CHAPTER 2

SYSTEM PERFPRMANCE

2.1 System Details

2.1.1 Binary PSK System

In general the modulating signal in a PSK system is pulse-code-modulated (PCM) waveform. Considering a modulating binary signal of v(t), which becomes +V or -V, the PSK waveform is given by [18]

$$v_{PSK}(t) = A \cos \{\omega_o t + \psi(t)\} \quad \ldots \quad \ldots \quad ..2.1$$

where A is a constant amplitude and $\psi = 0$ for $v(t) = +V$ and $\psi = \pi$ for $v(t) = -V$. The Eq.(2.1) can also be written as

$$v_{PSK}(t) = \frac{v(t)}{V} A \cos \omega_o t \quad \ldots \quad \ldots \quad 2.2$$

$$= \pm A \cos \omega_o t \text{ depending on the sign of } v(t).$$

For a constant amplitude sinusoid carrier its phase-shift keying with a sharp keving transitions between two phase states separated by π radians provides an optimum signaling. The PSK signal, so generated, will have the waveform of plus-minus rectangular pulses of a continuously generated sinusoid carrier as illustrated in Fig 2.1[19-21]

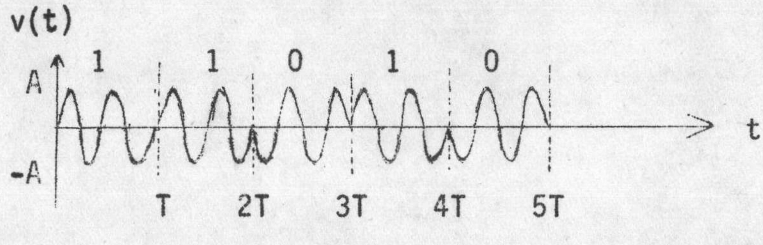


Fig 2.1 PSK Signal

This type of signal is produced by means of DSB-SC modulation of a carrier by a bipolar rectangular waveform or by direct phase modulation of a carrier. However, with a view of avoiding the problem of critical frequency synchronization and intersymbol interference, a baseband signal of 100 percent sinusoid roll-off, better known as raised-cosine or cosine-squared pulse, is frequently used instead of a rectangular waveform[1]. For a binary message 101101001 represented by such pulses, as illustrated in Fig 2.2 (a), a binary PSK waveform[1] of Fig 2.2 (b) is obtained.

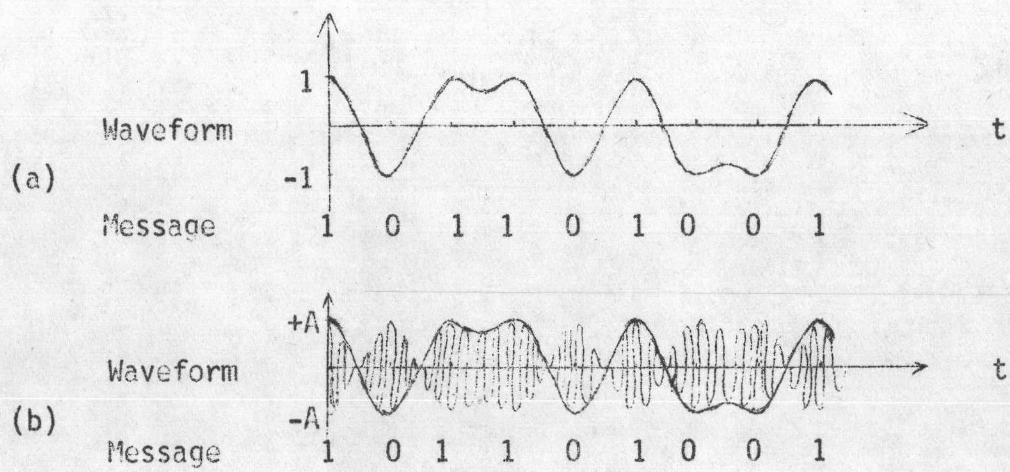


Fig 2.2 (a) Baseband Waveform for the Binary Message 101101001

(b) Binary PSK Waveform

In Fig 2.2(b) it can be seen that the amplitude variations and the phase reversals are distinct, of which amplitude variation can be removed by means of a limiter.

A PSK waveform is generated by applying a baseband signak v(t) and the carrier cos $\omega_o t$ as shown in Fig 2.3

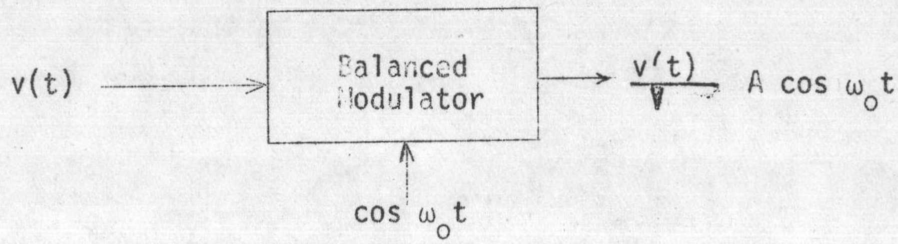


Fig 2.3 PSK Wave Generation

2.1.2 Four-phase PSK

So far the case discussed has been only for binary communication systems, where one of the two possible messages is transmitted in any interval $0 \leq t \leq T$. A system where many possible messages are transmitted in the interval T is called M-ary systems, where M being the number of messages.

Generally in a 4-phase PSK system, one of the 4 possible waveforms is transmitted in each interval. Such 4-phase signals are generated by the combination of two AM wavwes in quadrature[18-20]. The binary digits that are to be transmitted are grouped in two pairs, X and Y. Both X

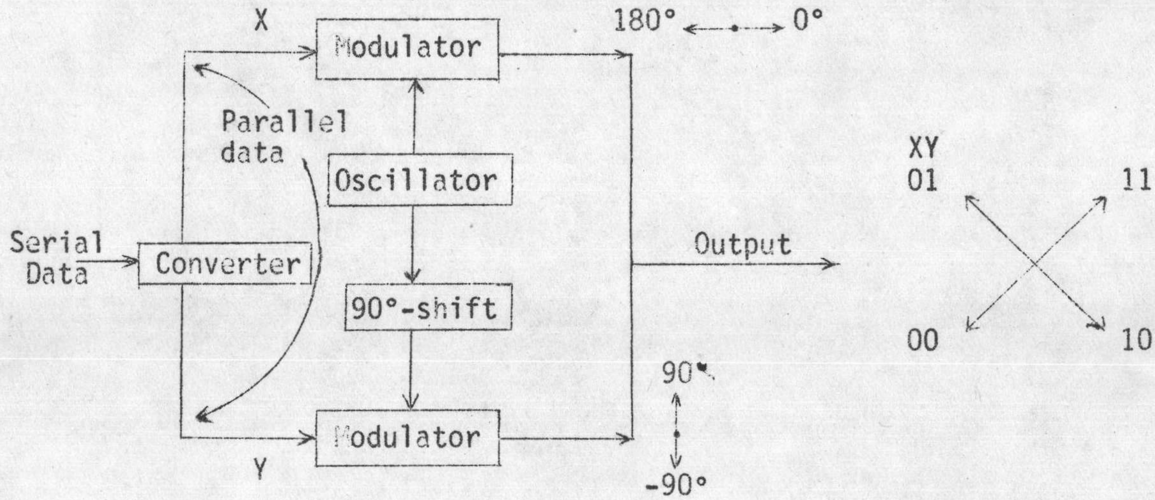


Fig 2.4 Four-phase Signal Generation

and Y digits are applied simultaneously to the linear amplitude modulators. The X modulator is fed with 0° carrier and the Y modulator with 90° carrier. The X modulator output will have 0° carrier when the X digit is a "1" and 180° carrier when the X digit is a "0". Likewise the Y modulator output will have plus or minus 90° carrier depending on whether Y is "1" or "0". Both the modulator outputs are summed to give a four-phase PSK signals as illustrated in Fig 2.4, where the two digit combinations are arranged in such a code that the adjacent combinations differ by one digit.

2.1.3 Armstrong Phase Modulator

As the modulator developed for the research is based on the Armstrong modulation technique, it is felt necessary to discuss the technique

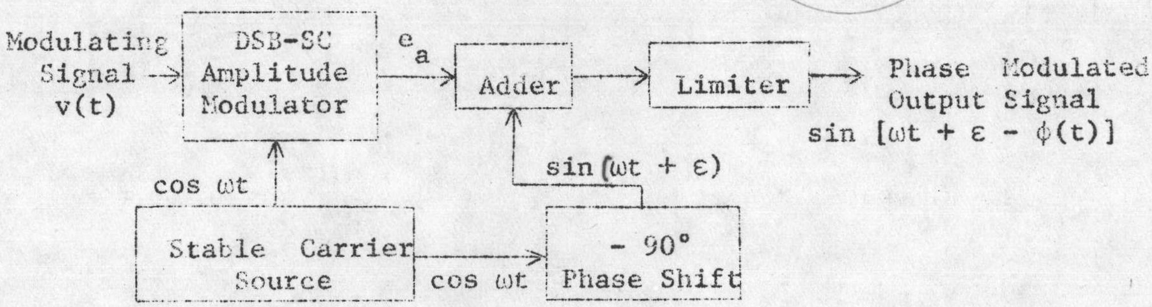


Fig 2.5 Armstrong Phase Modulator[15,18]

to certain details. In an Armstrong phase modulator as shown in Fig 2.5 the baseband signal is fed to a double-sideband suppressed-carrier amplitude modulator where modulation should preferably take place with low index of modulation to retain linearity. To the output of the modulator another carrier, 90° out of phase with the first one is added to the sidebands. The amplitude variation of the adder output is removed by a limiter which would finally yield a low-index phase-modulated signal.

