Chapter II

Theory

2.1 Mechanisms of Agitated Mixing System

In all liquid-mixing devices, it is necessary to have two elements. Firstly, there must be overall bulk or convective flow so that no stagnant regions exist within the device. Secondly, there must be an intensive or high-shear mixing region which is capable of providing the reduction in inhomogeneities or rate process enhancement required by the duty. Both these elements require energy to sustain them. The proportion of energy going to each depends on the particular application and, whatever its distribution, the mechanical energy is finally dissipated as heat.

As in most fluid flow situations, the regimes may be characterized as laminar or turbulent. In mixing vessels there is a substantial region between the two, representing a transition from one to the other. It is convenient to describe mixing mechanisms under either laminar or turbulent-flow conditions because they are quite different in most respects.

2.1.1 Laminar mixing

Laminar flow is associated normally with high-viscosity liquids. At typical rates of energy input, viscosities greater than about 1 Poise are required if the flow is to be truly laminar. Fluids of such a high viscosity are often rheologically complex too.

Under laminar flow conditions, inertial forces quickly die out under the action of the high viscosity. Therefore, rotating impellers must occupy a significant proportion of the vessel if adequate bulk motion is to be achieved. Close to these rotating surfaces, large velocity gradients exist. These are laminar regions of high shear rate which can cause fluid elements to be deformed and stretched. As they do so, they continually thin and elongate. Each time the elongated elements pass through such a high shear rate region, they undergo a similar process but on a smaller scale. Figure 2.1 shows these effects diagrammatically.

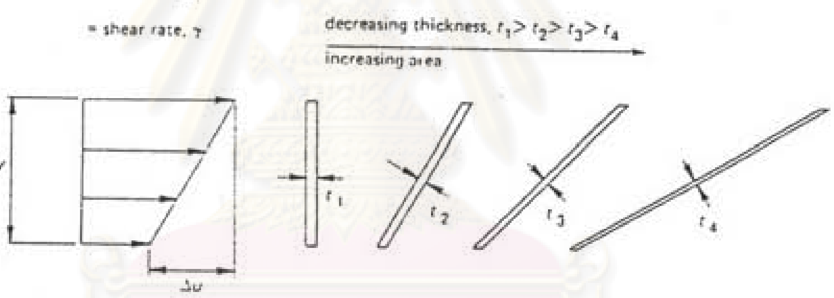

Figure 2.1 The thinning of fluid elements due to laminar shear flow

Associated with shear flows, in most practical situations, there are generally extensional flows, i.e. flows where the velocity in particular direction increases in that direction. Figure 2.2 shows a simple example of such a flow. Again, the process leads to an increase in surface area and a reduction in thickness of the element [5].

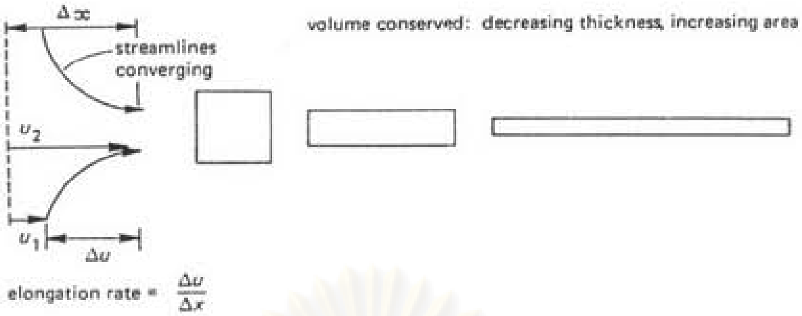

Figure 2.2 The thinning of fluid elements due to extensional flow

If there are solid agglomerates present within the high-viscosity fluid in either of the above flow fields, they experience high shear stresses which can lead to agglomerate break-up and homogenization. Similarly, in the creation of emulsions, foams and batters, it is these stresses resulting from shear and elongation which bring about a reduction in droplet or bubble size.

Of course, molecular diffusion is occurring all the time. However, until the fluid elements are sufficiently small, their specific surface area is not great enough for the rate of diffusion to be significant. On the other hand, for ultimate homogenization of miscible fluids to occur, molecular diffusion is required. With highly viscous fluids, this is a slow process as molecular diffusivity is itself inherently slow in such cases.

In laminar flow, a similar mixing mechanism arises when laminar shear occurs in a concentric cylinder geometry; this is indicated diagrammatically in Fig.2.3. Thus, each revolution leads to a further reduction in the thickness of smaller elements and redistributing them; Fig.2.4 indicates this process. Laminar mixing in in-line mixers is very largely a result of this mechanism.

In all the cases discussed above, the size or scale of pure fluid elements decreases progressively as mixing progresses due to flow or re-distribution. At the same time, the difference in concentration between the different elements is reduced due to molecular diffusion, especially as the area available for diffusion is increased with the decreasing size of the element.

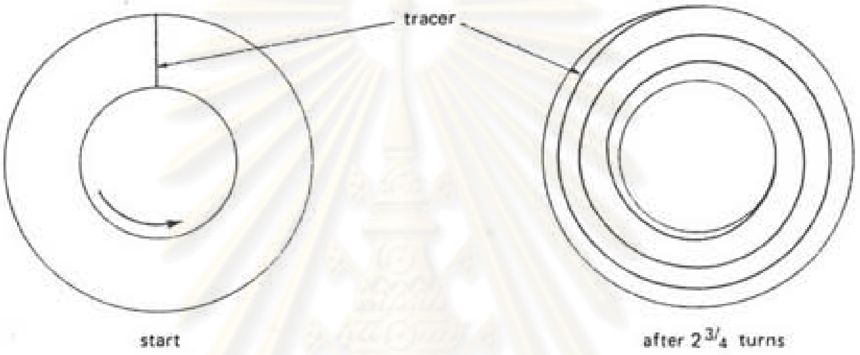

Figure 2.3 Fluid element thinning due to rotational shear

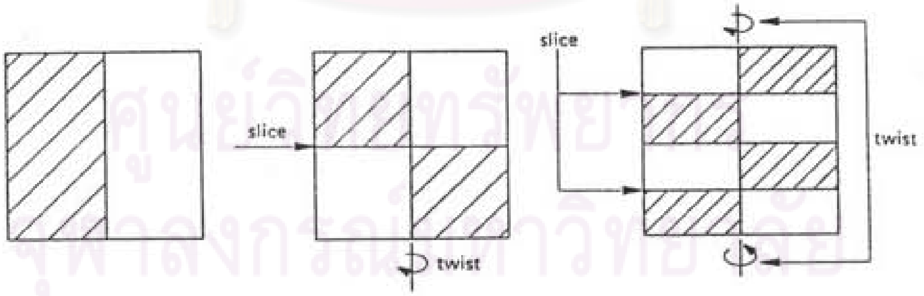

Figure 2.4 The concept of distributive mixing due to cutting and folding

2.1.2 Turbulent mixing

For all practical purposes, the bulk fluid flow in mixing vessels containing rotating impellers is turbulent if the fluid viscosity is less than about 1 mPoise. The inertia imparted to the fluid by the rotating impeller is sufficient to cause it to circulate readily throughout the vessel and back to the impeller again. During the fluid's passage, turbulent eddy diffusion takes place, though it is at its maximum in the impeller region. Eddy diffusion leads to mixing which is much more rapid than the rate of mixing associated with the mechanisms of laminar flow. Once again, for homogenization to occur at the molecular scale, molecular diffusion must occur. However, in these low-viscosity fluids, molecular diffusion is much faster than in high-viscosity materials. So the overall mixing process right down to the molecular level in turbulent flow is much more rapid than it is in laminar flow.

The rate of mixing in turbulent flow is greatest close to the impeller. Here, there is a high shear rate due to the trailing vortices associated with disc turbine impellers [6] and in addition there are large Reynolds stresses [7] in the radial discharge stream. Furthermore, a high proportion of the energy introduced by the impeller is dissipated here [8]. Thus the rate of homogenization of miscible liquids is greatest in this region and gas and liquid-liquid dispersion occurs predominantly here.

The complexity of the turbulent flow in mechanically agitated vessels, particularly because of the three-dimensional flow field, makes an analysis extremely difficult. However, provided the Reynolds number of the main flow is high enough, Kolmogoroff's theory of local isotropic turbulence can be used to give some insight into its structure [9]. Turbulent motion can be considered as a superposition of a spectrum of velocity fluctuations and eddy sizes on an overall mean flow. The large primary eddies have large velocity fluctuations of low frequency and are of a size comparable with the physical dimension of the impeller diameter, D. They are anisotropic and contain the bulk of the kinetic energy. Interaction of the large eddies with slow-moving streams produces smaller eddies of high frequency which further disintegrate until finally they are dissipated into heat by viscous forces. There is a transfer of kinetic energy down the scale from larger eddies to smaller eddies, the directional elements of the main flow being progressively lost in the process.

