Chapter 5

Summary

We have described the search for non-Gaussianity using wavelet-based methods. In this thesis, we review some parts of the works of Vielva et al. (V04) [27] and Mukherjee and Wang (MW04) [28]. In their works, the WMAP 1st-year data were analyzed by using the spherical Mexican hat wavelet (SMHW) as a statistical tool. The SMHW is an isotropic wavelet on the sphere which is derived from the inverse stereographic projection from the plane onto the sphere of the 2-D Mexican Hat wavelet where the scaling of wavelet is performed on the plane and then is projected on the sphere, and the translation along the sphere is the rotation around the center of the sphere. After transforming the data using this kind of spherical wavelet into 15 scales, the statistical analyzes of the wavelet coefficients were performed.

According to V04, the skewness and kurtosis spectra, the third and fourth moments of the wavelet coefficients in each wavelet scale, were measured on the all-sky data in order to characterize the deviation from Gaussian statistics of the one-point probability distribution in different wavelet scales. By comparing the 10000 Gaussian simulations, the excess of the kurtosis in the wavelet scale about $4^o - 5^o$, equivalent to about 10^o in the sky, was found outside the confidence interval at less than 1% significance level. In order to specify the region the non-Gaussian signature lives, the skewness and kurtosis spectra were also measured separately in each Galactic hemisphere, north and south, and then the kurtosis on the wavelet scale about 4^o was found outside the confidence interval at less than 1% significance level only in the southern Galactic hemisphere, but not in the northern Galactic hemisphere. From these results, we can conclude that the kurtosis on the scale about 4^o lives in the southern Galactic hemisphere.

After that, we described the work of MW04 which specified the non-Gaussian signature by other estimators measured on the wavelet coefficients. Note that the number of Gaussian simulations used in their work is 1000, which is less than 10000, the number of Gaussian simulations in V04. Using the extremum analysis in the southern Galactic hemisphere data, the non-Gaussianity was specified to be in the form of the too large number of the cold pixels around the wavelet scale 4^o. Furthermore, the scale-scale correlations which is an indicator of the deviation from Gaussian statistics were measured on both the all-sky data and the data in the southern Galactic hemisphere. Although the non-Gaussianity was not detected on the all-sky data, the scale-scale correlations were found outside the acceptance interval at the 0.3% significance level on the data in the southern Galactic hemisphere.

This review shows that the statistical analysis in the wavelet space, the space of the wavelet coefficients, is an optimal method. This is because using the wavelet transform the data is decomposed simultaneously into both the position and scale in the spherical sky. To understand this point, lets consider the characteristic of homogeneous Gaussian random field. If the CMB map is a homogeneous Gaussian random field, the distribution law, namely the Gaussian distribution law, of the temperature values does not depend on the position in the sky. In another view, a random field is Gaussian since there is no correlation between Fourier modes (scales), according to the central limit theorem. From these reasons, we can suspect that a small deviation from Gaussian statistics may be in the form of the presence of some special positions that do not share the same distribution law with others or the presence of the correlation between scales in a range of scales. In other words, a small deviation from Gaussian statistics can be localized in position or in a range of scales or in both. Consequently, the non-Gaussianity which is expected to be localized in the space-scale should be well detected using the tool like the wavelet analysis.

Beside the application of the isotropic spherical wavelet as the SMHW to detect the non-Gaussianity on the all-sky CMB data, there is an application of the directional spherical wavelet transform by McEwen et al. [36] to the WMAP 1st-year data and the strong non-Gaussianity was detected in the form of skewness of the spherical real Morlet wavelet coefficients on the effective size on the sky

$\sim 26^o$ at an azimuthal orientation of 72^o, outside the region of about 6 standard deviations. This is an interesting result. Moreover, there are some proposals to study the morphology of the CMB anisotropy as a form of finding the deviation from Gaussian statistics [37].

Moreover, the non-linear coupling parameter f_{NL} was constrained by the methods following MW04. To constrain f_{NL}, the skewness which is most sensitive to this form of non-Gaussianity on all scales, 15 scales, were combined to perform the statistical tests. Two statistical tests which were used in their works are the χ^2 test and the Fisher discriminant function. In the χ^2 test, f_{NL} is constrained to be 50 ± 80 at 68% confidence, and less than 220 and 280 at 95% and 99% confidence levels, respectively. The results obtained using the Fisher discriminants are comparable with those obtained using the χ^2 test, namely f_{NL} values are 120 and 250 with 75% and 95% confidence, respectively.

The non-Gaussianity is expected to give important information about the early universe complementary to that from the study of the temperature and polarization power spectrum in CMB. This motivates the cosmologists to improve the statistical techniques for the detection of non-Gaussianity. Furthermore, it is expected that the data from the Planck satellite (to be launched about 2007) with higher resolution and better foreground cleaning will help to disentangle the non-Gaussian signal and give us a clearer picture and better understanding of this matter. And the wavelet analysis may help in that works.