Chapter 3

1-Dimensional Neutronic & Thermal-Hydraulic model

In nuclear reactor physics, all basic understanding is coming from how the fission neutron behave when it is released from the fission reaction. The well-known theory which is used in all particle movement is transport theory. But due to the complexity of differential equations in transport equations, simplified model called diffusion theory is applied and satisfied at a certain level.

If we start with one fission reaction induced by one neutron react with fissile material, the new-born nucleus will mostly not stable and finally split in 2 fragments with 2-3 free neutron emitted. These fragments normally are not stable. They will change their nucleus structure by emitting their excess electron calling Beta, high electromagnetic energy called Gamma or neutron called delayed neutron. Either 1 or 2 or 3 of the emission could occur in changing from unstable fission fragments to the final stable elements. It sometimes require a few successive emission before reaching the stable state.

The nucleus stabilization is not immediately after fission reaction. It would have delay to get to the stable state which is up to probability of changing and element. It can be just a few nano second up to millions years to change half of radioactive element to another element. This time is known as half-life and is defined as the average time to change element from one isotope to another isotope by emit either of the 3 emission.

Fission neutron energy is pretty high and distributed in continuous spectrum named as Maxwellian distribution. It has an average about 2 MeV. The high energy particle need to be moderated before it is absorbed by any nucleus.

Moderator is used for neutron moderation. The lighter the element, the better moderation it is. Water is probably the usual and cheap moderator being used in the present day. It contains with 2 hydrogen atom and one oxygen atom in one molecule.

The by-product of fission reaction is the energy released in heat form. It in fact converts from the kinetic energy coming with fission fragments. But the fission fragments are big particles and contain many positive particles, so the traveling path is not much more than a millimeter. All kinetic energy will finally dissipate to the heat form.

Heat removal system is normally the main concern in nuclear reactor design. The generated-heat density in nuclear reactor is far higher than the normal fossil-fuel heat generator. And again, water is normally used as coolant. It have many advantages on high specific heat, well-known on physical properties and cheap.

There is another factor which will effect to the reactor operation named as reactivity feedback. This feedback is coming from operation of the reactor such as fuel temperature, coolant temperature, fission products poisoning, etc.

Numerical method will ease us to cope with the differential equation. Computer program is programmed and run to repeat the boring and tough calculation work within a few milliseconds. So, the results can be calculated and shown in real time.

When we do calculation by using any numerical methods, error seem to be an important factor which need to be concerning. If we want to get the higher accuracy, the elements need to be very small and will cause the calculation load to machine. We can play around with the error and calculation time. Then select the proper elements size to match our need.

This chapter consist of 3 sections; one is the neutronic model concerning about neutron behavior, second is thermal hydraulic model dealing with heat removal system and the last is reactivity feedback.

3.1 Neutronic Model

Neutronic model will start with the initial equation mentioned about the diffusion of neutron particle. Let's assume in the simplified model. Fuel pin in one dimensional along the length is considered. The materials in cross section plane have to be homogenized such as fuel, clad, coolant, etc.

The general procedure is to rewrite the differential diffusion equation in finite difference form and then solve the resulting system of difference equations on a computer. We will write the general differential equation in one dimensional Cartesian coordinate which is the selected system.

$$\frac{1}{v}\frac{\partial \Phi(z)}{\partial t} - \nabla . D\nabla . \Phi(z) + \Sigma_a \Phi(z) = S(z) \qquad (1)$$

where

Φ	= neutron flux, neutron/cm^2.s
v	= neutron speed, cm/s
t	= time, s
D	= diffusion coefficient, cm
Σ_a	= macroscopic absorption cross section, cm^{-1}
S(z)	= neutron source term, neutron/cm^3

Subject to the boundary conditions characterizing a finite element of length a:

$$\Phi(0) = \Phi(a) = 0 \qquad (2)$$

where

$\Phi(0)$ = neutron flux at the beginning point

$\Phi(a)$ = neutron flux at length "a"