Kolmogoroff [10] argued that, for large Reynolds numbers, the smaller eddies are independent of the bulk motion and are isotropic. The properties of these eddies are firstly a function of the local energy dissipation rate/unit mass, ε_T. Below the eddy size, λ_k, at which viscous dissipation occurs, their properties also depend on viscosity. There is therefore an equilibrium established which contains a very wide range of eddy sizes, the universal equilibrium range. With the larger of these eddies ($D_T >> \lambda_T >> \lambda_k$), energy passes from bigger to smaller eddies without dissipation. This is the inertial sub-range. The size below over which viscous dissipation occurs is the viscous sub-range. Figure 2.5 shows this structure. An eddy Reynolds number, Re_k, is now defined for the Kolmogoroff length scale as

$$Re_k = \lambda_k V_k / \nu \qquad (2\text{-}1)$$

Conceptually the Reynolds number represents the balance of inertial to viscous forces; and within the spectrum of sizes the Kolmogoroff scale is the size where these two forces are in balance. Therefore, it is convenient to quantify eddy Reynolds numbers in general in relation to Re_k by the definition $Re_k = 1$

From dimensional reasoning, the Kolmogoroff length scale is defined as

$$\lambda_k = \left(\nu^3 / \varepsilon_T \right)^{1/4} \tag{2-2}$$

and the velocity scale v_k associated with it is

$$v_k = \left(\nu \, \varepsilon_T \right)^{1/4} \tag{2-3}$$

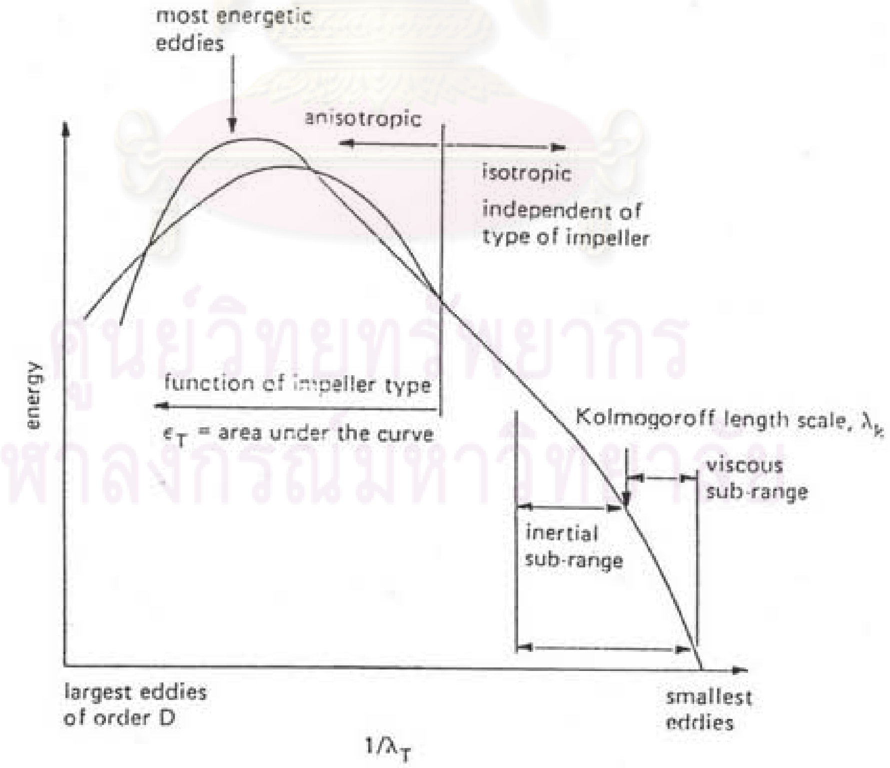

Figure 2.5 Spectrum of eddy sizes and their energy in turbulent flow

By implication, therefore, processes which are particularly dependent on turbulent eddies and their associated forces are likely to be well correlated by energy dissipation rate. Bubble information and micro mixing phenomena, for instance, fall into this category. However, non-homogeneous nature of stirred tanks turbulence is significant, e.g. solid suspension and solid-liquid mass transfer, are not well correlated that way.

Measurements have been made of turbulence structure by a number of worker, e.g. Molen *et al.* [11] using laser-Doppler methods and Rao and Brodkey [12] using hot-film anemometry. However, little progress has been made to date for design purposes.

2.2 Equipment and Operation

The hydrodynamics of agitated mixing system may be affected from many variables. However the main factors can be described as follows,

2.2.1 Agitator

2.2.1.1 Types of Impeller

Impellers can be classified as two general types: axial flow and radial flow. An axial-flow impeller is one in which the principal locus of flow occurs along the axis of the impeller (parallel to the impeller shaft). A radial-flow impeller discharges flow along the impeller radius in distinct patterns.

a) Axial-flow Impellers

Axial-flow impellers include the propeller, often designed on the basis of the screw theory, which requires a constant pitch across the face of the blade. This mean there is a continuous increase in blade angle from the blade tip to the hub (Fig.2.6)

Figure 2.6 Typical marine-type impeller

Pitch-to-diameter ratio (or simply "pitch ratio") is equal to the distance, in impeller diameters, that an impeller would advance for each revolution when rotated in a fluid body. For example, if the pitch ratio of a propeller is 1.0 (often called "square pitch"), it means that it would generate a path equal in length to its diameter for each propeller revolution. The pitch ratio of most axial-flow impellers is usually between 0.5 and 1.5 times the diameter. Figure 2.8 is a schematic of flow from another constant-pitch impeller (Fig.2.8 and Fig.2.9) has a constant blade angle. This means the pitch varies continuously from blade tip to hub. An impeller of this type produces a variable pitch-velocity discharge and variable shear rates across the impeller diameter, but it lends itself more easily to using fabricated materials. It also provides many production and maintenance economies, especially on full-scale equipment.

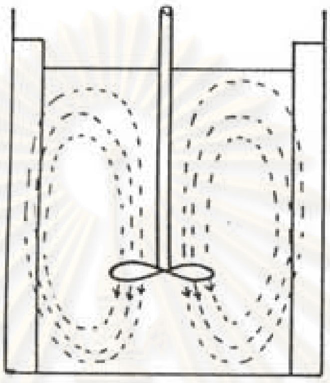

Figure 2.7 Axial flow pattern for propeller

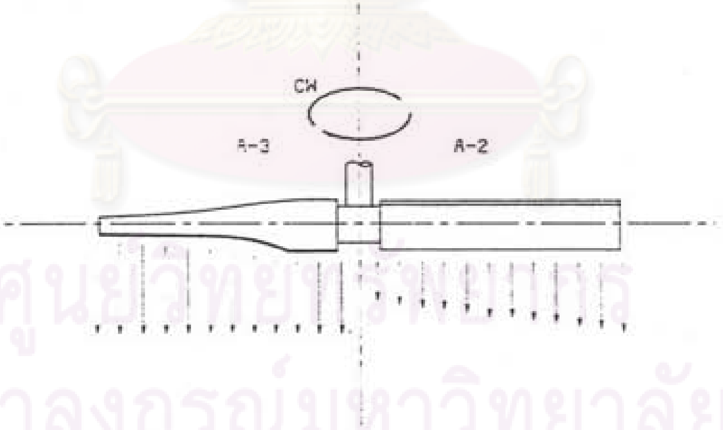

Figure 2.8 Schematic: pitch line velocity for A-2 type impeller (right), constant angle, variable pitch, and A-3 impeller (left), constant pitch, variable angle.

Propellers draw less power than most other impellers of the same diameter, running at the same speed. Therefore, compared with other impellers, propellers must run at much higher speeds to achieve a given horsepower and a particular pumping capacity. This results is low torque at a given power level and also results in a very economical series of mixers, called "portables." Portable mixers are so named because in the smaller sizes they are easily moved about; larger sizes, however, require mechanical means to carry them or to move them from one position to another. This kind of mixer is usually available in fractional and integral horsepower ranges, to a maximum of 5 Hp (Fig.2.10).

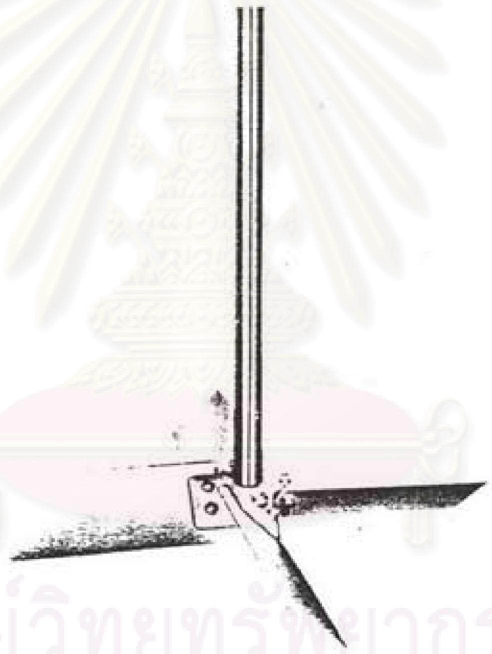

Figure 2.9 Typical axial-flow turbine

At higher horsepower, portables give way to fixed-mounted mixers with gear-boxes (speed reduced). These usually run at much lower output speeds than portables and at higher horsepower-to-speed ratios (higher torque for a given horsepower). This also means larger axial-flow impellers are required for a given horsepower. Higher torque and larger impellers notwithstanding, these mixers provide high process efficiencies and excellent overall mechanical and operating characteristics. A modification of the axial-flow impeller in Figure 2.9 is another axial-flow impeller shown in Figure 2.11. The blade is fabricated with a series of three different blade angles, approaching the shape of a propeller.

Qualitatively, mixing processes normally need both flow and various levels of fluid shear rate and turbulence. Thus, the design criterion which maximizes flow at the expense of all other head, shear, or turbulence losses is not applicable in every mixing situation. These other quantities may well be important for successful accomplishment of the process. This is in contrast to the design of pumps, where pumping capacity, system head developed, and hydraulic efficiency are the first criteria. The mixing impeller is not normally confined in a casing or a channel as with a pump, and rigorous definition of the fluid mechanics parameter around the mixing impeller is more complex.

It is also possible, and quite common, to put axial-flow impeller in draft tubes,[*] which define a total top-to-bottom circulation pattern in a mixing tank that can be used effectively for many kinds of flow-controlled processes. In this case, there is a defined channel or conduit; so measurements of impeller pumping efficiency are meaningful and can be used for overall design.

