CHAPTER IV

NUMERICAL SOLUTIONS

This chapter is concerned with the numerical results obtained from the solution scheme described in Chapter III. A computer program has been developed to investigate the interaction problem between a pile group and a multi-layered poroelastic medium. Convergence and stability of numerical solutions are investigated. The accuracy of the present solutions is verified by comparing with the existing solutions given in the literature. Numerical results are also presented in this chapter to investigate the influence of various parameters on the quasi-static behavior of vertically loaded pile group in multi-layered poroelastic medium.

4.1 Numerical Solution Scheme

The solution scheme described in Chapter III is implemented into a computer program. The tasks performed by the computer program can be summarized as follows:

1. The pile is discretized into *Ne* elements. The deformation of each pile is approximated by an exponential series given by equation (3.37). The Laplace transform is then applied to the equation (3.37).

2. The strain energy of the fictitious pile group is computed from equation (3.39)-(3.43).

3. The submatrix of vertical influence function of the extended half-space, equation (3.44), is determined to establish the flexibility matrix, equation (3.45).

4. The strain energy of the extend half-space, Uh_g, is computed by using equation (3.55) to (3.59).

5. The system of simultaneous linear equations is solved for the generalized coordinates in the Laplace domain, $\overline{\alpha}_m^i(s)$, for the i^{th} pile, $i = 1, 2, ..., Np$ and $m = 1, 2, ..., Nt$.

6. Appropriate numerical Laplace inversion scheme is employed to obtain time domain solutions of the generalized coordinates, $\overline{\alpha}_m^i(s)$.

7. The pile deformation and the unknown body force are obtained by back substituting the generalized coordinates $\alpha_m^i(t)$ into equations (3.37) and (3.51) respectively.

The major computation effort performed by the computer program is the evaluation of the influence functions. The influence functions appear in terms of semi-finite integrals with respect to ξ and s given by equation (3.14). The semi-finite integrals with respect to ξ can be evaluated by applying numerical quadrature. The scheme subdivides the interval of the integral and employs Simpson's rule to evaluate the integral over each subinterval. The error for each subinterval is estimated, and the bisection procedure is continued until the error criterion is reached.

4.2 Convergence and Numerical Stability of Present Solution

The convergence and stability of the numerical solution scheme described in the previous section are investigated with respect to the following parameters:

1. The upper limit of integration, ξ_L, used in the numerical integration of equation (3.14) to determine the flexibility matrix of the multilayered half-space.

2. The number of terms Nt used in the displacement of each pile given by equation (3.37).

3. The number of elements Ne used in the discretization of the piles as shown in Figure 2(b).

Table 1 presents the convergence of axial stiffness, Kv, where $Kv = V_0/\mu a\Delta_0$ for an axially loaded single pile and a pile group in a homogeneous elastic half-space with respect to ξ_L. The geometry of the piles-half-space system and properties are shown in Figure 3. The pile length, $L = 40a$, and the spacing between center of piles, $d = 5a$, are considered in this figure. In addition, three values of pile stiffness are shown, i.e., $Ep/\mu = 100$, 6000 and ∞. It is found from the results shown in Table 1 that the axial stiffness for all values of pile stiffness converges for $\xi_L \geq 40$.

The influence of number of terms in the displacement approximate equation (3.37), Nt, on the axial stiffness is shown in Table 2. The geometry and properties of the piles-half-space system are the same as those employed in Table 1. It appears that the accurate numerical results are obtained when $Nt \geq 8$. Therefore, subsequence numerical solutions presented in this chapter are then computed by employing $\xi_L = 40$ and $Nt = 8$.

Figure 4 shows the convergence of axial stiffness with respect to Ne, the number of elements used for discretizing piles. Four different values of pile length, i.e., $L/a = 40$, 60, 80 and 100, with $Ep/\mu = 6000$ and ∞ are considered. In addition, the properties of the half-space are the same as those in Figure 3. It is found that the numerical solutions converge for Ne equal to 30 and 40 for $20 < L/a \leq 60$ and $60 < L/a \leq 100$ respectively.

4.3 Comparison with Existing Solutions

The accuracy of the present solution scheme is verified by comparing the solutions obtained from the present scheme with the existing solutions. Three types of elastic media, namely a homogeneous half-space, nonhomogeneous half-space and a multi-layered half-space are considered in the comparison.

Figures 5, 6 and 7 present a comparison of the axial stiffness of the piles-homogeneous half-space system of figure 3 with those reported in existing studies (Butterfield and Banerjee 1971). In Figure 5 the axial stiffness is plotted with respect to the ratio of L/a for $Ep/\mu = 6000$ and ∞, with the spacing between center of piles, d, being equal to $5a$ for P1, P2, P3 and P4. Figure 6 presents a comparison of the axial stiffness for different values of the ratio of L/a for P5. It is evident that the present solutions are in very good agreement with the existing solutions for both ratios of Ep/μ. Figure 7 presents a comparison of the group reduction factor, R_G, for different values of d/a for $L = 40a$. Once again, it can be clearly seen that both solutions agree very closely for all ratios of d/a.

Figure 8 presents a comparison of the distribution of shear stress along pile, $T_z 2a\pi L/V_0$, for a single pile embedded in a homogeneous elastic half-space obtained from the present scheme with those given by Poulos and Davis (1980). It can be seen from this figure that the two sets of solutions agree very closely for both values of pile stiffness.

