinear analysis (ω = 1.4)			Difference (%)		
		CE8	CE12	CE16	CE8	CE12	CE16	CE8	CE12	CE16
1	1	-	-	-	-	-	-	-	-	-
2	1	-	-	-	-	-	-	-	-	-
3	1	815.8	-	-	815.9	-	-	0.01	-	-
4	1	3091.1	-	-	3075.4	-	-	-0.51	-	-
5	1	1360.4	1164.8	-	1362.4	1160.9	-	-	-	-
	2	1694.8	1152.1	-	1752.8	1148.6	-	-	-	-
	3	1903.8	1076.2	-	1971.2	1050.0	-	3.42	-2.50	-
6	1	2557.8	3326.5	-	2634.2	3289.5	-	2.90	-1.12	-
7	1	1650.6	1589.9	1380.6	1688.2	1757.6	1245.0	-	-	-
	2	2128.5	1965.7	1190.4	2179.2	2177.0	1098.6	-	-	-
	3	2303.9	2135.6	1044.4	2280.9	2269.7	970.2	-	-	-
	4	2359.3	2232.0	987.5	-	-	-	-3.44	1.66	-1.78
8	1	2298.2	2500.1	2223.6	2322.2	2835.7	1852.7	-	-	-
	2	2207.7	2471.2	2494.8	2142.2	2855.2	2285.7	-	-	-
	3	2151.3	2406.6	2499.3	1980.8	2664.7	2360.0	-	-	-
	4	-	-	-	1952.3	2577.4	2429.7	-	-	-
	5	-	-	-	1962.2	2520.5	2466.0	-9.64	4.52	-1.35

Remarks: NSI – number of shape iteration. CE – cable element.

4.2 Case Study 2: The Quincy Bayview Bridge

For case study 2, the Quincy Bayview Bridge crossing the Mississippi River at Quincy, Illinois (Wilson and Gravelle, 1991) as shown in Figure 4.6 is chosen for the analysis. The bridge is a three span double-tower double-cable-plane structure with semi-harp cable arrangement. However, since only two-dimensional cable-stayed bridges are under consideration in this study, the bridge will be analyzed as a single-cable-plane structure. The tower height is 70 m and the main span of the bridge is 274 m and there are two equal side spans of 134 m. The bridge is constructed by the cantilever method with 16 construction stages where the construction sequence is similar to that of case study 1. The physical properties and weight of the bridge structure are summarized in Table 4.5 and Figure 4.7.

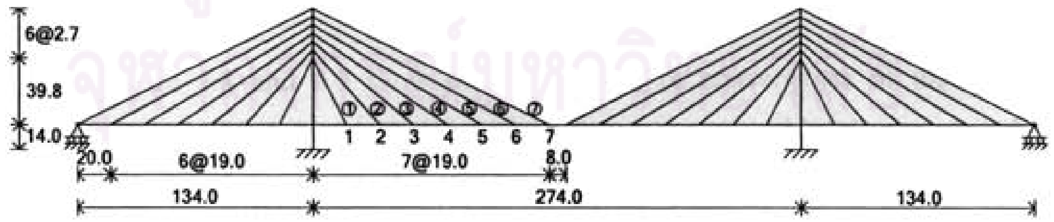


Figure 4.6 The Quincy Bayview bridge

Table 4.5 Physical properties and weight of the bridge structure

	Section number*	E (kN/m^2)	A (m^2)	I (m^4)	Weight (kN/m)
Girder	-	2.06×10^8	0.83	0.34	101.0
Tower	1	3.08×10^7	17.88	16.4	-
	2	3.08×10^7	7.06	14.03	-
Cable	1	2.06×10^8	0.009	-	0.89
	2	2.06×10^8	0.0068	-	0.77
	3	2.06×10^8	0.0054	-	0.708
	4	2.06×10^8	0.0035	-	0.52

*See Figure 4.5

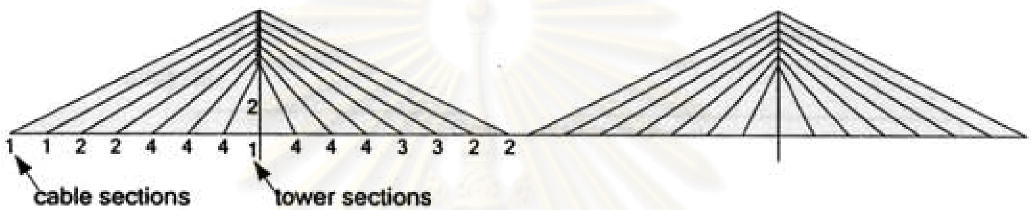


Figure 4.7 Tower and cable section numbers

The comparison of NSI at construction stages of odd numbers (3, 5, 7,..., 15) and the last stage (16) with the variation of the over-relaxation factor from 1 to 2 is shown in Table 4.6 and Table 4.7 for linear and nonlinear analysis, respectively. It can be seen from both linear and nonlinear analysis that $\omega = 1.2$ is the optimum value. The total number shape iteration used, for the construction stages where the shape iteration is performed, with the optimum over-relaxation factor ($\omega = 1.2$) for linear analysis and nonlinear analysis are less than those with $\omega = 1.0$ about 12% and 20%, respectively. It should be noted in the case of nonlinear analysis with $\omega = 2.0$ at construction stages 11, 13, 15, and 16 that the results are invalid since compression occurs in the cables.

Figure 4.8 and 4.9 show the comparison of convergence of the vertical displacement at Node4 and Node5 and tension forces in cable element 4 and 5, respectively, at construction stage 11 as shown in Figure 4.10 during the shape iteration for linear analysis between the results with $\omega = 1.0$ and $\omega = 1.2$. As can be seen in those figures, the convergence rate of the shape iteration with $\omega = 1.2$ is slightly better than that with $\omega = 1.0$.

Table 4.6 NSI used with the variation of the over-relaxation factor from 1 to 2 for linear analysis

	Over-relaxation factor ω										
	1.0	1.1	*1.2*	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
Stage3	1	1	*1*	1	1	1	1	1	1	1	1
Stage5	2	2	*2*	2	2	2	2	2	2	2	2
Stage7	2	2	*2*	3	3	3	3	3	4	5	5
Stage9	3	3	*3*	3	3	4	5	5	7	9	15
Stage11	4	4	*3*	3	4	5	5	7	9	14	28
Stage13	4	4	*3*	4	5	5	7	8	11	18	47
Stage15	5	5	*5*	5	5	5	7	9	13	21	71
Stage16	4	3	*3*	2	3	4	4	4	4	6	8
$\sum$	25	24	*22*	23	26	29	34	39	51	76	177

Remarks: Italicized data means the optimum over-relaxation factor.

Table 4.7 NSI used with the variation of the over-relaxation factor from 1 to 2 for nonlinear analysis.

	Over-relaxation factor ω										
	1.0	1.1	*1.2*	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
Stage3	1	1	*1*	1	1	1	1	1	1	1	1
Stage5	2	2	*2*	2	2	2	2	2	2	2	2
Stage7	2	2	*2*	3	3	3	3	3	3	5	5
Stage9	3	3	*3*	3	3	4	5	5	7	9	14
Stage11	4	4	*3*	3	4	5	5	7	9	14	-*
Stage13	4	3	*3*	4	5	5	7	8	11	18	-
Stage15	5	5	*3*	5	5	5	7	9	13	21	-
Stage16	4	3	*3*	2	3	4	4	4	4	6	-
$\sum$	25	23	*20*	23	26	29	34	39	50	76	-

*The results are invalid since compression occurs in the cables. Italicized data means the optimum over-relaxation factor.

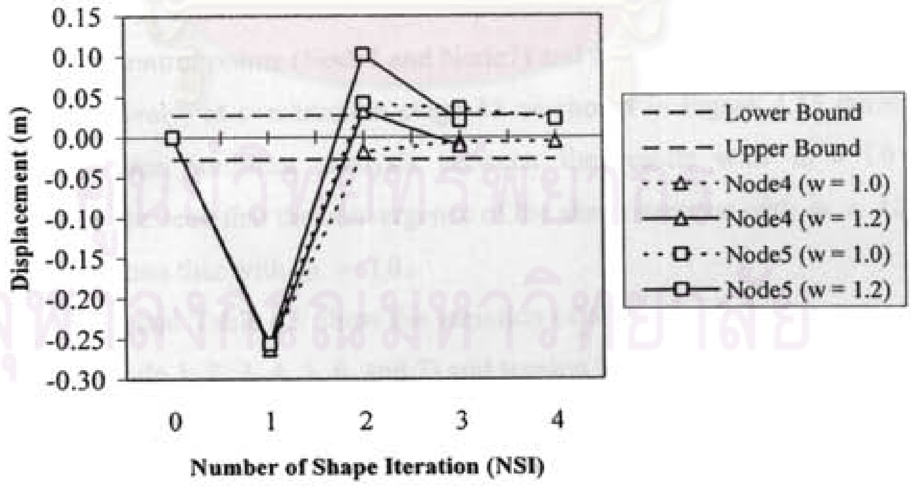


Figure 4.8 Convergence of vertical displacement at control points at construction stage 11 during the shape iteration for linear analysis

