CHAPTER 3

SYSTEM MODEL AND SYSTEM ANALYSIS

Introduction

The basic transmission scheme of a RMA system is the transformation of each binary bit into a set of time-frequency (TF) symbols. These symbols form a code sequence which is uniquely assigned to a particular user. The receiver of the system, then, performs the inverse transformation from a sequence of symbols back to the corresponding binary bit.

The procedures for RMA code sequence generation, which utilizing Euclidean Geometry (EG) Difference Set {De} and Projective Geometry (PG) Difference Set {Dp}, were presented in Chapter 2.

Our main interest in this chapter is in the application of the RMA code sequences, the system model as well as the analysis of the interaction between RMA signals in a multiple access transmission environment. The multiple access interference (MAI) and the channel noise in this proposed RMA system is modeled as an additive white Gaussian noise (AWGN) process with variance equal to the MAI variance plus the variance of channel noise.

The performance parameters, which are considered in this chapter, are the signal-to-noise ratio (SNR) at the output of each correlation receiver and the total SNR at the threshold comparator input.

Part of the system model and analysis in the next two sections have some similarity with the Spread Spectrum Multiple Access (SSMA) system presented in (Pursley, 1977), (Pursley, Garber, and Lenhnert, 1980), (Anderson, and Wintz, 1969), and (Morrow, and Lehnert, 1989). This is because users in both RMA and SSMA access the satellite in a random manner and the systems are operating in an environment where the cochannel interference is dominated noise.

RMA System Model

A model for a RMA transmission system is presented in Figure 3.1. The L active users simultaneously transmit through the satellite transponder to L receivers. Each RMA transmitter contains a set of frequency generators (FG), a set of delay units, biphase modulators, and assembler. Assume that each transmitter has the same power and there is only one overlapping symbol between any two sequences (X=1).

The # k user's data signal $b_k(t)$ is a sequence of unit amplitude, positive and negative, rectangular pulse of duration T. The # k user is assigned a RMA code sequence, which contains M symbols, to represent each of it's binary information bit. Each FG generates a carrier frequency with proper delay by the delay unit to represent a symbol in the code sequence. The biphase modulator phase modulates the data signal $b_k(t)$ of the # k user onto the carrier. The number of such FG and delay units depends on the number of symbols/sequence (M). After passing through the assembler, the corresponding code sequence $S_k(t)$ is formed for transmission. The remaining (L - 1) transmitters are identical to the ≠ k transmitter except that different FG and delay units are used to generate different code sequence for each of the transmitter.

At the receiving end, the corresponding FG and delay units, and correlation detections are used to reverse the encoding process from the received signal W(t). The total output from all M correlation receivers of any #k user (C_k) will include the desired signal $b_k(t)$, channel noise n(t), and multiple-access interference (MAI) signals. The recovery of the data bit b_k is done by the threshold comparator which provides at T interval a +1 or -1 output state, depending on whether C_k is larger or smaller than zero. All notations in the RMA system model are described as followed:

$$b_k(t) = \sum_{j=-\infty}^{\infty} b_{k,j} p_T(t-jT) \quad \text{where } b_{k,j} = \{-1,1\} \tag{3-1}$$

$$p_T(t) = \begin{cases} 1 & , 0 \leqslant t \leqslant T \\ 0 & \text{elsewhere} \end{cases}$$

ω_u = carrier generated by FG unit ($f_u = f_0, f_1, \ldots, f_{max}$)

$\theta_{k,u}$ = random phase shift of the f_u carrier at the # k user

t_p = time delay cause by delay unit ($t_p = t_0, t_1, \ldots, t_{max}$)

τ_k = channel delay between desired receiver (# i) and transmitter # k which includes the differences in propagation delay and message start time.

$S_k(t)$ = transmitted signals from # k user

n(t) = channel noise process which is assumed to be an additive white Gaussian Process with spectral density No/2 and has zero mean.

