CHAPTER 4

SYSTEM PERFORMANCE AND SYSTEM DESIGN

Introduction

In Chapter 3, we presented and analyzed the RMA transmission system, which utilized the RMA code sequences discussed in Chapter 2. The exact expression for the signal-to-noise ratio (SNR) has also been derived. In this chapter a simple derivation of the average SNR and the probability of error, Pe, which determine the performance of the RMA system is presented.

Because the statistic of the simultaneous users is time varying, the exact evaluations of SNR and Pe are difficult. Instead , an approximation is used, based on the assumption that all noise is additive white Gaussian noise and large number of users are involved. The system is operating in the environment where there are strong interfering signals, hence, the system performance is not sensitive to the variation in the thermal noise. Consequently, allowing the use of very large number of small aperture terminals for both transmission and reception.

Approximation of the Average Signal-to-Noise Ratio

We have shown in the previous chapter that the output from each correlation receiver includes the desired signal, channel noise, and the multiple-access interference (MAI). This MAI is caused by all simultaneous users whose sequence symbols are represented by the same carrier frequencies as the desired symbols. The total number of cochannel symbols for each carrier is simply the number of time-delay units in the TF matrix, or T_d.

The expression for the total SNR at the threshold comparator input, which is the summation of all the output's from M correlation receivers, is given by Eq.(3-28) as

$$SNR_i = \left[\frac{M}{\frac{1}{3M} U_i + \frac{N_o}{2E_b}} \right]^{1/2}$$

where $U_i = \sum_{n=1}^{M} \hat{R}_{i,n} + \sum_{n=1}^{M} R'_{i,n}(T_d-1)$

and $\frac{E_b}{N_o} = \frac{PT}{2N_o}$

The precise value of U_i can be calculated only when the statistic of the simultaneous users are known at a particular time for a particular system determined by the corresponding TF matrix . If we assume worst case performance, i.e. all users are simultaneously transmitting, then we have

$$\max.(\hat{R}_{i,n}) = R-1$$

$$\max.(R'_{i,n}) = R$$

Therefore, $\max.(U_i) = \sum_{n=1}^{M}(R-1) + \sum_{n=1}^{M} R(T_d-1)$

$$= M(RT_d-1) \qquad (4-1)$$

From Eqs.(4-1) and (2-16), the maximum MAI for the system which utilized PG set is

$$\max.(U_i)_{PC} = M(MT_d-1) \qquad (4-2)$$

From Eqs.(4-1) and (2-17), similar result is given for the case of EG set as

$$\max.(U_i)_{EG} = M[(M+1)T_d-1] \tag{4-3}$$

In order to simplify notations, we define SNR = SNR_i and $U = U_i$ from here on.

The average value of multiple access interference and consequently the signal-to-noise ratio, which is useful for preliminary system design where the statistic of each user is not available but the sequence parameters and TF matrix are known, can be expressed as

$$\overline{SNR} = \left[\frac{M}{\frac{1}{3M}\bar{U} + \frac{N_0}{2E_b}}\right]^{1/2} \tag{4-4}$$

where $\bar{U} = \bar{a} + c\bar{d}$ (4-5)

The parameters $\bar{a}$ and $\bar{d}$ are the part of MAI of any user , which is caused by his own transmitter, and any other simultaneous user accordingly.

$\bar{a}$ = average number of designated user cochannel symbols ($0 \leq a \leq M$)

$\bar{d}$ = average number of cochannel symbols between designated user and any other user $(0 \leqslant d \leqslant M)$

c = total number of simultaneous users $(0 \leqslant c < L)$

Both $\bar{a}$ and $\bar{d}$ can be expressed in term of sequence length, as followed :

$$\bar{a} = \bar{y}M \qquad \text{where } 0 \leqslant y \leqslant 1 \tag{4-6}$$

$$\bar{d} = \frac{\max.(U) - \bar{y}M}{\max.(c)} \tag{4-7}$$

Parameter $\bar{y}$ is a user's cochannel symbols-to-sequence symbols ratio, or

$$\bar{y} = \text{average}\left(\frac{\text{number of cochannel symbols in a sequence}}{\text{total number of symbols in that sequence}}\right)$$

