Chapter 2
Background Knowledge

To prepare for the later chapters, we would like to start with background knowledge regarding cosmic rays and related topics. The first section will afford a general outlook on topics studied in this thesis. It is a general introduction about cosmic rays and their origins. The second gives some explanation on interactions between charged particles and magnetic fields. This is important for understanding cosmic ray transport in space plasmas as detailed in Chapter 3. Next the concept of shock acceleration will be explained.

2.1 Cosmic Rays and Their Spectra and Origins

2.1.1 Cosmic Rays

In 1912, V. Hess discovered cosmic rays by experimenting in a balloon at various heights up to 17,500 feet (about 5.25 km) maximum (Friedlander 1989). In the work, he found that three electroscopes, which are used to measure the intensity of radiation, discharged more rapidly when the balloon moved higher. He concluded that the origin of the kinds of rays which made electroscopes discharged should be extra-terrestrial (Rigden 1996; Friedlander 1989; Schlickeiser 2002). Afterwards, the rays were named "cosmic rays." At present, we know that cosmic rays are high energy particles and gamma rays, originating and coming to Earth from outer space. Nevertheless, the term "cosmic rays" is usually used for just the energetic particles, as we use it in this thesis.

The observed cosmic ray particles are not exotic. They are just ordinary particles that are accelerated to high energy. Most of them, approximately 85 percent, are protons. About 12 percent are helium nuclei. About 2 percent are electrons. The remainder are heavy nuclei (Longair 1997). Nevertheless, they are at very much higher energy than ordinary particles originating on the Earth.

2.1.2 Cosmic Ray Spectra and Origins

Cosmic ray energy spectra span a very wide range from 10^5 eV/nucleon up to above 10^{20} eV/nucleon (Schlickeiser 2002). Furthermore, for various cosmic ray species, the spectra (see Figure 2.1) can be well represented by power-law distributions over the energy range $E > 10^{10}$ eV/nucleon. Power-law energy spectra can be written mathematically as follows: In a normal scale, the flux of cosmic rays is proportional to energy to some power,

$$j(E) \propto E^{-\gamma_e}. \tag{2.1}$$

Another relation is in a log scale: there is a linear relationship between $\log j(E)$ and $\log E$ with slope $-\gamma_e$,

$$\log j(E) = -\gamma_e \log E + \text{const}, \tag{2.2}$$

where $j(E)$ is particle flux/(time · area · solid angle · kinetic energy/nucleon), for an ion species E is expressed in terms of the kinetic energy/nucleon (i.e., kinetic energy divided by the mass number of an ion), and γ_e is a constant called the spectral index. Apart from describing spectra in terms of energy, the spectra are often described in terms of momentum in this thesis:

$$n(p) \propto p^{-\gamma}, \tag{2.3}$$

and in a log scale

$$\log n(p) = -\gamma \log p + \text{const}, \tag{2.4}$$

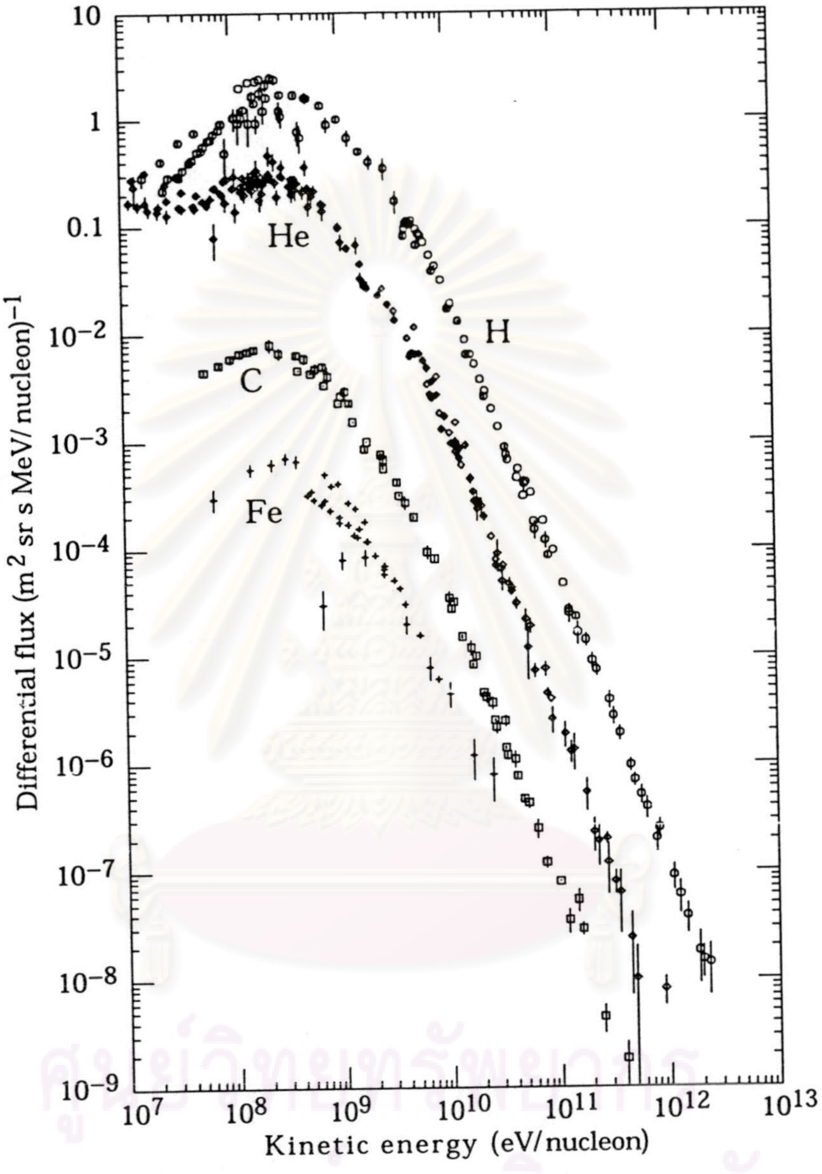

Figure 2.1: Energy spectra of various cosmic ray species in the energy range above 10^7 eV (Picture credit: Simpson 1983)

where $n(p)$ is particle number/(volume · solid angle · momentum/nucleon) and for an ion species p is expressed in terms of momentum/nucleon. Actually, $n(p)$ and $j(E)$ are the same quantity, but the γ and γ_e values are not the same. For nonrelativistic particles, E is proportional to p^2. Therefore, $\gamma = 2\gamma_e$. However, γ and γ_e are the same for the case of ultrarelativistic particles, because p is proportional to E (see Appendix A). In addition, since the rest mass energy of a nucleon is approximately 10^9 eV, ions that have an energy smaller than 10^8 eV/nucleon ($<$ 10% of a nucleon rest mass energy) could be classified as nonrelativistic particles, and particles with more than 10^{10} eV/nucleon ($>$10 times of a nucleon rest mass energy) could be classified as ultrarelativistic particles.