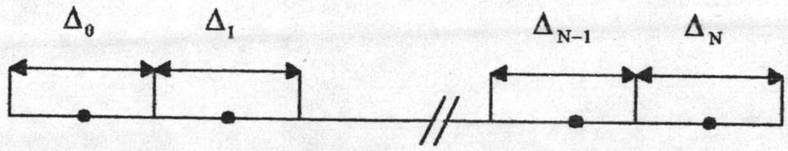

Figure 3.1 Divided regions along fuel pin length

We define spacing between each node as

$$\Delta_0 + \Delta_1 + \ldots\ldots\ldots\ldots + \Delta_{N-1} + \Delta_N = a \tag{3}$$

and

Δ_i = space occupied by node i

Discretized the length of element to N+1 nodes with unequal spacing. Just in case of steady state condition, our general equation reduce to:

$$-\nabla.D\nabla.\Phi(z)+\Sigma_a\Phi(z)=S(z) \tag{4}$$

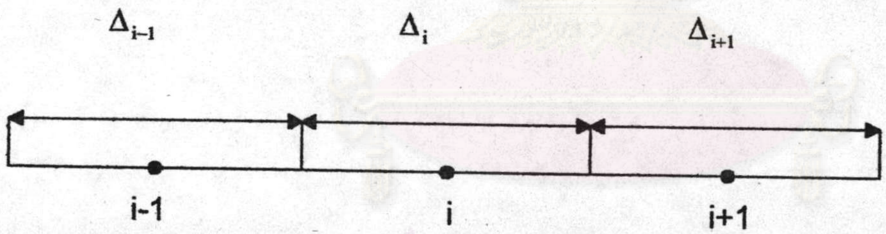

Figure 3.2 Discretized region at node I and its neighbor in neutronic model

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Considering the first term first at node i.

$$\frac{\partial}{\partial x} D \frac{\partial \Phi}{\partial x}\bigg|_i = \frac{D\frac{\partial \Phi}{\partial x}\Big|_{i+\frac{\Delta_i}{2}} - D\frac{\partial \Phi}{\partial x}\Big|_{i-\frac{\Delta_i}{2}}}{\Delta_i}$$

$$\left.\frac{\partial}{\partial x} D \frac{\partial \Phi}{\partial x}\right|_i = \frac{1}{\Delta_i}\left\{ D_{i+\frac{\Delta_i}{2}} \frac{\Phi_{i+1}-\Phi_i}{\frac{\Delta_i+\Delta_{i+1}}{2}} - D_{i-\frac{\Delta_i}{2}} \frac{\Phi_i-\Phi_{i-1}}{\frac{\Delta_i+\Delta_{i-1}}{2}} \right\}$$

$$= \frac{1}{\Delta_i}\left\{ \frac{D_i \frac{\Delta_i}{2} + D_{i+1}\frac{\Delta_{i+1}}{2}}{\frac{\Delta_i+\Delta_{i+1}}{2}} \times \frac{\Phi_{i+1}-\Phi_i}{\frac{\Delta_i+\Delta_{i+1}}{2}} - \frac{D_i \frac{\Delta_i}{2} + D_{i-1}\frac{\Delta_{i-1}}{2}}{\frac{\Delta_i+\Delta_{i-1}}{2}} \times \frac{\Phi_i-\Phi_{i-1}}{\frac{\Delta_i+\Delta_{i-1}}{2}} \right\}$$

$$= \frac{2}{\Delta_i}\left\{ \frac{(D_i\Delta_i + D_{i+1}\Delta_{i+1})}{(\Delta_i+\Delta_{i+1})^2}(\Phi_{i+1}-\Phi_i) - \frac{(D_i\Delta_i + D_{i-1}\Delta_{i-1})}{(\Delta_i+\Delta_{i-1})^2}(\Phi_i-\Phi_{i-1}) \right\} \quad (5)$$

Replace this equation in (4), the result can simplified in such a series of equation as

$$a_{i,i-1}\Phi_{i-1} + a_{i,i}\Phi_i + a_{i,i+1}\Phi_{i+1} = S_i \quad (6)$$

where

$$a_{i,i-1} = -\frac{2}{\Delta_i}\left(\frac{D_i\Delta_i + D_{i-1}\Delta_{i-1}}{(\Delta_i+\Delta_{i-1})^2}\right)$$

$$a_{i,i} = \Sigma_i^a + \frac{2}{\Delta_i}\left(\frac{D_{i+1}\Delta_{i+1} + D_i\Delta_i}{(\Delta_i+\Delta_{i+1})^2} + \frac{D_{i-1}\Delta_{i-1} + D_i\Delta_i}{(\Delta_{i-1}+\Delta_i)^2}\right) \quad (7)$$

$$a_{i,i+1} = -\frac{2}{\Delta_i}\left(\frac{D_{i+1}\Delta_{i+1} + D_i\Delta_i}{(\Delta_i+\Delta_{i+1})^2}\right)$$