L = total number of users (or sequences)

W(t) = received signal at any receiver

$$= \sum_{k=1}^{L} S_k(t-\tau_k) + n(t) \qquad (3-2)$$

$Z_{k,n}(t)$ = input signal to the correlation receiver # n (n = 1,2,....,M) of the # k user

$C_{k,n}$ = output signal from the # n correlation receiver of the # k user.

$\sqrt{P}$ = carrier amplitude

M = number of correlation receivers of each user, or total number of symbols/sequence

An Example of RMA System

Figure 3.2 illustrates a RMA transmission system which involves 12 users (L = 12). A unique code sequence of length 3 is assigned to every user as shown in Table 3.1. The sequence parameters of those in Table 3.1 are listed as followed: M = 3 symbols/sequence, R = 4 sequences, X = 1 symbol, and Le = 12 sequences

Each symbol in all sequences is an entry in a discrete time-frequency matrix in Table 3.2. For examples, symbol "6" is represented by frequency f_0 and time-delay t_2 , symbol "7" is represented by frequency f_1 and time-delay t_2. The data signal $b_1(t)$ of the # 1 user is encoded into sequence {1 6 7} while sequence {3 7 8} is represented data signal $b_{12}(t)$ of the # 12 user.

# User	Sequence
1	1 6 7
2	2 7 0
3	3 0 1
4	4 1 2

# User	Sequence
5	5 2 3
6	6 3 4
7	7 4 5
8	0 5 6
9	0 4 8
10	1 5 8
11	2 6 8
12	3 7 8

<u>Table 3.1</u> Sequences from EG Set

$\{ D_e \} = \{ 1,6,7 \}$

	f_0	f_1	f_2
t_0	0	1	2
t_1	3	4	5
t_2	6	7	8

<u>Table 3.2</u> Time-Frequency Matrix

$T_d = 3,\ F = 3$

where T_d can be defined as number of cochannel symbols

From Fig.3.2, the transmitted signal from # 1 user can be written as

$$S_1(t)=\sqrt{P}\, b_1(t)\Big\{\cos\,[\omega_1(t-t_0)+\Theta_{1,1}] + \cos\,[\omega_0(t-t_2)+\Theta_{1,0}] + \cos\,[\omega_1(t-t_2)+\Theta_{1,1}]\Big\}$$

signal from # 2 user,

$$S_2(t)=\sqrt{P}\, b_2(t)\Big\{\cos\,[\omega_0(t-t_0)+\Theta_{2,0}] + \cos\,[\omega_2(t-t_0)+\Theta_{2,2}] + \cos\,[\omega_1(t-t_2)+\Theta_{2,1}]\Big\}$$

signal from # 12 user,

$$S_{12}(t)=\sqrt{P}\, b_{12}(t)\Big\{\cos\,[\omega_0(t-t_1)+\Theta_{12,0}] + \cos\,[\omega_1(t-t_2)+\Theta_{12,1}] + \cos\,[\omega_2(t-t_2)+\Theta_{12,2}]\Big\}$$

Thus, the received signal W(t) in Eq.(3-2) can be written as

$$\begin{aligned}W(t)&=\sum_{k=1}^{12} S_k(t-\tau_k)+n(t)\\
&= \sqrt{P}\, b_1(t-\tau_1)\Big\{\cos\,[\omega_1\,(t-t_0)-(\omega_1\tau_1-\Theta_{1,1})]\\
&\quad + \cos\,[\omega_0(t-t_2)-(\omega_0\tau_1-\Theta_{1,0})] + \cos\,[\omega_1\,(t-t_2)-(\omega_1\tau_1-\Theta_{1,1})]\Big\}\\
&\quad + \sqrt{P}\, b_2(t-\tau_2)\Big\{\cos\,[\omega_0\,(t-t_0)-(\omega_0\tau_2-\Theta_{2,0})]\\
&\quad + \cos\,[\omega_2(t-t_0)-(\omega_2\tau_2-\Theta_{2,2})] + \cos\,[\omega_1\,(t-t_2)-(\omega_1\tau_2-\Theta_{2,1})]\Big\}\\
&\quad + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\end{aligned}$$

