CHAPTER 0

INTRODUCTION

Let $\mathbb{R}^n$ denote the Euclidean space of dimension $n \geq 1$. The set

$$H = \mathbb{R}^n \times \{t / t \geq 0\}$$

is called a half-space. Points (x,t) in the half-space are simply in the form of $(x_1, x_2, \ldots, x_n, t)$, $t \geq 0$.

Suppose now that u is a real valued function defined on an open set Ω and has continuous partial derivatives thereon. The Laplacian of u, Δu, is defined by

$$\Delta u = \sum_{i=1}^{n} \frac{\partial^2}{\partial x_i^2} u.$$

If u is a function of variables other than x and it is necessary to clearify the meaning of the Laplacian, we shall use Δ to signify that the Laplacian is relative to the coordinates of x.

A real valued function $u(x,t)$ is said to be "Temperature" on an open set $\Omega \subseteq H$ if it has second partial derivatives thereon on Ω and satisfies the heat equation

$$(1) \qquad \Delta u = \frac{\partial}{\partial t} u, \qquad \text{on } \Omega.$$

In this thesis, we construct the Poisson integral for a temperature on the half-space. First, we observe that

$$K(x,t) = \prod_{i=1}^{n} k(x_i,t) = (4\pi t)^{-n/2} \exp\left(-\frac{|x|^2}{4t}\right),$$

where

$$k(x_i,t) = (4\pi t)^{-\frac{1}{2}} \exp\left(\frac{-x_i^2}{4t}\right), \quad |x|^2 = \sum_{i=1}^{n} x_i^2,$$

is a Temperature on the half-space. Next, we are going to prove that the integral

$$u(x,t) = \int_{\mathbb{R}^n} K(y-x,t)\phi(y)dy.$$

is a Temperature on the half-space with the given initial temperature $\phi(y)$ at time 0.

Throughout this thesis, some knowledge of real analysis are assumed.

Brieftly, the structure of this thesis is as follows:

Chapter I introduces some properties of $K(x,t)$ and recalls some facts which are going to be used.

Chapter II is dealt with the Poisson integral of a function.

Chapter III shows the uniqueness of a positive temperature with a prescribed initial condition.

Chapter IV ends the thesis with a representation of a solution of the equation (1) in the form of the Poisson integral of a measure. It is hoped that this study will enough to provide tools for further studies by others who feel interested in this area.