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APPENDICES

APPENDIX A

The Matrices $\mathbf{R}(\xi,z,s)$ and $\mathbf{S}(\xi,z,s)$ in equations (3.15) and (3.16) respectively, are given by

$$\mathbf{R}(\xi,z,s)=\begin{bmatrix} -\xi\delta e^{\gamma z} & -\xi\delta e^{-\gamma z} & a_1 z e^{\xi z} & -a_1 z e^{-\xi z} & e^{\xi z} & e^{-\xi z} \\ \gamma\delta e^{\gamma z} & -\gamma\delta e^{-\gamma z} & -\left(a_1 z-\dfrac{a_2}{\xi}\right)e^{\xi z} & -\left(a_1 z+\dfrac{a_2}{\xi}\right)e^{-\xi z} & -e^{\xi z} & e^{-\xi z} \\ 2\mu a_3\eta e^{\gamma z} & 2\mu a_3\eta e^{-\gamma z} & -2\mu a_4\eta e^{\xi z} & -2\mu a_4\eta e^{-\xi z} & 0 & 0 \end{bmatrix}$$

(A-1)

$$\mathbf{S}(\xi,z,s)=2\mu\begin{bmatrix} -\gamma\xi\delta e^{\gamma z} & \gamma\xi\delta e^{-\gamma z} & \left(a_1\xi z-\dfrac{1}{2}\right)e^{\xi z} & \left(a_1\xi z+\dfrac{1}{2}\right)e^{-\xi z} & \xi e^{\xi z} & -\xi e^{-\xi z} \\ \xi^2\delta e^{\gamma z} & \xi^2\delta e^{-\gamma z} & \left(a_4-a_1\xi z\right)e^{\xi z} & \left(a_4+a_1\xi z\right)e^{-\xi z} & -\xi e^{\xi z} & -\xi e^{-\xi z} \\ -a_3\kappa\gamma\dfrac{\delta}{c}e^{\gamma z} & a_3\kappa\gamma\dfrac{\delta}{c}e^{-\gamma z} & a_4\kappa\xi\dfrac{\delta}{c}e^{\xi z} & -a_4\kappa\xi\dfrac{\delta}{c}e^{-\xi z} & 0 & 0 \end{bmatrix}$$

(A-2)

where $$a_1=\frac{1}{2(1-2\nu_u)} \quad \text{(A-3)}$$

$$a_2=\frac{(3-4\nu_u)}{2(1-2\nu_u)} \quad \text{(A-4)}$$

$$a_3=\frac{B(1+\nu_u)(1-\nu)}{3(\nu_u-\nu)} \quad \text{(A-5)}$$

$$a_4=\frac{(1-\nu_u)}{(1-2\nu_u)} \quad \text{(A-6)}$$

$$c=2\mu a_3\kappa\eta \quad \text{(A-7)}$$

$$\eta=\frac{B(1+\nu_u)}{3(1-\nu_u)} \quad \text{(A-8)}$$

$$\delta=\frac{\eta}{s/c} \quad \text{(A-9)}$$

$$\gamma=\sqrt{\xi^2+\frac{s}{c}} \quad \text{(A-10)}$$

APPENDIX B

This appendix presents the derivation of the strain energy of fictitious elastic pile i^{th} in Laplace domain, as shown in Figure 2(b). The axial strain corresponding to equation (3.38) can be expressed as

$$\bar{\varepsilon}^i(z,s) = \sum_{m=1}^{Nt} \frac{-(m-1)\bar{\alpha}_m^i(s)}{L} e^{-(m-1)z/L} \tag{B-1}$$

In view of the conventional constitutive relation the strain energy of fictitious elastic pile i^{th} can be written as

$$Up^i = \frac{1}{2}\int_L \int_A \bar{\sigma}^i \, \bar{\varepsilon}^i \, dA dz = \frac{\pi (a^i)^2}{2} \int_0^L E^{i^*} \left(\bar{\varepsilon}^i\right)^2 dz \tag{B-2}$$