$$+ \sqrt{P}\, b_{12}(t-\tau_{12})\Big\{\cos[\omega_0 (t-t_1) - (\omega_0\tau_{12} - \theta_{12,0})]$$

$$+ \cos[\omega_1 (t-t_2) - (\omega_1\tau_{12} - \theta_{12,1})] + \cos[\omega_2 (t-t_2) - (\omega_2\tau_{12} - \theta_{12,2})]\Big\}$$

$$+ n(t) \qquad (3\text{-}3)$$

The input signal to the first correlation receiver of the # 1 user is given by

$$Z_{L1} = W(t) \cos \omega_1(t-t_0)$$

$$= \sum_{k=1}^{12} S_k(t-\tau_k) \cos \omega_1 (t-t_0) + n(t) \cos \omega_1 (t-t_0)$$

and the output from the first correlation receiver of the # 1 user is

$$C_{L1} = \int_0^T Z_{L1} dt$$

Let $\phi_{k,u} = \omega_u \tau_k - \theta_{k,u}$ (3-4)

As we are concerned only with relative phase shifts modulo 2π and relative time delays modulo T, any desired # i user will have $\theta_i = 0$ and $\tau_i = 0$ and $0 \leq \tau_k \leq T$ and $0 \leq \theta_{k,u} \leq 2\pi$ for $k \neq i$. In this particular example, we let i=1. Therefore, $\theta_1 = 0$, $\tau_1 = 0$ and $\phi_{1,u} = 0$.

If $b_{k,-1}$ and $b_{k,0}$ are two consecutive binary data (+1 or -1) of # k user, currently transmitting during the period T, then

$$
\begin{aligned}
\int_0^T b_k(t-\tau_k)\,dt &= \int_0^{\tau_k} b_k(t-\tau_k)\,dt + \int_{\tau_k}^{T} b_k(t-\tau_k)\,dt \\
&= b_{k-1}(\tau_k) + b_{k,0}(T-\tau_k) \\
&= B_k(\tau_k) \qquad (3\text{-}5)
\end{aligned}
$$

and

$$
\begin{aligned}
\int_0^T b_i(t)\,dt &= b_{i,j}(T) \\
&= B_i(T) \qquad (3\text{-}6)
\end{aligned}
$$

where $b_{i,j}$ is the current data bit of #i user.

It is shown in Appendix C.1 that the output from the first correlation receiver of # 1 user becomes

$$
\begin{aligned}
C_{1,1} = \frac{\sqrt{P}}{2}\, B_1(T) + \int_0^T n(t)\cos\omega_1(t-t_0)\,dt \\
+ \frac{\sqrt{P}}{2}\Big\{ B_3(\tau_3)\cos\phi_{3,1} + B_4(\tau_4)\cos\phi_{4,1} + B_{10}(\tau_{10})\cos\phi_{10,1} \\
+ B_1(T)\cos[\omega_1(t_2-t_0)] + B_2(\tau_2)\cos[\phi_{2,1}+\omega_1(t_2-t_0)] \\
+ B_7(\tau_7)\cos[\phi_{7,1}+\omega_1(t_2-t_0)] + B_{12}(\tau_{12})\cos[\phi_{12,1}+\omega_1(t_2-t_0)] \\
+ B_4(\tau_4)\cos[\phi_{4,1}+\omega_1(t_1-t_0)] + B_6(\tau_6)\cos[\phi_{6,1}+\omega_1(t_1-t_0)] \\
+ B_7(\tau_7)\cos[\phi_{7,1}+\omega_1(t_1-t_0)] + B_9(\tau_9)\cos[\phi_{9,1}+\omega_1(t_1-t_0)]\Big\}
\end{aligned}
$$