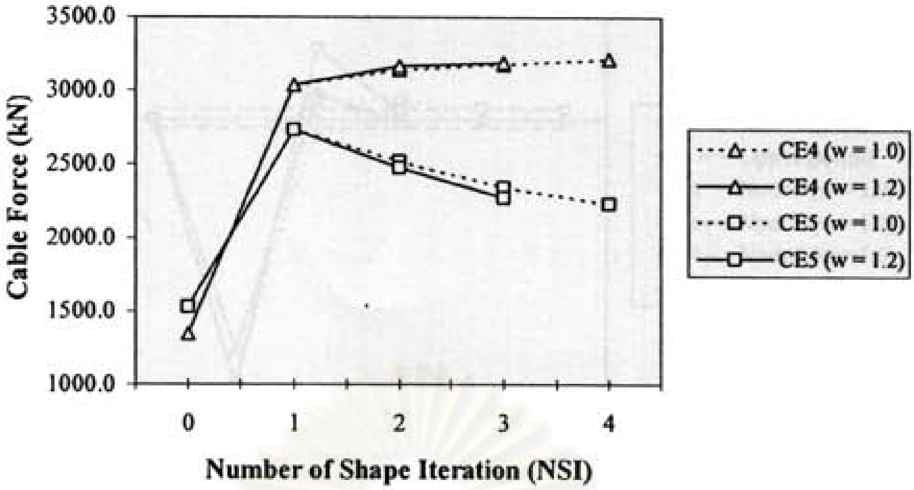


Figure 4.9 Convergence of cable forces at construction stage 11 during the shape iteration for linear analysis

Figure 4.10 Construction stage 11

Figure 4.11 and 4.12 show the comparison of convergence of the vertical displacement at control points (Node5 and Node7) and tension forces in cable element 5 and 7, respectively, at construction stage 15 as shown in Figure 4.13 during the shape iteration for nonlinear analysis between the results with $\omega = 1.0$ and $\omega = 1.2$. It can be seen that the convergence of the shape iteration with $\omega = 1.2$ is 2 iterations faster than that with $\omega = 1.0$.

Table 4.8 and Table 4.9 show the variation of the vertical displacement at the control points (node 1, 2, 3, 4, 5, 6, and 7) and tension forces in cable element 1, 2, 3, 4, 5, 6 and 7 at each construction stage calculated during the shape iteration with $\omega = 1.2$ for linear analysis and nonlinear analysis, respectively. There exists the maximum difference about 5.09% in the results of the cable forces computed from linear and nonlinear analysis.

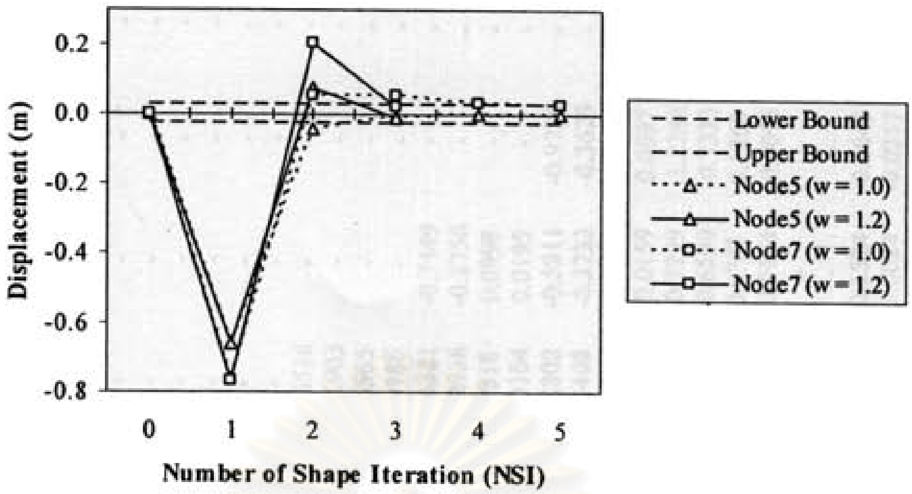


Figure 4.11 Convergence of vertical displacement at control points at construction stage 15 during the shape iteration for nonlinear analysis

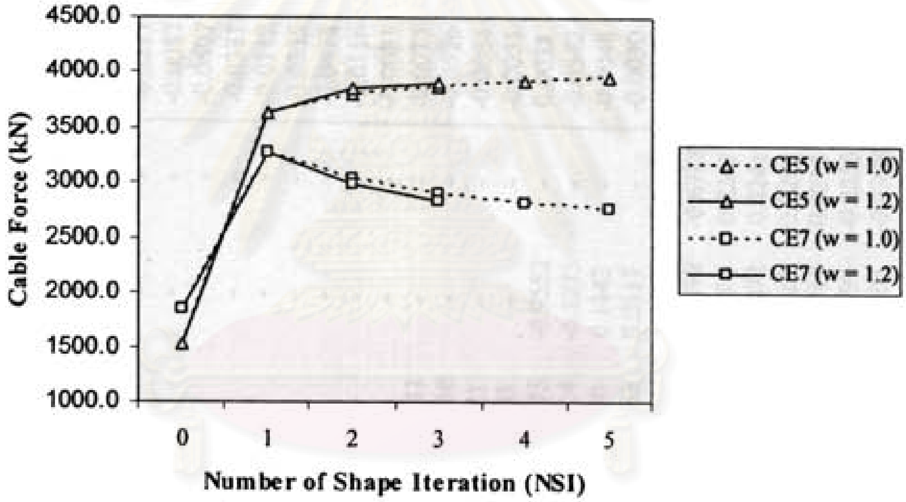


Figure 4.12 Convergence of cable forces at construction stage 15 during the shape iteration between for nonlinear analysis

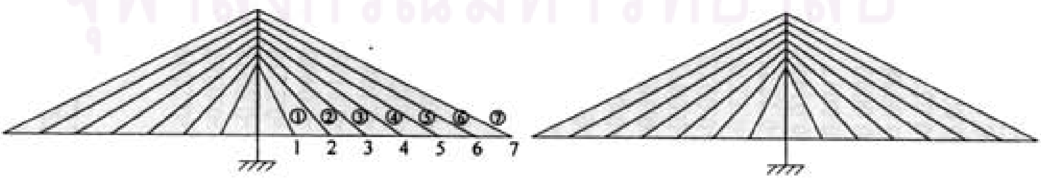


Figure 4.13 Construction stage 15

Table 4.8 Vertical displacement (m) at control points calculated during the shape iteration at each construction stage

Stage	NSI	Linear analysis (ω = 1.2)							Nonlinear analysis (ω = 1.2)						
		Node1	Node2	Node3	Node4	Node5	Node6	Node7	Node1	Node2	Node3	Node4	Node5	Node6	Node7
1	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-
2	1	-0.0236	-	-	-	-	-	-	-0.0236	-	-	-	-	-	-
3	1	-0.0002	-	-	-	-	-	-	-0.0002	-	-	-	-	-	-
4	1	-0.0769	-0.2349	-	-	-	-	-	-0.0771	-0.2356	-	-	-	-	-
5	1	-0.0209	-0.0388	-	-	-	-	-	-0.0211	-0.0394	-	-	-	-	-
	2	-0.0086	-0.0038	-	-	-	-	-	-0.0087	-0.0041	-	-	-	-	-
6	1	-0.0603	-0.2238	-0.4542	-	-	-	-	-0.0607	-0.2251	-0.4565	-	-	-	-
7	1	-0.0378	-0.0900	-0.1168	-	-	-	-	-0.0381	-0.0912	-0.1194	-	-	-	-
	2	-0.0113	-0.0109	0.0195	-	-	-	-	-0.0112	-0.0110	0.0189	-	-	-	-
8	1	-0.0334	-0.1488	-0.3647	-0.6487	-	-	-	-0.0335	-0.1495	-0.3665	-0.6516	-	-	-
9	1	-0.0474	-0.1238	-0.1724	-0.1782	-	-	-	-0.0478	-0.1259	-0.1786	-0.1903	-	-	-
	2	-0.0120	-0.0196	0.0010	0.0581	-	-	-	-0.0116	-0.0187	0.0013	0.0565	-	-	-
	3	-0.0084	-0.0177	-0.0127	0.0189	-	-	-	-0.0081	-0.0169	-0.0121	0.0186	-	-	-
10	1	-0.0075	-0.0578	-0.1955	-0.4309	-0.7331	-	-	-0.0075	-0.0585	-0.1986	-0.4381	-0.7449	-	-
11	1	-0.0525	-0.1472	-0.2274	-0.2634	-0.2568	-	-	-0.0526	-0.1484	-0.2319	-0.2738	-0.2750	-	-
	2	-0.0088	-0.0160	-0.0072	0.0316	0.1021	-	-	-0.0084	-0.0148	-0.0057	0.0318	0.0998	-	-
	3	-0.0052	-0.0127	-0.0191	-0.0108	0.0203	-	-	-0.0051	0.0122	-0.0182	-0.0104	0.0195	-	-
12	1	0.0038	-0.0076	-0.0873	-0.2745	-0.5793	-0.9552	-	0.0038	-0.0080	-0.0892	-0.2802	-0.5911	-0.9735	-
13	1	-0.0545	-0.1592	-0.2622	-0.3312	-0.3524	-0.3275	-	-0.0542	-0.1594	-0.2651	-0.3408	-0.3733	-0.3629	-
	2	-0.0052	-0.0075	-0.0028	0.0162	0.0590	0.1342	-	-0.0048	-0.0060	0.0001	0.0191	0.0593	0.1294	-
	3	-0.0028	-0.0055	-0.0124	-0.0196	-0.0163	0.0117	-	-0.0030	-0.0055	-0.0116	-0.0182	-0.0159	0.0097	-
14	1	-0.0527	-0.1235	-0.2378	-0.4362	-0.7532	-1.1842	-1.6847	-0.0550	-0.1290	-0.2475	-0.4525	-0.7799	-1.2248	-1.7400
15	1	-0.0795	-0.2225	-0.3770	-0.5137	-0.6163	-0.6699	-0.6788	-0.0832	-0.2314	-0.3932	-0.5407	-0.6590	-0.7333	-0.7662
	2	0.0008	0.0110	0.0291	0.0520	0.0824	0.1375	0.2247	-0.0008	0.0082	0.0258	0.0482	0.0763	0.1260	0.2055
	3	-0.0013	0.0017	0.0031	-0.0022	-0.0119	-0.0072	0.0241	-0.0022	0.0001	0.0017	-0.0028	-0.0124	-0.0093	0.0185
	4	-0.0003	0.0049	0.0093	0.0074	-0.0008	0.0022	0.0292	-	-	-	-	-	-	-
	5	-0.0006	0.0038	0.0078	0.0062	-0.0022	-0.0014	0.0213	-	-	-	-	-	-	-
16	1	-0.0194	-0.0371	-0.0596	-0.0950	-0.1476	-0.1994	-0.2253	-0.0201	-0.0379	-0.0608	-0.0980	-0.1524	-0.2045	-0.2298
	2	-0.0037	0.0008	0.0088	0.0115	0.0010	-0.0203	-0.0308	-0.0042	0.0009	0.0092	0.0110	-0.0024	-0.0217	-0.0314
	3	-0.0041	-0.0027	0.0026	0.0064	-0.0006	-0.0131	-0.0198	-0.0044	-0.0025	0.0033	0.0064	-0.0013	-0.0136	-0.0195

Remarks: NSI – number of shape iteration.