Note that since the medium is multilayered, equation (B-3) rewritten in view of equation (B-2) as

$$\begin{aligned}
Up_g &= \frac{\pi (a^i)^2}{2} \int_0^{\Delta t_1^i} E_1^{i^*} \sum_{m=1}^{Nt}\sum_{n=1}^{Nt} (m-1)(n-1)\bar{\alpha}_m^i(s)\bar{\alpha}_n^i(s) e^{-(m+n-2)(z/L)} dz \\
&+ \frac{\pi (a^i)^2}{2} \int_{\Delta t_1^i}^{\Delta t_1^i + \Delta t_2^i} E_2^{i^*} \sum_{m=1}^{Nt}\sum_{n=1}^{Nt} (m-1)(n-1)\bar{\alpha}_m^j(s)\bar{\alpha}_n^j(s) e^{-(m+n-2)(z/L)} dz \\
&\quad \vdots \\
&+ \frac{\pi (a^i)^2}{2} \int_{\Delta t_1^i + \Delta t_2^i + \ldots + \Delta t_{Ne-1}^i}^{\Delta t_1^i + \Delta t_2^i + \ldots + \Delta t_{Ne}^i} E_{Ne}^{i^*} \sum_{m=1}^{Nt}\sum_{n=1}^{Nt} (m-1)(n-1)\bar{\alpha}_m^i(s)\bar{\alpha}_n^i(s) e^{-(m+n-2)(z/L)} dz
\end{aligned} \tag{B-3}$$

By integrating equation (B-3), the strain energy of fictitious elastic pile i^{th} can be obtain in the following form

$$Up^i = \sum_{m=1}^{Nt}\sum_{n=1}^{Nt} \bar{\alpha}_m^i(s) D_{mn}^i \bar{\alpha}_n^i(s) \tag{B-4}$$

$$D_{mn}^i = \frac{\pi (a^i)^2 (m-1)(n-1)}{2L(m+n-2)} \sum_{k=1}^{Ne} E_k^{i^*} \left[\left(e^{-(m+n-2)(z_k^i - \Delta t_k^i/2)} - e^{-(m+n-2)(z_k^i + \Delta t_k^i/2)}\right)\right] \tag{B-5}$$

where z_k^i and Δt_k^i denote the distance from the top to the middle of the k^{th} element and the thickness of the k^{th} element of the i^{th} pile respectively.

APPENDIX C

This appendix presents the relation between the matrices $\boldsymbol{\beta}^j$ and $\boldsymbol{\omega}^j$. The relationship between unknown body forces and the vertical displacements of the extended half-space can be expressed as

$$\begin{bmatrix} \mathbf{F}^{1,1} & \mathbf{F}^{1,2} & \cdots & \mathbf{F}^{1,j} & \cdots & \mathbf{F}^{1,Np} \\ \mathbf{F}^{2,1} & \mathbf{F}^{2,2} & \cdots & \mathbf{F}^{2,j} & \cdots & \mathbf{F}^{2,Np} \\ \vdots & \vdots & \ddots & \vdots & \cdot^{\cdot^{\cdot}} & \vdots \\ \mathbf{F}^{i,1} & \mathbf{F}^{i,2} & \cdots & \mathbf{F}^{i,j} & \cdots & \mathbf{F}^{i,Np} \\ \vdots & \vdots & \cdot^{\cdot^{\cdot}} & \vdots & \ddots & \vdots \\ \mathbf{F}^{Np,1} & \mathbf{F}^{Np,2} & \cdots & \mathbf{F}^{Np,j} & \cdots & \mathbf{F}^{Np,Np} \end{bmatrix} \begin{Bmatrix} \mathbf{B}^1 \\ \mathbf{B}^2 \\ \vdots \\ \mathbf{B}^j \\ \vdots \\ \mathbf{B}^{Np} \end{Bmatrix} = \begin{Bmatrix} \mathbf{w}^1 \\ \mathbf{w}^2 \\ \vdots \\ \mathbf{w}^j \\ \vdots \\ \mathbf{w}^{Np} \end{Bmatrix} \tag{C-1}$$