(3-7)

The desired symbol at the # 1 receiver of the first user is symbol "1" (represented by (f_1, t_0)), which is transmitted from the same user, and will result in recovery of the data signal $b_1(t)$ at the output of the # 1 receiver. However, there are interference signals from other users who share the same symbol "1" and users who share those symbols that represented by the same frequency (f_1) but have different time-delay. Hence, the above equation can be rewritten as

$C_{1,1}$ = [desired symbol "1"(f_1, t_0) from # 1 user]

+ [unwanted symbol "1"(f_1, t_0) from # 3,# 4,# 10 user

+ unwanted symbol "7"(f_1, t_2) from # 1,# 2,# 7,# 12 user

+ unwanted symbol "4"(f_1, t_1) from # 4,# 6,# 7,# 9 user]

+ channel noise

Generalized RMA System Analysis

The user # i (i=1,2,....,L) has receiver # n (n=1,2,....,M) designed to match the symbol which represented by (f_u, t_m). Hence, the output from this receiver will be as derived in the previous section.

$C_{i,n}$ = [desired signal from # i user] + [channel noise]

+ [MAI signals]

or

$$C_{i,n} = \frac{\sqrt{P}}{2} B_1(T) + \int_0^T n(t) \cos \omega_u (t - t_m)\, dt$$

$$+ \frac{\sqrt{P}}{2} \left\{ \sum_{\hat{k}=1}^{L} B_{\hat{k}}(\tau_{\hat{k}}) \cos \phi_{\hat{k},u} \right.$$

$$\left. + \sum_{\substack{t_p = t_0 \\ t_p \neq t_m}}^{t_{max}} \sum_{k'=1}^{L} B_{k'}(\tau_{k'}) \cos [\phi_{k',u} + \omega_u (t_p - t_m)] \right\} \qquad (3\text{-}8)$$

where $\hat{k}$ = any simultaneous user that shares the desired symbol (f_u, t_m) with # i user

k' = any simultaneous user that shares the symbol (f_u, t_p) which represented by the same frequency (f_u) as the desired symbol but has different time delay

From Eq.(3-4), $\phi_{k,u}$ is a random variable which depends on random channel delay (τ_k) and phase shift $\Theta_{k,u}$, and uniformly distributed on the interval $[0, 2\pi]$. Therefore,

$$E[\cos \phi_{k,u}] = 0 \qquad (3\text{-}9)$$

where E[.] is the expectation operator.

$b_{k,j}$ is also a random variable independent to $\phi_{k,u}$, which takes on the value +1 or -1 with equal probability. Because of the symmetry involved, we need to consider only $b_{i,j} = 1$. Hence, from Eq.(3-8) and (3-9) the mean of $C_{i,n}$ can be derived for the case of current data signal $b_{i,j} = 1$ as

$$\bar{C}_{i,n} = E[C_{i,n}]$$

$$= \frac{\sqrt{P}}{2}\{E[b_{i,j}(T)]\} = \frac{\sqrt{P}\,T}{2} \qquad (3\text{-}10)$$

Because the channel noise n(t) has zero mean and spectral density $N_o/2$, its variance is derived as

$$\mathrm{Var}\left[\int_0^T n(t)\cos\omega_u(t-t_m)dt\right] = E\left[\left\{\int_0^T n(t)\cos\omega_u(t-t_m)dt\right\}^2\right]$$

$$= \frac{N_o T}{4} \qquad (3\text{-}11)$$

The channel noise and all multiple-access interference components are mutually independent, thus, the noise-cross-unwanted signal term is zero and the variance of all noise component of $C_{i,n}$ can be written as

$$\mathrm{Var}[C_{i,n}] = \mathrm{Var}[MAI] + \mathrm{Var}[channel\ noise]$$

$$= \mathrm{Var}[Q_{i,n}] + \mathrm{Var}[X_{i,n}] + N_o T/4 \qquad (3\text{-}12)$$