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Table 4.9 Cable forces (kN) calculated during the shape iteration at each construction stage

Stage	NSI	Linear analysis (ω = 1.2)							Nonlinear analysis (ω = 1.2)						
		CE1	CE2	CE3	CE4	CE5	CE6	CE7	CE1	CE2	CE3	CE4	CE5	CE6	CE7
1	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-
2	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-
3	1	794.5	-	-	-	-	-	-	794.6	-	-	-	-	-	-
4	1	1915.8	-	-	-	-	-	-	1906.8	-	-	-	-	-	-
5	1	1090.9	1319.8	-	-	-	-	-	1090.5	1316.7	-	-	-	-	-
	2	1277.7	1427.7	-	-	-	-	-	1278.0	1426.2	-	-	-	-	-
6	1	2135.8	3513.4	-	-	-	-	-	2138.5	3505.8	-	-	-	-	-
7	1	1328.8	1792.4	1860.4	-	-	-	-	1331.3	1796.6	1852.6	-	-	-	-
	2	1603.8	2062.5	1882.7	-	-	-	-	1606.2	2068.6	1877.5	-	-	-	-
8	1	2048.5	3425.6	4096.5	-	-	-	-	2052.0	3434.1	4087.2	-	-	-	-
9	1	1457.9	2099.4	2192.4	2491.7	-	-	-	1461.5	2115.8	2223.0	2450.6	-	-	-
	2	1768.1	2510.5	2392.1	2335.0	-	-	-	1766.4	2522.0	2427.7	2302.6	-	-	-
	3	1924.9	2711.6	2467.3	2131.1	-	-	-	1918.4	2714.6	2498.5	2107.5	-	-	-
10	1	1974.3	3204.8	3628.0	4920.8	-	-	-	1968.3	3214.1	3673.7	4877.4	-	-	-
11	1	1518.3	2307.2	2518.4	3034.7	2732.3	-	-	1518.7	2316.3	2540.5	3061.5	2685.7	-	-
	2	1795.0	2726.5	2834.6	3162.8	2475.8	-	-	1789.3	2725.9	2852.3	3195.1	2439.7	-	-
	3	1894.9	2874.2	2959.0	3189.9	2275.4	-	-	1886.6	2866.4	2969.1	3218.2	2250.0	-	-
12	1	1761.1	2877.3	3434.9	4917.3	5016.6	-	-	1753.0	2872.6	3455.5	4968.6	4960.1	-	-
13	1	1533.1	2405.9	2718.0	3460.1	3175.3	3168.3	-	1529.4	2406.1	2732.3	3504.9	3221.8	3081.9	-
	2	1760.6	2765.7	3047.8	3776.0	3218.0	2829.9	-	1750.7	2751.4	3047.4	3811.3	3275.2	2762.6	-
	3	1814.8	2828.2	3123.8	3878.4	3235.0	2642.1	-	1805.6	2810.2	3114.2	3899.6	3288.3	2592.9	-
14	1	1401.5	2469.4	3070.1	4548.9	4829.2	5789.1	-	1383.8	2447.5	3063.7	4591.1	4914.5	5689.5	-
15	1	1418.5	2372.4	2794.2	3743.5	3592.4	3757.7	3377.0	1397.7	2358.0	2793.9	3768.1	3640.4	3797.3	3283.4
	2	1645.5	2695.7	3087.5	4090.5	3802.5	3754.9	3054.8	1619.0	2673.2	3075.3	4098.7	3847.2	3809.4	2981.6
	3	1712.7	2723.9	3098.3	4131.7	3869.9	3764.9	2894.8	1693.0	2707.8	3084.3	4127.8	3902.5	3816.3	2839.3
	4	1747.8	2709.1	3071.9	4136.5	3932.7	3816.9	2819.3	-	-	-	-	-	-	-
	5	1779.1	2692.1	3040.2	4126.3	3981.9	3866.2	2759.3	-	-	-	-	-	-	-
16	1	1734.3	2653.1	3031.8	4218.6	4208.8	4240.2	3087.4	1654.6	2670.4	3077.4	4230.2	4142.1	4196.9	3157.3
	2	1760.1	2611.2	2949.1	4122.3	4221.66	4365.3	3224.1	1686.3	2626.1	2990.4	4136.0	4161.8	4325.4	3288.3
	3	1805.1	2609.9	2899.3	4034.1	4197.5	4414.4	3292.2	1737.5	2622.7	2936.1	4048.0	4142.3	4376.7	3352.9

Remarks: NSI – number of shape iteration. CE – cable element.

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4.3 Case Study 3: A Double-Tower Symmetric Fan Cable-Stayed Bridge

For case study 3, a double-tower symmetric fan cable-stayed bridge taken from Adeli and Zhang (1995) as shown in Figure 4.14 is analyzed. The dimensions of the bridge structure are the same as those of Quincy Bayview Bridge in case study 2. The bridge is constructed by the cantilever method with 16 construction stages where the construction sequence is similar to the manner of the case study 1. The physical properties and weight of the bridge structure are shown in Table 4.10 and Figure 4.15.

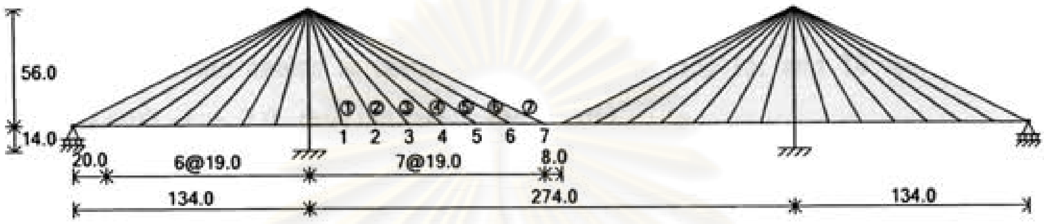

Figure 4.14 The double-tower symmetric fan cable-stayed bridge

Table 4.10 Physical properties and weight of the bridge structure

	Section number*	E (kN/m^2)	A (m^2)	I (m^4)	Weight (kN/m)
Girder	-	2.06×10^8	0.83	0.34	101.0
Tower	1	3.08×10^7	17.88	16.4	-
	2	3.08×10^7	7.06	14.03	-
Cable	1	2.06×10^8	0.009	-	0.89
	2	2.06×10^8	0.0068	-	0.77
	2	2.06×10^8	0.0054	-	0.708
	3	2.06×10^8	0.0035	-	0.52

*See Figure 4.7

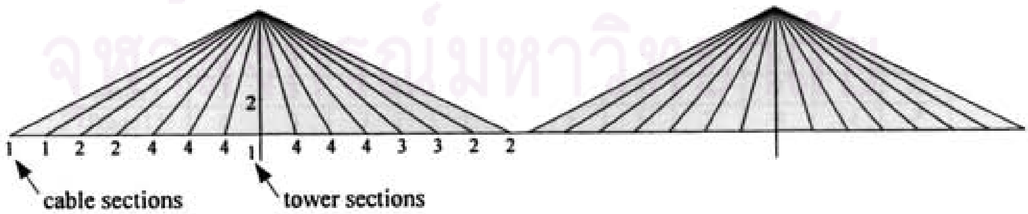

Figure 4.15 Tower and cable section numbers

The comparison of number of shape iteration (NSI) at construction stages of odd numbers (3, 5, 7,..., 15) and the last stage (16) with the variation of the over-relaxation factor from 1 to 2 is shown in Table 4.11 and Table 4.12 for linear and nonlinear analysis, respectively. Although the cable arrangement is different, the NSI used in this case study is quite similar to those in case study 2 where the value of $\omega = 1.2$ is optimum for both linear and nonlinear analysis. The total number shape iteration used, for the construction stages where the shape iteration is done, with the optimum over-relaxation factor ($\omega = 1.2$) for linear analysis and nonlinear analysis are less than those with $\omega = 1.0$ about 12.5% and 20%, respectively. The compression also occurs in the cables at construction stages 11, 13, 15, and 16 for nonlinear analysis with $\omega = 2.0$, thus the results are invalid.