Suppose the modulating baseband signal is

$$e = v(t) \quad \dots \quad \dots \quad |v(t)| \leqq 1 \quad \dots \quad \dots \qquad 2.3$$

Then the DSB-SC amplitude modulator output is

$$e_a = mv(t) \cos \omega t \quad \dots \quad \dots \quad \dots \qquad 2.4$$

where $m \leqq 1$ is the modulation index.

Upon adding a quadrature carrier of correct phase to the e_a, the sum becomes

$$e_p = mv(t) \cos \omega t + \sin (\omega t + \varepsilon) \ \ldots \quad \ldots. \quad \ldots \quad 2.5$$

where ε is small and represents an error in carrier phase. The phasor relationship of this Eq (2.5) is shown in Fig 2.6.

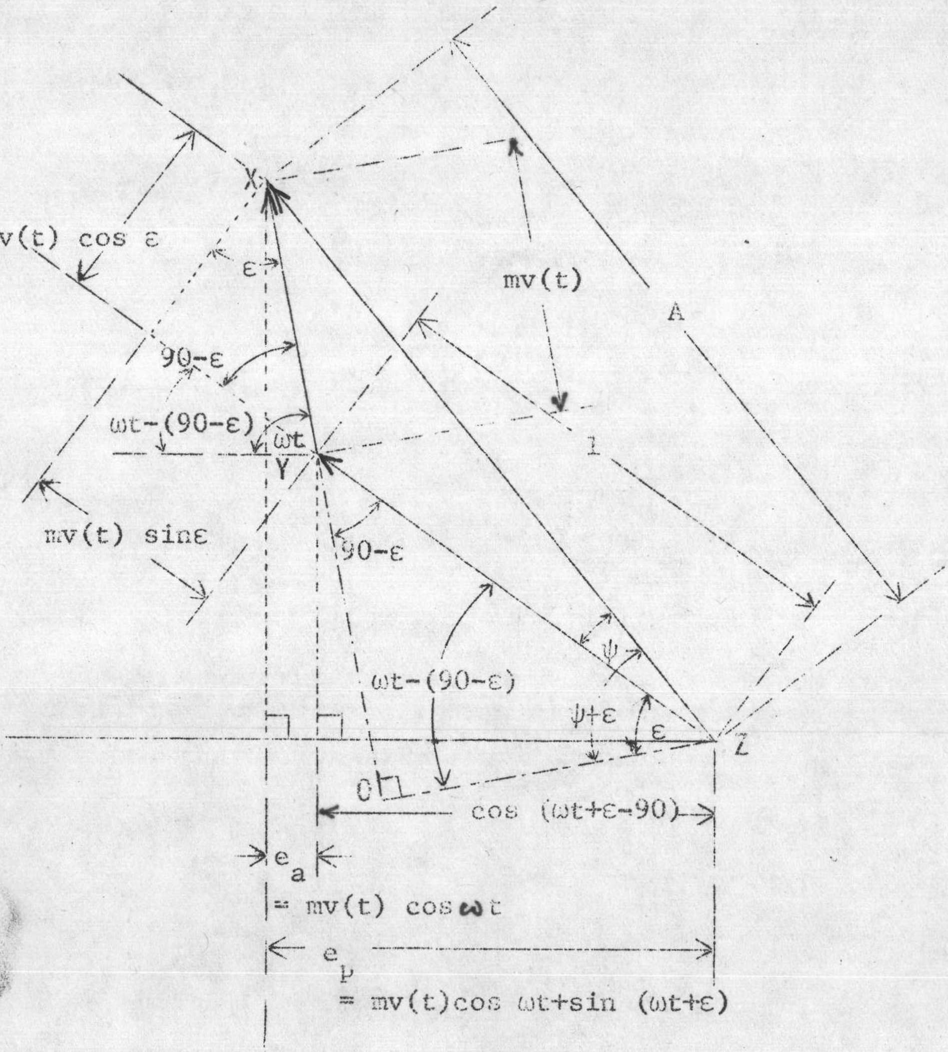


Fig 2.6 Phasor Diagram

From Fig 2.6, the magnitude of A is given by

$$A = \sqrt{[m^2v^2(t)\cos^2\varepsilon + \{1 + mv(t)\sin\varepsilon\}^2]}$$

$$= \sqrt{[m^2v^2(t)\{\cos^2\varepsilon + \sin^2\varepsilon\} + 2mv(t)\sin\varepsilon + 1]}$$

$$= \sqrt{[m^2v^2(t) + 2mv(t)\sin\varepsilon + 1]} \quad \ldots \quad \ldots \quad \ldots \quad 2.6$$

and the direction is given by

$$\psi = \tan^{-1}\left[\frac{mv(t)\cos\varepsilon}{1 + mv(t)\sin\varepsilon}\right] \quad \ldots \quad \ldots \quad \ldots \quad 2.7$$

$$\therefore e_p = A\cos(\psi + \omega t + \varepsilon - 90°)$$

$$= A\cos[-\{90° - (\omega t + \varepsilon + \psi)\}]$$

$$= A\cos\{90° - (\omega t + \varepsilon + \psi)\}$$

$$\therefore e_p = A\sin(\omega t + \varepsilon + \psi) \quad \ldots \quad \ldots \quad \ldots \quad 2.8$$

i.e;

$$e_p = \{\sqrt{[1 + 2mv(t)\sin\varepsilon + m^2v^2(t)]}\}\sin\left(\omega t + \varepsilon + \tan^{-1}\left[\frac{mv(t)\cos\varepsilon}{1 + mv(t)\sin\varepsilon}\right]\right) \quad \ldots \quad 2.9$$

The amplitude of this signal becomes constant when it is passed through a perfect limiter, whereby the phase modulated signal is obtained whose phase modulation is given by ψ as expressed by Eq (2.7), which can further be expanded as :

$$\psi(t) = mv(t)\cos\varepsilon - m^2v^2(t)\sin\varepsilon\cos\varepsilon + m^3v^3(t)\sin^2\varepsilon\cos\varepsilon - \frac{m^3}{3}v^3(t)\cos^3\varepsilon + \ldots \quad \ldots \quad 2.10$$

In the expansion, for small non-linear distortion, it is controlled by second and third order terms, so those of higher order are omitted.

For an ideal case $\varepsilon = 0$, which yields the first term of the expansion as the desired modulating signal, while the second and third terms vanish, remaining the last term as third order distortion. But when $\varepsilon \neq 0$, second order distortion occurs and the required output signal amplitude is reduced by the factor cos ε. Besides that, the proper choice of m also results in reduction of distortion to some extent. The choice of m which is proportional to the phase distortion depends upon the specific value of v(t).

The further development of this Armstrong phase modulator is resulted in a new PSK modulator which is described in the following section.

2.1.4 A New Phase Modulator

Inthe last section, the output phase of the Armstrong modulator is shown to be proportional to the inverse tangent of the baseband signal. For such a case good linearity is obtained only by the small phase deviation. Now a new phase modulator[17] with an improved linearity will be discussed.

In this modulator, two Armstrong modulators are arranged in balanced configuration. The inphase carrier is added to the quadrature carrier at the output of the DSB-SC modulators. A proper adjustment of the added inphase carrier will eliminate the third order nonlinearity and the linearity will further be improved. For this reason, a modulation index of unity is used for the two Armstrong modulators.