Cosmic Rays from Outside the Solar System

There is evidence that cosmic rays above $\sim 10^8$ eV/nucleon that arrive continuously (are not associated with transient solar or heliospheric events) come from outside the solar system (with a few exceptions). The spectral index of these cosmic rays is not constant for the whole energy range (see Figure 2.2). From observations at Earth, the spectral index is about 2.5-2.7 for energies of approximately 10^{10} eV/nucleon. The spectral index is constant up to an energy of about 10^{15} eV/nucleon; a change in the spectral index there is called the "knee." For the energy range above the knee, the spectra are slightly steeper with a spectral index of about 3.1. Beyond the knee, the spectra are flatter again at the "ankle," with a change in the spectral index at the energy of around 10^{19} eV/nucleon (Kirk et al. 1994).

Nowadays, scientists do not fully understood the spectra over the energy range above the knee yet. Perhaps cosmic rays above the ankle might not be produced in our galaxy. Nevertheless, there are many reasons to believe that

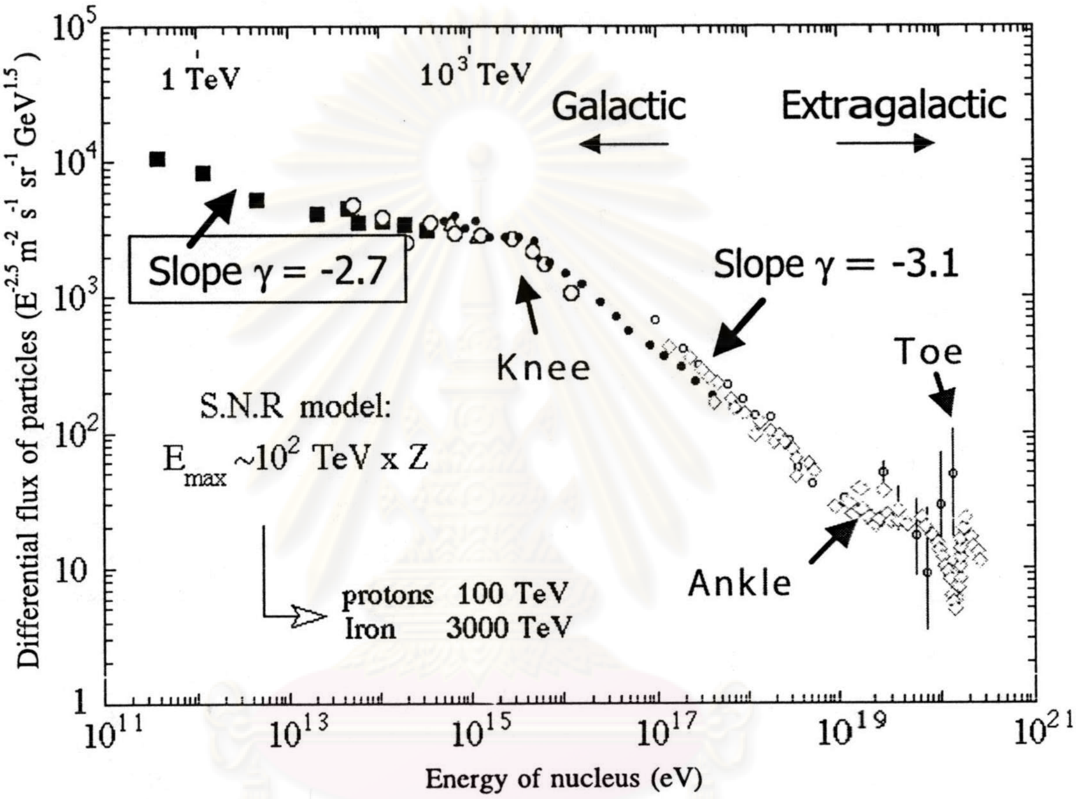

Figure 2.2: All-cosmic-ray-particle spectrum at high energy with different spectral indices for different energy ranges (the flux is scaled by $E^{2.5}$ to see the slope differences more easily) (Picture credit: http://www.phys.washington.edu/∼walta)

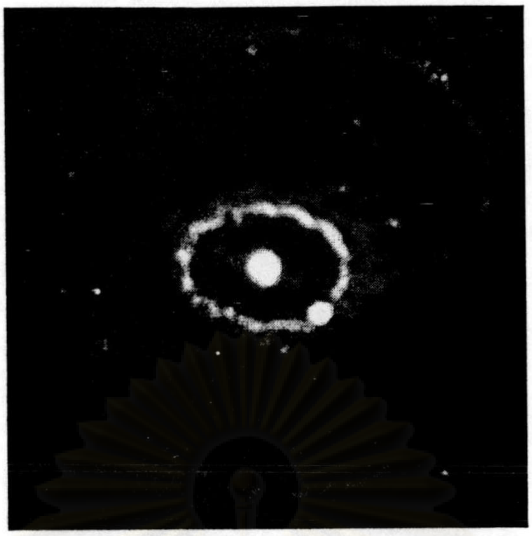

Figure 2.3: A supernova remnant, which looks like an expanding bubble with shock waves around the outer rim. (Picture credit: http://www.astro.columbia.edu/~ben)

cosmic rays in the energy range above 10^{10} eV/nucleon and up to the knee ($\approx 10^{15}$ eV/nucleon) should mainly originate from the shock acceleration process at "shocks" in remnants of supernova explosions (see in Figure 2.3) in our galaxy (see the detailed reasons in Appendix B). Thus, these cosmic rays are usually called "galactic cosmic rays" (GCRs). Note that a shock is a discontinuity in fluid (here, space plasma) properties such as flow velocity, pressure, density, etc., occurring in fluids colliding together or colliding with obstacles with a relative velocity greater than the sound speed in the fluid.

In fact, GCRs span into the energy range below 10^{10} eV/nucleon also, but they cannot maintain the power-law spectra shape anymore. However, this does not contradict the success of shock acceleration theory, which (in its simplest form) predicts power-law shapes of spectra, because galactic cosmic rays of $E < 10^{10}$ eV/nucleon are subject to some effects from the solar system, called "solar modulation."

Solar Energetic Particles

Apart from supernova remnant shocks, which are well known as the main origin of GCRs, there are many other places in our universe that can accelerate particles. In particular, in our solar system, many energetic particles are produced from many kinds of origins. However, the energies of cosmic rays produced in our solar system are not as high as GCR energies; rather their energy range is from $\sim 10^5$ eV/nucleon to $\sim 10^8$ eV/nucleon.

Figure 2.4 shows the total time-integrated intensity of oxygen from ≈ 300 eV/nucleon to ≈ 300 MeV/nucleon. Furthermore, it shows the time-integrated fluence of oxygen from some events providing high energy particles also. In addition, this spectral shape seem to be common for various types of particles (see Figure 2.5).