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[*] A draft tube is a circular duct which is used to direct flow to and from the impeller. It is usually a vertical, cylindrical tube with diameter slightly larger than the impeller diameter and a height less than the height of the fluid in the tank.

Figure 2.10 Portable mixer (direct drives)

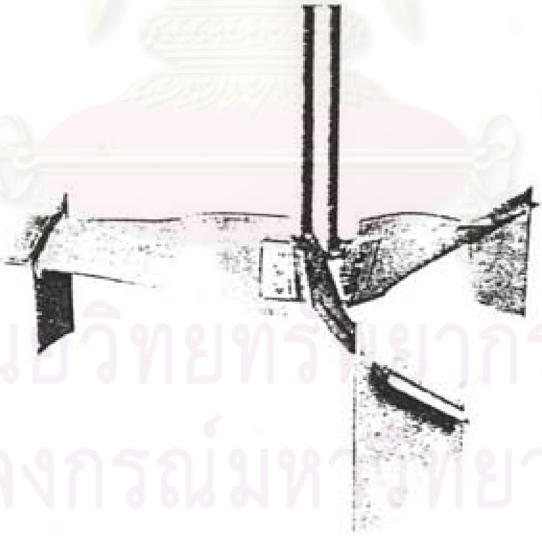

Figure 2.11 Fabricated A-3 impeller with stabilizing fins, variable angle, essentially constant pitch.

b) Radial-flow Impellers

Radial-flow impellers may either have a disc (Fig.2.12) or be open (Fig.2.13) and may have either flat or curved blades. Open impeller types (without the disc) do not normally pump in a true radial direction since there is a pressure difference between each side of the impeller. They tend to pump upward or downward while discharging radically.

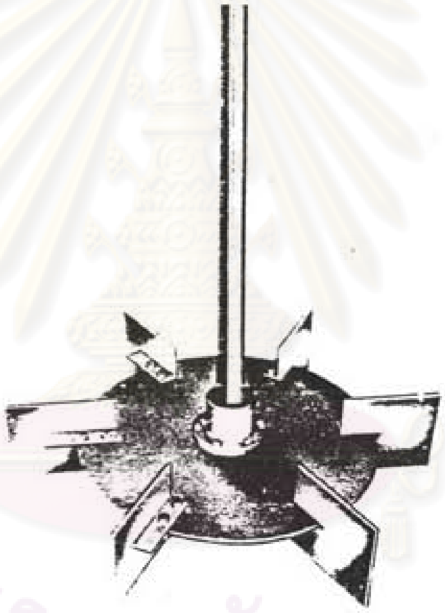

Figure 2.12 Flat-blade turbine

Disc-type radial impellers do tend to pump in a more radial direction, although at close clearances at the bottom of the tank or at close proximity to the liquid surface, and with close spacing to adjacent disc impellers, their radial pumping capacities are modified.

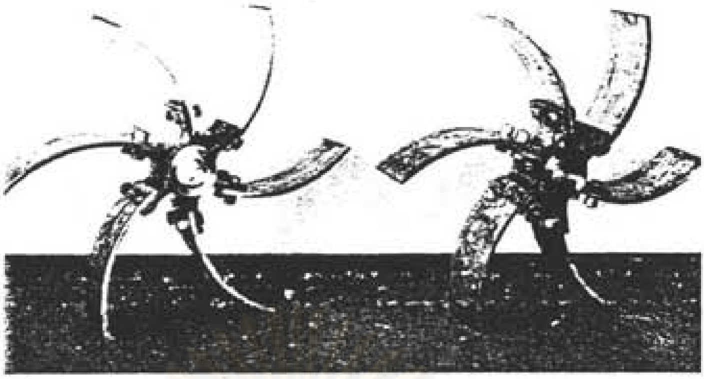

Figure 2.13 Spiral backswept turbines

Because of the more uniform radial-flow pattern, disc impellers tend to draw more power than open impellers, which affects the economy of their application. The disc impeller also characteristically prevents gas bubbles from passing through the low shear zone around the hub. Therefore, they have been used essentially for gas-liquid-type mixing processes. The use of large-diameter radial-flow impellers is best typified in the two- blade paddle, which is typical of solid suspension or blending applications where high flow and low shear rates are the requirement. These impellers normally operate at low speeds, because that is what the process condition usually requires. Low speed is further necessary because the two-blade impeller is mechanically more unstable than the more common four- to eight-blade impellers.

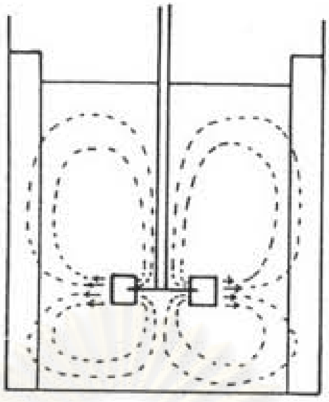

Figure 2.14 Radial flow pattern for disc turbine

2.2.2 Entering mixers

2.2.2.1 Side-entering mixers

Side-entering mixers are special types of mixers which have very exacting specifications and applicability. For homogeneous fluids, the flow pattern shown in Fig.2.15 is excellent; however, the mixer must be placed at the correct angle for the top-to-bottom flow pattern to properly develop. In large petroleum storage tanks (either gasoline or oil) power level are low, on the order of 0.05 hp / 1,000 gal (10 W / m^3). Under these conditions, the mixer shaft should be angled about 7 to 10° from the tank diameter. When looking from behind the mixer, if the impeller is rotating clockwise, the mixer must be angled to the left (Fig.2.16).

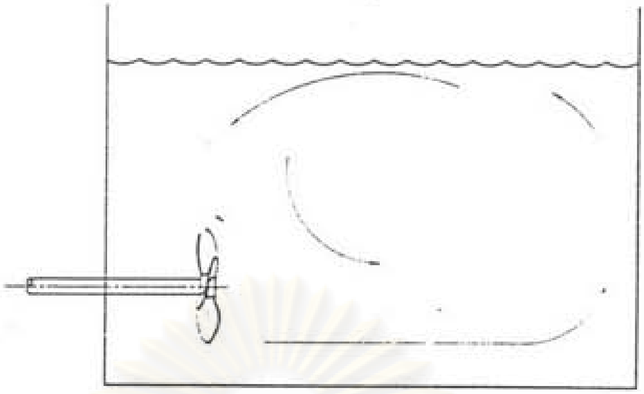

Figure 2.15 Typical side-entering flow pattern

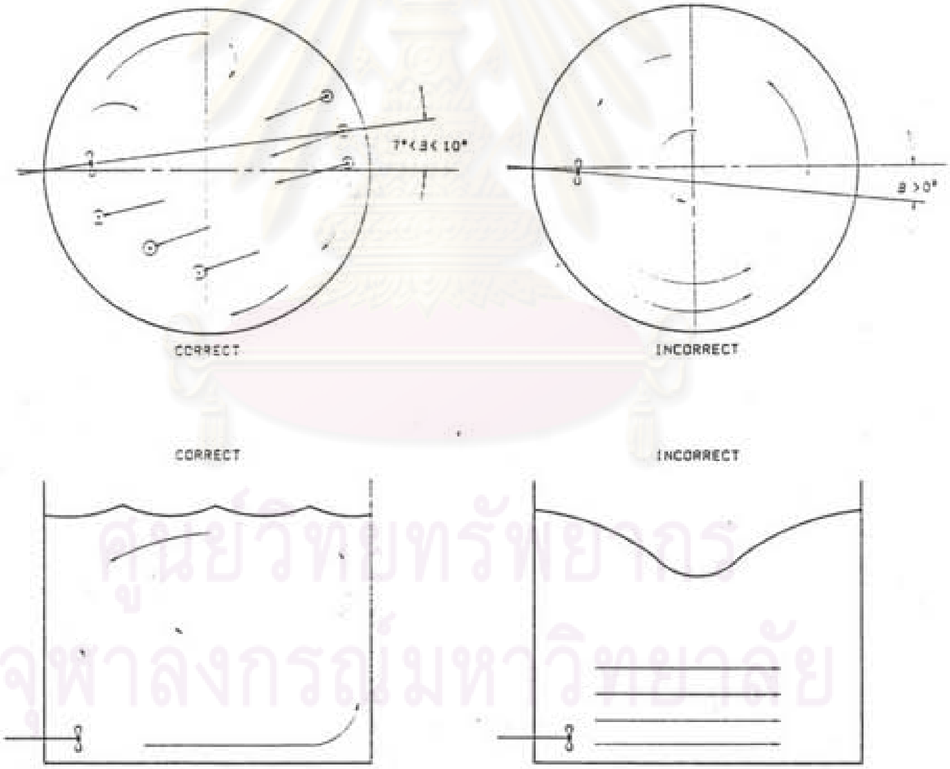

Figure 2.16 Correct angle to achieve flow pattern shown in Fig.2.15 vs. incorrect angle.

2.2.2.2 Top-entering mixers

The top-entering mixers are generally used as standard configuration. Axial-flow or Radial-flow impellers in unbaffled tanks containing low-viscosity fluids tend to swirl and produce vortices which are almost always undesirable. Installing baffles destroys the vortices and promotes flow pattern conductive to good mixing.

2.2.3 Baffle

If either the marine propeller or any of the impeller agitators are used in an unbaffled vessel containing low viscosity liquid(s), vortexing develops. The liquid swirls in the direction of the agitation rotation, causing a drop in liquid level around the agitator shaft. Vortexing increases with impeller speed until eventually the vortex passes through the agitator. The mixing efficiency of vortexing systems is usually lower than for geometrically similar nonvortexing systems.

Baffles must be provided for low-viscosity mixing when top-entry mixers are located on the tank centerline. This includes all types of mixers and impellers/ propellers.