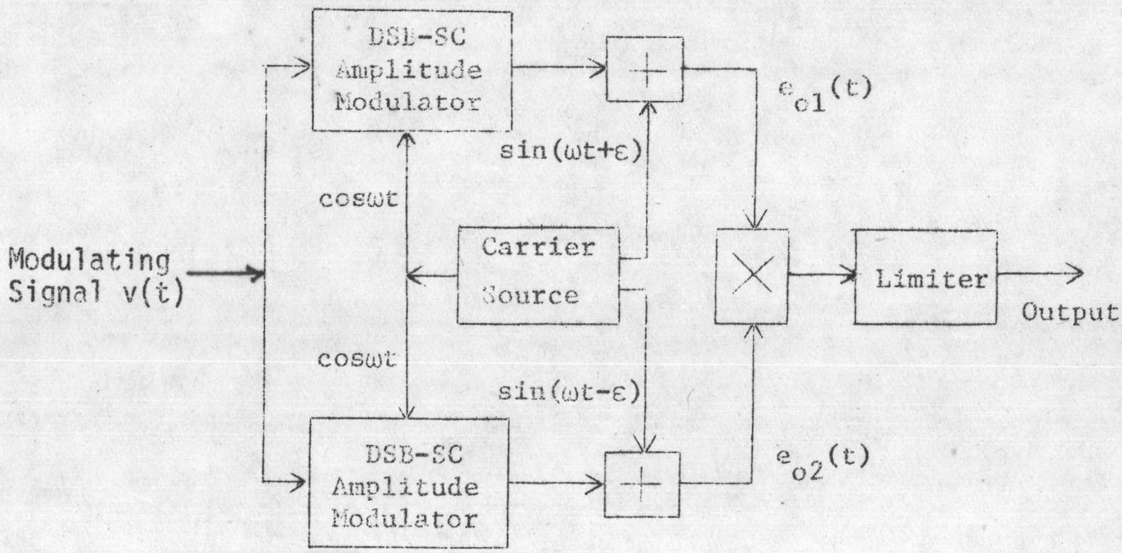


Fig 2.7 New Phase Modulator

Just as in the case of Armstrong Modulator, here is v(t) as modulating signal which is fed to the DSB-SC AM along with the carrier cos ωt. With the exclusion of index of modulation factor compared to the former case, each DSB-SC AM will yield an output of v(t) cos ωt. Upon adding this output to the two inphase quadrature carriers in the two adders, two sums are obtained. They are

$$e_{o1}(t) = v(t)\cos \omega t + \sin(\omega t + \varepsilon) \quad \ldots \ldots \ldots |v(t)| \leq 1 \ldots \quad 2.11$$

and $$e_{o2}(t) = v(t)\cos \omega t + \sin(\omega t - \varepsilon) \quad \ldots \ldots \ldots \quad \ldots \quad \ldots \quad 2.12$$

The phasor relationship for the Eq (2.11) can be established the same way by the Fig 2.6. However, the carrier phase error ε can be included in the phase modulation by slight extension in the phasor diagram Fig 2.6. Extending XY towards Z and dropping a normal line to

XY extended, it is evident that $\angle ZYO = 90° - \varepsilon$ and $\angle XZO = \psi + \varepsilon$. From Eq (2.8), Eq (2.11) becomes

$$e_{o1}(t) = A \sin (\omega t + \varepsilon + \psi) \quad \dots \quad \dots \quad \dots \quad \dots \quad 2.13$$

$$\text{where } A = \sqrt{\{v^2(t) + 2v(t) \sin \varepsilon + 1\}} \quad \dots \quad \dots \quad 2.14$$

From Fig 2.6, YZ = 1, OY = sin ε and OZ = cos ε;

$$\therefore \psi + \varepsilon = \tan^{-1}\left[\frac{OY + YX}{OZ}\right] = \tan^{-1}\left[\frac{v(t) + \sin \varepsilon}{\cos \varepsilon}\right] \dots \quad 2.15$$

Hence the Eq (2.13) becomes

$$e_{o1}(t) = \left[\sqrt{\{1 + 2v(t) \sin \varepsilon + v^2(t)\}}\right] . \sin\left(\omega t + \tan^{-1}\left[\frac{v(t) + \sin \varepsilon}{\cos \varepsilon}\right]\right) \quad \dots \quad 2.16$$

Similarly, for the case of $e_{o2}(t)$, another phasor diagram is drawn as in Fig 2.8, from which e_{o2} is obtained as

$$e_{o2}(t) = \left[\sqrt{\{1 - 2v(t) \sin \varepsilon + v^2(t)\}}\right] \sin\left(\omega t + \tan^{-1}\left[\frac{v(t) - \sin \varepsilon}{\cos \varepsilon}\right]\right) \quad \dots \quad \dots \quad 2.17$$

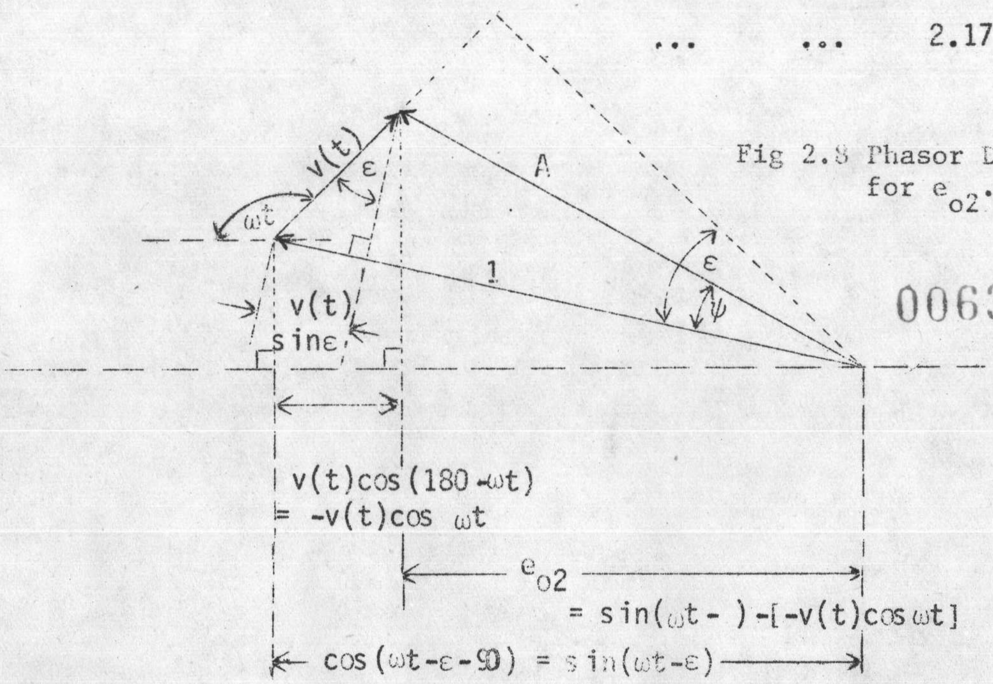


Fig 2.8 Phasor Diagram for e_{o2}.

When the two outputs, e_{o1} and e_{o2} are multiplied in the multiplier, the products becomes

$$e_{o1}(t).e_{o2}(t) = [\sqrt{\{1 + 2v(t)\sin\varepsilon + v^2(t)\}}.\sin\{\omega t + \tan^{-1}\frac{v(t) + \sin\varepsilon}{\cos\varepsilon}\}]$$

$$\times[\sqrt{\{1 - 2v(t)\sin\varepsilon + v^2(t)\}}.\sin\{\omega t + \tan^{-1}\frac{v(t) - \sin\varepsilon}{\cos\varepsilon}\}]$$

and using the trigonometrical relationship $2\sin A.\sin B = \cos(A-B) - \cos(A+B)$

$$e_{o1}(t).e_{o2}(t) = [\sqrt{\{1+v^2(t)\}^2 - 4v^2(t)\sin^2\varepsilon}]\tfrac{1}{2}[\cos\{\tan^{-1}\frac{v(t)+\sin\varepsilon}{\cos\varepsilon} -$$

$$-\tan^{-1}\frac{v(t)-\sin\varepsilon}{\cos\varepsilon}\} - \cos\{2\omega t + \tan^{-1}\frac{v(t)+\sin\varepsilon}{\cos\varepsilon} + \tan^{-1}\frac{v(t)-\sin\varepsilon}{\cos\varepsilon}\}]$$

.... 2.18

Now considering only the phase relationship and assuming that

$$\tan^{-1}\frac{v(t) + \sin\varepsilon}{\cos\varepsilon} = A \quad \text{and} \quad \tan^{-1}\frac{v(t) - \sin\varepsilon}{\cos\varepsilon} = B$$

From trigonometry, $\tan(A - B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

and $\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

Now substituting the values of A and B, tan (A-B) and tan (A+B) become

$$\tan(A-B) = \frac{\sin 2\varepsilon}{v^2(t) + \cos 2\varepsilon} \quad \text{and } \tan(A+B) = \frac{2v(t)\cos\varepsilon}{1 - v^2(t)}$$

$$\therefore e_{o1}(t)\, e_{o2}(t) = \sqrt{\{1 + v^2(t)\}^2 - 4v^2(t)\sin^2\varepsilon}.\ \tfrac{1}{2}\{\cos[\tan^{-1}\frac{\sin 2\varepsilon}{v^2(t)+\cos 2\varepsilon}]$$

$$- \cos[2\omega t + \tan^{-1}\frac{2v(t)\cos\varepsilon}{1 - v^2(t)}]\} \ldots. \qquad 2.19$$

Here $\cos[\tan^{-1}\frac{\sin 2\varepsilon}{v^2(t) + \cos 2\omega}]$ is negligible compared to

$\cos[2\omega t + \tan^{-1}\frac{2v(t)\cos\varepsilon}{1 - v^2(t)}]$ because ε is small and $v(t)$ lies between ± 1.