Although Figure 2.4 shows the oxygen spectrum over a wide energy range, not all of the particles shown in Figure 2.4 can be called cosmic rays. Some of them, called the solar wind, are just thermal particles from the Sun that form a plasma with a bulk flow speed. The average bulk speed of the "slow solar wind" is ≈ 327 km/s (Foukal 1990). Moreover, there is another kind of solar wind called the "fast solar wind," which streams out from "coronal hole regions" of the Sun. The fast solar wind has an average speed of ≈ 702 km/s (Foukal 1990). Though the energy of the fast solar wind is more than the slow solar wind energy, the fast solar wind is still not included in the term "cosmic rays."

Cosmic rays, or "energetic particles," have an energy greater than the solar wind, above ~ 0.1 MeV/nucleon, because these particles have been accelerated by some mechanism. As in the case of galactic cosmic rays, the main mechanism is shock acceleration. Note that in Figure 2.4, three of four kinds of

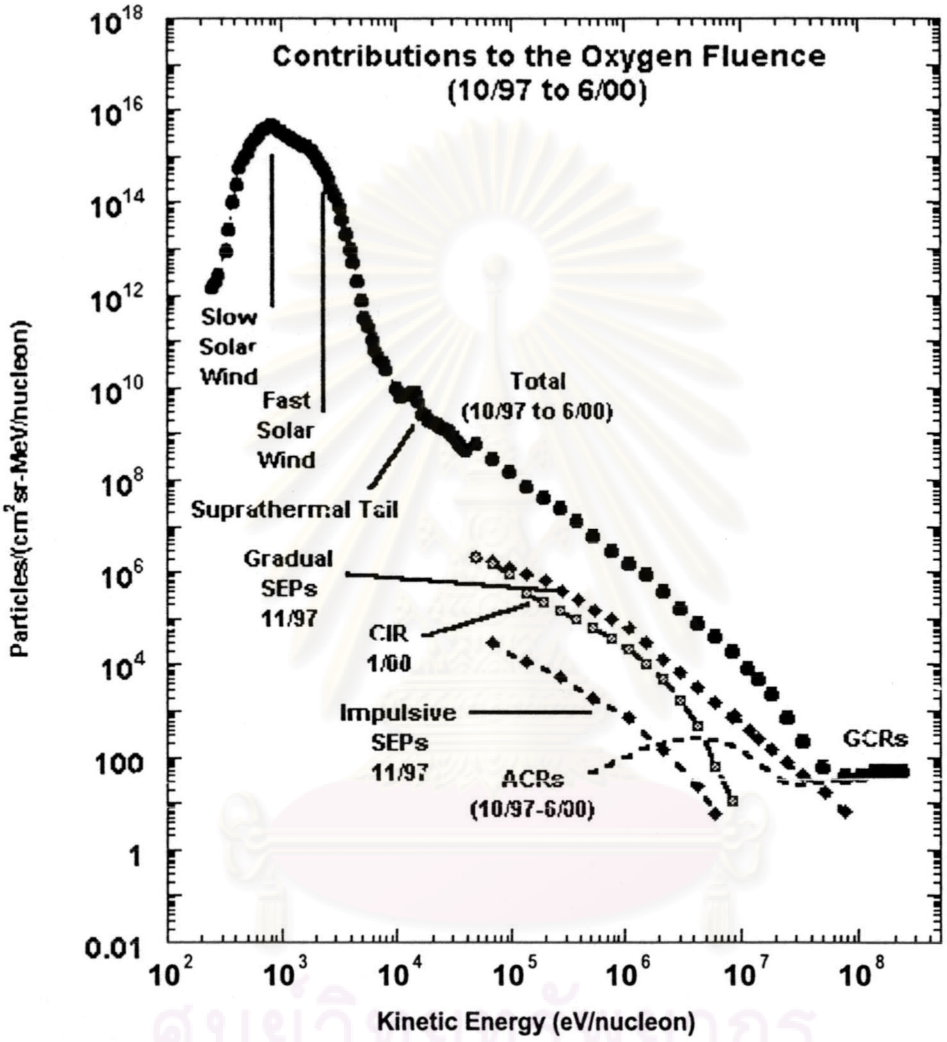

Figure 2.4: Time-integrated intensities of oxygen from ≈ 300 eV/nucleon to ≈ 300 MeV/nucleon (Picture credit: Mewaldt et al. 2001)

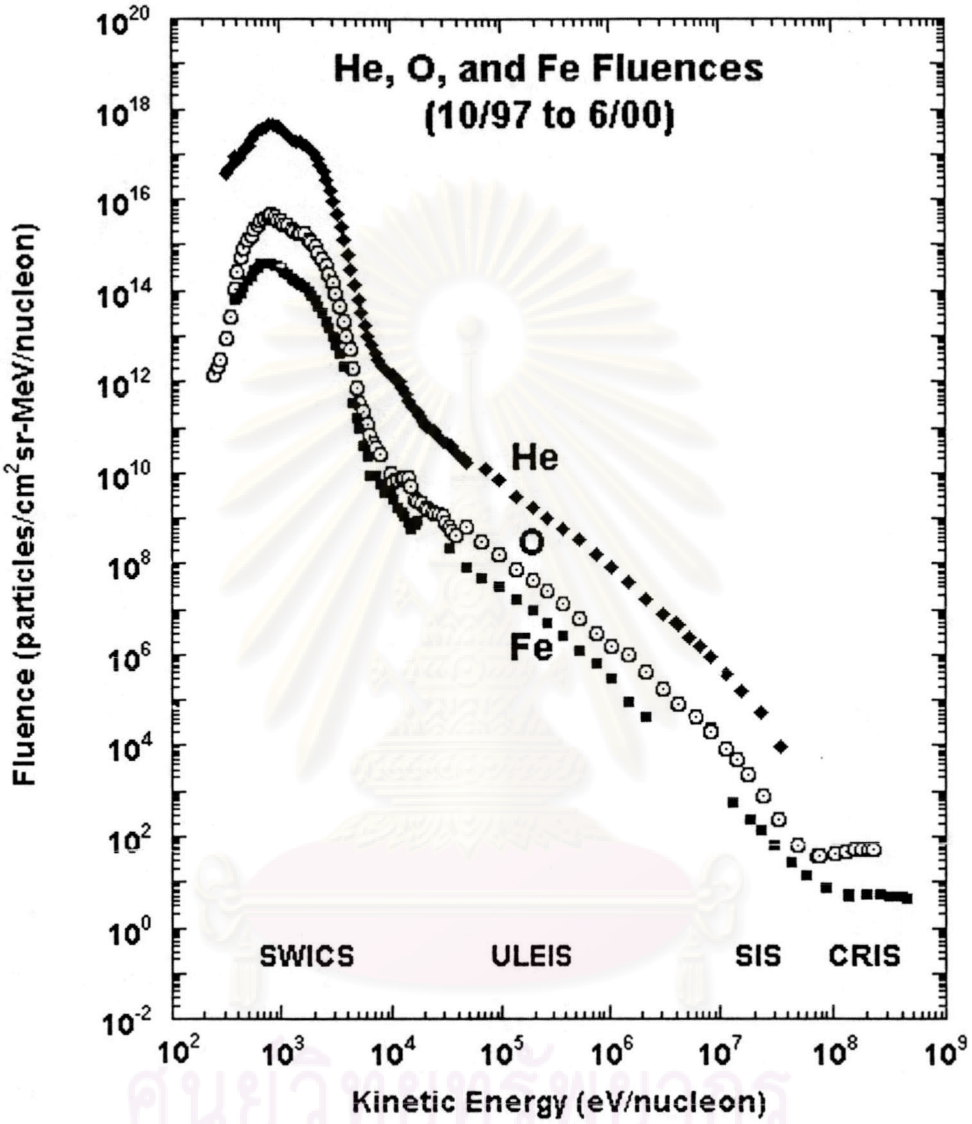

Figure 2.5: Time-integrated intensities of helium, oxygen, and iron from ≈ 300 eV/nucleon to ≈ 300 MeV/nucleon (Picture credit: Mewaldt et al. 2001)