When all users are actived simultaneously, we have

$$\max.(c) = L-1 \tag{4-8}$$

For PG set sequences, substituting Eqs.(2-11) and (4-2) into Eq.(4-7) produces

$$(\bar{d})_{PG} = \frac{M(MT_d-1) - \bar{y}M}{[M(M-1) + 1] - 1}$$

$$= \frac{(MT_d - 1) - \bar{y}}{M - 1} \tag{4-9}$$

For EG set sequences, similar result can be given by substituting Eqs.(2-14) and (4-3) into Eq.(4-7)

$$(\bar{d})_{EG} = \frac{M[(M+1)T_d - 1] - \bar{y}M}{M(M+1) - 1} \qquad (4\text{-}10)$$

Due to the fact that the sequence length is normally much greater than one for the application in a RMA transmission system, Eqs.(4-9) and (4-10) reduce to

$$\bar{d} = T_d \qquad (4\text{-}11)$$

From Eqs.(4-5),(4-6) and (4-11), we have the following result for both cases of difference sets

$$\bar{U} = \bar{y}M + cT_d \qquad (4\text{-}12)$$

Finally substituting $\bar{U}$ into Eq.(4-4), we have the following desired expression

$$\overline{SNR} = \left[\frac{M}{\frac{1}{3M}[\bar{y}M+cT_d] + \frac{N_o}{2E_b}}\right]^{1/2} \qquad (4\text{-}13)$$

In terms of energy per data bit, $\overline{\mathrm{SNR}}$ can be expressed as

$$\overline{\mathrm{SNR}} = \left[\frac{M}{\frac{1}{(E_b/N_o)_{MAI}} + \frac{1}{(E_b/N_o)_{th.}}} \right]^{1/2} \qquad (4\text{-}14)$$

where $\left(\frac{E_b}{N_o}\right)_{th.} = \frac{2E_b}{N_o}$

or $\left(\frac{E_b}{N_o}\right)_{th.}$ = energy-per-bit/channel thermal noise density ratio

$$\left(\frac{E_b}{N_o}\right)_{MAI} = \frac{3M}{yM + cT_d} \qquad (4\text{-}15)$$

or $\left(\frac{E_b}{N_o}\right)_{MAI}$ = energy-per-bit/multiple-access-interference density ratio

Probability of Error of RMA System

In this case of a communication system with additive white Gaussion noise (AWGN), which includes multiple access interference noise and channel noise, a data bit error occurs at the threshold comparator output when the integrated amplitude of the noise is larger than the integrated amplitude of the desired signal in the opposite direction.

The threshold comparator provides, at T intervals, a +1 or -1 output state, depending on whether C_i is larger or smaller than zero, respectively. Therefore, the probability of error is given by

$$\begin{aligned} Pe &= P[C_i>0/b_i= -1]\ P[b_i= -1] \\ &\quad + P[C_i<0/b_i= +1]\ P[b_i= +1] \end{aligned} \tag{4-16}$$

where $P[C_i>0/b_i= -1]$ = probability of having sampled C_i larger than zero, given that a -1 b_i is being transmitted

For equa-probable signaling stated in Chapter 3, Eq.(4-16) becomes

$$Pe = 0.5\{\ P[C_i>0/b_i= -1] + P[C_i<0/b_i= +1]\ \}$$

or $\quad Pe = 0.5\ P[$(total noise at the sampling instant) > (total desired signals at the sampling instant)$]$

The probability of error can be approximated by (Feher, 1984)

$$Pe = Q[\overline{SNR}] \tag{4-17}$$

From here on, we will denote $\overline{SNR}$ as SNR and $\overline{y}$ as y.

where

$$Q(SNR) = \int_{SNR}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \, dx$$

When SNR>4, Q(SNR) can be approximated by

$$Q(X) = \frac{1}{\sqrt{2\pi}\,X} e^{-x^2/2}$$

In general, the error probability will not be exactly Q(SNR) but this is typically a very good approximation for practical large RMA system of which Gaussian approximation for MAI can be assumed.