This equation also shows that the output phase modulation, which is the sum of the phase modulations on $e_{o1}(t)$ and $e_{o2}(t)$, has been superimposed on the frequency 2ω. Eliminating the explicit time dependence, the phase modulation is written as

$$\phi(v,\varepsilon) = \tan^{-1} \frac{2v \cos \varepsilon}{1 - v^2} \quad \ldots \quad \ldots \; -1 \leqq v \leqq 1 \; \ldots \; 2.20$$

So the required output phase of the modulator is expressed in terms of input voltage, v, and the small error in carrier phase, ε, whose proper choice optimizes the modulator for specific application. For the values of v, 0 and ± 1, the Eq (2.20) becomes

$$\phi(0,\varepsilon) = 0 \quad \text{and} \quad \phi(\pm 1,\varepsilon) = \pm \tfrac{1}{2}\pi \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad 2.21$$

which shows that Eq (2.20) is independent of ε. Hence the Eq (2.20), for an ideal modulator becomes

$$\phi_{ideal}(v) = v\pi/2 \quad \ldots \quad \ldots \quad -1 \leqq v \leqq 1 \quad \ldots \quad \ldots \quad 2.22$$

The value of ε for which the maximum difference between $\phi(v,\varepsilon)$ and $\phi_{ideal}(v)$, as given by Eqs (2.20) and (2.22) respectively, is determined by direct computation and the results is shown in Fig 2.9 and in tables 2.1 to 2.3.

v	$\phi(v,\varepsilon) - \phi_{ideal}(v)$ in degrees				
volts	ε=0°	ε=15°	ε=30°	ε=45°	ε=60°
0.1	2.42	2.04	0.92	-0.87	- 3.23
0.2	4.61	3.92	1.84	-1.58	- 6.23
0.3	6.39	5.49	2.72	-2.00	- 8.75
0.4	7.60	6.61	3.52	-2.04	-10.53
0.5	8.13	7.17	4.10	-1.69	-11.31
0.6	7.92	7.09	4.37	-1.02	-10.85
0.7	6.98	6.34	4.18	-0.26	- 9.08
0.8	5.32	4.89	3.44	+0.35	- 6.23
0.9	2.97	2.76	2.05	+0.21	- 2.92

Table 2.1

A plot for this table is shown in Fig 2.9, where the curve for $\varepsilon = 45°$ is close to the linearity. So further computation is done and they are tabulated in table 2.2.

v	$\phi(v, \varepsilon) - \phi_{ideal}(v)$ in degrees			
volts	ε=40°	ε=42°	ε=43°	ε=45°
0.1	-0.20	-0.46	-0.60	-0.87
0.2	-0.30	-0.80	-1.05	-1.58
0.3	-0.20	-0.90	-1.23	-2.00
0.4	+0.11	-0.71	-1.14	-2.04
0.5	+0.60	-0.26	-0.72	-1.69
0.6	+1.15	+0.33	-0.10	-1.02
0.7	+1.56	+0.87	+0.52	-0.26
0.8	+1.63	+1.15	+0.90	+0.35
0.9	+1.15	+0.92	+0.78	+0.21

Table 2.2

v	$\phi(v,\varepsilon) - \phi_{ideal}(v)$ in degrees		
volts	ε=42.3°	ε=42.4°	ε=42.5°
0.1	-0.50	-0.52	-0.53
0.2	-0.87	-0.90	-0.93
0.3	-1.00	-1.04	-1.08
0.4	-0.84	-0.88	-0.92
0.5	-0.40	-0.45	-0.49
0.6	+0.20	+0.16	-0.12
0.7	+0.78	+0.74	+0.70
0.8	+1.08	+1.05	+1.03
0.9	+0.88	+0.86	+0.85

Table 2.3

As seen from the table 2.2, the linearity lies between those for the values 42 and 43 of ε. Further computation is carried out to search for the best possible linearity and they are given in table 2.3 above.

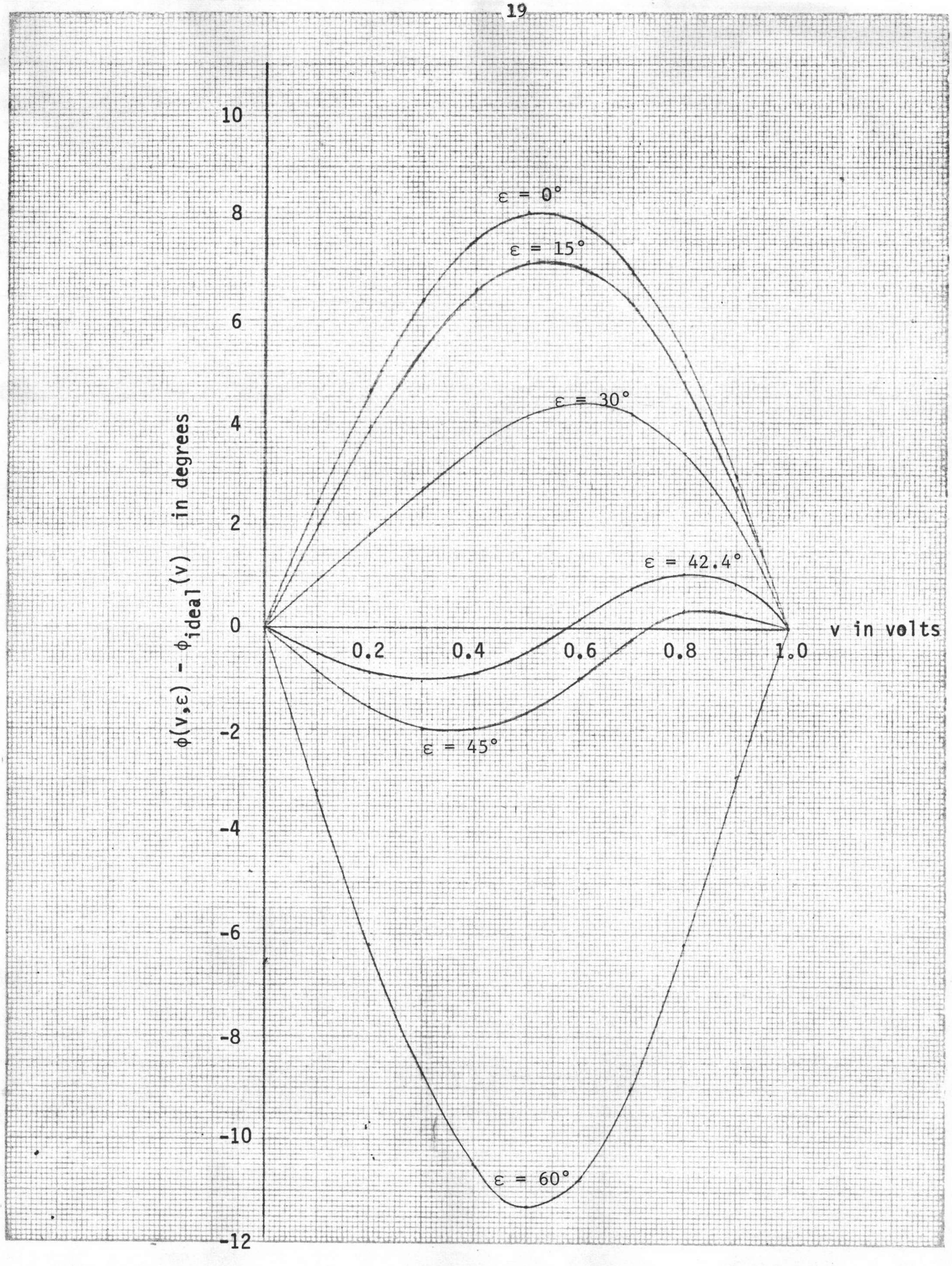


Fig 2.9 Deviation from Perfect Linearity

Finally it is found that the deviation from perfect linearity is not more than 1.05°(degrees) for the range $-\frac{1}{2}\pi \leq \phi \leq +\frac{1}{2}\pi$ when $\varepsilon = 42.4°$ which is also plotted in Fig 2.9. This Fig 2.9 also proves that the maximum phase error for the balanced Armstrong modulator, for which $\varepsilon = 0$, is reduced by an approximate factor of 8 (correct factor is 8.13).

The amplitude of the product of the signals e_{o1} and e_{o2} is $|e_{o1}(v).e_{o2}(v)|$ and is written as

$$|e_{o1}(v).e_{o2}(v)| = \frac{1}{2}\sqrt{(1+v^2)^2 - 4v^2 \sin^2\varepsilon} \quad \ldots \quad \ldots \quad 2.23$$

This amplitude increases with v and becomes maximum at v = 1

$$\begin{aligned} |e_{o1}(1).e_{o2}(1)| &= \frac{1}{2}\sqrt{4-4\sin^2\varepsilon} \\ &= \sqrt{1-\sin^2\varepsilon} \\ &= \cos\varepsilon \quad \ldots \quad \ldots \quad \ldots \quad 2.24 \end{aligned}$$

Any amplitude variation is smoothed by means of a limiter, whereby the final output is obtained. Circuit details of various functions will be treated henceforth.