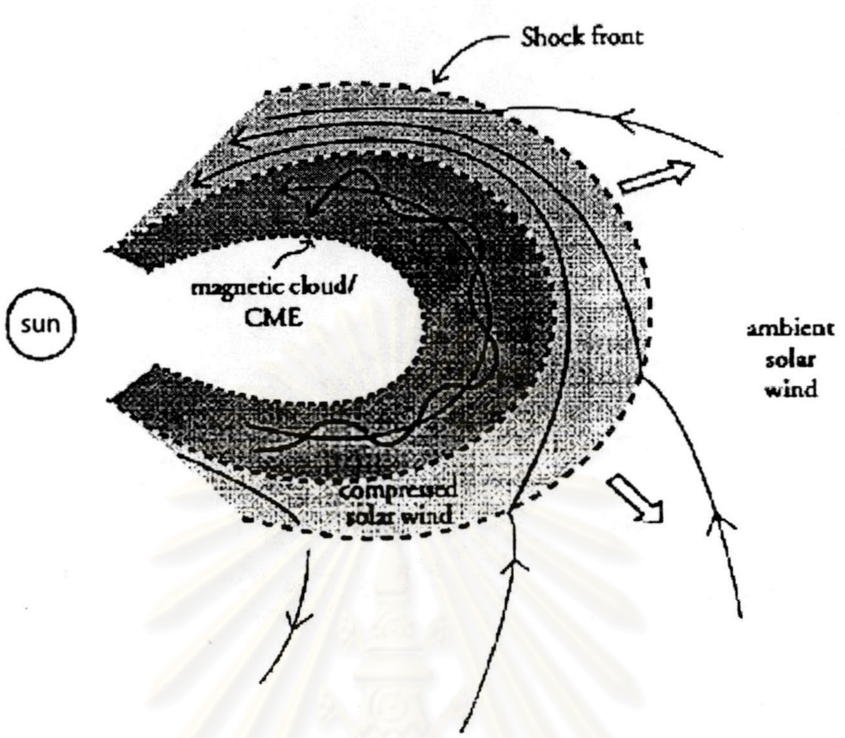

Figure 2.6: A shock due to a CME (Picture credit: Cravens 1997)

cosmic rays originating in the solar system can be connected with shock acceleration as follows: The first are solar energetic particles accelerated by shocks taking place when mass is ejected from the solar corona and collides with the solar wind ahead of it (see Figure 2.6). This event is called a "coronal mass ejection" (CME). Usually, a major CME event will also involve a gradual solar flare, which can be called a gradual solar energetic particle (SEP) event.

Particles are also accelerated by shocks at regions of interaction between slow solar wind and fast solar wind. These regions are called "corotating interaction regions" (CIRs). The collisions between the slow solar wind and fast solar wind, streaming out from different regions of the Sun, are due to rotation of the Sun (see Figure 2.7).

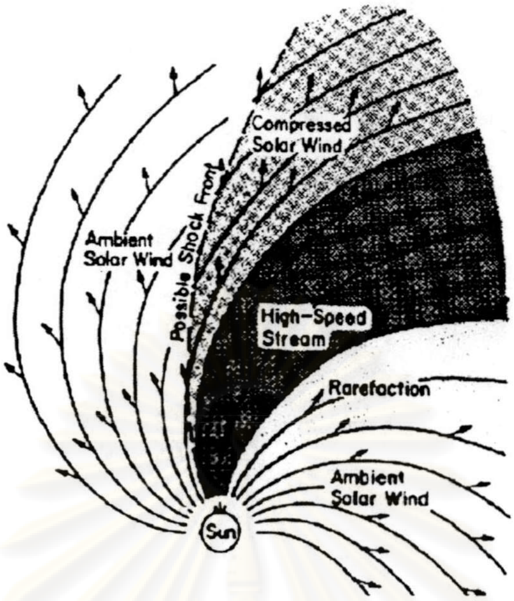

Figure 2.7: A shock at a CIR (Picture credit: Hundhausen 1972)

Finally, there are "anomalous cosmic rays" (ACRs). ACRs are cosmic rays that originate from the solar wind termination shock, a shock caused by the collision between the solar wind and interstellar medium at the boundary of the solar system (see Figure 2.8).

2.2 Magnetic Fields and Charged Particles in Space

Apart from many kinds of charged particles and electromagnetic waves, space is also full of magnetic fields produced by many objects such as the Earth, the Sun, planets and stars. In order to describe cosmic ray transport through space in the presence of a magnetic field, we should understand the interaction between the magnetic field and charged particles.

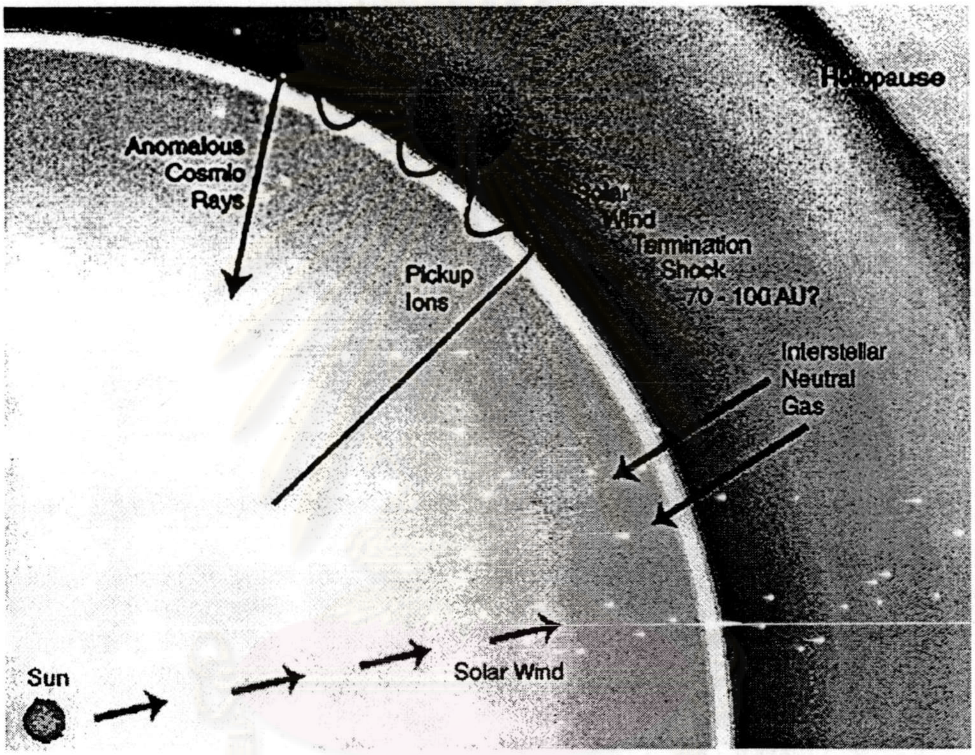

Figure 2.8: The solar wind termination shock and anomalous cosmic rays ACRs (Picture credit: http://helios.gsfc.nasa.gov)