Table 4.11 NSI used with the variation of the over-relaxation factor from 1 to 2 for linear analysis

	Over-relaxation factor ω										
	1.0	*1.1*	*1.2*	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
Stage3	1	*1*	*1*	1	1	1	1	1	1	1	1
Stage5	2	*2*	*2*	2	2	2	2	2	2	2	2
Stage7	2	*2*	*2*	3	3	3	3	3	5	5	6
Stage9	3	*3*	*3*	3	3	4	5	5	7	9	15
Stage11	4	*3*	*3*	3	4	5	5	7	9	14	30
Stage13	4	*3*	*3*	4	4	5	7	8	11	19	51
Stage15	5	*4*	*4*	5	5	5	7	9	13	21	74
Stage16	3	*3*	*3*	2	3	3	4	4	4	6	10
$\sum$	24	*21*	*21*	23	25	28	34	39	52	77	189

Remarks: Italicized data means the optimum over-relaxation factor.

Table 4.12 NSI used with the variation of the over-relaxation factor from 1 to 2 for nonlinear analysis

	Over-relaxation factor ω										
	1.0	1.1	*1.2*	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
Stage3	1	1	*1*	1	1	1	1	1	1	1	1
Stage5	2	2	*2*	2	2	2	2	2	2	2	2
Stage7	2	2	*2*	3	3	3	3	3	4	5	6
Stage9	3	3	*3*	3	3	4	5	5	7	9	12
Stage11	4	3	*3*	3	4	5	5	7	9	14	-*
Stage13	4	3	*3*	4	4	5	6	8	11	18	-
Stage15	5	4	*3*	5	5	5	7	9	13	21	-
Stage16	4	3	*3*	2	3	4	4	4	4	6	-
$\sum$	25	21	*20*	23	25	29	33	39	51	76	-

*The results are invalid since compression occurs in the cables. Italicized data means the optimum over-relaxation factor.

Figure 4.16 and 4.17 show the comparisons of convergence of the vertical displacement at Node4 and Node5 and tension forces in cable element 4 and 5, respectively, at construction stage 11 as shown in Figure 4.18 during the shape iteration for linear analysis between the results with $\omega = 1.0$ and $\omega = 1.2$. The convergence patterns in this case study is almost the same as those of case study 2 where the convergence rate of the shape iteration is slightly improved with $\omega = 1.2$.

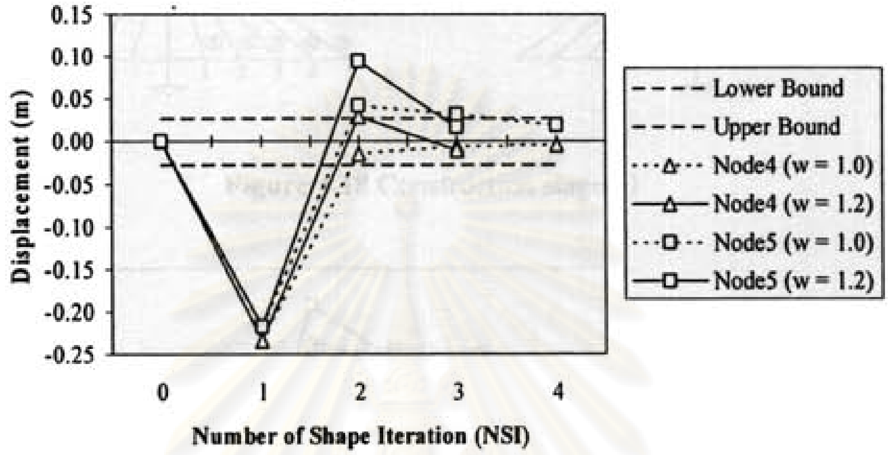

Figure 4.16 Convergence of vertical displacement at control points at construction stage 11 during the shape iteration for linear analysis

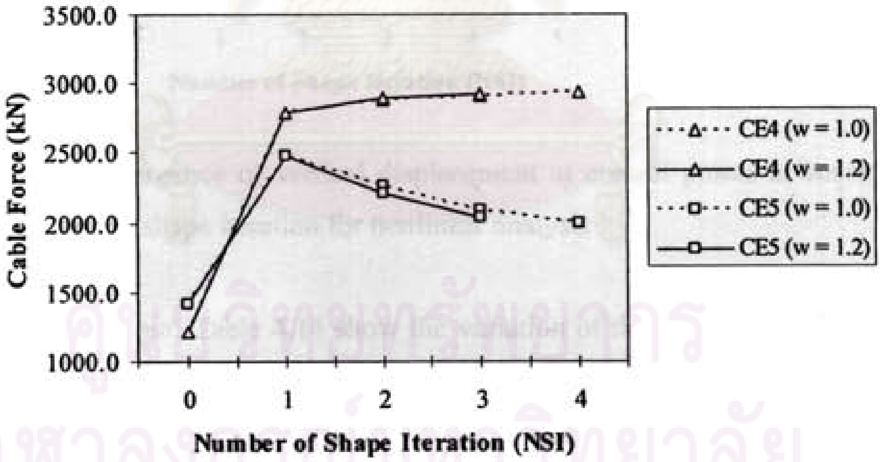

Figure 4.17 Convergence of cable forces at construction stage 11 during the shape iteration for linear analysis

Figure 4.19 and 4.20 show the comparisons of convergence of the vertical displacement at Node5 and Node7 and tension forces in cable element 5 and 7, respectively, at construction stage 15 as shown in Figure 4.21 during the shape iteration for nonlinear analysis between those with $\omega = 1.0$ and $\omega = 1.2$. It is seen that with $\omega = 1.2$ the NSI used is 3, whereas 5 shape iterations are required for $\omega = 1.0$.

Figure 4.18 Construction stage 11

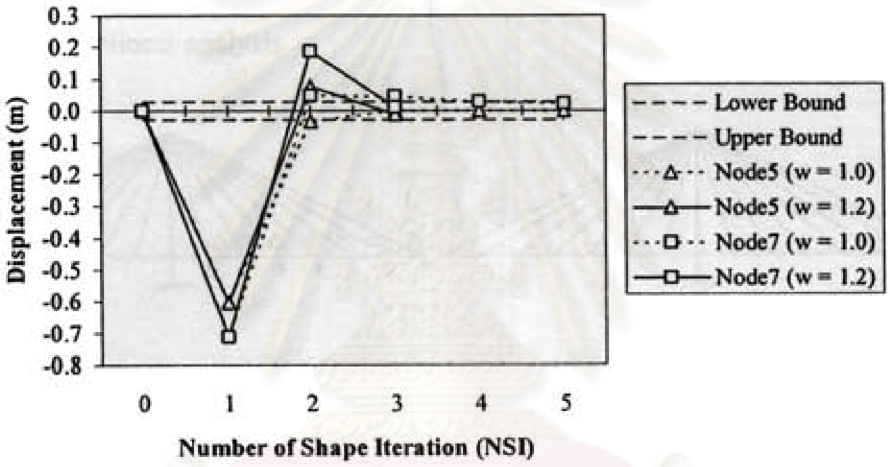

Figure 4.19 Convergence of vertical displacement at control points at construction stage 15 during the shape iteration for nonlinear analysis

Table 4.13 and Table 4.14 show the variation of the vertical displacement at the control points (node 1, 2, 3, 4, 5, 6, and 7) and tension forces in cable element 1, 2, 3, 4, 5, 6 and 7 at each construction stage calculated during the shape iteration with $\omega = 1.2$ for linear analysis and nonlinear analysis, respectively. The result of the cable forces determined from linear and nonlinear analysis is about 5.02% (maximum) different.

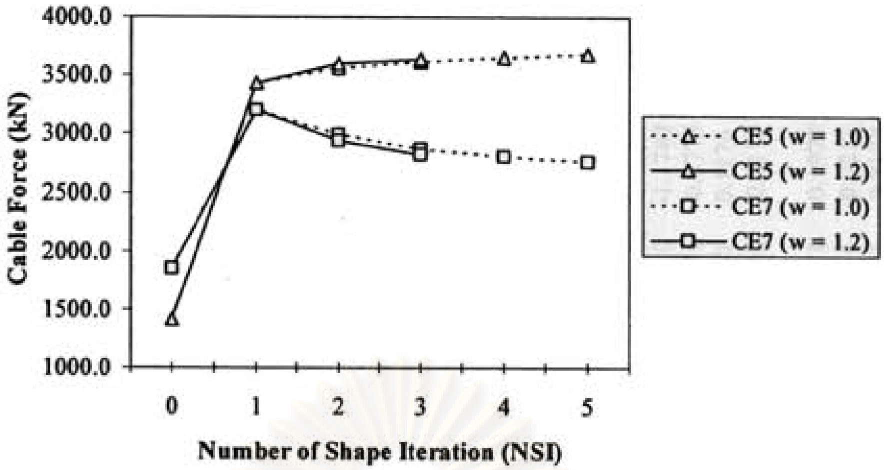

Figure 4.20 Convergence of cable forces at construction stage 15 during the shape iteration for nonlinear analysis

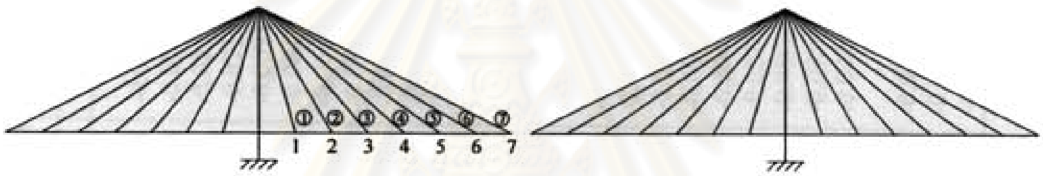

Figure 4.21 Construction stage 15

4.4 Case Study 4: A Single-Tower Harp Cable-Stayed Bridge

For the last case study, a single-tower prestressed concrete cable-stayed bridge located in Ningbo City, China (Chen et al., 2000) as shown in Figure 4.22 is selected. The tower is 54 m high and is stepped with the biggest section below the deck and the smallest section on the top. The main and back span lengths of the bridge are 105 and 90 m, respectively. There are 22 cables arranged in the harp pattern and three sizes of cables are used. There are 24 construction stages following the same construction sequence as case study 1. The physical properties and weight of the bridge structure are summarized in Table 4.15 and Figure 4.23.