2.2 Circuit Details

2.2.1 Balanced Modulator

A balanced modulator is widely used to generate DSB-SC signal. The discussion of the theory of a balanced modulator is therefore felt necessary before going into circuit details of the DSB-SC Amplitude Modulator. The basic type of balanced modulator is the bridge type circuit shown in Fig 2.10. In this bridge circuit the carrier portion of the signal is cancelled in the push-pull output circuit. Because of their non-linear transfer characteristics the field-effect transistors (FET) are used sothat the output is the product of the input voltages. The transfer curve (I_d vs V_{gs}) of such FET is parabolic[22], whose partial power-series expansion is

$$i_d = i_o + av_{gs} + bv_{gs}^2 + \ldots \qquad \ldots \qquad \ldots \quad 2.25$$

where i_o is the current for zero gate-source voltage, and a, b, ... are constants.

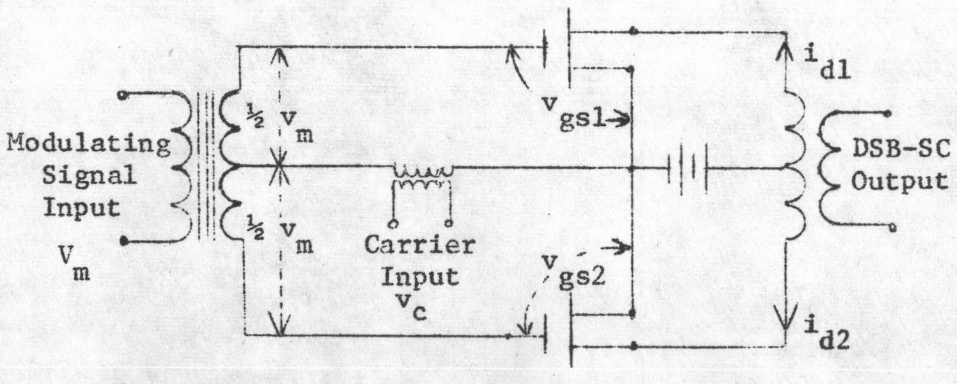


Fig 2.10 A Balanced Modulator Circuit

As the drain currents i_{d1} and i_{d2} flow in opposite directions in the primary winding of the output transformer, the effective primary current i_p is their difference.

$$i_p = i_{d1} - i_{d2}$$

$$= a(v_{gs1} - v_{gs2}) + b(v_{gs1}^2 - v_{gs2}^2) + \dots \dots$$

$$= a(v_{gs1} - v_{gs2}) + b(v_{gs1} + v_{gs2})(v_{gs1} - v_{gs2}) + \dots \quad 2.26$$

From the Kirchhoff's voltage law for the input loops,

$$v_{gs1} = \tfrac{1}{2} v_m + v_c \quad \dots \quad 2.27$$

$$v_{gs2} = -\tfrac{1}{2} v_m + v_c \quad \dots \quad 2.28$$

Substituting the last two equations into Eq (2.26),

$$i_p = a(v_m) + b(2v_c)(v_m) + \dots \quad 2.29$$

The low-frequency term v_m is blocked by the output (RF) transformer which passes only the terms of higher frequency order like $2v_c v_m$ which contains both the upper and lower sidebands. With the advent of integrated circuit (IC) technology, this basic function of balanced modulator has been put into one chip of IC and named it MC 1596.

The DSB-SC Amplitude Modulator circuit detail made up of the IC MC 1596[23] is illustrated in Fig 2.11. The AM operation of this circuit demands the adjustment of the carrier null potentiometer, 50K, for the proper amount of carrier insertion in the output signal.

The schematic diagram[24] of the MC 1596 is given in Fig 2.12.

Two assumptions have been made to analyse this circuit, Fig 2.12. The first assumption is that the similar devices within the monolithic chip

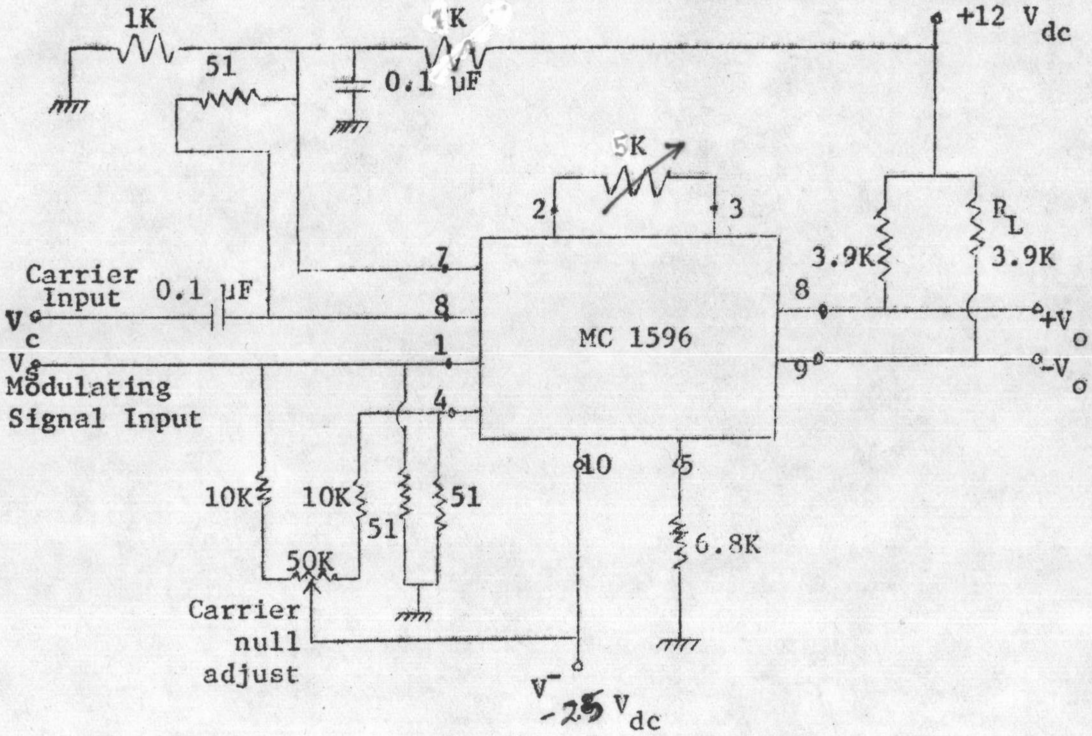


Fig 2.11 DSB-SC Amplitude Modulator

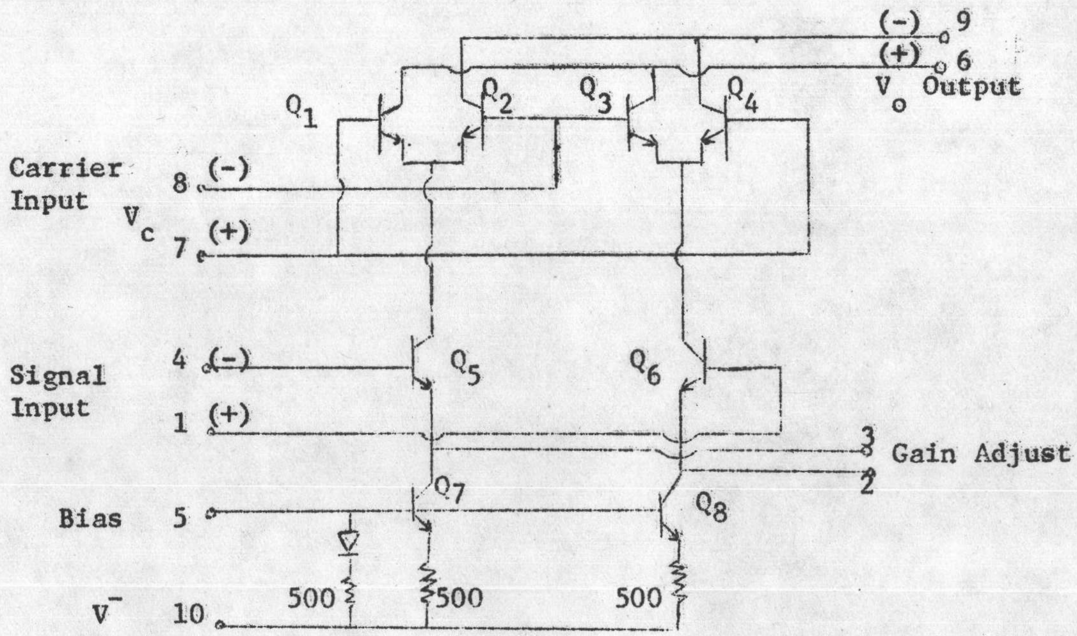


Fig 2.12 Schematic Diagram of MC 1596

are assumed to be identical and matched; and the second one is that the transistor base currents are negligible compared with the magnitude of the collector currents, thereby the collector and emitter currents are assumed equal.

There are three differential amplifiers, Q_1-Q_2, Q_3-Q_4, and Q_5-Q_6 where the last one drives the dual differential amplifiers Q_1-Q_2, and Q_3-Q_4. The bias circuitry along with the transistors Q_7 and Q_8 provides constant current sources for the lower differential amplifier Q_5-Q_6. The MC 1596 schematic diagram is simplified and illustrated in Fig 2.13.