Baffles assure that the entire batch will pass through that zone of the impeller where there is maximum agitation. Baffles also:

- Promote the flow pattern required for the process.
- Direct flow from the impeller, producing the required vertical, top-to-bottom currents.

- Change the flow from a rotary to a mixing pattern.
- Avoid excess swirling, vortexing and air induction.
- Greatly improve mixer-loading accuracy. The use of baffles yields more accurate data, resulting in better mixing correlations and predictable process results. Thus mixing loading-specifying any two of the parameters power, speed and tank diameter- is improved.
- Assure a stable, consistent power draw.
- Produce more uniform radial shaft loads, allowing for longer shaft lengths.

Vertical cylindrical tanks should be equipped with four baffles, 1/12 the tank diameter in width, extending vertically along the straight side of the tank and located 90° apart. Wider baffles provide slightly stronger vertical mixing currents, but may act as flow dampeners by reducing mass flow and rotary motion.

Fewer and narrower baffles allow for more rotary motion but also reduce impeller draw. This limits the energy that can be applied to the batch, because of swirling at high power levels.

When carryover from one batch to the next is undesirable, baffles should be self-cleaning. In waterlike batches, offsetting standard width baffles one-third of the baffle width from the tank wall permits the flow to scour the area behind the baffle. Unless the baffle is offset from the tank wall, the area immediately behind the baffle tends to be a dead spot. For higher viscosity batches, (i.e., $\mu \geq 10,000$ centipoise), the offset distance is one-half the baffle width. As a rule, for open top tanks, baffles should be made of the same thickness as the tank wall, but not less than 1/4 inch. Flow induced loads act on baffles requiring proper support structure. These loads are a function of the type of mixing impeller, tank diameter (D_T), speed (N), Hp, etc., and must be calculated by the mixer supplier. Once the supplier provides the loading on the baffles, the user can properly design an adequate baffle support structure.

Baffles can be eliminated for propeller mixers by mounting the units in an angular off-center position. Off-center top-entry mixers achieve good top-to-bottom and eliminate swirling. This is particularly effective for non-Newtonian fluids, such as paper stock, starch slurries, etc.

There is a limit to how much power can be applied to a top-entry mixer before vortexing begins, so such configurations are unsuitable for high-power applications. Power levels exceeding 10-15 Hp per 1,000 gallons of tank contents will produce vortexing, even in a baffled tank, in low viscosity fluids. Also, as the shaft rotates, an unbalanced unidirectional fluid force develops on the impeller. This causes stress reversal on the shaft, and the shaft diameter must be made larger as a result. As the shaft rotates, it goes through compression and tension on each revolution, subjecting it to fatigue.

Vortexing flow patterns will occur with top-entry mixers, mounted on the tank centerline, without baffles in waterlike viscosities. Vortexing will also occur in baffled tanks with highly powered machines. Some process results are best suited for vortexing flow regimes. These include certain slurry takedown applications with difficult to wet out solids (i.e., clays, talcs, starches, etc.), gas liquid mass transfer applications, where induction of the gas from above the liquid mass transfer applications, where induction of the gas from above the liquid level back into the liquid via vortexing will enhance the mass transfer.

Portable or fixed mount top-entry agitators, with or without stuffing boxes or mechanical seals, still allow for operating above or below the first natural frequency. However, for a given shaft diameter such as setup will not allow for the use of all shaft lengths.

Larger mixers employing radial and axial flow turbines are usually mounted in rigid support bases. These structures can be beams spanning the top of tanks, mounting flanges, or combinations of the two. Axial and radial flow impellers are typically larger in diameter than propellers and hence run at slower speeds in larger tanks. These impellers are more stable mechanically, but do require a larger motor.

A single radial flow turbine will produce two distinct mixing zones in a tank. Multiple radial flow impellers on a single shaft will produce a greater number of zones or stages in the tank. This type of flow distribution is beneficial to plug flow reactors or to promote holdup of reactants (particularly gas) in a specific zone. Zone agitation leads to non uniform distribution of reactants and temperature gradients throughout the stirred vessel. In a continuous flow scheme in a baffled tank being mixed with multiple radial flow turbines, where some degree of control from inlet to outlet is required, this configuration can be used along with proper location of the inlet and outlet to provide control of residence time and reduce short circuiting. Axial and radial flow patterns in baffled and unbaffled tank are shown in Fig. 2.17 and 2.18.

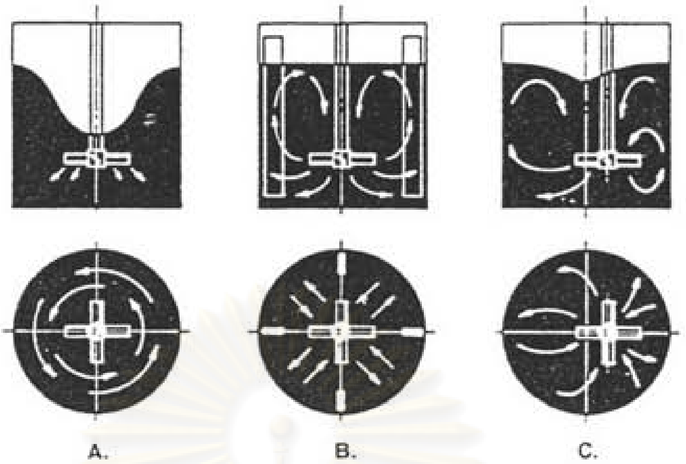

Figure 2.17 Axial turbine flow patterns

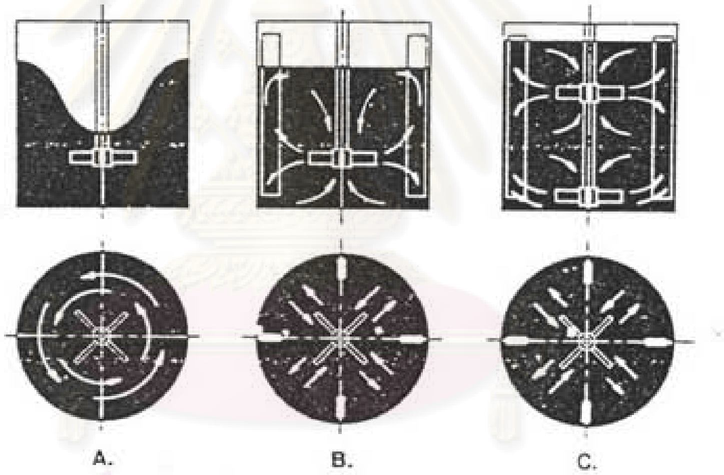

Figure 2.18 Radial turbine flow patterns

As liquid viscosity increases, the need for baffles to reduce vortexing decreases. When high viscosity liquids are mixed, baffle widths may be reduced to 1/20 tank diameter. With viscous liquids, baffles are most effective when positioned away from the tank wall or at an angle to the axis of symmetry of the vessel. A space of 1.0 baffle width is sufficient to allow liquid to move along the tank wall, thus avoiding stagnant areas behind the baffle. When turbines and marine propellers are used to mix liquids of viscosities greater than 20,000 cP, baffles are not required. The liquid's natural resistance to flow becomes self-baffling and dampens vortexing due to liquid swirl.

2.2.4 Power

A mixer is essentially a pump, although not a very efficient one, and has many of the same characteristics of a pump. The most fundamental of these is that the power applied by any mixing impeller produces a pumping effect and a velocity head which is expressed as

$$P \propto QH\rho \tag{2-4}$$

where P = impeller power input

Q = flow rate or pumping capacity

H = velocity head

ρ = density of the fluid

In Eq. (2-4) either the flow component Q or the component H can be stressed such that a large flow, small head, or a large head, small flow, may be produced for the same power input P. (In mixing technology the impeller head can be used synonymously with shear or turbulence.) All of the power a mixer supplies to a fluid produces flow and head (or shear).

Impeller power consumption is a function of the impeller speed N, diameter D, the impeller design itself, and a number of mixing environmental factors including:

1. Physical properties of the fluid medium.

2. Vessel size and geometry, or shape.
3. Impeller location relative to vessel and fluid boundaries and relative to other impellers or obstructions in the mixing vessel.
4. The presence or absence of baffles, their design, and location.

These are the major variables of practical significance, although some investigators have reported other important factors in special cases. For example, Flynn and Treybal found that the absence of an interface (where a closed vessel is a completely filled with fluid) required wider baffles to produce the same power response obtained in a vessel containing the same fluid with an air-liquid interface, or free surface.

The calibration of geometrically similar series or family of mixing impellers determines the effect of the mixing environment on the horsepower drawn for that particular impeller design. This is necessary for the evaluation of pilot plant studies and to assure proper loading of plant-size (full-scale) mixers. In order to minimize the number of variables, a convenient system configuration is selected. Thus, the power consumption of any impeller design can be readily compared to any other design on the basis of similar systems.