where

$$Q_{i,n} = \frac{\sqrt{P}}{2} \sum_{\hat{k}=1}^{L} B_{\hat{k}}(\tau_{\hat{k}}) \cos \phi_{\hat{k},u} \tag{3-13}$$

$$X_{i,n} = \frac{\sqrt{P}}{2} \sum_{\substack{t_p=t_0 \\ t_p\neq t_m}}^{t_{max}} \sum_{k'=1}^{L} B_{k'}(\tau_{k'}) \cos[\phi_{k',u} + \omega_u(t_p - t_m)] \tag{3-14}$$

From Eqs.(3-9),(3-13) and (3-14), we find that the mean of all unwanted signal components are zero.

$$E[Q_{i,n}] = E[X_{i,n}] = 0$$

Thus,

$$Var[Q_{i,n}] = E[(Q_{i,n})^2] \tag{3-15}$$

$$Var[X_{i,n}] = E[(X_{i,n})^2] \tag{3-16}$$

Appendix C.2 and C.3 show the derivation of $E[(Q_{i,n})^2]$ and $E[(X_{i,n})^2]$ which have the following results

$$Var[Q_{i,n}] = \frac{P}{8T}\left[\frac{2T^3}{3}\right] \hat{R}_{i,n}$$

$$Var[Q_{i,n}] = \frac{PT^2}{12} \hat{R}_{i,n} \tag{3-17}$$

$$Var[X_{i,n}] = \frac{PT^2}{12} \sum_{\substack{t_p=t_0 \\ t_p\neq t_m}}^{t_{max}} R'_{i,n}(t_p) \tag{3-18}$$

where $\hat{R}_{i,n}$ = total number of $\hat{k}$ users, $0 \leq \hat{R}_{i,n} \leq R-1$

$R'_{i,n}(t_p)$ = total number of k' users, $0 \leq R'_{i,n} \leq R$

Generally, the TF matrix is a rectangular or square matrix, therefore, $R'_{i,n}(t_0) = R'_{i,n}(t_1) = \ldots\ldots = R'_{i,n}(t_{max}) = R'_{i,n}$, then Eq.(3-18) becomes

$$Var[X_{i,n}] = \frac{PT^2}{12}(T_d-1)\,R'_{i,n} \qquad (3\text{-}19)$$

where T_d = number of time-delay in the TF matrix

By utilizing Eqs.(3-10),(3-17) and (3-19) we can derive the signal-to-noise ratio ($SNR_{i,n}$) of the output of each correlation receiver ($C_{i,n}$) as

$$SNR_{i,n} = \frac{\text{rms desired signal}}{\sqrt{\text{Variance of the noise component of } C_{i,n}}}$$

$$SNR_{i,n} = \frac{\sqrt{P}\ T/2}{\sqrt{Var[C_{i,n}]}}$$

$$SNR_{i,n} = \left\{ \frac{PT^2/4}{\frac{PT^2}{12}[\hat{R}_{i,n} + R'_{i,n}(T_d-1)] + \frac{N_0T}{4}} \right\}^{1/2}$$

Finally, we have the simplified expression for the SNR at the output of #i user's correlation receiver # n as

$$SNR_{i,n} = \left\{ \frac{1}{3}[\hat{R}_{i,n} + R'_{i,n}(T_d-1)] + \frac{N_o}{2E_b} \right\}^{-1/2} \qquad (3\text{-}20)$$

where $E_b = \frac{PT}{2}$ is the energy per data bit

The ideal case would be when only # i user is actived, which will reduce Eq.(3-20) to

$$\max.[SNR_{i,n}] = \left\{ \frac{2E_b}{N_o} \right\}^{1/2} \qquad (3\text{-}21)$$

If the statistic of the simultaneous are known, then $\hat{R}$ and R' are also known and can be substituted into Eq.(3-20).

