CHAPTER 2

CODE SEQUENCES FOR RANDOM MULTIPLE ACCESS

Introduction

As discussed in Chapter 1 that the basic transmission scheme of RMA system is the transformation of each binary bit into a sequence of preassigned nonbinary symbols. For a given number of access stations, which generally is very large, the system performance are greatly affected by the sequence parameters, such as sequence length and the number of overlapping symbols. Therefore, it is important to find ways to construct a large number of code sequences for a given length with specified number of overlapping symbols allowed between any pair of sequences.

The difference sets are particularly useful to provide the solution to the problem. We can easily obtain the solution by matching the structure of the sets to the physical parameters of the problem as follows:

Parameter	Code Sequence	Difference Set
M	-Sequence length (symbols/sequence)	-Number of distinct positive integer in the set
V	-Total number of symbols	-Maximum value of pairwise differences of all M integers
X	-Number of overlapping symbols allowed between any pair of sequences	-Number of times that the differences between any two integers in the set appear
L	-Total number of usable sequences	N/A

Difference Sets

Two types of difference sets, Euclidean Geometry (EG) Difference Set {De} and Projective Geometry (PG) Difference Set {Dp}, are particularly useful for the generation of these sequences. For application purposes, the algorithms for set generation and the proof of the algorithms are omitted here and they can be found in (Baumert, 1971), (Berman, 1972), (Mann, 1965), (Raghavarao, 1971), and (Wu, 1984).

The following properties are common to all difference sets {D}.(Wu, 1984) (Baumert, 1971)

If $\{D\} = \{d_0, d_1, \ldots, d_{M-1}\}$ with parameters V,M,X then

A) for any positive integer $\delta \neq 0$ (mod V), which is not necessary an element of {D}

$$d_i - d_j = \delta \pmod{V} \tag{2-1}$$

has X solution pairs (d_i, d_j) with d_i and d_j in {D}

B) $\{D_i\} = \{d_0+i, d_1+i, \ldots\ldots d_{M-1}+i\} \pmod{V}$ (2-2)

with $i = 0,1,2,\ldots\ldots,V-1$

is also a difference set with same parameters as {D}.

A) Sequences Generated from {Dp}

The following relation has to be satisfied for PG set as a result from Eq.(2-1). (Wu, 1984)(Baumert, 1971)

$$M(M-1) = X(V-1) \tag{2-3}$$

When $X \geq 1$, each sequence can be derived from $\{D_p\}$ as followed

$S_0 = d_0, d_1, \ldots\ldots, d_{M-1}$ mod V

$S_1 = d_0+1, d_1+1, \ldots, d_{M-1}+1$ mod V

$S_2 = d_0+2, d_1+2, \ldots, d_{M-1}+2$ mod V

. . . . .

$S_{M-1} = d_0+(M-1), d_1+(M-1), \ldots, d_{M-1}+(M-1)$ mod V

Table of some PG sets, $\{D_p\}$, from (Baumert, 1971), are shown in Appendix A. As can be observed that there exists an infinite number of difference sets with X = 1 but for every X > 1, there exists only a finite number of difference sets. For example, if we want to derive sequences with M = 6 symbols in each sequence, when X = 1, we can utilize $\{D_p\}$ = {1,5,11,24,25,27} but there is no $\{D_p\}$ available with X > 1 for the same value of M. In the case when M = 5, there are two difference sets with different X and V available. All usable sequences for those two sets are shown in Appendix B.1.

B) Sequences Generated from {De}

In contrast to Eq.(2-3), the relation for {De} is:

$$M(M-1) < X(V-1) \tag{2-4}$$

Equation (2-4) is an inequality which has to be held among these three set parameters. The following are all usable sequences which can be derived from $\{D_e\}$ in the special case when X = 1 and V = M^2 where M is a prime or prime power (Wu, 1984).

$S_0 = d_0, d_1, \ldots, d_{M-1} \quad \mathrm{mod}(M^2-1)$

$S_1 = d_0+1, d_1+1, .., d_{M-1}+1 \quad \mathrm{mod}(M^2-1)$

. . . .

. . . .

$S_{M^2-2} = d_0+(M^2-2), d_1+(M^2-2), \ldots, d_{M-1}+(M^2-2) \qquad \mathrm{mod}(M^2-1)$

$S_{M^2-1} = 0(M+1), 1(M+1), 2(M+1), \ldots, (M-2)(M+1), M^2-1$

$S_{M^2} = 0(M+1)+1, 1(M+1)+1, 2(M+1)+1, \ldots, (M-2)(M+1)+1, M^2-1$

$S_{M^2+1} = 2, 1(M+1)+2, 2(M+1)+2, \ldots, (M-2)(M+1)+2, M^2-1$

. . . .

. . . .