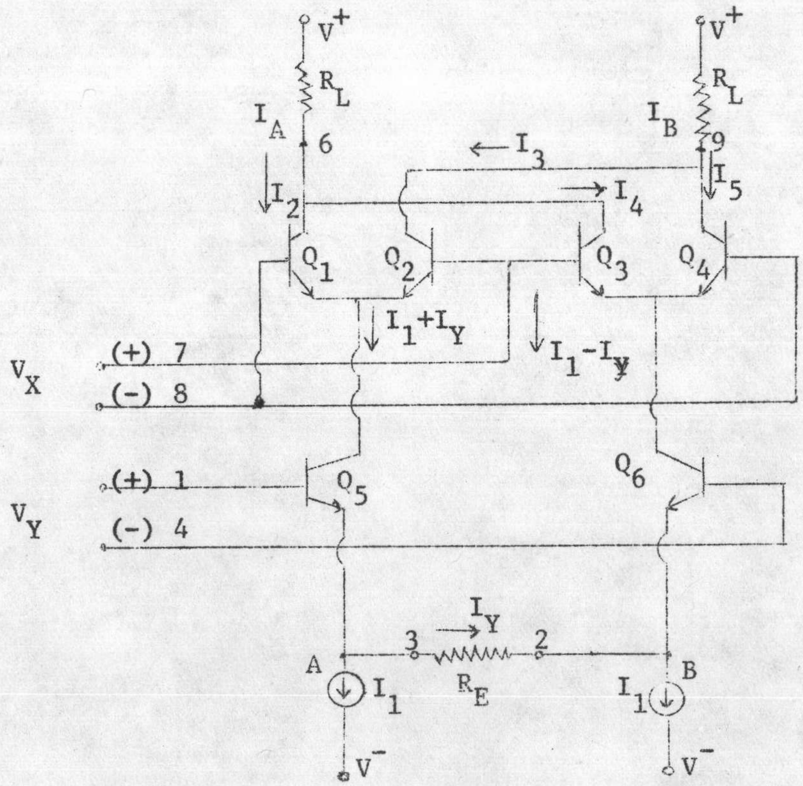


Fig 2.13 Analytic Model of MC 1596.

The output is proportional to the product of the input voltages V_X and V_Y, provided the magnitudes of V_X and V_Y are maintained within the limits of linear operation of the three differential amplifiers, and is given by V_O.

$$V_O = K\, V_X\, V_Y \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad 2.30$$

where the constant K depends on the selection of external components.

Usually a high level input signal is applied to the dual differential amplifiers Q_1, Q_2, Q_3 and Q_4 (carrier input port) and a low level input signal to the lower differential amplifier Q_5 and Q_6 (modulating signal input port) for the linear operations of all the differential amplifiers. The output contains only the sum and difference frequency components and amplitude information of the modulating signal. If the signal input level is too high, its harmonics are generated and appear in the output as spurious sidebands of the suppressed carrier. If the carrier input of high level is used, it has the advantage of maximizing device gain and ensures that any amplitude variations present on the carrier do not appear on the output sidebands.

Two types of operation is considered for mathematical analysis. One type is low level sine waves for both V_X and V_Y, and other is low level sine wave for V_Y and large signal square wave for V_X.

For sine wave inputs,

$$V_X = E_X \cos \omega_x t \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \; 2.31$$

$$V_Y = E_Y \cos \omega_Y t \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \; 2.32$$

where E_X and E_Y are the peak values of V_X and V_Y respectively. Hence

$$V_O = KE_XE_Y \cos\omega_X t \cos \omega_Y t$$

$$= \tfrac{1}{2} KE_XE_Y\{ \cos (\omega_X + \omega_Y)t + \cos (\omega_X - \omega_Y)t \} \quad \dots \quad \dots \; 2.33$$

For high level square wave in the second case, switching function is assumed in the upper differential amplifiers. The Fourier series for the symmetrical square signal is

$$f(t) = 2E_X \sum_{n=1,3,5,\dots}^{\infty} \frac{\sin \frac{1}{2} n \pi}{\frac{1}{2} n \pi} \cos n\omega_X t \quad \dots \quad \dots \quad 2.34$$

Therefore the output voltage is

$$V_O = \tfrac{1}{2} K\, 2E_XE_Y \Big\{\sum_{n=1,3,5,\dots}^{\infty} \frac{\sin\frac{1}{2} n\pi}{\frac{1}{2} n\pi} \cos n\omega_X t \cos\omega_Y t\Big\} \quad \dots \quad 2.35$$

which is

$$V_O = K E_XE_Y \sum_{n=1,3,5,\dots}^{\infty} \frac{\sin \frac{1}{2}n\pi}{\frac{1}{2} n\pi} \{\cos(n\omega_X+\omega_Y)t + \cos(n\omega_X - \omega_Y)t\} \quad \dots \quad 2.36$$

In the first case, for low level input signals, it is noticeable from Eq (2.33) that the output signal consists of the sum and difference frequencies $(\omega_X \pm \omega_Y)$ only, whereas in the second case, for the operation with a high level input at the V_X inputs, Eq (2.36) reveals that the outputs obtained are sum and difference frequencies at various harmonics, $(\omega_X \pm \omega_Y)$, $(3\omega_X \pm \omega_Y)$, $(5\omega_X \pm \omega_Y)$ etc.

It is therefore evident that the second case with the square wave carrier signal encounters many odd harmonics and that the first case should be chosen for its better system performance as the carrier input level is at its low and linear range.

2.2.2 Phase Shifter

The phase shifter illustrated in Fig 2.14(b) used for this research is based on a standard phase-shifting circuit shown in Fig 2.14(a) which develops a constant-amplitude variable-phase voltage e_{vp}. This e_{vp} can be varied in phase from 0° to 180° by adjusting R_3 from zero to infinity. The open-circuit-gain of Q_1 can be expressed as

$$G = \frac{e_o}{e_{in}} = \left(1 + \frac{R_2}{R_1}\right) = 2 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad 2.37$$

Since the collector-to-emitter output impedance Z_o is low, the phase of e_o is practically independent of the phase of Z_L[25].

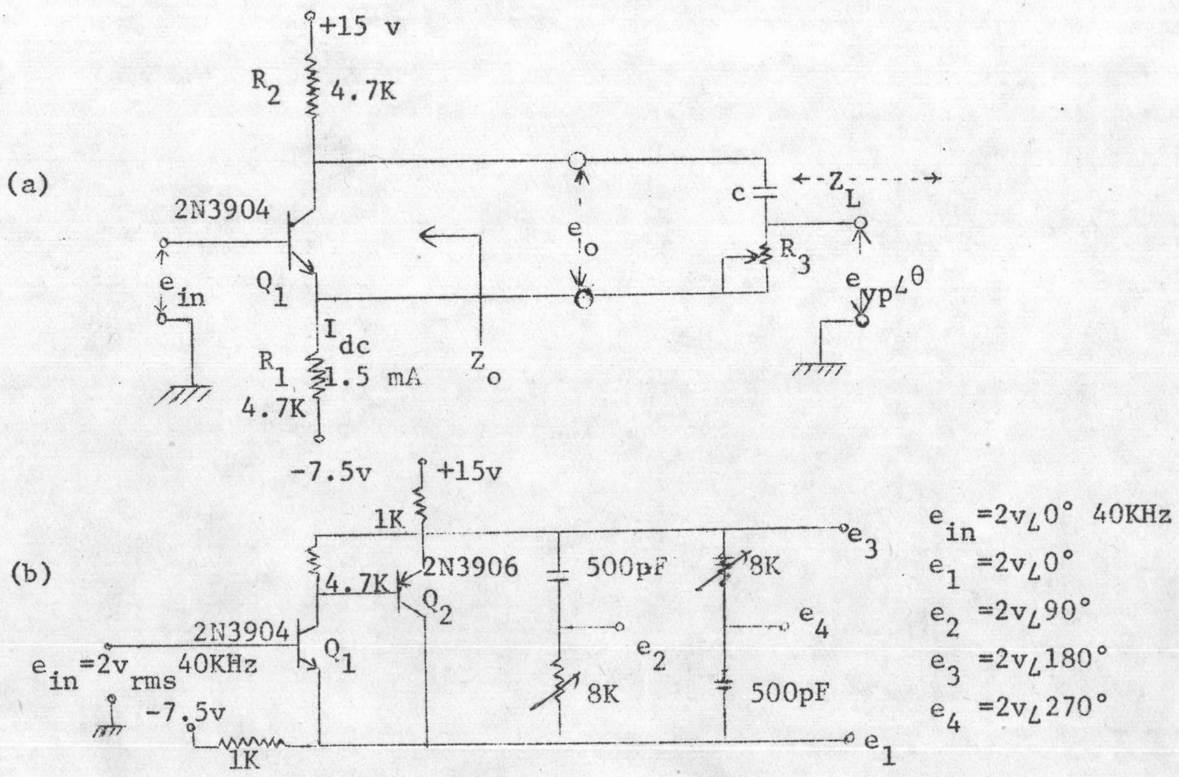


Fig 2.14 (a) Standard Phase-shifter

(b) Four-output Phase-shifter

Z_o is given by

$$Z_o = (1 + \frac{R_2}{R_1})R_e = 2R_e \cong 40\,\Omega \quad \dots \quad \dots \quad \dots \quad \dots \; 2.38$$

where R_e is the dynamic resistance[26,27] which is the reciprocal of the slope of the volt-ampere characteristics of emitter-base junction for which I is related to V by

$$I = I_o(\varepsilon^{V/\eta V_T} - 1) \quad \dots \quad \dots \quad \dots \quad 2.39$$

where I_o: reverse saturation current of the junction

η : a constant

= 1 for large currents and 2 for small currents (rated)