Table 4.13 Vertical displacement (m) at control points calculated during the shape iteration at each construction

Stage	NSI	Linear analysis (ω = 1.2)							Nonlinear analysis (ω = 1.2)						
		Node1	Node2	Node3	Node4	Node5	Node6	Node7	Node1	Node2	Node3	Node4	Node5	Node6	Node7
1	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-
2	1	-0.0236	-	-	-	-	-	-	-0.0236	-	-	-	-	-	-
3	1	-0.0002	-	-	-	-	-	-	-0.0002	-	-	-	-	-	-
4	1	-0.0813	-0.2460	-	-	-	-	-	-0.0816	-0.2467	-	-	-	-	-
5	1	-0.0213	-0.0392	-	-	-	-	-	-0.0216	-0.0401	-	-	-	-	-
	2	-0.0092	-0.0043	-	-	-	-	-	-0.0093	-0.0047	-	-	-	-	-
6	1	-0.0613	-0.2226	-0.4496	-	-	-	-	-0.0619	-0.2243	-0.4527	-	-	-	-
7	1	-0.0377	-0.0865	-0.1073	-	-	-	-	-0.0381	-0.0881	-0.1107	-	-	-	-
	2	-0.0127	-0.0119	0.0204	-	-	-	-	-0.0126	-0.0121	0.0198	-	-	-	-
8	1	-0.0322	-0.1365	-0.3329	-0.5959	-	-	-	-0.0324	-0.1374	-0.3352	-0.5996	-	-	-
9	1	-0.0471	-0.1181	-0.1575	-0.1532	-	-	-	-0.0474	-0.1206	-0.1648	-0.1676	-	-	-
	2	-0.0139	-0.0218	-0.0008	0.0553	-	-	-	-0.0133	-0.0206	-0.0002	0.0538	-	-	-
	3	-0.0097	-0.0184	-0.0129	0.0172	-	-	-	-0.0094	-0.0176	-0.0121	0.0169	-	-	-
10	1	-0.0068	-0.0475	-0.1670	-0.3810	-0.6614	-	-	-0.0068	-0.0485	-0.1711	-0.3905	-0.6769	-	-
11	1	-0.0516	-0.1394	-0.2082	-0.2333	-0.2174	-	-	-0.0517	-0.1406	-0.2131	-0.2447	-0.2376	-	-
	2	-0.0105	-0.0172	-0.0073	0.0291	0.0939	-	-	-0.0100	-0.0158	-0.0056	0.0295	0.0917	-	-
	3	-0.0064	-0.0125	-0.0166	-0.0096	0.0169	-	-	-0.0063	-0.0120	-0.0157	-0.0091	0.0162	-	-
12	1	0.0034	-0.0020	-0.0676	-0.2372	-0.5241	-0.8823	-	0.0034	-0.0023	-0.0698	-0.2443	-0.5388	-0.9050	-
13	1	-0.0530	-0.1491	-0.2390	-0.2976	-0.3141	-0.2880	-	-0.0527	-0.1491	-0.2418	-0.3072	-0.3352	-0.3238	-
	2	-0.0069	-0.0078	0.0000	0.0185	0.0550	0.1202	-	-0.0065	-0.0064	0.0029	0.0214	0.0553	0.1158	-
	3	-0.0045	-0.0059	-0.0091	-0.0151	-0.0149	0.0076	-	-0.0047	-0.0060	-0.0086	-0.0140	-0.0146	0.0056	-
14	1	-0.0549	-0.1248	-0.2300	-0.4134	-0.7136	-1.1281	-1.6123	-0.0576	-0.1312	-0.2411	-0.4320	-0.7440	-1.1742	-1.6748
15	1	-0.0765	-0.2080	-0.3452	-0.4670	-0.5624	-0.6160	-0.6284	-0.0802	-0.2170	-0.3615	-0.4937	-0.6041	-0.6775	-0.7126
	2	-0.0018	0.0089	0.0308	0.0555	0.0820	0.1284	0.2048	-0.0034	0.0058	0.0273	0.0515	0.0760	0.1176	0.1871
	3	-0.0036	-0.0006	0.0038	-0.0007	-0.0102	-0.0093	0.0169	-0.0046	-0.0023	0.0022	-0.0002	-0.0108	-0.0115	0.0118
	4	-0.0022	0.0031	0.0098	0.0096	0.0012	0.0022	0.0264	-	-	-	-	-	-	-
16	1	-0.0202	-0.0366	-0.0558	-0.0889	-0.1407	-0.1923	-0.2179	-0.0215	-0.0388	-0.0582	-0.0920	-0.1451	-0.1977	-0.2237
	2	-0.0044	0.0001	0.0095	0.0128	0.0002	-0.0193	-0.0295	-0.0052	-0.0008	0.0090	0.0121	-0.0014	-0.0217	-0.0322
	3	-0.0046	-0.0030	0.0034	0.0076	0.0005	-0.0120	-0.0183	-0.0052	-0.0035	0.0034	0.0075	-0.0001	-0.0132	-0.0197

Remarks: NSI – number of shape iteration.

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Table 4.14 Cable forces (kN) calculated during the shape iteration at each construction stage

Stage	NSI	Linear analysis (ω = 1.2)							Nonlinear analysis (ω = 1.2)						
		CE1	CE2	CE3	CE4	CE5	CE6	CE7	CE1	CE2	CE3	CE4	CE5	CE6	CE7
1	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-
2	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-
3	1	756.7	-	-	-	-	-	-	756.7	-	-	-	-	-	-
4	1	1683.1	-	-	-	-	-	-	1674.2	-	-	-	-	-	-
5	1	992.1	1203.8	-	-	-	-	-	992.2	1199.8	-	-	-	-	-
	2	1146.4	1309.5	-	-	-	-	-	1147.2	1307.5	-	-	-	-	-
6	1	1824.0	3246.4	-	-	-	-	-	1827.2	3237.3	-	-	-	-	-
7	1	1169.2	1609.7	1687.4	-	-	-	-	1171.8	1615.6	1677.6	-	-	-	-
	2	1399.7	1865.2	1692.1	-	-	-	-	1402.1	1873.2	1685.7	-	-	-	-
8	1	1725.0	3028.1	3764.4	-	-	-	-	1728.3	3038.9	3753.5	-	-	-	-
9	1	1264.0	1876.7	1993.6	2246.3	-	-	-	1266.9	1894.1	2030.7	2198.9	-	-	-
	2	1525.6	2269.3	2190.3	2062.9	-	-	-	1523.2	2280.0	2232.0	2026.3	-	-	-
	3	1667.8	2468.2	2271.4	1863.3	-	-	-	1660.7	2469.1	2306.9	1837.1	-	-	-
10	1	1684.6	2832.8	3281.1	4466.1	-	-	-	1678.3	2841.7	3337.2	4413.4	-	-	-
11	1	1302.6	2050.7	2301.8	2787.5	2483.0	-	-	1302.7	2059.6	2326.5	2810.4	2436.5	-	-
	2	1534.9	2438.4	2602.0	2897.7	2213.8	-	-	1529.6	2437.1	2621.3	2924.5	2180.4	-	-
	3	1630.9	2576.3	2714.1	2921.6	2028.8	-	-	1623.4	2568.1	2725.4	2943.8	2007.0	-	-
12	1	1514.8	2522.6	3076.7	4489.9	4635.5	-	-	1507.2	2517.5	3101.5	4544.4	4571.3	-	-
13	1	1304.6	2122.7	2483.0	3216.4	2957.3	2956.4	-	1301.3	2122.0	2497.0	3261.5	2993.9	2875.0	-
	2	1496.8	2443.2	2773.8	3487.4	2983.9	2636.9	-	1488.6	2429.4	2773.0	3522.4	3029.4	2577.0	-
	3	1561.3	2505.3	2827.7	3561.6	2998.9	2477.7	-	1553.8	2488.9	2818.7	3582.9	3040.9	2435.5	-
14	1	1196.0	2129.7	2705.6	4131.4	4529.2	5543.8	-	1178.3	2106.5	2698.9	4178.1	4609.8	5440.7	-
15	1	1194.6	2066.0	2535.1	3485.6	3388.7	3591.3	3286.2	1175.4	2050.4	2533.5	3509.2	3434.2	3616.6	3204.1
	2	1393.6	2360.2	2784.4	3762.9	3556.8	3580.3	3001.7	1369.5	2338.4	2772.6	3771.9	3599.9	3618.4	2940.1
	3	1476.3	2405.8	2782.2	3770.2	3607.0	3594.1	2868.4	1457.7	2391.0	2770.1	3768.5	3639.1	3629.7	2822.6
	4	1530.9	2412.6	2751.0	3754.0	3659.0	3646.8	2805.2	-	-	-	-	-	-	-
16	1	1492.3	2375.0	2730.4	3830.8	3883.6	4020.0	3122.7	1427.8	2362.6	2753.6	3850.2	3867.0	4004.5	3129.8
	2	1535.4	2354.1	2648.1	3725.9	3896.3	4149.3	3258.5	1478.1	2346.4	2670.0	3743.6	3883.4	4136.3	3263.2
	3	1590.6	2367.4	2600.2	3633.7	3873.4	4203.0	3328.0	1539.6	2362.7	2619.6	3648.2	3861.1	4192.3	3333.0