$S_{M^2+(M-1)} = M, 1(M+1)+M, 2(M+1)+M, \ldots, (M-2)(M+1)+M, M^2-1$

An example of usable sequences derived from a $\{D_e\}$ are shown in Appendix B.2.

Usable Sequences

From the concept that a difference set is a set of M distinct positive integers such that the pairwise differences of all M integers appear exactly X times and from the properties of difference sets described in the previous section, we can derive the number of usable sequences (L) as followed:

The total number of different pairs of symbols available from all V symbols is $\frac{V(V-1)}{2}$ (2-5)

The total number of different pairs of symbols in a sequence of M symbols is $\frac{M(M-1)}{2}$ (2-6)

When there is only one symbol overlapping between any two sequences, the total number of usable sequences can be written as

$$L = \frac{V(V-1)}{M(M-1)} \qquad (2\text{-}7)$$

When every V(V-1)/2 pairs is shared among X sequences, we have

$$L = \frac{V(V-1)X}{M(M-1)} \qquad (2\text{-}8)$$

For PG Sets, from Eq.(2-3)

$$1 = \frac{(V-1)X}{M(M-1)} \qquad (2\text{-}9)$$

Let the total number of usable sequences = L_p , then from Eqs.(2-8) and (2-9), when X > 0 we have

$$L_p = V_p \qquad (2\text{-}10)$$

When X = 1, $L_p = M(M-1) + 1$ (2-11)

For EG Sets, from Eq.(2-4)

$$1 < \frac{X(V-1)}{M(M-1)} \tag{2-12}$$

The total number of usable sequences = L_e

From Eqs.(2-8) and (2-12), when X > 0 we have

$$L_e > V_e \tag{2-13}$$

In the special case of M is a prime number or prime power , X = 1 and V = M^2

$$L_e = M(M+1) \tag{2-14}$$

Complete orthogonality among the set of sequences, the case when X = 0, provides the maximum cross-correlation property and thus, minimize the probability of error. However, such complete orthogonality also limits the number of usable sequences to V/M. In order to increase the number of code sequence we allow one or more overlapping symbol between any two sequences which is the case for X > 0.

Table 2.1 shows some examples of number of sequences, each of which contains M symbols, which can be constructed from V symbols when X = 1.

M	Number of Symbols		Number of Sequences	
	$V_p = M(M-1)+1$	$V_e = M^2$	$L_p = V_p$	$L_e = M(M+1)$
3	7	9	7	12
4	13	16	13	20
5	21	25	21	30
6	31	#	31	#
8	57	64	57	72
9	73	81	73	90
17	273	289	273	306
30	871	#	871	#
97	##	9,409	##	9,506
98	9,507	#	9,507	#

No EG Set available when M is not a prime or prime power

PG Set is not available for that value of M

Table 2.1. Number of Symbols and Sequences for Sequence Length from 3 to 98

We can observed from Table 2.1 that for the same length of sequence and allowing only one overlapping symbol, EG set always requires more number of symbols(V) than PG set and provides more number of usable sequences.

Other Parameters

Due to the availability of the difference sets when X>1 (Baumert, 1971)(Wu, 1984), from now on we will limit the parameter X to 1, although the general approach may be accommodated for the case of X > 1.

As we will show in Chapter 3, there is a parameter R which will effect the RMA system performance. R is defined as the number of sequences that share the same symbol

or R = (total symbols from all sequences, including repeated symbols)/(total number of symbols available from TF matrix)

$$= \frac{\text{(total sequences)*(sequence length)}}{T_d * F}$$

$$R = \frac{L*M}{V} \qquad (2\text{-}15)$$

(where * denotes a multiple sign)

In the case of PG set, substituting the result in Eq.(2-10) into (2-15) we have

$$R_p = M \qquad (2\text{-}16)$$

For EG set, we substitute Eq.(2-14) into Eq.(2-15) to obtain

$$R_e = \frac{M(M+1)*M}{M^2}$$

$$= M+1 \qquad (2-17)$$

At this point, we would like to relate the sequence parameter V to the TF matrix mentioned earlier in Chapter 1. For X = 1, from Eqs. (2-9) and (2-14), we have

$$V_p = M(M-1) + 1 \qquad (2-18)$$

$$V_e = M^2 \qquad (2-19)$$

Total available symbols from the TF matrix is T_d*F where the parameters T_d and F are as defined in Chapter 1.

In order to provide enough symbols for the construction of sequences, we must have

EG set, $T_d*F = M^2$ (2-20)

PG set, $T_d*F = M(M-1)+1$ (2-21)

As the total number of symbols required in many PG set (Appendix A) are odd numbers, there will be situations when

$$V_p = T_dF + 1$$

or

$$V_p = T_dF - 1$$

In other word, one of the V_p code symbols is not an entry in the TF matrix but can be used to construct code sequences. The transmitter will transmit a zero level signal to represent this additional symbol. But in the latter case, there will be an unused symbol in the matrix.