The Q(SNR) function is tabulated in Appendix D (Table D.1). The sketch in Fig.D.1 shows the probability of error (Pe) as a function of SNR in the frequently interested range of $10^{-3} < Pe < 10^{-12}$.

From Eqs.(4-13) and (4-17), we can conclude that the probability of error of the RMA system depends on the following parameters:

- sequence length (M)
- user's cochannel symbols-to-sequence symbols ratio (y)
- number of simultaneous users (c)
- number of time-delay in the TF matrix or number of cochannel symbols (T_d)
- energy-per-bit/channel-thermal-noise density ratio $(E_b/N_o)_{th.}$

Numerical Results of the Probability of Error

Note that the parameters M, y and T_d depend on the structure of the sequence and the size of the TF matrix, but the $(E_b/N_o)_{th.}$ can be calculated from the link budget parameters, such as transmitted carrier power, bit rate, etc. Tables in Appendix E show numerical results of the evaluation of the probability of error for different values of these five parameters. The results in all 15 tables are plotted in Figure E.1 to E.12. It is of interest to know how the system will perform when the number of simultaneous users are varying, as this is the only parameter which can not be predetermined or fixed due to the random access nature of the system. Thus, the graphs in Appendix E are of Pe versus c for different values of M, y, T_d and $(E_b/N_o)_{th.}$.

Our objective is to keep the value of $(E_b/N_o)_{th.}$ to a minimum as the higher $(E_b/N_o)_{th.}$ may result in higher uplink carrier power, or larger antenna at the received station or a reduced data rate. Futher discussion on this issue can be found in the next section. In Chapter 3 we assume that all stations have the same power, thus, any increase in energy per data bit from the desired user also means an increase in the MAI. Therefore, $(E_b/N_o)_{th.}$ has little effect on the Pe as we can observe in Figs.(E.4), (E.6), (E.8) and (E.10). The Pe is also not sensitive to the change of parameter y as shown in the plots in Figs.(E.5), (E.7), (E.9), and (E.11). On the contrary, Figs.(E.1), (E.2), (E.3) and (E.12) demonstrate that the sequence length and the number of cochannel symbols in the TF matrix greatly affect the probability of error, especially when the number of simultaneous users are relatively very small compared to the total number of users.

The sequence length will determine the size of the system as derived in Chapter 2 [(Eqs.(2-11) and (2-14)]

EG set, $L_e = M(M+1)$

PG set, $L_p = M(M-1) + 1$

Examples in Appendix E are for sequences of length 59, 97, 107, 127, and 169 which can be constructed from EG sets. The corresponding number of users are calculated as followed:

Sequence Length (M)	Total Number of Users (L_e)
59	3,540
97	9,506
107	11,556
127	16,256
169	28,730
289	83,810

Table 4.1 Total Number of Users for Each Sequence Length

Preliminary System Design

For preliminary system design it is useful to be able to carry out a tradeoff between the parameters M, T_d, and c as they all have significant effect on Pe. Such a tradeoff can be based on the Eq.(4-18) as followed:

From Eqs.(4-13) and (4-17), we can express Pe as

$$Pe = Q\left(\left[\frac{M}{\frac{1}{3}\left[\ y + \frac{cT_d}{M}\ \right] + \frac{N_o}{2E_b}}\right]^{1/2}\right) \qquad (4-18)$$

Given y = 0.2, E_b/N_o = 0.2 dB, Eq.(4-18) becomes

$$Pe = Q\left(\left[\frac{M}{\frac{1}{3}\left[\ 0.2 + \frac{cT_d}{M}\ \right] + 0.477}\right]^{1/2}\right) \qquad (4-19)$$

The effects of these three parameters can be summarized as followed:

Case A

If we want a certain number of simultaneous users, we have to increase the sequence length in order to improve the system performance. An example is shown in Table 4.2

M (symbols/sequence)	Pe
289	5.95E-09
169	6.71E-06

M (symbols/sequence)	Pe
127	8.00E-05
107	2.65E-04
97	4.72E-04
59	4.90E-03

Table 4.2 Pe for Different Sequence Length When Number of Simultaneous Users is Fixed (c = 25), and T_d = M

Case B

If a certain percentage of users is expected to transmit simultaneously, we'll find that smaller size system will have a better performance than the bigger one. This can be observed from the Table 4.3.

Total users	c = 0.001% of Total users	Pe
83,810	84	7.4E-04
28,730	29	2.5E-05
16,256	16	1.7E-06
11,556	12	6.3E-07
9,506	10	2.9E-07

Total users	c = 0.001% of Total users	Pe
3,540	4	1.0E-08

Table 4.3 Pe for Different System Size When c is a Fixed Percentage of Total Users(c = 0.001% of L), and T_d= M

Case C

Since the noise in this RMA system is dominated by cochannel interference, we can improve the system performance by reduce the number of cochannel symbols. In other words, we have to reduce T_d while increasing F accordingly , so that the total number of symbols ($V = T_d*F$) can remain constant.

Table 4.4 and Fig.E.12 show how we can significantly reduce the probability of error by increasing the carrier frequency. In this specific example, the sequences are derived from a EG set, hence, $T_d*F = M^2$. When the TF matrix is a square matrix, we have $F = T_d = M$. If we reduce the cochannel symbols by half, the TF matrix becomes a rectangular matrix and F = 2M.

c (users)	Pe	
	T_d = M	T_d = 0.5M
14	4.48E-09	3.38E-15
16	3.45E-08	1.03E-13
18	1.64E-07	1.45E-12
20	5.84E-07	1.29E-11
22	1.68E-06	8.12E-11
24	4.12E-06	3.89E-10
26	8.87E-06	1.50E-09
28	1.72E-05	4.88E-09
30	3.0E-05	1.38E-08
32	4.9E-05	3.45E-08
34	8.0E-05	7.84E-08

Table 4.4 Pe for Different Cochannel Symbols(T_d)

(M = 169)

Case D

If the system is designed to provide a fixed nominal bit error rate performance, the number of simultaneous users can not exceed a certain limit. An example is shown in Table 4.5 for a bit error rate performance equal to or better than 10^{-7}.

M	Maximum c	Max.c as % of Total Users
289	31	0.037%
169	16	0.056%
127	13	0.08%
107	11	0.095%
97	10	0.1%
59	5	0.14%

Table 4.5 Maximum Number of Simultaneous Users for Different Sequence Length When $Pe = 10^{-7}$ and $T_d = M$

Examples of Link Budget for Very Small Aperture Antennas

In this section some link budgets are presented to illustrate that the proposed RMA technique is of use to a system with very small aperture terminals (VSAT).

We have demonstrated in the previous section that the $(E_b/N_o)_{th.}$ has some contribution to the system performance (Pe) but it will required a big increase in $(E_b/N_o)_{th.}$ to have any noticeable effect on Pe. In addition, the carrier power varies with $(E_b/N_o)_{th.}$, thus, it is important to keep the value of this parameter at some minimum level. Since a level of $(E_b/N_o)_{th.}$ assumed in Appendix E is 0.4 dB, which corresponds to (E_b/N_o) of 0.2 dB, we will use this same value for conveniently relating the link budgets to the Pe already calculated . The link budgets are designed for the system which the transmit stations and the receive stations are of the same size.

The following system parameters are used in the link budgets.