Remarks: NSI – number of shape iteration. CE – cable element

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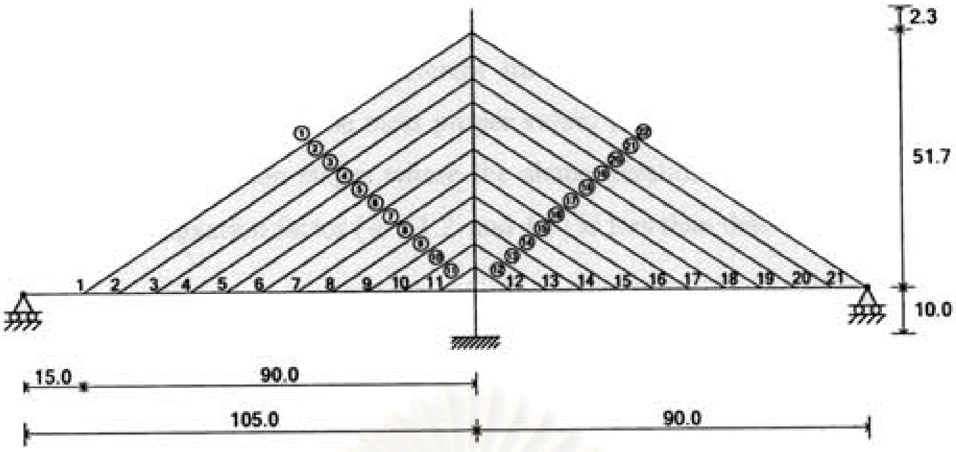

Figure 4.22 The single-tower harp cable-stayed bridge

Table 4.15 Physical properties and weight of the bridge structure

	Section number*	E (kN/m^2)	A (m^2)	I (m^4)	Weight (kN/m)
Girder	-	3.5×10^{7}	12.145	4.706	286.0
Tower	1	3.0×10^{7}	45.0	79.688	-
	2	3.0×10^{7}	19.0	19.939	-
	3	3.0×10^{7}	14.46	11.212	-
Cable	1	2.1×10^{8}	0.0208	-	16.016
	2	2.1×10^{8}	0.0166	-	12.782
	3	2.1×10^{8}	0.0130	-	10.010

*See Figure 4.9

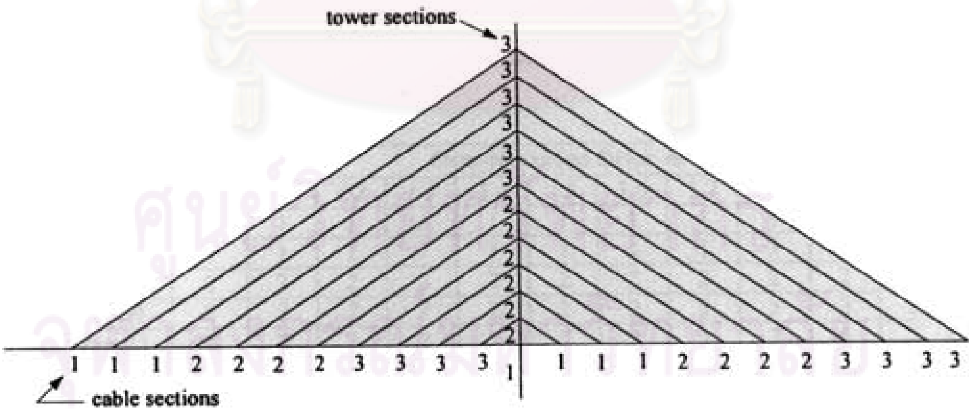

Figure 4.23 Tower and cable section numbers

The comparison of number of shape iteration at construction stages of odd numbers (3, 5, 7,..., 23) and the last stage (24) with the variation of the over-relaxation factor from 1 to 2 is shown in Table 4.16 and Table 4.17 for linear and nonlinear analysis, respectively. For linear analysis, the NSI used in various construction stages is not much different from those of the previous case studies where ω = 1.2 and 1.3 are the most proper value. The total number shape iteration used, for the construction stages where the shape iteration is carried out, with the optimum over-relaxation factor ($\omega = 1.2$ and 1.3) is less than that with $\omega = 1.0$ about 3.03%.

On the other hand, very large NSI is used in nonlinear analysis. Hence, larger value of ω is required in order to accelerate the convergence rate of the shape iteration. With the value of ω = 1.9 which is the optimum value for nonlinear analysis, the NSI used for most construction stages of odd numbers is considerably reduced. For example, the NSI used for construction stage 24 with ω = 1.0 is 114. Meanwhile, only 51 shape iterations is needed when ω = 1.9 is used. Figure 4.24 and 4.25 show, for instance, the comparison of the convergence of the vertical displacement at Node3 and tension force in the cable element 3 at construction stage 19 as shown in Figure 4.26 during the shape iteration for nonlinear analysis between ω = 1.0 and ω = 1.8. It can be seen for ω = 1.0 that the NSI required to achieve the allowable tolerance is 22 while only 8 iterations is used for ω = 1.8. The total number shape iteration used with the optimum over-relaxation factor ($\omega = 1.9$) is less than that with $\omega = 1.0$ about 56.52%. This clearly indicates that the use of the SOR technique can accelerate the convergence rate of the shape iteration effectively.

Table 4.16 NSI used with the variation of the over-relaxation factor from 1 to 2 for linear analysis

	Over-relaxation factor ω										
	1.0	1.1	*1.2*	*1.3*	1.4	1.5	1.6	1.7	1.8	1.9	2.0
Stage3	1	1	*1*	*1*	1	1	1	1	1	1	1
Stage5	1	1	*1*	*1*	1	1	1	1	1	1	1
Stage7	2	2	*2*	*2*	2	2	2	2	2	2	2
Stage9	2	2	*2*	*2*	2	2	2	2	3	3	3
Stage11	2	2	*2*	*2*	3	3	3	3	3	5	5
Stage13	2	2	*3*	*3*	3	3	4	5	5	7	9
Stage15	3	3	*3*	*3*	3	4	5	5	7	10	16
Stage17	3	3	*3*	*3*	3	5	5	7	9	13	24
Stage19	3	3	*3*	*3*	4	5	5	7	10	15	37
Stage21	3	3	*3*	*3*	4	5	6	8	11	18	53
Stage23	4	4	*3*	*3*	5	5	7	9	11	19	75
Stage24	7	7	*6*	*6*	6	5	5	5	4	4	2
$\sum$	33	33	*32*	*32*	37	41	46	55	67	98	228

Remarks: Italicized data means the optimum over-relaxation factor.

Table 4.17 NSI used with the variation of the over-relaxation factor from 1 to 2 for nonlinear analysis

	Over-relaxation factor ω										
	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	*1.9*	2.0
Stage3	1	1	1	1	1	1	1	1	1	*1*	1
Stage5	1	1	1	1	1	1	1	1	1	*1*	1
Stage7	3	3	3	3	3	3	3	3	2	*2*	2
Stage9	4	3	3	3	3	3	2	2	2	*2*	2
Stage11	4	3	3	3	3	2	2	2	2	*3*	3
Stage13	5	4	4	3	3	2	2	4	4	*4*	4
Stage15	9	8	8	7	7	7	7	6	6	*6*	6
Stage17	12	11	10	10	9	8	8	6	6	*8*	10
Stage19	22	20	19	17	16	15	13	12	8	*10*	14
Stage21	44	40	36	34	31	29	26	22	19	*12*	16
Stage23	103	94	87	80	74	69	65	61	56	*40*	-*
Stage24	114	104	95	88	82	77	72	69	66	*51*	-
$\sum$	322	292	270	250	233	217	202	189	173	*140*	-

*The results are invalid since compression occurs in the cables. Italicized data means the optimum over-relaxation factor.

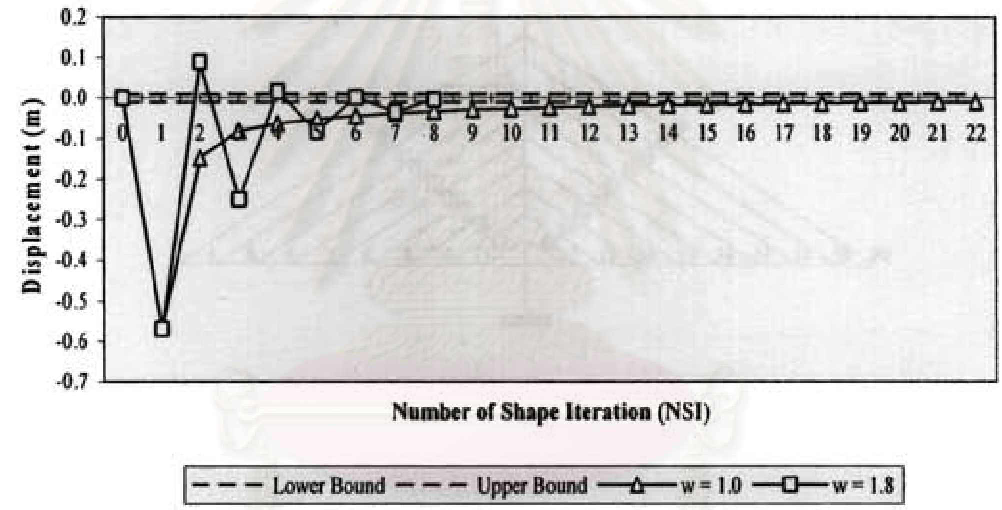

Figure 4.24 Convergence of vertical displacement at Node3 at construction stage 19 during the shape iteration for nonlinear analysis