or

$$\begin{aligned} \mathbf{F}^{1,1}\mathbf{B}^1 + \mathbf{F}^{1,2}\mathbf{B}^2 + \ldots + \mathbf{F}^{1,j}\mathbf{B}^j + \ldots + \mathbf{F}^{1,Np}\mathbf{B}^{Np} &= \mathbf{w}^1 \\ \mathbf{F}^{2,1}\mathbf{B}^1 + \mathbf{F}^{2,2}\mathbf{B}^2 + \ldots + \mathbf{F}^{2,j}\mathbf{B}^j + \ldots + \mathbf{F}^{2,Np}\mathbf{B}^{Np} &= \mathbf{w}^2 \\ \vdots \qquad\qquad &\quad \vdots \\ \mathbf{F}^{i,1}\mathbf{B}^1 + \mathbf{F}^{i,2}\mathbf{B}^2 + \ldots + \mathbf{F}^{i,j}\mathbf{B}^j + \ldots + \mathbf{F}^{i,Np}\mathbf{B}^{Np} &= \mathbf{w}^j \\ \vdots \qquad\qquad &\quad \vdots \\ \mathbf{F}^{Np,1}\mathbf{B}^1 + \mathbf{F}^{Np,2}\mathbf{B}^2 + \ldots + \mathbf{F}^{Np,j}\mathbf{B}^j + \ldots + \mathbf{F}^{Np,Np}\mathbf{B}^{Np} &= \mathbf{w}^{Np} \end{aligned} \tag{C-2}$$

where $\mathbf{F}^{i,j}$ denotes the Laplace transform of the vertical displacement of the i^{th} pile due to a vertical body force of the j^{th} pile and $\mathbf{w}^j$ vertical displacement of the j^{th} pile. For each equation in the system in equation (C-2) yields

$$\sum_{i=1}^{Np}\mathbf{F}^{i,1}\mathbf{B}^1 + \sum_{i=1}^{Np}\mathbf{F}^{i,2}\mathbf{B}^2 + \ldots + \sum_{i=1}^{Np}\mathbf{F}^{i,j}\mathbf{B}^j + \ldots + \sum_{i=1}^{Np}\mathbf{F}^{i,Np}\mathbf{B}^{Np} = \sum_{j=1}^{Np}\mathbf{w}^j \tag{C-3}$$

Substituting $\mathbf{w}^j = \boldsymbol{\omega}^j\boldsymbol{\alpha}^j$ and $\mathbf{B}^j = \boldsymbol{\beta}^j\boldsymbol{\alpha}^j$ into equation (C-3) results in

$$\sum_{i=1}^{Np}\mathbf{F}^{i,1}\boldsymbol{\beta}^1\boldsymbol{\alpha}^1 + \sum_{i=1}^{Np}\mathbf{F}^{i,2}\boldsymbol{\beta}^2\boldsymbol{\alpha}^2 + \ldots + \sum_{i=1}^{Np}\mathbf{F}^{i,j}\boldsymbol{\beta}^j\boldsymbol{\alpha}^j + \ldots + \sum_{i=1}^{Np}\mathbf{F}^{i,Np}\boldsymbol{\beta}^{Np}\boldsymbol{\alpha}^{Np} = \sum_{j=1}^{Np}\boldsymbol{\omega}^j\boldsymbol{\alpha}^j \tag{C-4}$$

The above equation can be rewritten in the following matrix form

$$\left[\sum_{i=1}^{Np}\mathbf{F}^{i,1}\boldsymbol{\beta}^1 \quad \sum_{i=1}^{Np}\mathbf{F}^{i,2}\boldsymbol{\beta}^2 \quad \cdots \quad \sum_{i=1}^{Np}\mathbf{F}^{i,j}\boldsymbol{\beta}^j \quad \cdots \quad \sum_{i=1}^{Np}\mathbf{F}^{i,Np}\boldsymbol{\beta}^{Np}\right]\begin{Bmatrix}\boldsymbol{\alpha}^1\\ \boldsymbol{\alpha}^2\\ \\ \boldsymbol{\alpha}^j\\ \\ \boldsymbol{\alpha}^{Np}\end{Bmatrix}=\left[\boldsymbol{\omega}^1 \quad \boldsymbol{\omega}^2 \quad \cdots \quad \boldsymbol{\omega}^j \quad \cdots \quad \boldsymbol{\omega}^{Np}\right]\begin{Bmatrix}\boldsymbol{\alpha}^1\\ \boldsymbol{\alpha}^2\\ \\ \boldsymbol{\alpha}^j\\ \\ \boldsymbol{\alpha}^{Np}\end{Bmatrix}$$

(C-5)

Finally, are obtained the following relations

$$\sum_{i=1}^{Np}\mathbf{F}^{i,j}\boldsymbol{\beta}^j = \boldsymbol{\omega}^j \tag{C-6}$$

BIOGRAPHY

Mr. Napadon Sornpakdee was born in Prajuab Kiree Khun in 1975. He obtained the Bachelor Degree in Civil Engineering from Chulalongkorn University in 1997. He continued his study for Master Degree in Civil Engineering at Chulalongkorn University in 2000.