The effect of impeller speed and diameter If the power response of a given impeller is measured at various speeds in water, the resulting data can be plotted as in Fig.2.19. The slope of the line is 3.0, which indicates that

$$P \propto N^3 \tag{2-5}$$

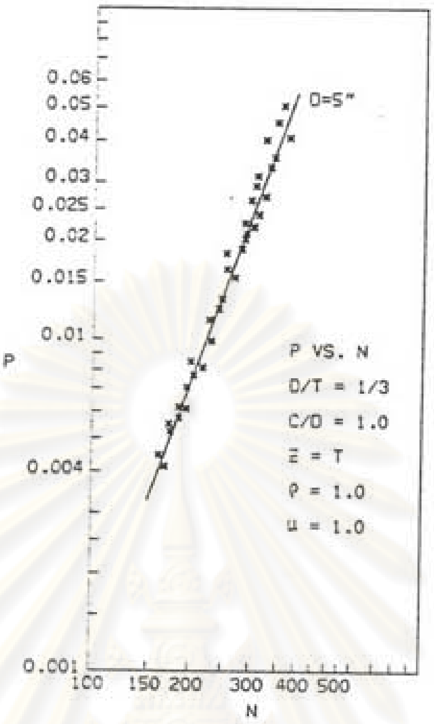

Figure 2.19 Power vs. rpm for typical 5-in. Impeller

If different impellers of the same design are run in water and in proportionally large tanks. An analysis of these data shows that *P* varies as impeller diameter to the fifth power:

$$P \propto D_i^5 \qquad (2\text{-}6)$$

Combining Eqs. (2-5) and (2-6), then

$$P \propto N^3 D_i^5 \qquad (2\text{-}7)$$

It can be shown that impeller power draw varies directly with specific gravity or density of the fluid:

$$P \propto \rho \tag{2-8}$$

This was originally done by operating a convenient size impeller at constant speed in various fluids of similar viscosity and widely different densities. Combining Eqs. (2-7) and (2-8),

$$P \propto N^3 D^5 \rho \tag{2-9}$$

Power Number-Reynolds Number Relationships

A Number of investigators have reported impeller power characteristics in terms of the dimensionless quantities power number N_p and Reynolds number N_{Re}. Equations for these quantities may be written

$$N_p = \frac{1.523 \times 10^{13} P}{N^3 D^5 \rho} \tag{2-10}$$

$$N_{\mathrm{Re}} = \frac{10.754 N D^2 \rho}{\mu} \tag{2-11}$$

where P = impeller horsepower

N = impeller speed, rpm

D = impeller diameter, in.

ρ = fluid specific gravity

μ = fluid viscosity, cP

The constants in Eqs. (2-10) and (2-11) are required to produce absolute values of N_p and N_{Re} when the appropriate units are used as defined above. Similarly, using SI derived units,

$$N_p = \frac{2.158 \times 10^{17} P_1}{N^3 D^5 \rho} \tag{2-12}$$

$$N_{\mathrm{Re}} = \frac{1.667 \times 10^{-5} N d^2 \rho}{\mu_0} \tag{2-13}$$

where P_1 = impeller power, W

N = impeller speed, rpm

d = impeller diameter, mm.

ρ = fluid specific gravity

μ_0 = fluid viscosity, Pa sec

The mixing Reynolds number N_{Re} was developed from the Reynolds number relationship for flow in pipes:

$$\mathrm{Re} = \frac{DV\rho}{\mu} \tag{2-14}$$

where V = fluid velocity

D = inside pipe diameter

Typical power number-Reynolds number relationships for two different impeller designs in a baffled tank are provide in Fig.2.20. The shapes of the curves are similar to the friction-factor curve, the N_p - N_{Re} relationship for mixing impellers can be divided into three fluid zones: laminar, transitional, and turbulent.

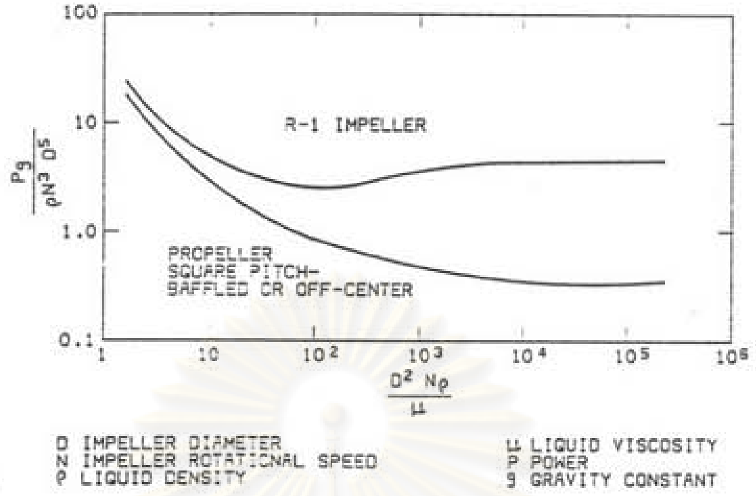

Figure 2.20 Typical power number vs. Reynolds number curve

2.2.5 The Age Distribution of Fluid Leaving a Vessel and The Mixing Time

It is evident that elements of fluid taking different routes through the reactor may require different lengths of time to pass through the vessel. The distribution of these times for the stream of fluid leaving the vessel is called the exit age distribution E, or the residence time distribution RTD of fluid.

It is convenient to represent the RTD in such a way that the area under the curve is unity, or

$$\int_0^\infty \mathrm{E}\, dt = 1 \qquad (2\text{-}15)$$

This procedure is called normalizing the distribution, and Fig.2.21 shows this distribution in normalized form.

With this representation the fraction of exit stream of age* between t and $t+dt$ is E dt. The fraction younger than age t_1 is $\int_0^{t_1} \mathrm{E}\,dt$.

Whereas the fraction of material older than t_1, shown as the shaded area in Fig.2.21, is

$$\int_{t_1}^{\infty} \mathrm{E}\,dt \quad = \quad 1-\int_0^{t_1} \mathrm{E}\,dt \qquad (2\text{-}16)$$

The E curve is the distribution needed to account for nonideal flow. Other related distributions are mentioned later.

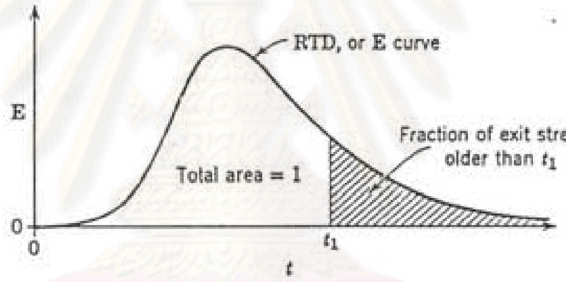

Figure 2.21 The exit age distribution curve E for fluid flowing through a vessel; also called residence time distribution, or RTD.

The F Curve

With no tracer initially present anywhere impose a step input of tracer of concentration C_o on the fluid stream entering the vessel. Then a time record of tracer in the exit stream from the vessel, measured as C/C_o, is called the F curve. Figure 2.22 sketches this curve and shows that it always rises from 0 to 1.

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* The term "age" for an element of the exit stream refers to the time spent by that element in the vessel.

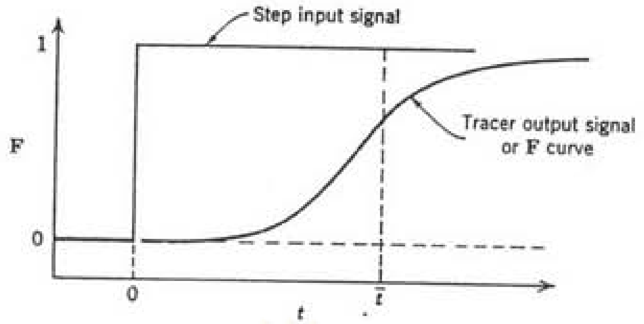

Figure 2.22 Typical downstream signal, called the F curve, in response to an upstream step input signal.

The C Curve

With no tracer initially present anywhere impose an idealized instantaneous pulse of tracer on the stream entering the vessel. Such an input is often called a delta function or impulse. The normalized response is then called the C curve.

To perform this normalization we divide the measured concentration by Q, the area under the concentration-time curve. Thus we have on normalization.

$$\int_0^\infty \mathrm{C}\,dt = \int_0^\infty \frac{C}{\mathrm{Q}}\,dt = 1 \qquad \text{where} \qquad \mathrm{Q} = \int_0^\infty C\,dt \quad \text{(2-17)}$$

Figure 2.23 shows the C curve and its properties.

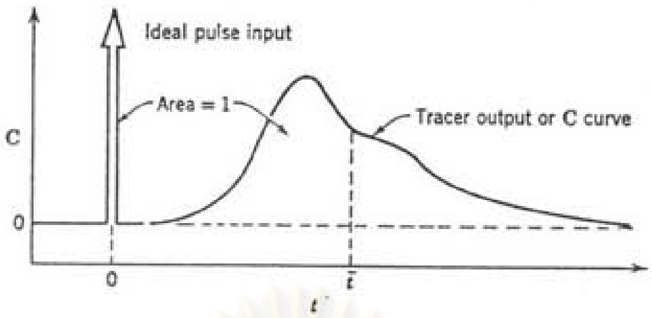

Figure 2.23 Typical downstream signal, called the C curve, in response to an upstream δ-function input signal.

Relation Among the F, C, and E Curves and "Mean Time" for Closed Vessels

The closed vessel being defined as one in which fluid enters and leaves solely by plug flow, thus with a flat velocity profile. Varying velocities, back diffusion, swirls, and eddies are not permitted at the entrance and exit. Real vessels often reasonably satisfy this assumption.

To relate E with C for steady-state flow note that the RTD for any batch of entering fluid must be the same as for any leaving batch. If this were not so, material of different ages would accumulate in the vessel, thus violating the steady-state assumption.

For the following experiment, at time t = 0 a pulse of tracer fluid is introduced into the stream entering the vessel. The C curve for the tracer then records when these molecules leave, in other words, their distribution of ages. Since the C curve represents the RTD for that particular batch of entering fluid, it must also be the RTD for any other batch, in particular, any batch in the exit stream. We have

$$C = E \tag{2.18}$$

Thus the C curve gives directly the exit age distribution.