So, we have arrived a series of N-1 equations which N+1 unknown flux. Fortunately, we have boundary condition from the edges of element which define flux at both as 0. These series of equation could be solved by Gauss elimination method (the forward elimination-backward substitution method). Let's first write equations in series as

$$\begin{aligned}
a_{11}\Phi_1 + a_{12}\Phi_2 &= S_1 \\
a_{21}\Phi_1 + a_{22}\Phi_2 + a_{23}\Phi_3 &= S_2 \\
a_{32}\Phi_2 + a_{33}\Phi_3 + a_{34}\Phi_4 &= S_3 \\
&\vdots \\
a_{N-1,N-2}\Phi_{N-2} + a_{N-1,N-1}\Phi_{N-1} &= S_{N-1}
\end{aligned} \quad (8)$$

When writing this equation in matrix form, we will get

$$\begin{pmatrix} a_{11} & a_{12} & & & & \\ a_{21} & a_{22} & a_{23} & & & \\ & a_{32} & a_{33} & a_{34} & & . \\ & & a_{43} & a_{44} & a_{45} & \\ & & . & . & . & . \\ & & & & . & . \end{pmatrix} \begin{pmatrix} \Phi_1 \\ \Phi_2 \\ \Phi_3 \\ \vdots \\ \vdots \\ \Phi_{N-1} \end{pmatrix} = \begin{pmatrix} S_1 \\ S_2 \\ S_3 \\ \vdots \\ \vdots \\ S_{N-1} \end{pmatrix} \qquad (9)$$

After Using forward elimination, we eventually get the form

$$\begin{pmatrix} 1 & A_1 & 0 & 0 & 0 & . \\ 0 & 1 & A_2 & 0 & 0 & . \\ 0 & 0 & 1 & A_3 & 0 & . \\ 0 & 0 & 0 & 1 & A_4 & . \\ . & . & . & . & . & . \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \Phi_1 \\ \Phi_2 \\ \Phi_3 \\ \Phi_4 \\ \vdots \\ \Phi_{N-1} \end{pmatrix} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \vdots \\ \alpha_{N-1} \end{pmatrix} \qquad (10)$$

where

$$A_n = \frac{a_{n,n+1}}{a_{n,n} - a_{n,n-1}A_{n-1}} \quad ; \quad A_1 = \frac{a_{12}}{a_{11}}$$

$$\alpha_n = \frac{S_n - a_{n,n-1}\alpha_{n-1}}{a_{n,n} - a_{n,n-1}A_{n-1}} \quad ; \quad \alpha_1 = \frac{S_1}{a_{11}} \qquad (11)$$

The solution of these equations will be defined by back substitute

$$\Phi_{N-1} = \alpha_{N-1}$$
$$\Phi_{N-2} = -A_{N-2}\Phi_{N-1} + \alpha_{N-2} \qquad (12)$$

Let's go back to our initial condition (1) which is not in the steady state condition. We can rewrite the finite element equations as:

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$$\frac{1}{v}\left(\frac{\Phi_i^{t+\Delta t}-\Phi_i^{t}}{\Delta t}\right)+a_{i,i-1}\Phi_{i-1}^{t+\Delta t}+a_{i,i}\Phi_i^{t+\Delta t}+a_{i,i+1}\Phi_{i+1}^{t+\Delta t}=S_i \tag{13}$$

Rearrange the equations, then we will get the new feature:

$$a_{i,i-1}\Phi_{i-1}^{t+\Delta t}+\left(a_{i,i}+\frac{1}{v\Delta t}\right)\Phi_i^{t+\Delta t}+a_{i,i+1}\Phi_{i+1}^{t+\Delta t}=S_i+\frac{1}{v\Delta t}\Phi_i^{t} \tag{14}$$