1. Operating Frequency = 6/4 GHz and 14/11 GHz
2. Satellite Parameters
    - Antenna Gain-to-Noise Temperature Ratio (G/T)
        = -2 dB/K (6/4 GHz)
        = 1.7 dB/K (14/11 GHz)
    - Saturation Flux Density
        = -81 dBW/m^2 (6/4 GHz)
        = -90.5 dBW/m^2 (14/11 GHz)
    - Saturation EIRP = 36 dBW (6/4 GHz)
        = 41 dBW (14/11 GHz)
3. Earth Station Parameters
    - Antenna Diameter 1.2 m., 1 m., 0.8 m.

- Antenna G/T = 10 dB/K (1.2 m. at 4 GHz)
= 15.5 dB/K (1.2 m. at 11GHz)
= 13.8 dB/K (1.0 m. at 11 GHz)
= 11.8 dB/K (0.8 m. at 11 GHz)

4. Information Bit Rate (R_b) 1200 bit/second

The link budget are given as followed:

	14/11 GHz			6/4 GHz
Antenna Diameter (m.)	1.2	1	0.8	1
Antenna Gain (dB)	42.3	40.8	38.8	35.0
HPA (dBW/carrier)	-24.1	-21.6	-16.9	-1.4
EIRP/carrier (dBW)	18.2	19.2	21.9	33.6
Free Space Loss (dB)	207.0	207.0	207.0	199.7
Atmospheric Loss + Antenna Pointing Loss (dB)	1.0	1.0	1.0	1.0
Satellite G/T (dB/K)	1.7	1.7	1.7	-2.0
Up Link (C/T)thermal(dBW/K)	-188.1	-186.4	-184.4	-169.1
Antenna Gain 1 m^2 (dB-m^2)	44.0	44.0	44.0	37.0
Saturation Flux Density (dBW/m^2)	-90.5	-90.5	-90.5	-81.0
Input Backoff/carrier (dB)	-53.6	-51.9	-49.9	-51.1

Output Backoff/carrier (dB)	-48.1	-46.4	-44.4	-46.6
Saturation EIRP (dBW)	41.0	41.0	41.0	36.0
EIRP Downlink (dBW)	-7.1	-5.4	-3.4	-10.6
Free Space Loss (dB)	205.0	205.0	205.0	196.0
Atmospheric Loss and Antenna Pointing Loss (dB)	1.0	1.0	1.0	1.0
Earth Station G/T (dB/K)	15.5	13.8	11.8	10.0
Downlink (C/T)thermal(dBW/K)	-197.6	-197.6	-197.6	-197.6
Total (C/T)thermal (dBW/K)	-197.6	-197.6	-197.6	-197.6
Boltzman's Const.(dBW/K-Hz)	-228.6	-228.6	-228.6	-228.6
Total (C/N_o)thermal (dBW/Hz)	31.0	31.0	31.0	31.0
Information Bit Rate (dB)	30.8	30.8	30.8	30.8
(E_b/N_o) (dB)	0.2	0.2	0.2	0.2

Some equations related to the link budget are given.

$$\frac{1}{\text{total }(C/T)} = \frac{1}{\text{uplink }(C/T)} + \frac{1}{\text{downlink }(C/T)}$$

where (C/T) = (C/T)thermal

= carrier-to-thermal noise temperature ratio

$$(C/N_o)\text{thermal} = (E_b/N_o) + 10\log(R_b) \quad \text{dB}$$

More details in the calculation and parameters of the link budget can be found in (Feher, 1984) and (Ha, 1990).

In order to increase the speed of information bit, we have to increase the transmitted EIRP or the received antenna, i.e., increase G/T of the earth terminal. Table 4.6 shows the required (C/N_o)thermal for each bit rate while keeping (E_b/N_o) unchange.

R_b		Total (C/N_o)thermal
bit/sec.	dB	dBW/Hz
1200	30.8	31
2400	33.8	34
4800	36.8	37
9600	39.8	40

Table 4.6 $(C/N_o)_{th.}$ Required for Different Bit Rate

As an example, if we increase the information bit rate of 1200 bit/sec. to 4800 bit/sec., we have to increase the EIRP proportionally. That is to raise the uplink EIRP level up 6 dB. Another alternative is to introduce error correction coding in order to improve the bit error rate.