To relate E with F, for a steady flow of fluid entering the vessel, at time t = 0 switch to tracer fluid and record the rising concentration of tracer fluid in the exit stream, the F curve. At any time t > 0 tracer fluid and only red fluid in the exit stream is younger than age t. Thus we have

$$\begin{pmatrix}\text{fraction of tracer fluid}\\ \text{in the exit stream}\end{pmatrix} = \begin{pmatrix}\text{fraction of exit stream}\\ \text{younger than age } t\end{pmatrix}$$

But the first term is simply the F value, while the second is given by $\int_0^{t_1} E\,dt$. So we have, at time t,

$$F = \int_0^{t} E\,dt \tag{2-19}$$

and on differentiating

$$\frac{dF}{dt} = E \tag{2-20}$$

Finally consider the term "mean time" of fluid in the vessel. Since this section only treats the steady state flow of constant density we have

$$\bar{t} = \frac{V}{v} = \text{holding time} = \text{mean residence time} = \text{space - time} \tag{2-21}$$

It may seem reasonable to expect that the mean of the E curve is given by $\bar{t}$ however this has yet to be shown. We do this by examining the contents of a vessel at time t = 0. We may thus write

$$\begin{pmatrix} \text{total volume of} \\ \text{fluid in the} \\ \text{vessel at } t = 0 \end{pmatrix} = \sum_{\text{all time}} \begin{pmatrix} \text{volume of fluid} \\ \text{which had entered} \\ t \text{ to } t + dt \\ \text{seconds earlier} \end{pmatrix} \begin{pmatrix} \text{fraction of this fluid} \\ \text{which stays more than} \\ \text{about } t \text{ seconds in the} \\ \text{vessel} \end{pmatrix}$$

In symbols and using Eq. (2-16) this word equation becomes

$$V = \int_0^\infty (v\,dt)\left(\int_t^\infty \mathrm{E}\,dt\right) \qquad (2\text{-}22)$$

or

$$\bar{t} = \frac{V}{v} = \int_0^\infty \left[\int_{t'}^\infty \mathrm{E}\,dt\right] dt' \qquad (2\text{-}23)$$

By changing the order of integration we find the desired result, or

$$\begin{aligned} \bar{t} &= \int_0^\infty \left[\int_0^t dt'\right] \mathrm{E}\,dt \\ &= \int_0^\infty t\,\mathrm{E}\,dt = \bar{t}_{\mathrm{E}} \end{aligned} \qquad (2\text{-}24)$$

We may summarize our findings as follows: at any time t

$$\mathrm{E} = \mathrm{C} = \frac{d\mathrm{F}}{dt} \quad \text{or} \quad \mathrm{F} = \int_0^t \mathrm{E}\,dt = \int_0^t \mathrm{C}\,dt$$

and

$$\bar{t} = \bar{t}_{\mathrm{C}} = \bar{t}_{\mathrm{E}} \qquad (2\text{-}25)$$

It is frequently desirable to characterize a distribution by a few numerical values. For this purpose the most important measure is the location of the distribution.

This is called the mean value or the centroid of the distribution. Thus for a C versus t curve the mean is given by

$$\bar{t} = \frac{\int_0^\infty t\, C\, dt}{\int_0^\infty C\, dt} \tag{2-26}$$

If the distribution curve is only know at a number of discrete time values t_i then

$$\bar{t} \cong \frac{\sum t_i C_i\, \Delta t_i}{\sum C_i\, \Delta t_i} \tag{2-27}$$

Properties of interest are examined as a function of the mixing time, and the required mixing time is the shortest time at which there is satisfactory assurance that the values of the properties are within selected limits. Decisions may be based on visual examination of a plot of time of sampling (abscissa and independent variable) for many operations. Evaluations may also be made at different operating conditions (for example, mixer speeds) and the effect of important operating variables determined by examining the required mixing time as a function of the operating variables.

AIChE Equipment Testing Procedure [13] recommended the selection of the mixing time of the curing-rate additive as the time required to achieve concentration of mixture within acceptable limit as shown in Figure 2.24 or to reach composition equilibrium by statistical analyzing as shown in Figure 2.25.

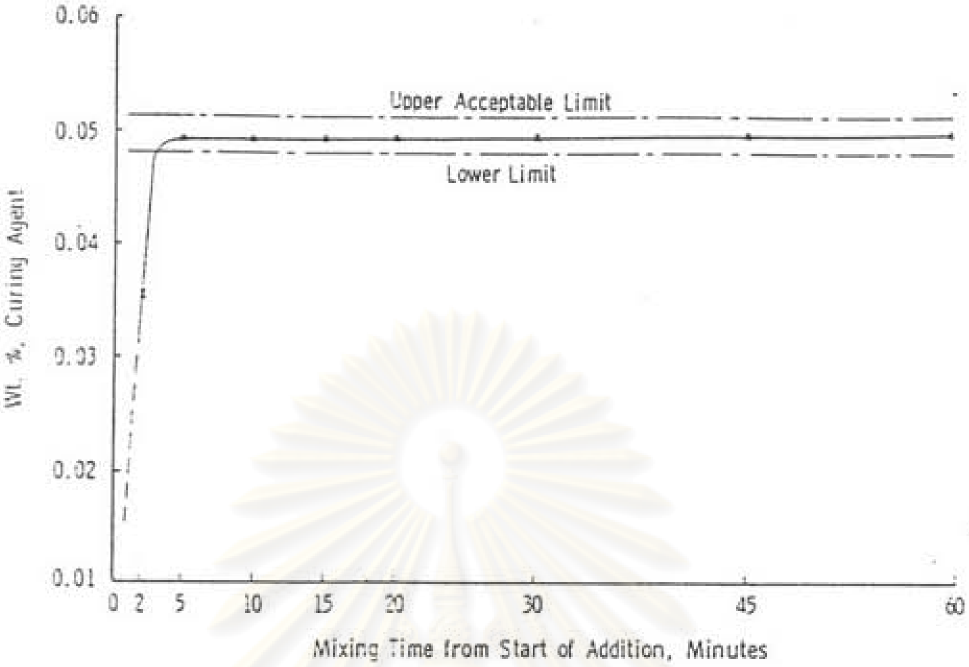

Figure 2.24 Composition vs. Mixing time in 150 gal. Dough mixer.

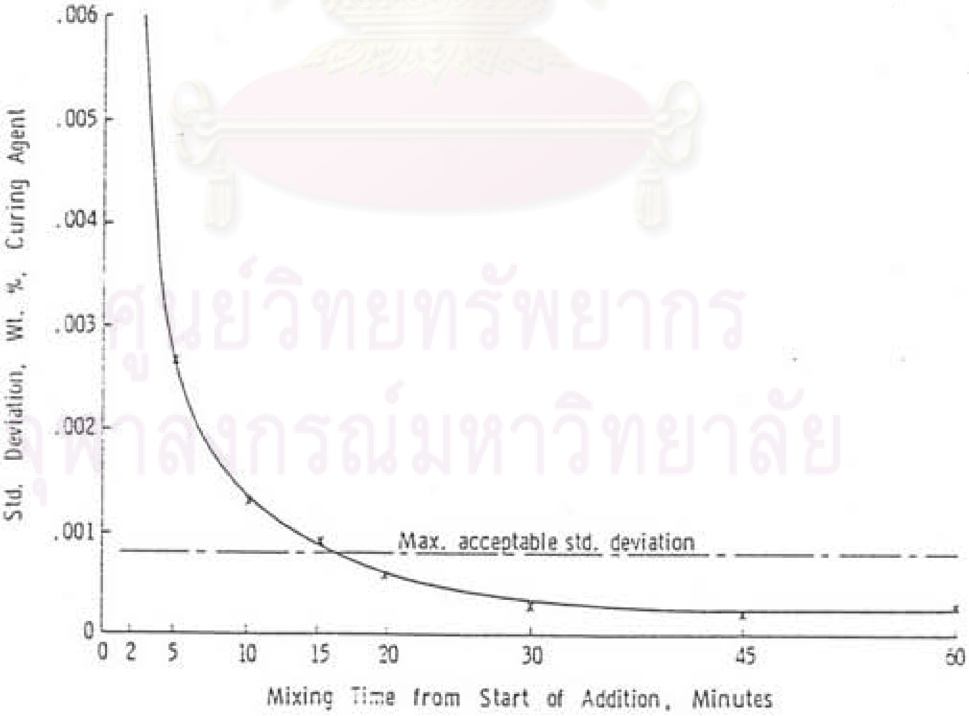

Figure 2.25 Standard variation vs. Mixing time in 150 gal. Dough mixer.

Kramers *et al.* [14] selected required mixing time as the time for concentration variations to become less than 0.1% of the average KCl concentration as shown in Figure 2.26.

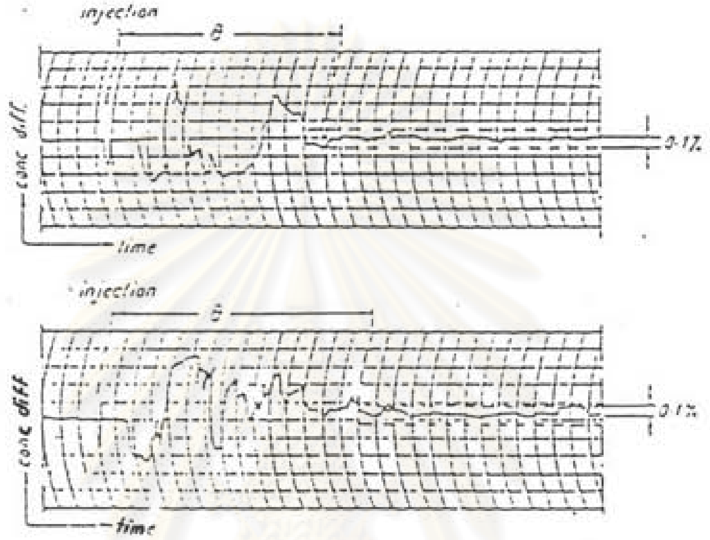

Figure 2.26 Method of determination mixing time

Noi *et al.* [15] selected required mixing time as the time to achieve variation of concentration of radioactive material (count rate) at both detectors less than 1 time standard diviation as shown in Figure 2.27 and Figure 2.28.