4.5 Optimum Value of the Over-Relaxation Factor

From the results of previous case studies the SOR technique can be used to improve the convergence rate of the shape iteration. However, the main difficulty in the use of this technique is to determine the optimum value of the over-relaxation factor. Carré (1961) proposed a method for estimating the optimum value of the over-relaxation factor as the iteration proceeds as follows:

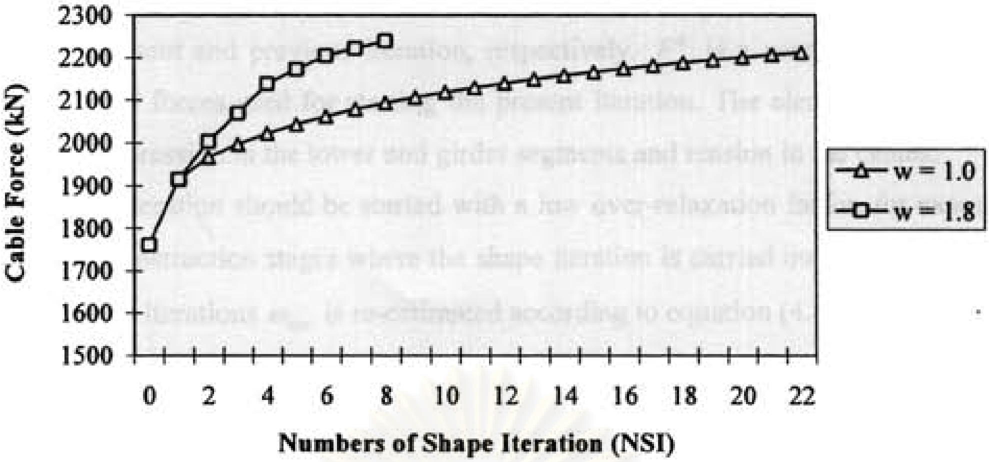

Figure 4.25 Convergence of tension force in cable element 3 at construction stage 19 during the shape iteration for nonlinear analysis

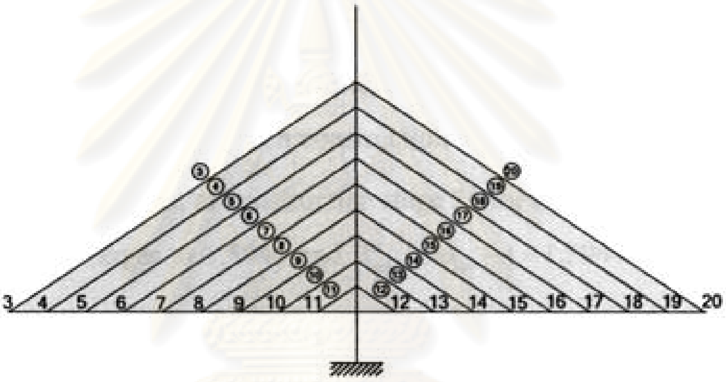

Figure 4.26 Construction stage 19

$$\omega_{opt} = \frac{2}{1+\sqrt{1-(\lambda_{max}+\omega-1)^2/\omega^2\lambda_{max}}} \tag{4.1}$$

where ω_{opt} is the optimum over-relaxation factor, ω is the over-relaxation factor used for starting the iteration, and λ_{max} is the maximum eigenvalue of the structure global stiffness matrix, $[K]$. Provided that all eigenvalues of $[K]$ are real, λ_{max} can be estimated by

$$\lambda_{max} = \frac{\left\| F^{k+1^*} - F^k \right\|}{\left\| F^k - F^{k-1} \right\|} \tag{4.2}$$

where F^{k+1*} and F^{k-1} are vectors containing all element axial forces determined from the present and previous iteration, respectively. F^k is a vector containing all element axial forces used for starting the present iteration. The element axial forces include compression in the tower and girder segments and tension in the cables.

The iteration should be started with a low over-relaxation factor, for example $\omega = 1$, for construction stages where the shape iteration is carried out, and after about every 5 or 10 iterations ω_{opt} is re-estimated according to equation (4.1). The estimated ω_{opt} will increase at each time of re-estimation. At the optimum over-relaxation factor λ_{max} will equal to $\omega_{opt} - 1$ (Young, 1954).

Here, nonlinear analysis of case study 4 is taken as an example for estimating ω_{opt} and the allowable tolerance $\varepsilon = 10^{-4}$ is used to terminate the shape iteration. $\omega = 1$ is used to start the estimation. ω_{opt} is then re-estimated every 5 iterations as the shape iteration proceeds until the convergence is accomplished. Table 4.18 shows the convergence of ω_{opt} and λ_{max} for each construction stage where the shape iteration is performed. It can be seen that, at construction stage 24 for example, after 75 iterations the value of ω_{opt} converges to 1.5944 and $\lambda_{max} = \omega_{opt} - 1 = 1.5944 - 1 = 0.5944$ which is different from λ_{max} calculated from equation (4.2) ($\lambda_{max} = 0.6321$). This might be because the number of shape iteration is not large enough. Thus, nonlinear analysis of case study 4 is performed again with the allowable tolerance $\varepsilon = 10^{-5}$ in order to see whether the value of $\lambda_{max} = \omega_{opt} - 1$ will continue converging to that estimated from equation (4.2) or not. Table 4.19 shows the convergence of ω_{opt} and λ_{max} which are estimated every 5 and 10 iterations during the shape iteration of construction stage 24. It can be obviously seen that the value of $\lambda_{max} = \omega_{opt} - 1$ is almost equal to that determined from equation (4.2) when the shape iteration is stopped.

The convergence of ω_{opt} for construction stage 24 during the shape iteration with the allowable tolerance $\varepsilon = 10^{-4}$ is plotted in Figure 4.27. The comparison of number of shape iteration at construction stages where the shape iteration is carried out between those with the variation of the over-relaxation factor from 1 to 2 and those with ω_{opt} is shown in Table 4.20. Figure 4.28 and 4.29 show the comparison of the convergence of the vertical displacement at Node3 and tension force in the cable element 3 at construction stage 19 as shown in Figure 4.26 during the shape iteration for nonlinear analysis between $\omega = 1.0$ and ω_{opt}. It can be seen that for $\omega = 1.0$ the NSI used is 22 while 18 iterations are needed for ω_{opt}.

Table 4.18 Convergence of ω_{opt} and λ_{max}

	Stage 3		Stage 5		Stage 7		Stage 9		Stage 11		Stage 13		Stage 15		Stage 17		Stage 19		Stage 21		Stage 23		Stage 24	
	NSI = 1		NSI = 1		NSI = 3		NSI = 4		NSI = 4		NSI = 5		NSI = 8		NSI = 10		NSI = 18		NSI = 32		NSI = 70		NSI = 76	
NSI	ω_{opt}	λ_{max}	ω_{opt}	λ_{max}	ω_{opt}	λ_{max}	ω_{opt}	λ_{max}	ω_{opt}	λ_{max}	ω_{opt}	λ_{max}	ω_{opt}	λ_{max}	ω_{opt}	λ_{max}	ω_{opt}	λ_{max}	ω_{opt}	λ_{max}	ω_{opt}	λ_{max}	ω_{opt}	λ_{max}
	1.0000	-	1.0000	-	1.0000	-	1.0000	-	1.0000	-	1.0000	-	1.0000	-	1.0000	-	1.0000	-	1.0000	-	1.0000	-	1.0000	-
5	-	-	-	-	-	-	-	-	-	-	-	-	1.6736	0.9620	1.4248	0.8370	1.3539	0.7723	1.3337	0.7504	1.3324	0.7490	1.3508	0.7690
10	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1.4548	0.6849	1.4593	0.7105	1.4533	0.7019	1.4559	0.6898
15	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1.4898	0.6307	1.4878	0.6162	1.4874	0.6268	1.5043	0.6653
20	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1.5052	0.6062	1.5084	0.6183	1.5301	0.6492
25	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1.5195	0.6108	1.5257	0.6253	1.5476	0.6459
30	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1.5366	0.6345	1.5433	0.6421	1.5596	0.6413
35	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1.5545	0.6338	1.5682	0.6376
40	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1.5618	0.6264	1.5747	0.6352
45	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1.5677	0.6257	1.5797	0.6327
50	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1.5721	0.6223	1.5836	0.6305
55	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1.5752	0.6181	1.5867	0.6286
60	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1.5785	0.6218	1.5892	0.6274
65	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1.5810	0.6193	1.5913	0.6259
70	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1.5930	0.6244
75	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1.5944	0.6231

46

Table 4.19 Convergence of ω_{opt} and λ_{max} of construction stage 24 with the allowable tolerlance $\varepsilon = 10^{-5}$

NSI	Every 5 iterations			Every 10 iterations		
	ω_{opt}	$\lambda_{max} = \omega_{opt} - 1$	λ_{max} from equation (4.2)	ω_{opt}	$\lambda_{max} = \omega_{opt} - 1$	λ_{max} from equation (4.2)
1	1.0000	-	-	1.0000	-	-
5	1.3598	0.3598	0.7784	1.0000	-	-
10	1.4561	0.4561	0.6815	1.6042	0.6042	0.9391
15	1.4997	0.4997	0.6539	-	-	-
20	1.5243	0.5243	0.6414	1.6043	0.6043	0.5974
25	1.5422	0.5422	0.6417	-	-	-
30	1.5547	0.5547	0.6385	1.6044	0.6044	0.6134
35	1.5639	0.5639	0.6355	-	-	-
40	1.5709	0.5709	0.6338	1.6047	0.6047	0.6176
45	1.5761	0.5761	0.6307	-	-	-
50	1.5802	0.5802	0.6285	1.6050	0.6050	0.6172
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
460	1.6128	0.6128	0.6184	1.6115	0.6115	0.6189
465	1.6129	0.6129	0.6185	-	-	-
470	1.6129	0.6129	0.6184	1.6116	0.6116	0.6190
475	1.6130	0.6130	0.6184	-	-	-
480	1.6130	0.6130	0.6183	converges	converges	converges
481	-	-	-			
482	converges	converges	converges			