Table 3 presents a settlement influence factor, I_w, for a single pile in a nonhomogeneous elastic medium with rigid base as shown in Figure 9(a). Note that $I_w = \Delta_0 Es(L)a/V_0$ where Δ_0 is the displacement at the top of the pile $(z = 0)$. Two values of the pile length, i.e., $L/a = 20$ and 50, with $Ep/Es(L) = 100$ and 1000 are considered. In addition, the degree of nonhomogeneity, ρ, is equal to 0.0 and $H/L = 2$ where $\rho = \mu(0)/\mu(L)$ and H denotes the depth of rigid base. Numerical results presented in Table 3 indicate that, the present solutions are in very good agreement with the solutions given by Chow(1987) and Poulos(1979).

Rajapakse(1990) presented solutions for an axially loaded single pile in a nonhomogeneous elastic half-space as shown in Figure 9(b). Figure 10 shows a comparison of the axial stiffness for different degree of nonhomogeneity obtained from the present scheme with those reported by

Rajapakse(1990). It can be seen from this figure that the two sets of solutions agree very closely for all values of ρ and Ep.

Finally, let consider a multi-layered system consisting of three layers with a rigid base as shown in Figure 11. The properties of each layer are given in Table 4. Table 5 presents a comparison of I_w for the case of a single pile. The present solutions are in good agreement for Case A and B with solutions from a finite element program ISOPE. But the solution from the program AXPIL5 (Poulos 1979) shows some significant differences from the other two schemes for case B due to the approximations made in the analysis. Table 6 shows the comparison of interaction factor for two piles from present scheme and that given by Chow (1987). The interaction factor is defined as the ratio of additional settlement due to adjacent pile to the settlement of a pile under its own weight. Numerical results presented in Table 6 indicate that interaction factor from both schemes agree very closely for all systems considered in the comparison. The accuracy of the present scheme is therefore confirmed through these independent comparisons.

4.4 Numerical Results and Discussion

Numerical results are presented in this section to demonstrate the influence of various parameters on the quasi-static response of axially loaded pile group embedded in a multi-layered poroelastic medium. A layered system consisting of the two poroelastic layers boned to an under lying poroelastic half-space is considered as shown in Figure 12(a). The properties of each layer are given in Table 7. The configuration of the pile groups in the numerical study is shown in Figure 3(b). A non-dimensional time, $t^* = c^{(2)}t/a^2$, in the range $10^{-4} \le t^* \le 10^5$ is considered in all numerical results presented in this section. Note that $c^{(2)}$ is the consolidation coefficient of the second layered given by equation (3.10). The influences of several parameters are presented from Figures 13 to 19.

The influence of spacing between the piles, d, on axial stiffness, Kv where $Kv = V_0 / \mu^{(2)} a\Delta_0$ is presented in Figure 13 for different pile stiffness,

($Ep^* = 1e+2$ and $Ep^* = 1e+6$). In addition, $Ep^* = Ep/\mu^{(2)}$, $\kappa^{(1)}/\kappa^{(2)} = 0.1$ and $L = 40a$. It is found that the values of axial stiffness increase for increasing the spacing between the piles for all types of pile group. This feature can be explained by the fact that the interaction between adjacent piles decreases when the spacing between piles is increased.

Figure 14 shows the non-dimensional pile settlement curves along the length of piles, w^*, where $w^* = \Delta_0\mu^{(2)}A/Va$ for various slenderness ratios, L/a. Consider the initial and final solutions of w^*, the difference between the two solutions decreases along the length of pile when the slenderness ratio of pile is increased. In addition, it is also found that the pile shortening, $w^*_{(z/L=0)} - w^*_{(z/L=1)}$, in a longer pile is greater than a shorter one.

Non-dimensional pile settlements at $z = 0$ and $z = L$ are presented in Figures 15 and 16 for different length of pile, ($L = 20a$, $40a$, $60a$ and $100a$) with $\kappa^{(1)}/\kappa^{(2)} = 0.1$. The numerical results indicate that the settlement at the top of the pile, $w^*_{(z=0)}$, decreases when the pile stiffness and the pile length are increased for all types of pile group. On the other hand, the settlement at pile tip, $w^*_{(z=L)}$, increases when Ep^* and number of piles in group, Np, are increased.

Time history of axial stiffness is shown in Figure 17 for $L = 40a$ and different values of $\kappa^{(1)}/\kappa^{(2)}$, i.e., $\kappa^{(1)}/\kappa^{(2)} = 0.1$, 1 and 10, to demonstrate the influence of permeability on Kv. Numerical results indicate that consolidation process is accelerated when the ratio of $\kappa^{(1)}/\kappa^{(2)}$ is increased as the permeability of the top layer is higher. This is also true for all types of pile and all values of Ep^*. Figures 18 and 19 show the effect of the ratio $\kappa^{(1)}/\kappa^{(2)}$ on the axial stiffness for $L = 40a$. Numerical results indicate that the time dependent responses of axial stiffness for different values of $\kappa^{(1)}/\kappa^{(2)}$ are different. It is also found that axial stiffness increases when the pile stiffness is increased for all values of $\kappa^{(1)}/\kappa^{(2)}$.

Figures 20 and 21 present the time history of axial stiffness for an axially loaded pile group in a multi-layered poroelastic medium with rigid base as shown in Figure 12(b) for different ratio of H/L. Note that H is the depth of rigid base and $\kappa^{(1)}/\kappa^{(2)} = 0.1$ and $L = 40a$ in the numerical results presented in these two figures. It is found that an increase in the depth of rigid base results in the reduction in the axial stiffness, Kv. It is also found that the depth of rigid base has significant influence on the time dependent behavior of axial stiffness for a rigid pile than that of the flexible one.