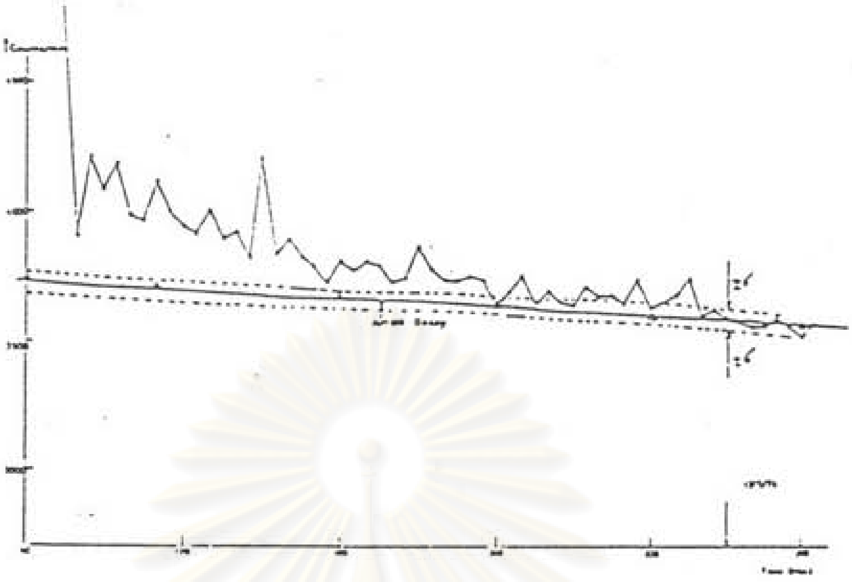

Figure 2.27 Scalar counts from detector No.1

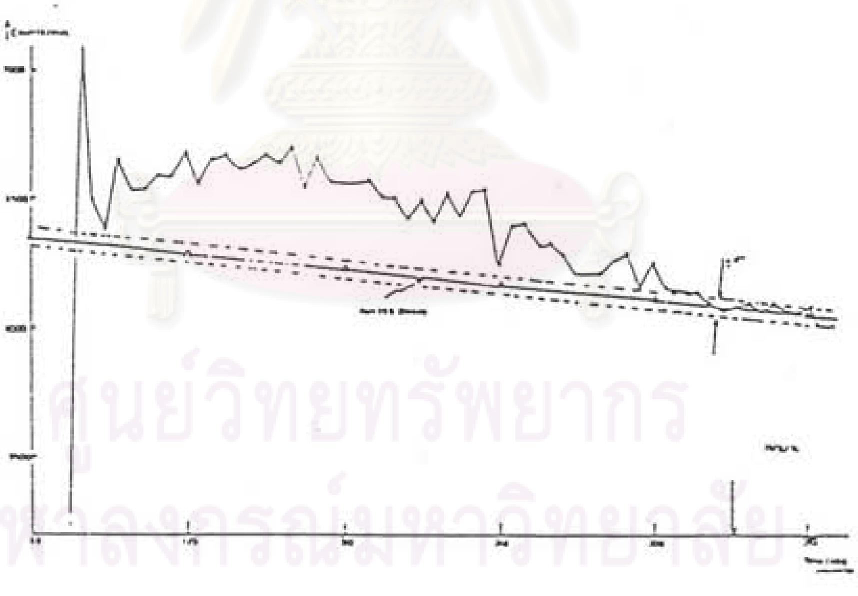

Figure 2.28 Scalar counts from detector No.2

2.2.6 Standard Tank Configuration

The vessel configuration, shown in Fig.2.29 and known as the Standard Tank Configuration [16], provides adequate mixing for most processing requirements found in industry.

However, it should be stressed that the Standard Tank Configuration is an arbitrary standard which in some circumstances is not the best configuration to use. For special processing conditions such as mixing liquids with a high solids content, a high viscosity or a shear sensitive character, this configuration may be impractical.

The Standard Tank Configuration has the following geometrical relationships:

1) The agitator is a 6 flat blade disc turbine impeller.
2) Impeller diameter $D_i = 1/3$ tank diameter D_T.
3) Impeller height from the tank bottom $H_i = 1.0$ impeller diameter.
4) Impeller blade width $q = 1/5$ impeller diameter.
5) Impeller blade length $r = 1/4$ impeller diameter.
6) Length of impeller blade mounted on the central disc $s = r/2 = 1/8$ impeller diameter.
7) Liquid height $H_l = 1.0$ tank diameter.
8) Baffle number = 4, vertical mounted at the tank wall and extending from the tank bottom to above the liquid surface.
9) Baffle width $W_b = 1/10$ tank diameter.

An analogous Standard Tank Configuration may be used with other agitator types.

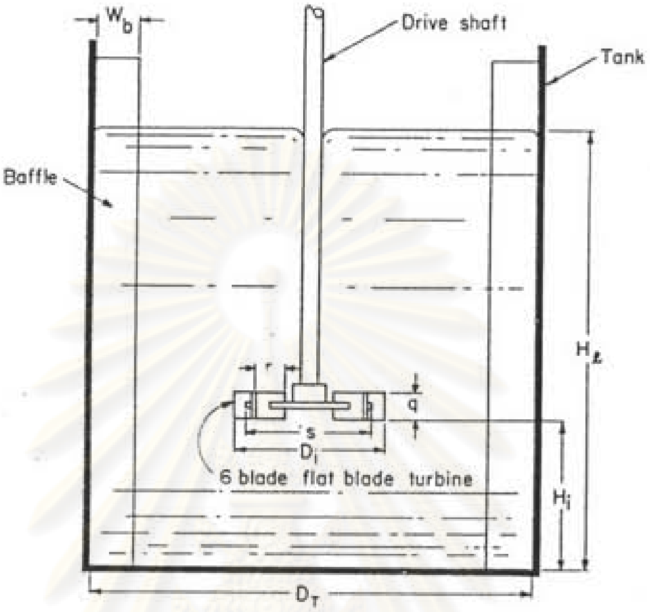

Figure 2.29 Standard tank configuration

2.2.7 Scale-up Analysis

In the process industries experimental data are often available on a laboratory-size or pilot-unit-size agitation system and it is desired to scale-up the results to design a full-scale unit. Since there is much diversity in processes to be scaled up, no single method can handle all types of scale-up problems, and many approaches to scale-up exist. Geometric similarity is, of course, important and simplest to achieve. Kinematic similarity can be defined in terms of ratios of velocities or of times. Dynamic similarity requires fixed ratios of viscous, inertial, or gravitational forces. Even if geometric similarity is achieved, dynamic and kinematic similarity cannot often be obtained at the same time. Hence, it is often up to the designer to rely on judgment and experience in the scale-up.

In many cases, the main objectives usually present in an agitation process are as follows: *equal liquid motion*, such as in liquid blending, where the liquid motion or corresponding velocities are approximately the same in both cases; *equal suspension of solids*, where the levels of suspension are the same ; and *equal rates of mass transfer*, where mass transfer is occurring between a liquid and solid phase, liquid-liquid phases, and so on, and the rates are the same.

A suggested step-by-step procedure to follow in the scale-up is detailed as follows for scaling up from the initial conditions where the geometric sizes given in standard configuration are D_{i1}, D_{T1}, H_1, q_1, and so on, to the final conditions of D_{i2}, D_{T2}, and so on.

1. Calculate the scale-up ratio R. Assuming that the original vessel is a standard cylinder with $D_{T1} = H_1$, the volume V_1 is

$$V_1 = \left(\frac{\pi D_{T1}^2}{4}\right)(H_1) = \left(\frac{\pi D_{T1}^3}{4}\right) \tag{2-28}$$

Then the ratio of the volumes is

$$\frac{V_2}{V_1} = \frac{\pi D_{T2}^3 / 4}{\pi D_{T1}^3 / 4} = \frac{D_{T2}^3}{D_{T1}^3} \tag{2-29}$$

The scale-up ratio is then

$$R = \left(\frac{V_2}{V_1}\right)^{1/3} = \frac{D_{T2}}{D_{T1}} \qquad (2\text{-}30)$$

2. Using this value of R, apply it to all of the dimensions in standard configuration to calculate the new dimensions.

3. Then a scale-up rule must be selected and applied to determine the agitator speed N_2 to use to duplicate the small-scale results using N_1. This equation is as follows:

$$N_2 = N_1\left(\frac{1}{R}\right)^n = N_1\left(\frac{D_{T1}}{D_{T2}}\right)^n \qquad (2\text{-}31)$$

where $n = 1$ for equal liquid motion, $n = 3/4$ for equal suspension of solids, and $n = 2/3$ for equal rates of mass transfer (which is equivalent to equal power per unit volume). This value of n is based on empirical and theoretical considerations.

2.2.8 Conductometric Method

Conductrometric method is the first electrical measurement applied to determine solubility product, dissociation constant and electrolyte's properties. Conductivity property is dependent on the number of ions because ions can conduct electricity. Ions have different capability in conducting electricity because of ions type, concentration and medium so the measured conductivity is not a specific amount for one ion in solution.