The variable R is related to the sequence length (M) and the total number of users (L) as followed:

<u>PG Set</u>, from Eq.(2-16) $\quad R_p = M$

from Eq.(2-11), $\quad L_p = M(M-1) + 1$

Thus, $\quad L_p = R_p(R_p-1) + 1 \qquad (3\text{-}22)$

<u>EG Set</u>, from Eq.(2-17) $\quad R_e = M+1$

from Eq.(2-14), $\quad L_e = M(M+1)$

Hence, $$L_e = R_e(R_e - 1) \tag{3-23}$$

Since each correlation receiver is performing independently, the total SNR from all M correlation receiver, which is also the SNR at the threshold comparator input, can be derived as followed:

$$SNR_i = \frac{\text{total rms desired signal}}{\sqrt{\text{total noise power at threshold comparator input}}}$$

$$= \frac{\sum_{n=1}^{M} \overline{C}_{i,n}}{\sqrt{Var\left[\sum_{n=1}^{M} C_{i,n}\right]}}$$

$$= \frac{\overline{C}_{i,n}}{\sqrt{Var(C_{i,n})}} \tag{3-24}$$

From Eq.(3-10), we can express the total rms desired signal at the threshold comparator input as $\sum_{n=1}^{M} \frac{\sqrt{P}\,T}{2}$

$$\text{total rms desired signal} = \frac{M\sqrt{P}\,T}{2} \tag{3-25}$$

From Eq.(3-12), we have that

$$\mathrm{Var}[C_{i,n}] = \mathrm{Var}[Q_{i,n}] + \mathrm{Var}[X_{i,n}] + \mathrm{Var}[\text{channal noise}]$$

Recall that the output from each correlation receiver are independent to each other, and the fact that the variance of the sum of zero-mean, independent random variable is the sum of their second moment, we have

$$\mathrm{Var}[\sum_{n=1}^{M} C_{i,n}] = \sum_{n=1}^{M} \left\{ E[(Q_{i,n})^2] + E[(X_{i,n})^2] + E[(\text{channel noise})^2] \right\}$$

$$\mathrm{Var}[C_i] = \sum_{n=1}^{M} \left\{ \mathrm{Var}[Q_{i,n}] + \mathrm{Var}[X_{i,n}] + \mathrm{Var}[\text{channel noise}] \right\} \qquad (3-26)$$

Substituting Eqs.(3-11),(3-17) and (3-18) into the above equation results in

$$\mathrm{Var}[\text{total noise and MAI signals}] = \frac{PT^2}{12} \left\{ \sum_{n=1}^{M} \hat{R}_{in} + \sum_{n=1}^{M} \sum_{\substack{t_p = t_0 \\ t_p \neq t_m}}^{t_{max}} R'_{in}(t_p) \right\} + \frac{MN_0T}{4} \qquad (3-27)$$

Finally, we will be able to obtain an expression for the SNR_i at the input of threshold comparator by substituting Eq.(3-27) and (3-25) into Eq.(3-24)

$$SNR_i = \frac{M\sqrt{P}\,\frac{T}{2}}{\sqrt{\frac{PT^2}{12}\left[\sum_{n=1}^{M}\hat{R}_{i,n} + \sum_{n=1}^{M}\sum_{\substack{t_p=t_0\\ t_p\neq t_m}}^{t_{max}} R'_{i,n}(t_p)\right] + \frac{MN_0T}{4}}}$$

$$= \left[\frac{\frac{M^2PT^2}{4}}{\frac{PT^2}{12}\left[\sum_{n=1}^{M}\hat{R}_{i,n} + \sum_{n=1}^{M}\sum_{\substack{t_p=t_0\\ t_p\neq t_m}}^{t_{max}} R'_{i,n}(t_p)\right] + \frac{MN_0T}{4}}\right]^{\frac{1}{2}}$$