For the source term on the right hand side, as we know it is the function of fission neutron which equal to

$$S_i=(1-\beta)v\Sigma_i^f\Phi_i^t+\sum_{j=1}^{6}\lambda_j C_i^j \tag{15}$$

$$\frac{\partial C_i^j}{\partial t}=-\lambda_j C_i^j+\beta_j v\Sigma_i^f\Phi_i^t \tag{16}$$

Equation (16) can be rewritten in the form of Implicit finite difference method as

$$\frac{C_i^{j,t}-C_i^{j,t-1}}{\Delta t}=-\lambda_j C_i^{j,t}+\beta_j v\Sigma_i^f\Phi_i^t$$

$$C_i^{j,t}=\frac{1}{1+\lambda_j\Delta t}\times\left(C_i^{j,t-1}+\beta_j v\Sigma_i^f\Phi_i^t\Delta t\right) \tag{17}$$

From (17) replacing delayed neutron precursor ($C_i^{j,t}$) for 6 groups in (16), then the source term will be done and matrix equation will be ready for solving.

Flux mentioned in the source term is flux in previous iteration.

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3.2 Thermal Hydraulic Model

The thermal hydraulic model would be also conducted in one-dimensional , but in different coordinate, radius in cylindrical coordinate. Firstly, consider the heat conduction equation and then apply the heat convection equation on the boundary condition. General equation look very similar to the neutron equation.

$$\rho\, Cp \frac{\partial T}{\partial t} - \nabla . k \nabla T = q''' \qquad (18)$$

where

ρ = material density

Cp = specific heat of material

k = thermal conductivity of material

We will use difference node point definition due to our purpose in this coordinate to find out the temperature on the edge instead of middle point.

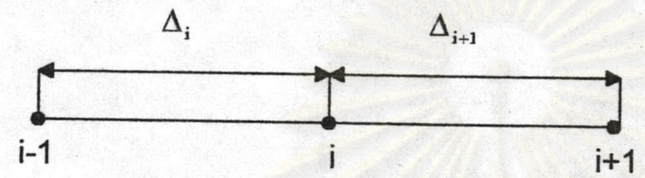

Figure 3.3 Discretized region at node I and its neighbor in thermal-hydraulic model

Considering the second term first

$$\nabla . k \nabla T = \frac{1}{r_i} \frac{d}{dr}\left(k(r).r.\frac{dT}{dr} \right)$$

$$= \frac{1}{r_i}\left[\frac{ k(r).r.\left.\frac{dT}{dr}\right|_{i+\frac{\Delta_{i+1}}{2}} - k(r).r.\left.\frac{dT}{dr}\right|_{i-\frac{\Delta_i}{2}} }{ \frac{\Delta_i + \Delta_{i+1}}{2} } \right]$$

$$= \frac{2}{r_i(\Delta_i + \Delta_{i+1})}\left\{ k_{i+\frac{\Delta_{i+1}}{2}} . r_{i+\frac{\Delta_{i+1}}{2}} . \frac{(T_{i+1} - T_i)}{\Delta_{i+1}} - k_{i-\frac{\Delta_i}{2}} . r_{i-\frac{\Delta_i}{2}} . \frac{(T_i - T_{i-1})}{\Delta_i} \right\}$$

$$= \frac{2}{r_i(\Delta_i + \Delta_{i+1})}\left\{ k_{i+1} . \left(\frac{r_i + r_{i+1}}{2}\right) . \frac{(T_{i+1} - T_i)}{\Delta_{i+1}} - k_i . \left(\frac{r_i + r_{i-1}}{2}\right) . \frac{(T_i - T_{i-1})}{\Delta_i} \right\}$$

$$= \frac{1}{r_i(\Delta_i + \Delta_{i+1})}\left\{ k_{i+1} . (r_i + r_{i+1}) . \frac{(T_{i+1} - T_i)}{\Delta_{i+1}} - k_i . (r_i + r_{i-1}) . \frac{(T_i - T_{i-1})}{\Delta_i} \right\} \qquad (19)$$

In the same style as the neutronic model, we can write the general equation in form of finite difference as