Conductance is the measured current occurred from the movement of positive ion to the cathode pole and negative ion to the anode pole have unit of electrical potential. The current occurs in according to the Ohm's law.

$$I = \frac{E}{R} \qquad (2\text{-}32)$$

when $E = 1$ $$I = \frac{1}{R} \qquad (2\text{-}33)$$

The measured I when $E = 1$ is Conductance (L);

$$L = \frac{1}{R} \qquad (2\text{-}34)$$

The flow of the current varies with the ion resistance in the solution. If the movement of the current flows highly, it means that the solution has low resistance so the conductance is inversed to the resistance (R). If the solution has the homogeneous substance, the resistance is varied to the length (l) and opposite to the cross sectional area (A) of the conducted matter.

$$R \propto \frac{l}{\mathrm{A}} \qquad (2\text{-}35)$$

$$R \propto \rho_{Resis} \frac{l}{\mathrm{A}} \qquad (2\text{-}36)$$

When ρ_{Resis} (specific resistance or resistivity (Ohm-cm.)) means the resistance of the matter with $l = 1$ cm. in length and 1 cm.^{2} in cross-sectional area.

Specific conductance (or conductivity, k) is the inversion of the specific resistance. The unit of conductivity is Mho/cm. or Siemen/cm. According to the following equations.

$$L = \frac{1}{R} = \frac{1}{\rho} \cdot \frac{\mathrm{A}}{l} \tag{2-37}$$

$$L = k \cdot \frac{\mathrm{A}}{l} \tag{2-38}$$

The length of the conducted matter or the solution (l) per cross-sectional area (A) means the distance between two poles of the cell per the cross-sectional area of the pole. This ratio is called cell constant (θ).

$$\theta = \frac{l}{\mathrm{A}} \tag{2-39}$$

$$L = \frac{1}{R} = \frac{k}{\theta} \tag{2-40}$$

The conductivity of the solution depends on the cell constant so the cell constant calibration is necessary for the conductivity determination.

2.2.9 Method of Measuring Mixer Performance

After injection of a small amount of tracer in an agitated liquid, concentration change and mixing time are measured in an appropriate location of the vessel. [17] The following methods were commonly used.

1. Using a small amount of electrolyte as a tracer, the concentration change was measured by a small probe. [18, 19, 20, 21]
2. After introducing warm liquid, the temperature change is measured with thermometers located in various positions. [22, 23]

3. After injection a dyed solution, the spreading patterns are followed by eye, by taking photographs or by using a detector comprised of photoelectric converter. [24]
4. After injection of a decolorizing agent as a tracer into a dyed liquid, the state of decolorization is traced. [25, 26]
5. Schlieren method which is a technique to trace the uniformity of solute concentration by a measuring the difference in refraction index. [27]

Figure 2.30 shows a measuring circuit for electric conductivity presented by Kramers *et al.* [27] and Figure 2.31 shows Schlieren technique which detects the concentration difference throughout the vessel.

It is essential to set the allowable degree of unevenness of solute concentration. Fig.2.32 shows the variation in observed mixing time θ_M with the difference of criteria for sufficient mixing. [28] From this diagram we can compare the sensivity of various methods in measuring mixing time.

In the decolorizing method, a reactant in the mother liquid and a reactant in a tracer undergo an instantaneous chemical reaction at their interface, so the observation of mixing process is replaced by the shift of the interface, and is different somewhat from other techniques.

A decolorizing method is conveniently used for detecting dead space, but is influenced markly by the ration in quantity of two reactants. In the case of neutralization reaction where alkaline solution with phenolphthalein is neutralized by acid, a tracer containing 20-30% excess acid compared with that equivalent to the base is adequate to give a steady mixing time. [29]

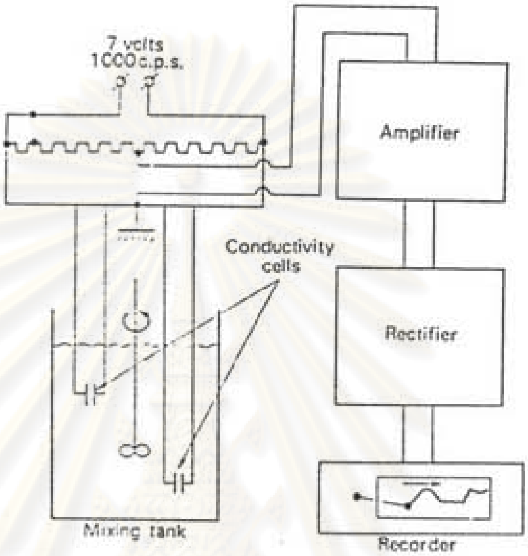

Figure 2.30 Method of measuring fluctuations by electrical conductivity.

The measuring method stated above is used for macro mixing. A direct method of measuring the micromixing state is hard to find. Merely by an improvement of electric probes, there is a limit of resolving power in 10-100 μ (scale), especially in high viscosity liquids the resolving scale becomes even coarser, because a stagnant layer may arise around the probes by adherence. Thus, for the measurement of micro mixing state, an optical method is recommended. As another approach, it is well-known that the rate of second order chemical reactions is influenced by the state of mixing. Using an inverse relation, we may be able to evaluate the degree of micro mixing from the rate of chemical reaction. [30, 31]

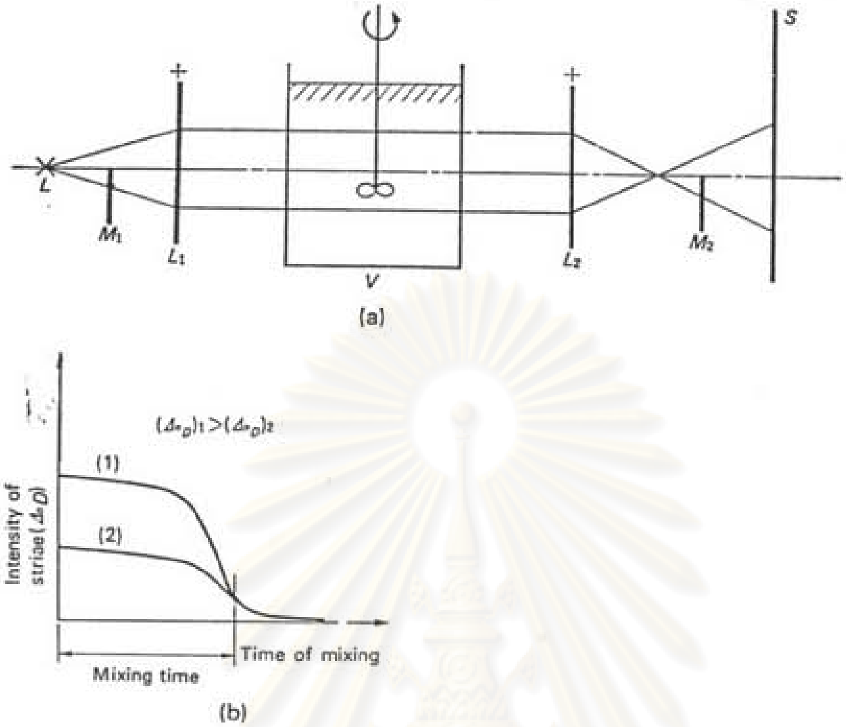

Figure 2.31 Experimental assembly by Schlieren method

Block diagram of instrument.

(a) L = Light source, L1 = First "Schlieren" lens, L2 = Second "Schlieren" lens, V = Mixing vessel with stirrer, S = Screen in plane of image of V, M1 = First knife-edge intercepting half of the beam, M2 = Second knife-edge (in plane of image of M1), intercepting the other half of the beam

(b) Measurement of mixing time
$(\Delta n_D)_1 > (\Delta n_D)_2$
Δn_D = Refractive index difference

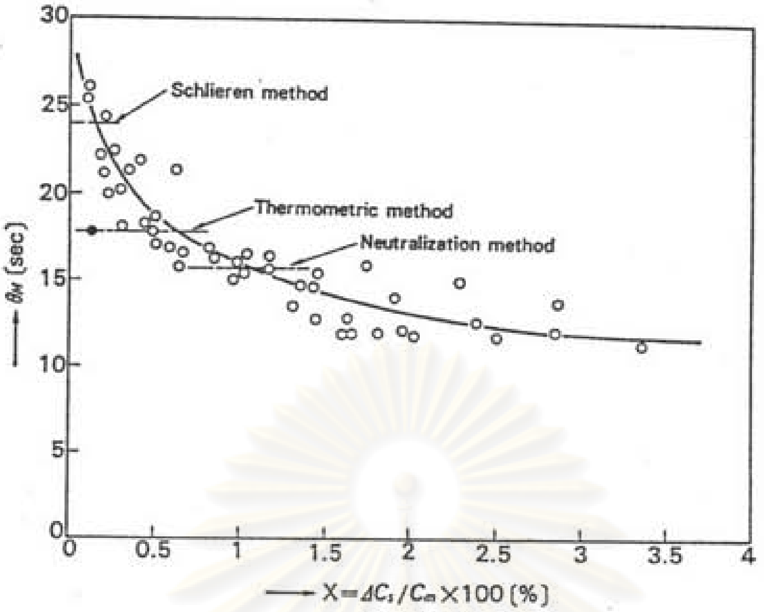

Figure 2.32 Variation of with change of criteria for sufficient mixing.

Conditions of agitation; standard type agitator D = H = 30 cm., 8 baffles; BW/D = 0.075, 8-flat-blade paddle; d/D = 1/2, b/D = 1/10, C/H = 1/2, N = 60 r.p.m., Re = 2.2×104 Agitated liquid; tap water (17 l)
O : Conductometric method, injected liquid: 0.6 cc of 2 N KCl aq. Soln./1 of tap water.
* Schlieren method; 6 cc of 56% saccharose soln./1 of tap water.
* Thermometric method; 15 cc of hot water (95°C)/1 of cold water (30°C): sensitivity; 1/30oC (X = 0.1 %; ●)
* Neutralization method; 2 cc of 3 N KOH aq.soln./1 of 6×10-3 N HCl aq.soln., indicator; phenolphthalein