V_T: volt-equivalent of temperature

$$\eta = \frac{KT}{q}$$

K : Boltzmann constant = 1.381×10^{-23} J/°K

T : temperature in degree Kelvin

q : electronic charge = 1.602×10^{-19} C

and $V_T = \frac{T}{11,600}$

and the dynamic resistance R_e is approximately related to V_T by

$$R_e = \frac{V_T}{I} \quad \dots \quad \dots\dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad 2.40$$

$$= 30/I$$

where R_e is in Ohms, I in milliamperes and $\eta = 1$

The emitter signal voltage is always a few percentage of e_{in} because of the low value of R_e[26]. Similarly, the collector voltage isalso a few percentage of its open-circuit value because the value of Z_o is low. So the output signal voltage e_o has its phase angle almost independent of Z_L. Consequently a number of phase-shifting networks, Z_L, depending on the amount of load current the transistor Q_1 is capable of delivering, can be connected in parallel across the output and the phase angle can be independently adjusted to any value without any interferences on the other circuits or networks.

Fig 2.14(b) illustrates a four-output phase shifter with output at every 90° apart. The two-transistors driver has very low internal impedance. Each output works into open-circuit loads and can be adjusted to the proper value without affecting other outputs. In practice the 8Kohm resistors are made variable to adjust the phase shift to the required value and an amplifier is added to its output so that any required amplitude is obtained or the amplitude of the output can be made equal to that of the input in case the output amplitude is decreased because of the circuit components flexible values. The phase shift is hence satisfactorily achieved.

2.2.3 Adders

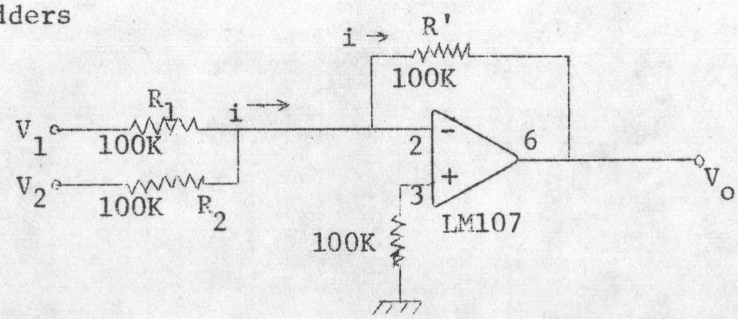


Fig 2.15 Adder

An adder shown in Fig 2.15 uses an operational amplifier as a summer. This operational amplifier yields an output which is an algebraic sum of the two input signals[26].

Since

$$i = \frac{V_1}{R_1} + \frac{V_2}{R_2} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad 2.41$$

then

$$V_o = - R'i = -(\frac{R'}{R_1}V_1 + \frac{R'}{R_2}V_2) \quad \ldots \quad \ldots \quad \ldots \quad 2.42$$

$$\therefore V_o = -(V_1 + V_2) \quad \text{when} \quad R_1 = R_2 = R' \quad \ldots \quad \ldots \quad 2.43$$

It is noted that the output is negative because an inverting summing amplifier is used. Hence, in order to obtain a positive output, one has to use a non-inverting amplifier. The Fig 2.16 can be changed to a non-inverting one by feeding the inputs to pin 3 and connecting the 100K resistor to pin 2 which is then earthed.

2.2.4. Multiplier

A general multiplier provides an output signal which is proportional to the product of two inputs. If the output always bears the correct algebraic sign the multiplier is said to be a four-quadrant multiplier.

A basic multiplier can be considered as a common-emitter amplifier which yields an output as

$$V_o = C\,V_{in}\,I_E \quad \ldots \quad \ldots \quad \ldots \quad 2.44$$

where C is gain constant.

The two variables, emitter current I_E and the input voltage V_{in}, are not independent of each other. The constant C should have a proper dimension to realize the equation dimensionally valid. This concept[28,29] has been adopted by Motorola to develop its MC 1595 monolithic

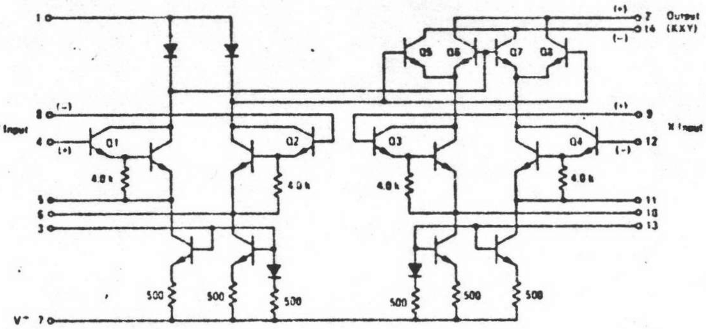


Fig 2.16 Schematic Diagram of MC 1595

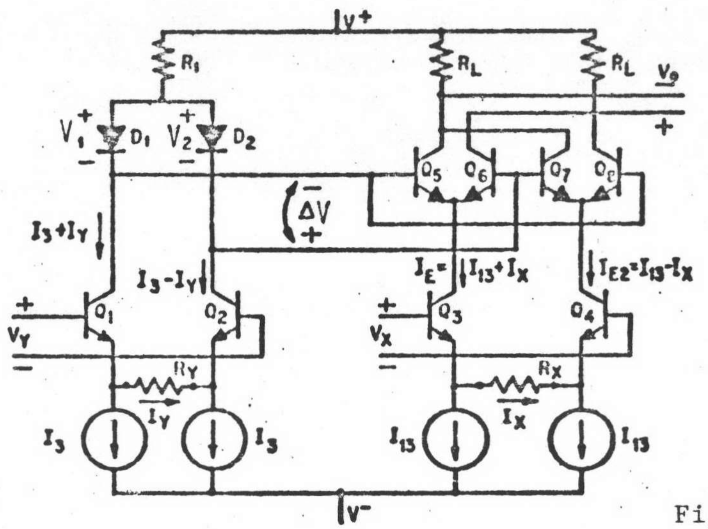


Fig 2.17 Analytic Model of MC 1595

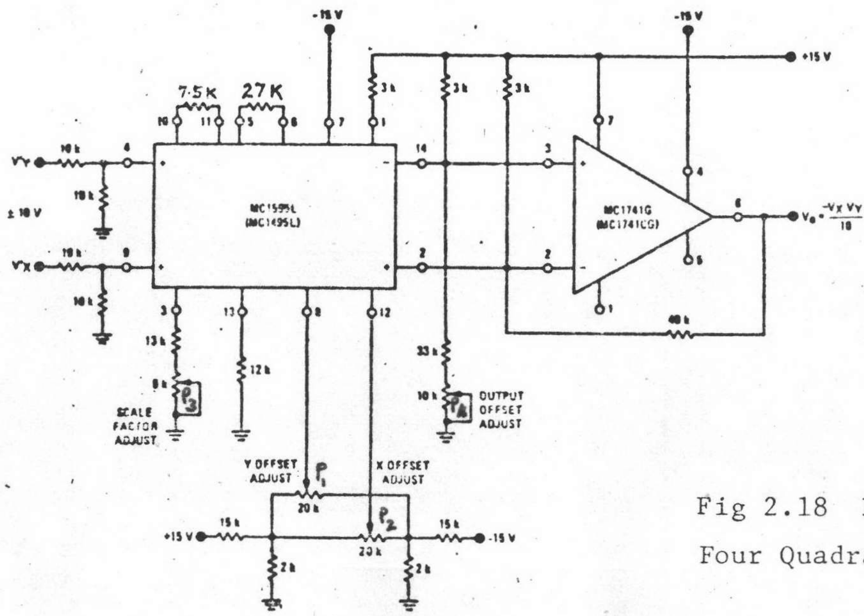
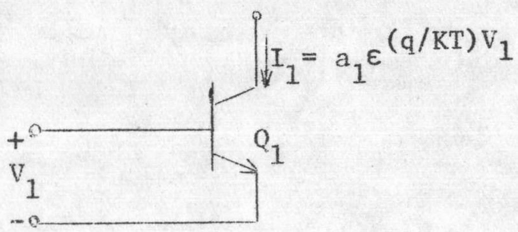


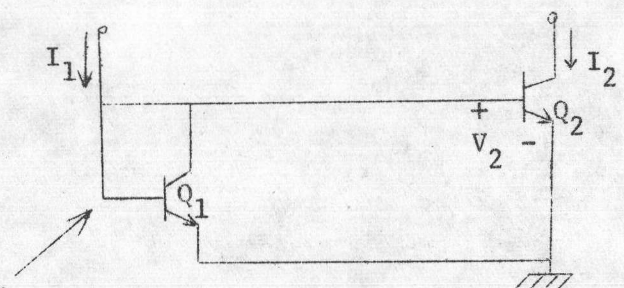
Fig 2.18 Practical Four Quadrant Multiplier

multiplier, whose schematic diagram is given in Fig 2.16 and analytic diagram in Fig 2.17. In Fig 2.17, the main amplifier is a cross-coupled quad differential unit, Q_5, Q_6, Q_7, and Q_8 whose functions are alike the modulator MC 1596 shown earlier in Fig 2.12 and Fig 2.13. The differential amplifier unit has common-emitter amplifier configuration, but has emitter currents and input voltages independent of each other. The cross coupling design allows inversion performances that are necessary to realize four-quadrant operations.