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$$\rho_i Cp_i\left(\frac{T_i^{t+\Delta t}-T_i^{t}}{\Delta t}\right)+a_{i,i-1}T_{i-1}^{t+\Delta t}+a_{i,i}T_i^{t+\Delta t}+a_{i,i+1}T_{i+1}^{t+\Delta t}=q''' \tag{20}$$

where

$$a_{i,i-1}=-\frac{k_i(r_{i-1}+r_i)}{r_i\Delta_i(\Delta_i+\Delta_{i+1})}$$

$$a_{i,i}=\frac{1}{r_i(\Delta_i+\Delta_{i+1})}\left(\frac{k_{i+1}(r_i+r_{i+1})}{\Delta_{i+1}}+\frac{k_i(r_i+r_{i-1})}{\Delta_i}\right) \tag{21}$$

$$a_{i,i+1}=-\frac{k_{i+1}(r_{i+1}+r_i)}{r_i\Delta_{i+1}(\Delta_i+\Delta_{i+1})} \tag{22}$$

re-arrange the equation

$$a_{i,i-1}T_{i-1}^{t+\Delta t}+\left(a_{i,i}+\frac{\rho_i\, Cp_i}{\Delta t}\right)T_i^{t+\Delta t}+a_{i,i+1}T_{i+1}^{t+\Delta t}=q'''+\frac{\rho_i Cp_i}{\Delta t}T_i^{t} \tag{23}$$

For q''' which is the heat generated term, It will refer to the neutron equation. We define φ as energy released per fission

$$q'''=\Sigma_i^f\Phi_i.\varphi \tag{24}$$

We replace q''' on the right hand side of temperature equation

$$a_{i,i-1}T_{i-1}^{t+\Delta t}+\left(a_{i,i}+\frac{\rho_i Cp_i}{\Delta t}\right)T_i^{t+\Delta t}+a_{i,i+1}T_{i+1}^{t+\Delta t}=\Sigma_f^t\times\Phi_i^t.ER+\frac{\rho_i Cp_i}{\Delta t}T_i^{t} \tag{25}$$

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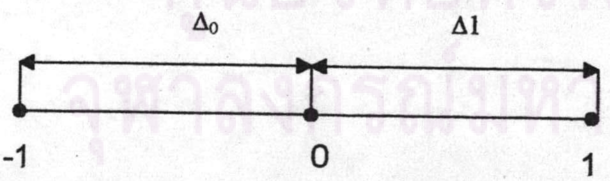

Figure 3.4 Region in central area of cylindrical rod

At inner boundary condition, node 0, temperature gradient is equal to 0 and the radius is too. So when considering the inner boundary, we shall go back to our general equation.

$$\rho\, Cp\frac{\partial T}{\partial t}-\nabla .\, k\nabla T=q''' \tag{18}$$

In the second term of (18), we are able to write in cylindrical coordinate as

$$\nabla .\, k\nabla T=\frac{1}{r}\frac{d}{dr}\left(k.r.\frac{dT}{dr}\right)$$

$$=k\frac{d^2T}{dr^2}+\frac{k}{r}\frac{dT}{dr}+\frac{dk}{dr}.\frac{dT}{dr} \tag{26}$$

If the thermal conductivity of materials are not change in the inner bound, the third term in (26) will disappear. In the second term, when substitute $\frac{dT}{dr}$ and r which equal to 0, we will get zero divide by 0. Apply the L'hospital's rule by differentiate upper and lower fraction.

$$k\frac{1}{\frac{dr}{dr}}\frac{d}{dr}\left(\frac{dT}{dr}\right)=k\frac{d^2T}{dr^2} \tag{27}$$

replace (27) in (26), we will get

$$\nabla .\, k\nabla T=2k\frac{d^2T}{dr^2} \tag{28}$$

Substitute (28) back into (18)

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$$\rho Cp\frac{T_i^{t+\Delta t}-T_i^{t}}{\Delta t}-2k\frac{d^2T}{dr^2}=q''' \tag{29}$$

$$\rho Cp\frac{T_i^{t+\Delta t}-T_i^{t}}{\Delta t}-2k\frac{\left(T_1^{t+\Delta t}-2T_0^{t+\Delta t}+T_{-1}^{t+\Delta t}\right)}{\Delta_0{}^2}=q''' \tag{30}$$