2.2.1 A Charged Particle in a Magnetic Field

It is a fundamental principle that charged particles must interact with a magnetic field. The motion of a charged particle under the influence of a magnetic field, **B**, is governed by the Lorentz force,

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B}, \tag{2.5}$$

where q is the charge of the particle and **v** is the velocity of the particle (Ruffolo 2002). This is an approach to consider the interaction between a charged particle and the magnetic field, called the "test charge approach."

Uniform Magnetic Field

Under a uniform magnetic field, a charged particle undergoes helical motion along the magnetic field line. The perpendicular component of the particle velocity, $\mathbf{v}_\perp$, leads to circular motion in the two dimensions perpendicuiar to the magnetic field. In the direction parallel to the magnetic field, there is no force acting on the particle because **F** is a cross product of **B**. In consequence, the particle moves in the parallel direction with a constant velocity $\mathbf{v}_\parallel$. Since the total motion is a superposition of the circular motion and parallel motion, the result will be helical motion along the magnetic field line (see Figure 2.9).

For the helical motion, there is a parameter used to describe the relation between the circular motion and the parallel motion along the field. This is the an angle between the velocity vector of the particle, **v**, and the magnetic field, **B**, called the "pitch angle" and denoted by θ (see Figure 2.10).

In this work, we often use the cosine of the pitch angle, denoted by μ, to describe the ratio of $\mathrm{v}_\parallel$ to v. Subsequently, μv and $(\sqrt{1-\mu^2})v$ are used to

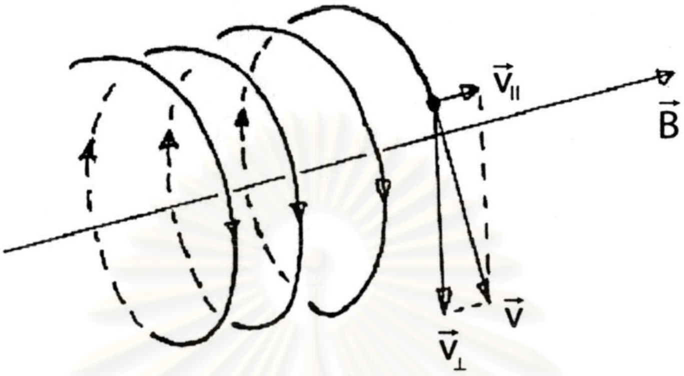

Figure 2.9: Helical motion of a charged particle in a uniform magnetic field

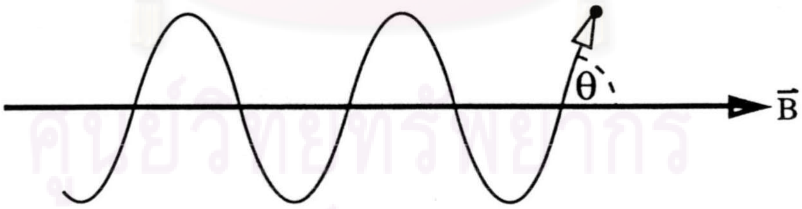

Figure 2.10: Side view of the helical motion, indicating the pitch angle

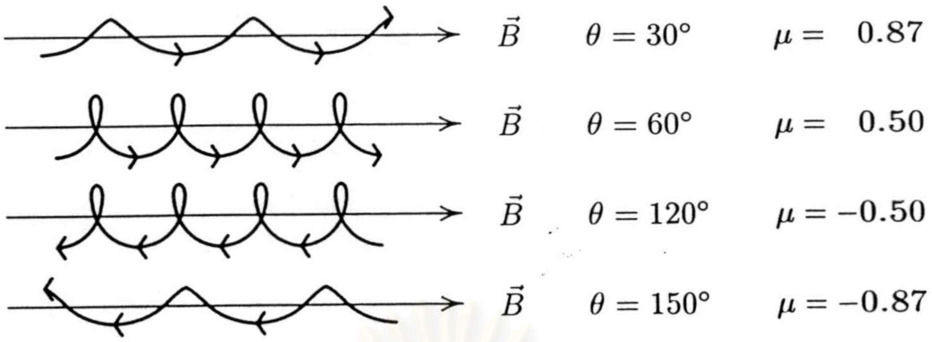

Figure 2.11: Helical motion of particles in a uniform magnetic field with various pitch angles

represent $|\mathbf{v}_\parallel|$ and $|\mathbf{v}_\perp|$ respectively, where v is the amplitude of $\mathbf{v}$. The cosine of the pitch angle has a value in the range of $-1 \leq \mu \leq 1$. If it is equal to 1, that means the particle has $\mathbf{v}_\parallel$ only; the circular motion does not exist and the particle moves in a straight line along the field. If $\mu = 0$, this means the particle has only $\mathbf{v}_\perp$; the motion is circular motion. If μ has a negative value, this means the particle has $\mathbf{v}_\parallel$ in the opposite direction with respect to the direction for a positive value. Helical motion of particles with various pitch angles is illustrated in Figure 2.11.

Converging and Diverging Magnetic Fields and Magnetic Mirroring

The pitch angle is constant for particle motion under a constant, uniform magnetic field, but it can change if the magnetic field changes or is not uniform. In particular, for a static magnetic field the rate of change of μ is

$$\dot{\mu} = \frac{v}{2L}(1-\mu^2), \tag{2.6}$$

where

$$\frac{1}{L} = -\frac{1}{B}\frac{dB}{dz}, \tag{2.7}$$

and z is the spatial coordinate along $\mathbf{B}$. In the case of a converging magnetic field ($dB/dz > 0$), the particle tends to accelerate backward, i.e., μ will decrease or may even change sign. Some forward-moving particle will turn around and move backward when the magnetic field converges. This phenomenon can be called "magnetic mirroring."