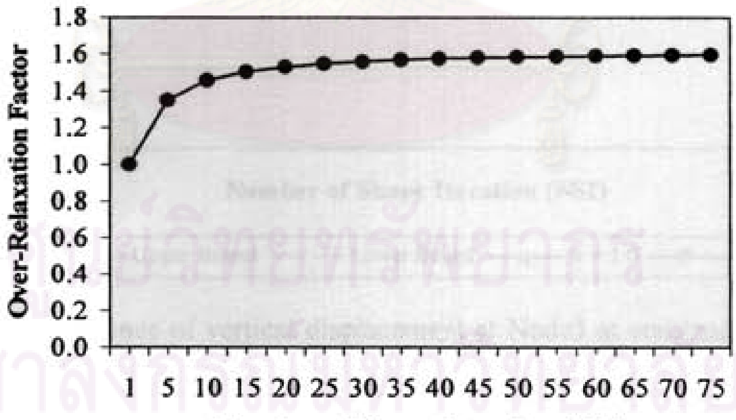

Figure 4.27 Convergence of the optimum over-relaxation factor of construction stage 24 during the shape iteration

Table 4.20 NSI used between those with the variation of the over-relaxation factor from 1 to 2 and ω_{opt} for nonlinear analysis

	Over-relaxation factor ω												
	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	*1.9*	2.0	ω_{opt}	NSI
Stage3	1	1	1	1	1	1	1	1	1	*1*	1	1.0000	1
Stage5	1	1	1	1	1	1	1	1	1	*1*	1	1.0000	1
Stage7	3	3	3	3	3	3	3	3	2	*2*	2	1.0000	3
Stage9	4	3	3	3	3	3	2	2	2	*2*	2	1.0000	4
Stage11	4	3	3	3	3	2	2	2	2	*3*	3	1.0000	4
Stage13	5	4	4	3	3	2	2	4	4	*4*	4	1.0000	5
Stage15	9	8	8	7	7	7	7	6	6	*6*	6	1.6736	8
Stage17	12	11	10	10	9	8	8	6	6	*8*	10	1.4248	10
Stage19	22	20	19	17	16	15	13	12	8	*10*	14	1.4898	18
Stage21	44	40	36	34	31	29	26	22	19	*12*	16	1.5365	32
Stage23	103	94	87	80	74	69	65	61	56	*40*	-*	1.5810	70
Stage24	114	104	95	88	82	77	72	69	66	*51*	-	1.5944	76
$\sum$	322	292	270	250	233	217	202	189	173	*140*	-	$\sum$	232

*The results are invalid since compression occurs in the cables.

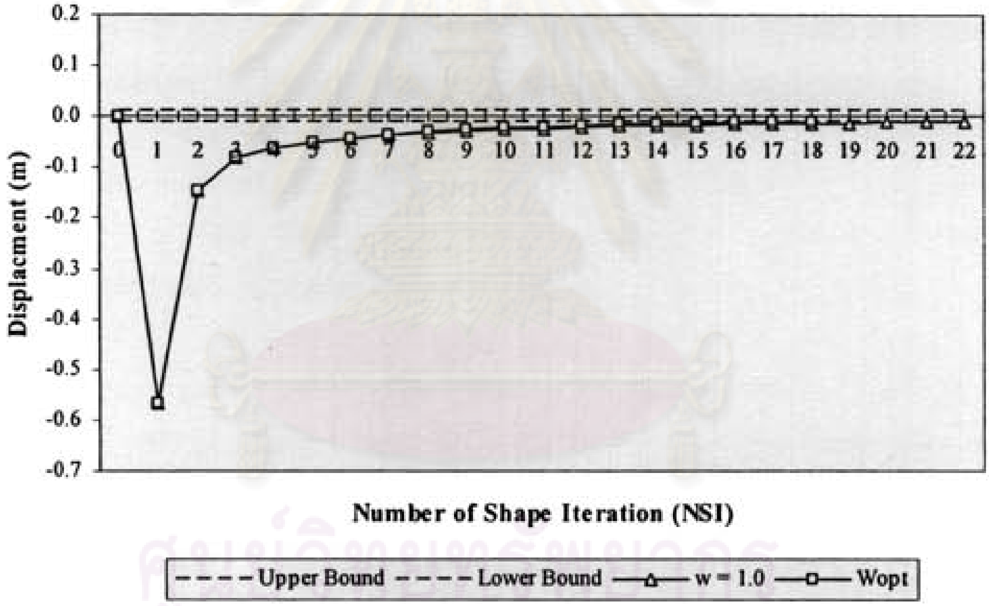

Figure 4.28 Convergence of vertical displacement at Node3 at construction stage 19 during the shape iteration for nonlinear analysis

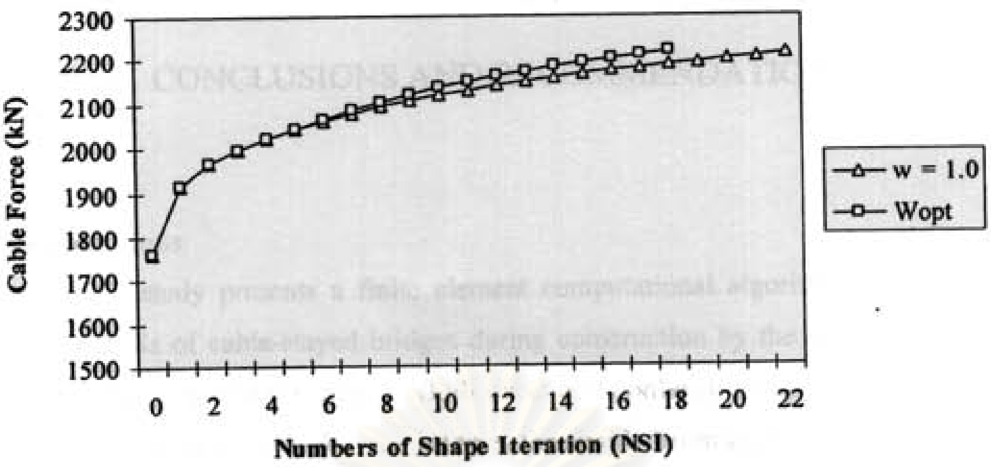

Figure 4.29 Convergence of tension force in cable element 3 at construction stage 19 during the shape iteration for nonlinear analysis

CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions

This study presents a finite element computational algorithm for the initial shape analysis of cable-stayed bridges during construction by the cantilever method using the substructuring technique. Both linear and nonlinear analyses are carried out. For the finite element model of the bridges, the simple beam element is used to model the tower and girder segments. The straight cable element without the sag effect is used to model the cables in linear analysis, whereas the cable element with the cable sag effect is employed for nonlinear analysis. The forward process analysis in accordance with the actual construction sequence is established to determine the tension forces in the stayed cables which yield the initial shape of the bridge at each stage of construction. For the forward process analysis of the bridges, structural members associated with each construction stage are grouped as a 'substructure'. Then substructures are assembled and analyzed according to the construction sequence of the bridge. The shape iteration is then carried out at each stage of cable installation and the last stage of construction in order to find the desired initial shape of the bridge.

The successive over-relaxation (SOR) technique is employed to accelerate the convergence rate of the shape iteration. During the shape iteration, the element axial forces determined from the present iteration are modified using the SOR technique before taken as the initial element forces for starting the next iteration. Four different types of cable-stayed bridges are analyzed with the over-relaxation factor varying from 1 to 2. Based on the numerical results from the case studies in the previous chapter, the following conclusions may be drawn:

1. Difference of less than 10% exists in the results calculated from linear and nonlinear analysis. The results from nonlinear analysis are theoretically more accurate, but more computational effort is needed especially for the bridges with a large number of cables. In practice, linear analysis may be used since the results are acceptable but the computational effort is less.

2. The SOR technique helps accelerate the convergence rate of the shape iteration. The optimum value of the over-relaxation factor, however, cannot exactly be determined since it varies from problem to problem and is often determined empirically (Chapra and Canale, 2006). Nevertheless, appropriate over-relaxation factor found in the case studies ranges between 1.1 and 1.9. For the bridges with a small number of cables, the value of the over-relaxation factor from 1.1 to 1.4 may be used. Large value of the over-relaxation factor tends to be suitable for the bridges with a large number of cables and significant improvement of the convergence of the shape iteration is achieved especially for nonlinear analysis.
3. The total number shape iteration used, found in the case studies for construction stages where the shape iteration is performed, with the optimum over-relaxation factor for linear analysis and nonlinear analysis are less than those with $\omega = 1.0$ (which is equivalent to those from the shape iteration in the previous study) about 3 - 12.5% and 20 - 57%, respectively.
4. The optimum value of the over-relaxation factor does not depend on types of cable arrangement. That is, the optimum value of the over-relaxation factor used for improving the convergence rate of the shape iteration for a bridge with semi-harp cable arrangement yields similar results for the same bridge with fan cable arrangement, for example.

5.2 Recommendations

In order to improve the benefit of this research, here some recommendations are made as follows:

1) As stated by Wang et al. (1996) geometric nonlinearities due to beam-column and large displacement effects are insignificant for the initial shape analysis, therefore they can be ignored. However, in case of a very accurate result is needed, they may be included in the analysis.
2) Due to the popularity of three-dimensional cable-stayed bridges, hence the program should be able to perform the analysis of the bridges in three dimensions.

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