In the symmetrical cylindrical coordinate, the T_{-1} is the mirror of T_1 which is equalized.

$$\rho Cp\frac{T_i^{t+\Delta t}-T_i^t}{\Delta t}-4k\frac{\left(T_1^{t+\Delta t}-T_0^{t+\Delta t}\right)}{\Delta_0^{\,2}}=q''' \tag{31}$$

At outbound condition we can't do on the same way, fuel clad transfer generated heat to coolant, water, by convection not conduction as in fuel pin.. Our equation had to relate with convection equation. Derive equation from energy balance equation.

heat conducted in + heat generated - heat convected = rate of change in internal energy

$$-k\frac{dT}{dr}+q'''.\frac{\Delta_i}{2}-h\left(T_i-T_\infty\right)=\rho Cp.\frac{\Delta_i}{2}\frac{dT_i}{dt} \tag{32}$$

Write (32) in finite difference form

$$-k_i\frac{\left(T_i^{t+\Delta t}-T_{i-1}^{t+\Delta t}\right)}{\Delta_i}+q'''.\frac{\Delta_i}{2}-h\left(T_i^{t+\Delta t}-T_\infty^{t+\Delta t}\right)=\rho Cp.\frac{\Delta_i}{2}\frac{\left(T_i^{t+\Delta t}-T_i^t\right)}{\Delta t}$$

$$-\frac{k_i}{\Delta_i}T_{i-1}^{t+\Delta t}+(h+\frac{k_i}{\Delta_i}+\frac{\rho Cp}{\Delta t}.\frac{\Delta_i}{2})T_i^{t+\Delta t}-hT_\infty^{t+\Delta t}=\rho Cp.\frac{\Delta_i}{2}\frac{T_i^t}{\Delta t}+q'''.\frac{\Delta_i}{2} \tag{33}$$

In fuel clad, we don't have any heat generated. So (33) will be reduced to

$$-\frac{k_i}{\Delta_i}T_{i-1}^{t+\Delta t}+(h+\frac{k_i}{\Delta_i}+\frac{\rho Cp}{\Delta t}.\frac{\Delta_i}{2})T_i^{t+\Delta t}-hT_\infty^{t+\Delta t}=\rho Cp.\frac{\Delta_i}{2}\frac{T_i^t}{\Delta t} \tag{34}$$

In water which interfaced with clad, we also apply the energy balance equation.

$$hA(T_i-T_\infty)-\dot{m}\,Cp(T_{\infty,out}-T_{\infty,in})=MCp\frac{dT}{dt} \tag{35}$$

And again, make (35) in our familiar form

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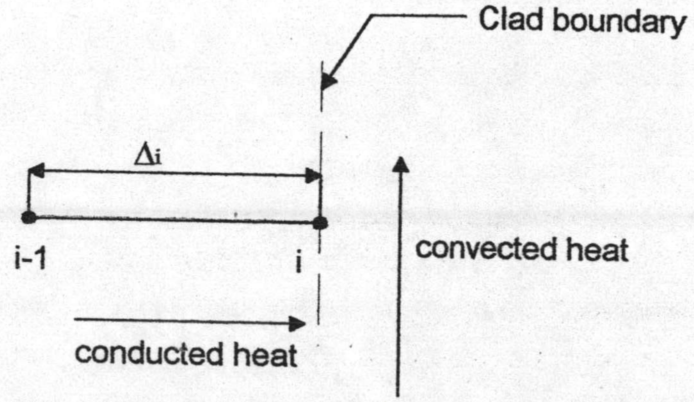