$$= \left[\frac{M}{\frac{1}{3M}\left[\sum_{n=1}^{M}\hat{R}_{i,n} + \sum_{n=1}^{M}\sum_{\substack{t_p=t_0\\ t_p\neq t_m}}^{t_{max}} R'_{i,n}(t_p)\right] + \frac{N_o}{PT}}\right]^{1/2}$$

$$SNR_i = \left[\frac{M}{\frac{1}{3M}U_i + \frac{N_o}{2E_b}}\right]^{1/2} \qquad (3\text{-}28)$$

where $$U_i = \sum_{n=1}^{M}\hat{R}_{i,n} + \sum_{n=1}^{M}\sum_{\substack{t_p=t_0\\ t_p\neq t_m}}^{t_{max}} R'_{i,n}(t_p) \qquad (3\text{-}29)$$

or $$U_i = \sum_{n=1}^{M}\hat{R}_{i,n} + \sum_{n=1}^{M} R'_{i,n}(T_d-1) \qquad (3\text{-}30)$$

and $\frac{E_b}{N_o} = \frac{PT}{2N_o}$ = energy-per-bit/channel noise-density ratio

Eqs.(3-28)and (3-30) shows that $SNR_{i,n}$ depends on number of simultaneous users, sequence length, the energy per data bit and the design of TF matrix which determines total numbers and values of time-delay units associated with any carrier frequency. Thus, in order to obtain a good system performance for a given energy per data bit, we have to optimize sequence parameters and the TF matrix. This is the subject to be discussed in the next chapter.