An adiabatic invariant under a slowly varying magnetic field, the magnetic moment, can be used to describe this. The magnetic moment for a charged particle, M, can be defined in terms of the kinetic energy of the particle due to motion perpendicular to $\mathbf{B}$ (Parks 1991),

$$M = \frac{1}{2}\frac{m|v_\perp|^2}{B}, \tag{2.8}$$

where m is the mass of the particle. The mass and the magnetic moment (in limit of a slowly varying $\mathbf{B}$) are constant. In addition, v is also constant for a static magnetic field (Ruffolo 2002). Using the expression of $|v_\perp|$ in terms of μ, we can conclude that

$$\frac{1-\mu^2}{B} = \text{const.} \tag{2.9}$$

When B is more intense, μ^2 must decrease. Then the particle will reflect at a mirror point where B is intense enough to make μ equal to zero. Furthermore, the invariance of the magnetic moment implies that the magnetic flux within the circular path is also constant. Thus, the motion of a particle in a converging magnetic field should be a spiral motion around a constant magnetic flux (a constant number of magnetic field lines), and the particle is reflected ("mirrored") backward along the guiding center when it encounters a strong enough magnetic field, as shown in Figure 2.12. If a particle moves into a less intense magnetic field, μ of the particle will not change sign and $|\mu|$ will tend toward one, the

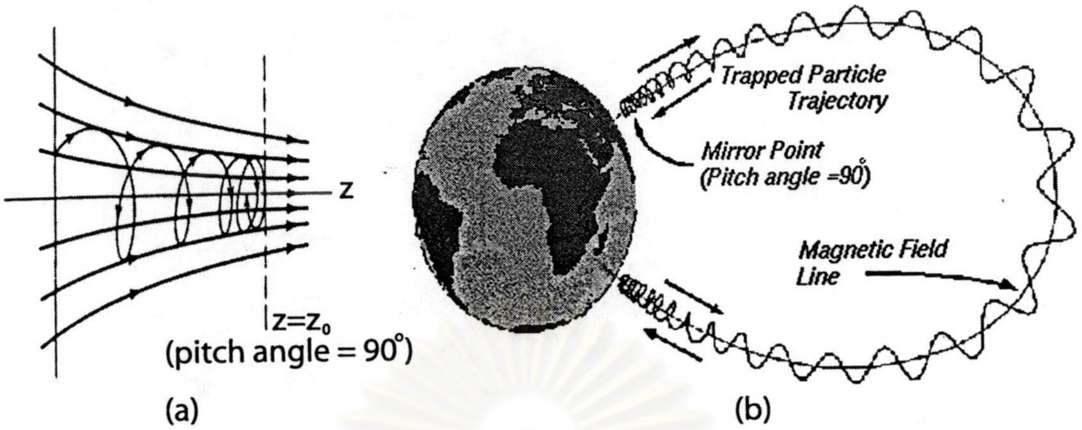

Figure 2.12: (a) Motion of a particle in a converging magnetic field and (b) magnetic mirroring in the Earth's radiation belts (Picture credit: Jackson 1975 and http://www.estec.esa.nl/wmwww/WMA/backgnd.html)

particle tend to move more directly forward, and will not encounter reflection by magnetic mirroring.

Irregular Magnetic Field Changes in the magnetic field lead to changes in motion of the charged particle. If the magnetic field gradually changes, the charged particle motion will also change systematically. In particular, in the case of a converging or diverging magnetic field, the pitch angle of the particle moving in the magnetic field is gradually changed to satisfy the magnetic invariant explained before. If the magnetic field is irregular, the motion of the particle moving through the irregular field will change randomly. That means the pitch angle of the particle changes randomly. This is called "pitch-angle scattering." Due to the pitch-angle scattering, the charged particle can change its direction of motion along the field line, i.e., the sign of μ can change (see Figure 2.13). This direction changing can also be treated as an elastic collision between the charged particle and the magnetic field. The strength of the pitch-angle scattering can

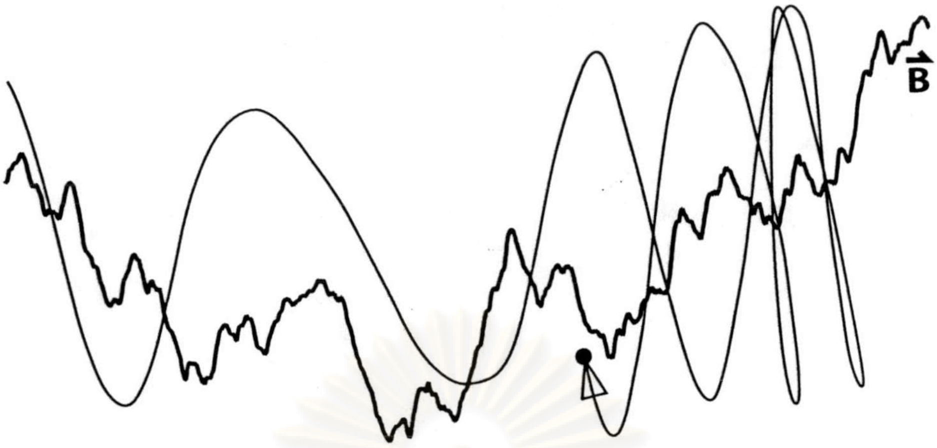

Figure 2.13: A charged particle changes its pitch angle along an irregular magnetic field. Furthermore, the direction of motion of the charged particle can also change.

be characterized by the "parallel mean free path," $\lambda_{\parallel}$. This is an average length that cosmic rays can move before changing their direction of motion along the magnetic field line. Furthermore, there is also a "perpendicular mean free path," $\lambda_{\perp}$, for describing particle diffusion across magnetic field lines.

2.2.2 Magnetic Field in a Plasma

Not only can the magnetic field affect charged particles, but on the other hand, charged particles can affect the magnetic field also when charged particles have enough energy density. In this case, the test charge approach mentioned before cannot be used.

The case of an interaction between high energy-density plasmas and magnetic fields is a case where charged particles (the plasma) can affect the magnetic field. A plasma is a balanced collection of ions and electrons, mixed together. It is a very high electric conductivity fluid. A consequence is that in the frame of the plasma, there is no electric field, **E**, inside the plasma. This is because if electric field is present, a lot of particles will suddenly move to cancel the internal electric field. Thus $\mathbf{E} = 0$ in a plasma and $\nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t$, where t is time, so there is no magnetic field change in the frame of the plasma. From another point of view, the magnetic field is stuck in the plasma and can be dragged when the plasma moves. This concept is usually called the "frozen-in-field" concept or "flux-freezing" concept. The plasma and field are tied together. If the ratio of magnetic energy density to kinetic energy density is much less than one, the field is dragged by the plasma. If the ratio is much greater than one, magnetic fields will determine the particle motions and the test charge approach can be used (Kallenrode 2001).