Figure 3.5 Clad and coolant interface

$$hA(T_i^{t+\Delta t} - T_\infty^{t+\Delta t}) - \dot{m}\,Cp(T_{\infty,out}^{t+\Delta t} - T_{\infty,in}) = MCp\frac{\left(T_\infty^{t+\Delta t} - T_\infty^{t}\right)}{\Delta t} \quad (36)$$

where

T_∞ = Average coolant temperature in region

$T_{\infty,out}$ = coolant outlet temperature from region

$T_{\infty,in}$ = coolant inlet temperature

We define the average temperature as the average of inlet and outlet temperature of coolant as

$$T_\infty = \frac{T_{\infty,out} + T_{\infty,in}}{2} \quad (37)$$

$$T_{\infty,out} = 2T_\infty - T_{\infty,in} \quad (38)$$

Substitute (38) into (36)

$$hA(T_i^{t+\Delta t} - T_i^{t+\Delta t}) - 2\dot{m}\,Cp(T_\infty^{t+\Delta t} - T_{\infty,in}) = MCp\frac{\left(T_\infty^{t+\Delta t} - T_\infty^{t}\right)}{\Delta t} \quad (39)$$

$$-hAT_i^{t+\Delta t} + (\frac{MCp}{\Delta t} + 2\dot{m}\,Cp + hA)T_\infty^{t+\Delta t} = \frac{MCp}{\Delta t}T_\infty^{t} + 2\dot{m}\,CpT_{\infty,in} \quad (40)$$

Coolant temperature which get out from one region will be the inlet temperature of the next region. The first region have a pool temperature as inlet temperature. The pool temperature could change with heat balance equation in pool.

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Total heat transfer in region can be calculated from second term of (39) which count on heat taken away by coolant from region.

In the same way as neutronic model, Gauss elimination method is used to solve these series of equation.

3.3 Reactivity Feed Back

In this model we are interested in 2 type of feed back from reactor operation; temperature feedback and fission products reactivity feedback.

3.3.1 Temperature feedback

As we know, fuel-moderator have remarkable prompt negative reactivity which directly effect the fission cross section in fuel itself. Starting from the definition of the temperature coefficient.

$$\alpha_T = \frac{d\rho}{dT} \tag{41}$$

where

ρ = reactivity and is defined as $\frac{k^{t+\Delta t} - k^{t}}{k^{t+\Delta t}}$; k is multiplication factor

Let's assume that our system operated at nearly critical, which result in $k \cong 1.00$. We can then rewrite the reactivity definition as

$$\rho = \frac{k-1}{k} \tag{42}$$

when substitute back in the equation of temperature coefficient

$$\alpha_T = \frac{1}{k^2}\frac{dk}{dT} \approx \frac{1}{k}\frac{dk}{dT} \tag{43}$$

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If we go further in the composition of k value, we firstly assume that the negative temperature coefficient affect only thermal utilization (f) and average of fission neutron emitted per thermal neutron absorbed by fuel (η_T) . The result will be

$$\alpha_T = \frac{1}{\frac{\Sigma_f^t}{\Sigma_{a,tot}^t}}\left(\frac{\frac{\Sigma_f^t}{\Sigma_{a,tot}^t} - \frac{\Sigma_f^0}{\Sigma_{a,tot}^0}}{dT}\right) \tag{44}$$

where

$\Sigma_{a,tot}$ = total absorption cross section, cm^{-1}

Σ_f^0 = fission cross section at reference temperature, cm^{-1}

If we define the total absorption cross section deduce by fission cross section as $\Sigma_{a,nf}$. Equation (44) could be written as

$$\alpha_T = \frac{1}{\frac{\Sigma_f^t}{\Sigma_{a,nf}^t + \Sigma_f^t}}\left(\frac{\frac{\Sigma_f^t}{\Sigma_{a,nf}^t + \Sigma_f^t} - \frac{\Sigma_f^0}{\Sigma_{a,nf}^0 + \Sigma_f^0}}{dT}\right) \tag{45}$$

We assume that the fission cross section will be the only parameter change with temperature, the others are constant . We could say that term $\Sigma_{a,nf}^t$ equals to $\Sigma_{a,nf}^0$. So, after rearrange, (45) can be re-written again as

$$\Sigma_f^t = \frac{\Sigma_{a,nf}^0 \Sigma_f^0}{\Sigma_f^0 + \Sigma_{a,nf}^0} \times \frac{1}{\left(1 - \alpha_T dT - \frac{\Sigma_f^0}{\Sigma_f^0 + \Sigma_{a,nf}^0}\right)} \tag{46}$$

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Once our reference temperature is set at 20 C. The above equation will look like

$$\Sigma_f^t = \frac{\Sigma_{a,nf}^0 \Sigma_f^0}{\Sigma_f^0 + \Sigma_{a,nf}^0} \times \frac{1}{\left(1 - \alpha_T(T-20) - \frac{\Sigma_f^0}{\Sigma_f^0 + \Sigma_{a,nf}^0}\right)} \quad (47)$$

3.3.2 Fission Products reactivity feedback

All fission products absorb neutrons with particular value of cross section. But there are 2 elements which out-standing from the others with very high absorption cross section and a remarkable fission yield; Xenon-135 and Samarium-149.