For Ebers-Moll Transistor:



For two identical transistors with $\beta \rightarrow \infty$



$$V_2 = \frac{KT}{q} \ln \frac{I_1}{a_1}$$

$$I_2 = a_1\ \varepsilon^{(q/KT)V_2} = a_2\ \varepsilon^{(q/KT)\{(KT/q)\ln(I_1/a_1)\}} = I_1 \quad \text{for } a_1 = a_2$$

Fig 2.19 Multiplier Preconditioning Principle
(Here q, K and T are same as in Eq (2.39))

For the input base-emitter voltage V_{in}, the output current of the amplifier is exponentially related to its input. If the voltage ΔV as shown in Fig 2.17 applied to the base terminals of the quad amplifier is a linear function of the signal voltage V_Y, the output current is an exponential function, i.e., non-linear. To avoid such a non-linearity the V_Y signal is preconditioned before it is applied to the main amplifier. In other words the use of a linear circuit has the disadvantage of distorting at least one input due to the non-linear processing involved. So the preconditioning of one input that cancels the non-linear processing distortion is necessary to overcome the distortion problem. Such preconditioning principle is illustrated in Fig 2.19.

The collector response of a transistor to an input base-emitter voltage is considered to explain the precondition principle briefly. In Fig 2.19, the two transistors are assumed identical. Output I_2 is produced by the input I_1. Their relationship is established, thereby they are seen approximately equal on condition that the constants a_1 and a_2 are equal. Such an operation is achieved by first converting the input current into a voltage V_1 with which I_1 has an exponential relationship as shown in Fig 2.19, and then converting V_1 into another current I_2, which is exponentially related to V_1 too. Since I_1 is exponentially related to V_1, V_1 is logarithmically related to I_1. Again since the transistors are identical and the logarithmic operation cancels the exponential operation, the output I_2 responds linearly to the input I_1, which has been nonlinearly processed inthe transistor junction.

In Fig 2.17 the preconditioning is achieved by passing the current resulting from V_Y signal through the diodes D_1 and D_2. As soon as the current from the input amplifiers Q_1 and Q_2 passes through the diodes, voltage develops across diodes logarithmically that is nonlinearly, which is the exact compliment of another nonlinear responses produced in the quad amplifier transistors.Hence the output current of the quad amplifier, in response to its base voltage, becomes linear. That is

$$V_o = K\, V_X V_Y \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad 2.45$$

which is the same as Eq (2.30), and where the constant K is the circuit scale factor[29]. The scale factor K is expressed as

$$K = \frac{2\, R_L}{R_X R_Y I_3} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad 2.46$$

where the resistors R_X and R_y are externally connected to adjust the gain of any specific value.

From Eq (2.45) it is noted that the constant K plays an important role to obtain V_o of significant magnitude. So K should be carefully chosen. In addition to that the externally connected resistors R_X and R_Y must be carefully chosen too to acquire the maximum input voltage swing without bringing about any nonlinearity. To meet such requirements, two following conditions are set to avoid any cut-off of the inputamplifiers $Q_1 - Q_2$ and $Q_3 - Q_4$.

$$V_{X(max)} < I_{13}\, R_X$$

and $$V_{Y(max)} < I_3\, R_y$$

The analysis of the MC 1595 for the rest of the circuit is the same as in the case of MC 1596 where the collector and emitter currents are assumed equal. The practical circuit[23] is shown in Fig 2.18.

2.2.5. Limiter

A simple limiter intended to use to smooth out the amplitude variation of the output of the multiplier is a double-ended clipper using two avalanche diodes as shown in Fig 2.20(a).

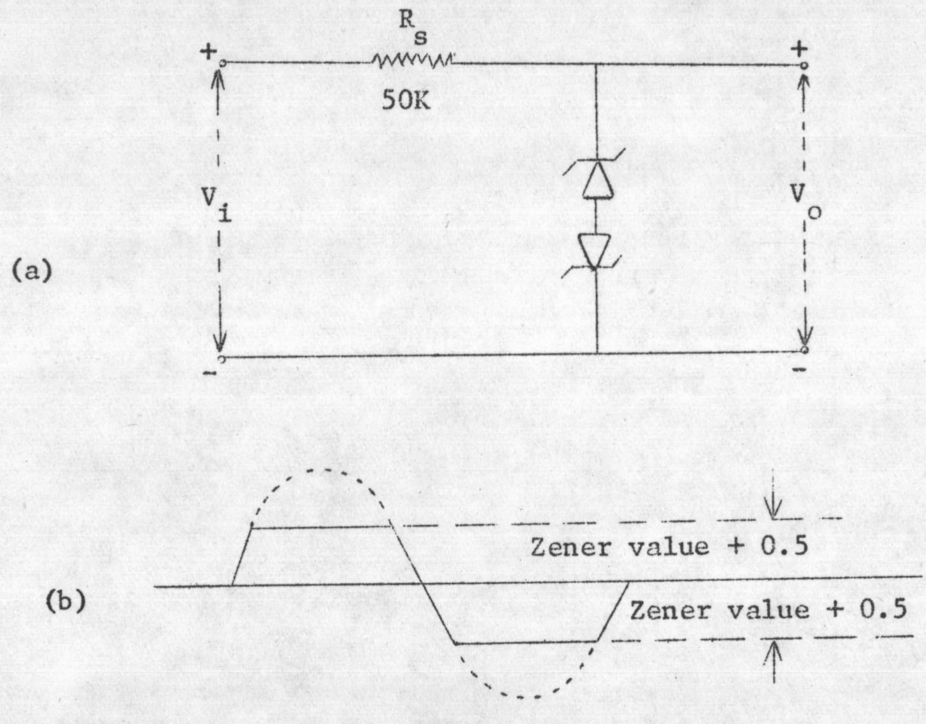


Fig 2.20 (a) Limiter and (b) Its Output

Zener voltage = 0.6 x input voltage

$$R_s(\text{minimum}) = \frac{\text{input voltage}}{\text{maximum current of Zener}}$$

The output waveform of the limiter is illustrated in Fig 2.20(b).

2.2.6. Built-in Carrier Frequency Source

The built-in self-starting oscillator used as the source of carrier frequency (see Fig 3.2 Detailed Circuit Diagram, p 41) for this research is the Wien-Bridge oscillator, whose general configuration is shown in Fig 2.21. The unique monolithic LM 370 is used as a constant amplitude oscillator. The RC networks, comprising R_1C_1 and R_2C_2, provide to the system the positive feedback whose transfer function becomes maximum when the lead produced by R_1C_1 equals the lag, in magnitude, produced by R_2C_2.

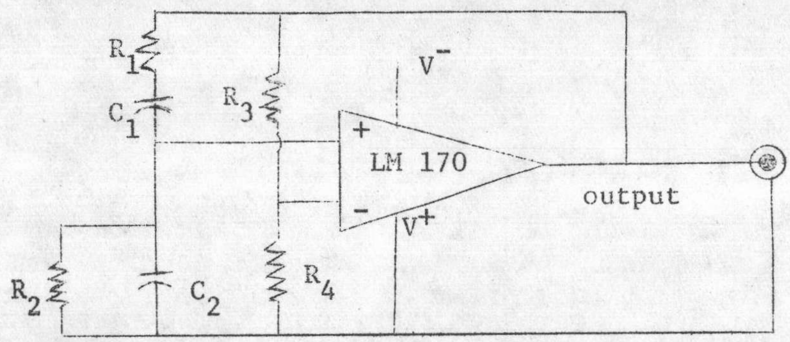


Fig 2.21 General Wien-Bridge Oscillator Configuration[30~32]

Under such condition the gain of the network is given by

$$A_V = \frac{1}{1 + R_1/R_2 + C_2/C_1} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \; 2.47$$

and the frequency of oscillation is given by

$$f = \frac{1}{2\pi \sqrt{R_1R_2C_1C_2}} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \; 2.48$$

The resistors R_3 and R_4, forming a negative feed-back loop, control the gain of the amplifier. To have a sustained oscillation, the loop of the system must be equal to unity, hence the condition of oscillation is

$$\frac{R_3 + R_4}{R_4} = 1 + \frac{R_1}{R_2} + \frac{C_2}{C_1} \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad 2.49$$

i.e.,

$$\frac{R_3}{R_4} = \frac{R_1}{R_2} + \frac{C_2}{C_1} \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad 2.50$$

In Fig 3.2, the potentiometers P_1 and P_2 in gang provide coarse tuning and P_3, fine tuning of the frequency of oscillation. Another potentiometer P_5 is used for setting the output level to the desired value.