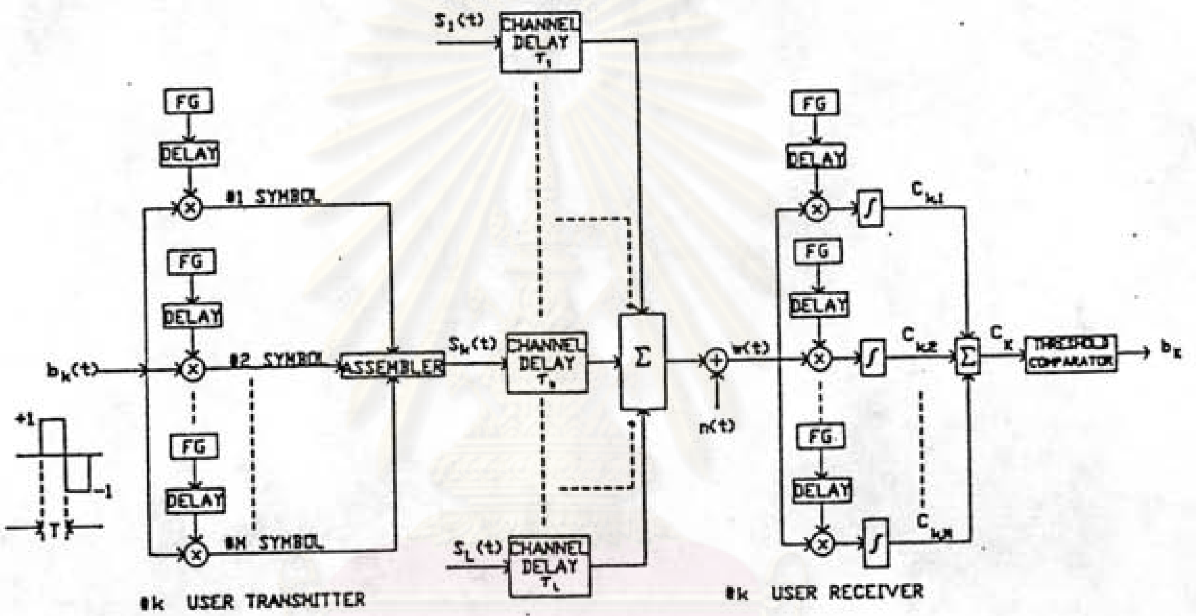

Figure 1. RMA System Model

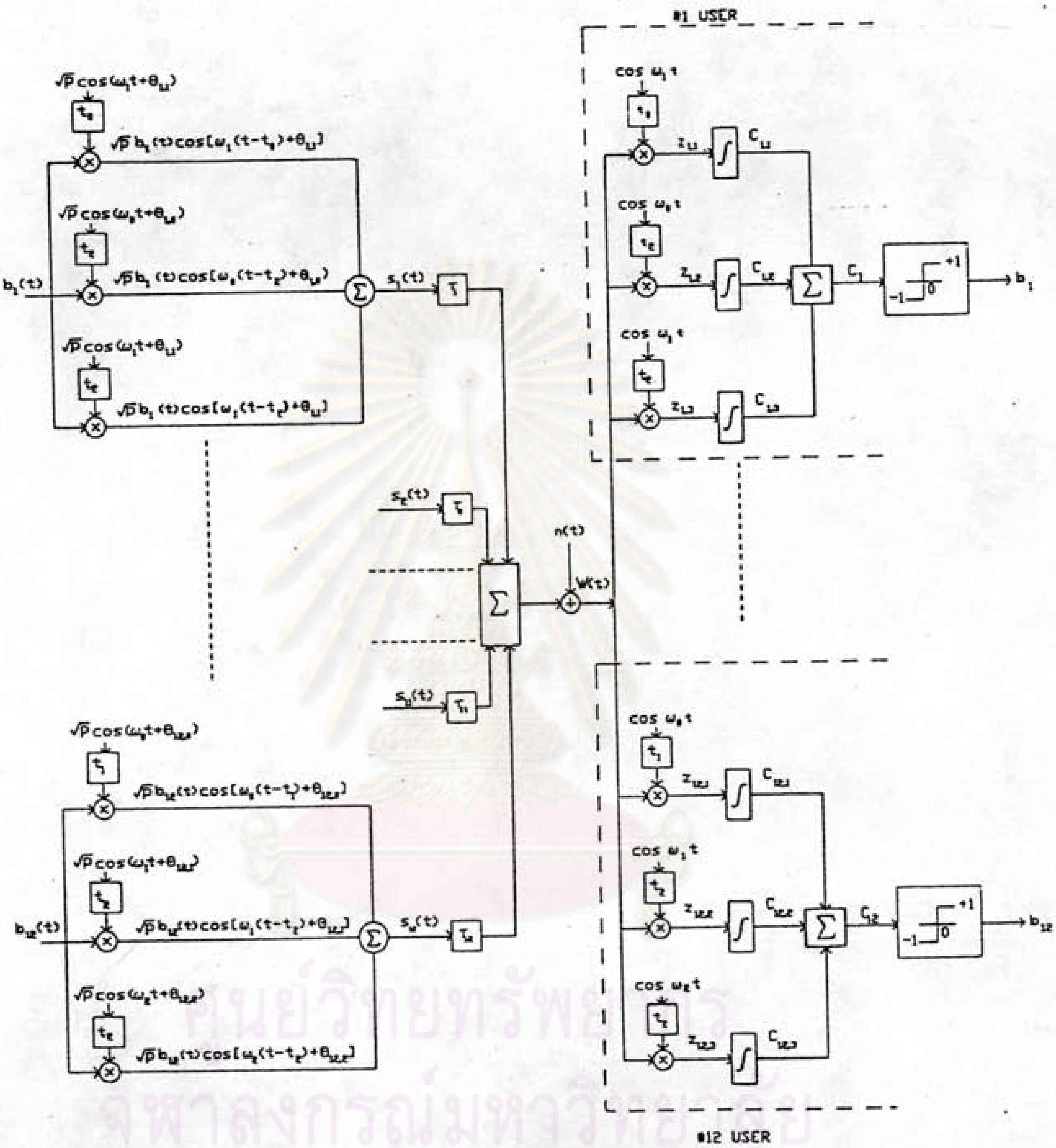

Figure 2. 12 Users RMA Transmission System