2.2.3 Cosmic Ray Motions in Space Plasmas

In order to explain cosmic ray motions in space plasmas, both the test charge approach and flux-freezing concept will be used in describing the interaction between charged particles and magnetic fields. In space, there are both cosmic rays, usually with low energy density, which can be described using the test charge approach, and plasmas, with high energy density, to which the flux-freezing concept can be applied. Due to the flux-freezing concept, motions of plasmas determine configurations of magnetic fields. Meanwhile cosmic rays move along the magnetic fields constrained by plasma motions, under the rules of the test charge approach. Therefore, we can say that the motion of cosmic rays is involved with space plasma motions. An example is the motion of a solar cosmic ray along a magnetic field line frozen in the solar wind, as shown in Figure 2.14. Since the solar wind is a plasma continuously flowing out from the Sun in all directions, while the Sun rotates once every 27 days or so, the solar magnetic field line is dragged out from the Sun and curved by the Sun's rotation. This magnetic field

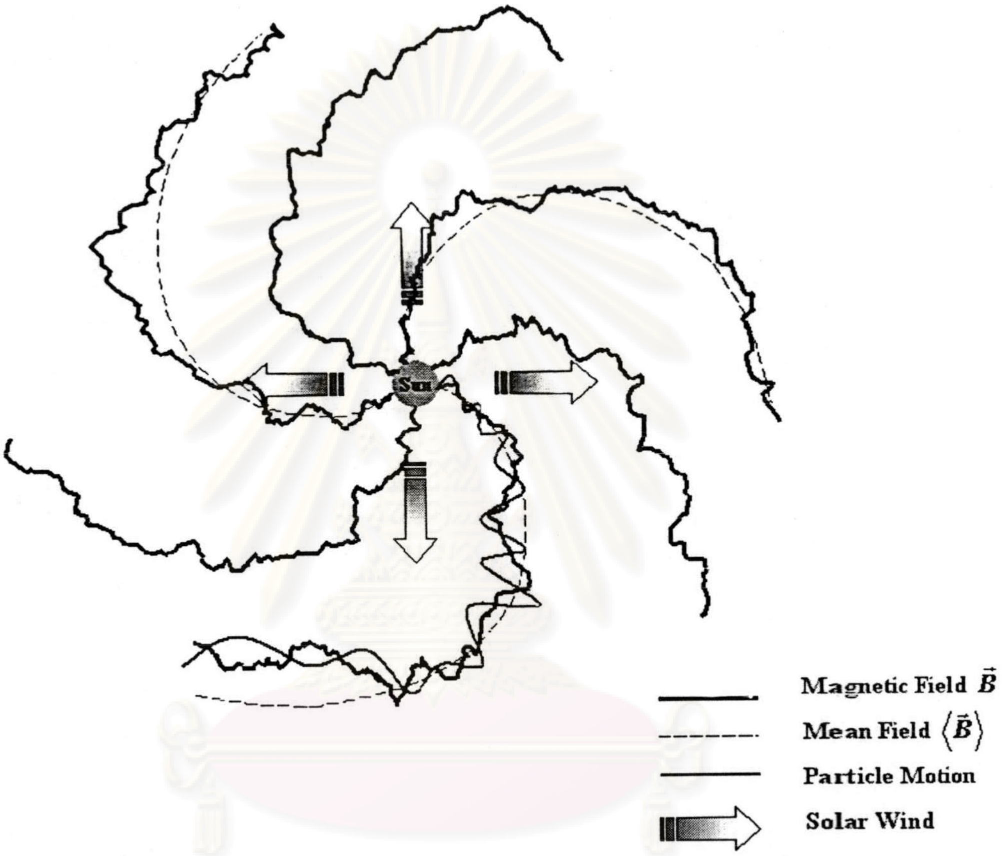

Figure 2.14: The motion of a cosmic ray along a magnetic field line frozen in the solar wind. (Picture credit: Chuychai et al. 2003)

line configuration is usually called an "Archimedean spiral." In addition, field lines dragged out by the solar wind are not smooth but very irregular, because the solar wind flowing out from the Sun is highly turbulent.

In summary, cosmic rays will move along magnetic field lines determined by plasma flow, as a superposition of two types of motions. The first is the cosmic ray motion along the mean magnetic field. This is a case of a gradual magnetic-field changing. The particle motions change systematically, satisfying the invariance of the magnetic moment. The second is the cosmic ray motion due to the irregularities of magnetic fields. The irregularities lead to pitch-angle scattering, a random change in the pitch angle characterized by a parallel mean free path. Because cosmic ray diffusion in the directions perpendicular to the magnetic field is small, we usually assume that cosmic rays do not move across the field lines.

2.3 Shock Acceleration Concepts and Mechanism

Studies of the origin of cosmic rays have been performed ever since Hess discovered cosmic rays in 1912. However, there was not a successful theory until the "shock acceleration theory" was proposed by Fermi in 1954. Before explaining concepts of the shock acceleration theory, some background about shocks is introduced.

2.3.1 Shocks

A shock wave, which astrophysicists usually called a "shock," is a very thin region in which fluid properties change rapidly (Choudhuri 1998). It can

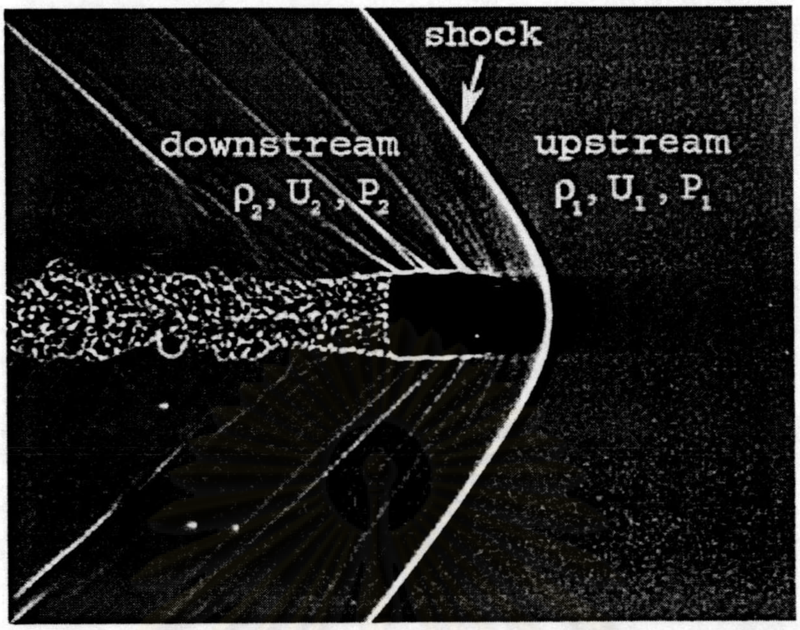

Figure 2.15: A shock, here produced by a bullet, divides a fluid into two regions, upstream and downstream. Fluid properties of the two regions are quite different, and change sharply at the shock.

occur when fluids collide together or collide with an obstacle, with a relative speed greater than the sound speed of the fluids. Usually, a shock is regarded as a discontinuity in fluid properties, such as density, pressure, fluid velocity, etc., dividing the fluid into two parts. One part is the fluid which has not passed the shock yet, called "upstream." The other part is fluid which has already passed the shock, called "downstream." In this thesis, subscripts "1" and "2" will be used to denote any variables involved with the upstream and downstream sides, respectively. As an example, a shock is produced by a bullet moving through any kind of fluid with a speed greater than the sound speed of the fluid, as shown in Figure 2.15. Some examples of astrophysical shocks have been already shown in Figures 2.6 - 2.8.