3.3.2.1 Xenon-135

Xe-135 is the most important fission product and sometime is a big concern in operation of reactor. Xe-135 is an element getting from decay of Iodine-135 and partial direct from fission itself. Microscopic absorption cross section of Xe-135 is $2.65\text{x}10^6$ barns at thermal energy. The diagram below show the decay scheme of Xe-135.

$$Te^{135} \xrightarrow{\beta^-} I^{135} \xrightarrow{\beta^-} Xe^{135} \xrightarrow{\beta^-} Cs135 \xrightarrow{\beta^-} Ba135(\text{stable})$$

↑ 11sec ↑ 6.7hr ↑ 9.2hr

fission fission fission

Te^{135} decays very rapidly to I^{135}. We may assume that Te^{135} fission yield will be add up with I^{135} yield. Element concentration can be mentioned as

$$\frac{dI}{dt} = \gamma_I \Sigma_f \Phi - \lambda_I I \quad (48)$$

$$\frac{dX}{dt} = \gamma_X \Sigma_f \Phi + \lambda_I I - \lambda_X X - \sigma_{a,X} X \Phi \quad (49)$$

Derive (46) and (47) in explicit form

$$I^{t+\Delta t} = \left(\gamma_I \Sigma_f \Phi - \lambda_I I^t\right) . \Delta t + I^t \quad (50)$$

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$$X^{t+\Delta t} = \left(\gamma_X \Sigma_f \Phi + \lambda_I I^{t+\Delta t} - \lambda_X X^t - \sigma_{a,X} X^t \Phi\right).\Delta t + X^t \quad (51)$$

Xe-135 direct production through fission will cease after shut down the reactor. But the decay of I-135 will continue with a higher rate than of Xe-135 decay. The Xe-135 concentration normally will increase for a while before I-135 concentration reduce to a certain concentration which the decay rate of Xe-135 is equal or higher. It may cause the less-excess-reactivity reactor unable to start till the concentration of Xe-135 decrease and make the reactor have enough excess reactivity re-start reactor again.

It also effect the reactor on changing power level from high to low while still operate. The accumulated Xe-135 will keep increasing and sometime over the core excess reactivity. Special care should be paid in less-excess-reactivity core.

3.3.2.2 Samarium-149

This element is not as much concern in calculation as Xe-135. It have average thermal cross section 58700 barns. Sm-149 is the decayed product of Nd-149 through Pm-149.

$$\text{fission} \longrightarrow Nd^{149} \xrightarrow[1.7hr]{\beta^-} Pm^{149} \xrightarrow[53hr]{\beta^-} Sm^{149}$$

Due to rapid decay of Nd-149 comparing with the successive Pm-149, we may assume that Pm-149 is a direct fission product. Elements concentration can be written as

$$\frac{dP}{dt} = \gamma_P \Sigma_f \Phi - \lambda_P P \quad (52)$$

$$\frac{dS}{dt} = \lambda_P P - \sigma_S S \Phi \quad (53)$$

Re-written in explicit form

$$P^{t+\Delta t} = \left(\gamma_P \Sigma_f \Phi - \lambda_P P^t\right).\Delta t + P^t \quad (54)$$

$$S^{t+\Delta t} = \left(\lambda_P P^{t+\Delta t} - \sigma_S S^t \Phi\right).\Delta t + S^t \quad (55)$$

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Sm -149 is a stable isotope and can only be produced via decay of Pm-149. So, after reactor shut down, the Sm-149 concentration will keep increasing till all the Pm-49 disappear. The only way that can eliminate Sm-149 from reactor core is to burn it with neutron.

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