2.3.2 Shock Acceleration Mechanism

Nowadays, it is commonly believed that most cosmic rays are accelerated and gain their energy at astrophysical shocks, according to the shock acceleration theory proposed by Fermi in 1954. This theory was based on the concept of a collision between a charged particle and magnetic field irregularities. Actually, the first presented acceleration mechanism using the concept of such collisions was not shock acceleration, but rather stochastic acceleration, also proposed by Fermi in 1949. However, the stochastic acceleration mechanism has a low acceleration efficiency, and cannot explain the cosmic ray spectrum.

As discussed in §2.1.1, a charged particle moving through an irregular magnetic field can be reflected if it encounters a sufficiently intense magnetic field. This event can be pictured as an elastic collision between the particle of mass m with an obstacle of very large (macroscopic) mass M. In one dimension, the collisions can be classified into two types. The first type is a "head-on collision," where the particle with initial velocity $\mathbf{v}_i$ collides face to face with an obstacle of velocity $\mathbf{U}$. We can assume that $U << v_i$, especially if there is an equipartition of energy between such objects. This collision type yields a particle energy gain with final particle velocity $\mathbf{v}_f = -(\mathbf{v}_i+2\mathbf{U})$, so the particle speed has increased by $2U$. The second type is a "following collision," where the particle is moving in the same direction as the obstacle, but is moving faster and collides with the obstacle when the particle catches up with it. In this case, the final particle velocity is $\mathbf{v}_f = -(\mathbf{v}_i - 2\mathbf{U})$, so the particle speed has decreased by $2U$ (see Figure 2.16).

At a shock, the plasma flows from upstream to downstream of the shock, and magnetic field irregularities are frozen in the plasma. Then the irregularities also move from upstream to downstream like the plasma. If there is a particle in

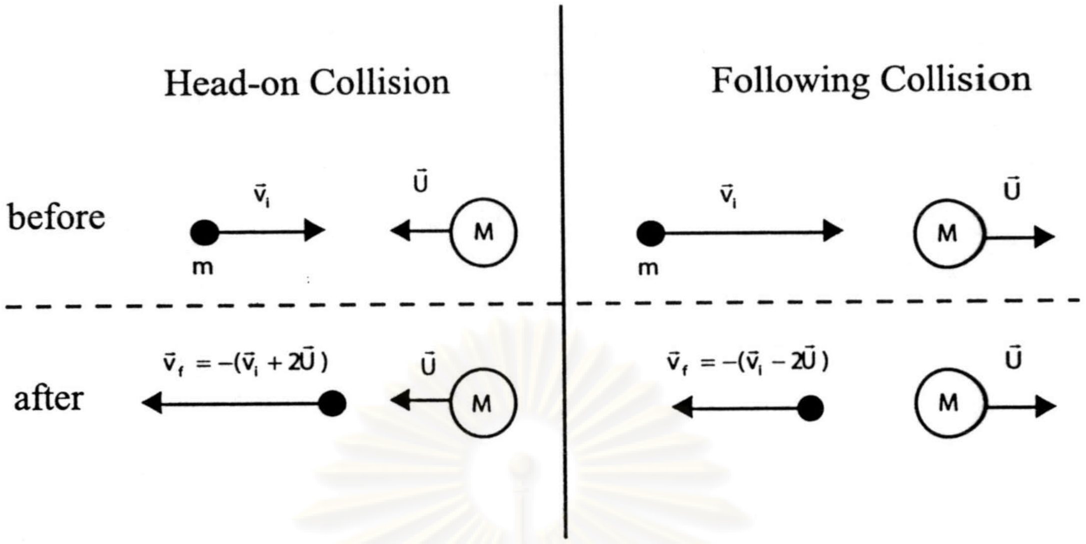

Figure 2.16: A head-on collision and a following collision between a particle of small mass and an obstacle of large mass

the upstream region near the shock, it may collide with an irregularity moving from far upstream on the upstream side, as a head-on collision (see Figure 2.17). Then it will bounce back, move across the shock, collide with another irregularity on the downstream side as a following collision, and move across the shock to the upstream side again. Therefore, for a cycle of collisions, the particle both gains and loses energy. However, overall it gains energy. The reason is that the plasma flow speed on the upstream side, U_1, is greater than plasma flow speed on the downstream side, U_2. As a consequence, from the head-on collision, the particle speed increases by $2U_1$, which is more than it loses by the following collision, when the particle speed decreases by $2U_2$. A demonstration of shock acceleration is shown in Figure 2.17.

The shock acceleration mechanism is often called the “first-order Fermi acceleration mechanism,” because this mechanism gives an energy change per particle energy proportional to first order in the small quantity $(U_1 - U_2)/v_i$.

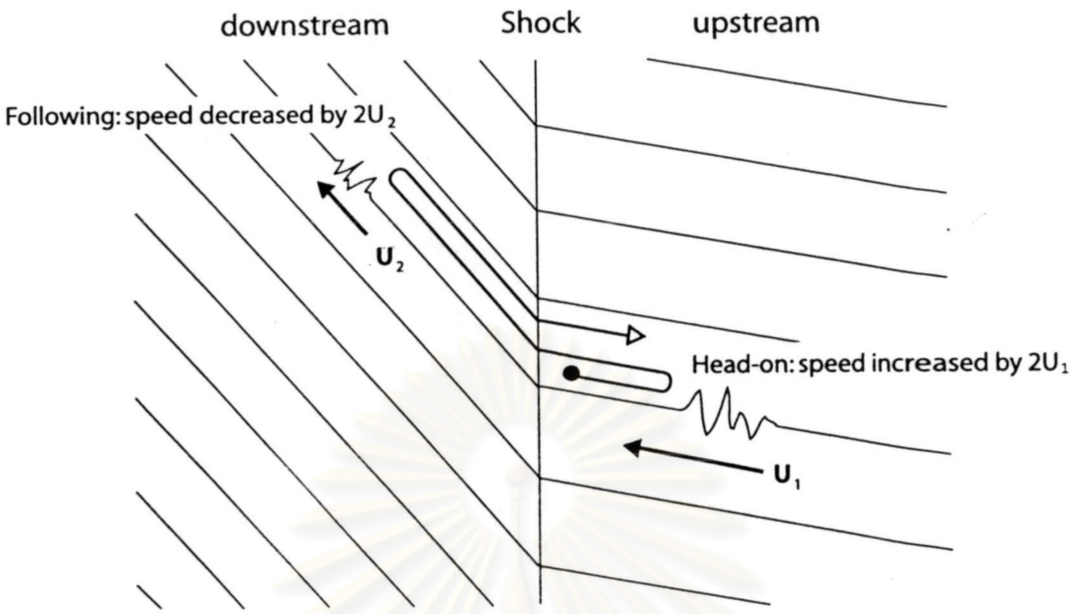

Figure 2.17: A demonstration of shock acceleration

An important point regarding this mechanism is that under certain conditions it gives a power-law spectrum corresponding with observations of cosmic rays, as will be shown in §4.1.