CHAPTER II
BACKGROUND AND LITERATURE REVIEW

This chapter presents background information required in this thesis work. Chapter II comprises the background of natural gas reservoir, simulation process and mathematical methods in this study and literature survey which reviews the related research work.

2.1 Types of Gas Reservoir (Abdel-Aal *et al.*, 2003)

"Gas reservoirs can be anything from a few hundred meters to tens of kilometres across in plane, and tens to hundreds of meters thick, with the gas trapped against an impermeable layer similar to oil traps. The gas will not flow right across the reservoir. So, gas wells have to be distributed through the reservoir to drain or sweep all the gas. The types of gas reservoirs can be classified into 6 categories according to the conditions of the formation as explained below:"

2.1.1 Dome-Shaped and Anticline Reservoir

"This reservoir is formed by the folding of the layers as shown in Figure 2.1. This formation is made up of porous layers, sedimentary rock and impermeable layer on top. The most common form is created when the impermeable sedimentary rock forms a 'dome' shape, like an umbrella. The dome is circular in outline, and the anticline is long and narrow. Oil and/or gas moved or migrated upward through the porous strata where it was trapped by the sealing cap rock and shape of structure."

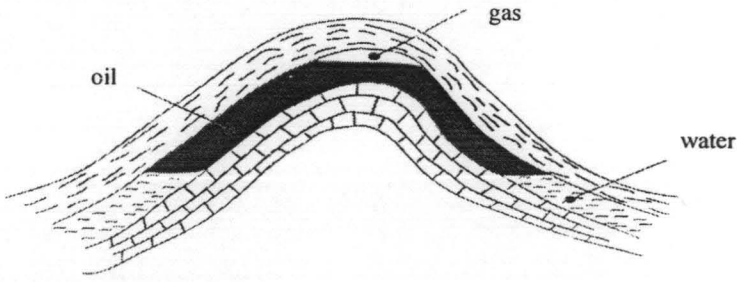

Figure 2.1 Dome-Shaped reservoir (Abdel-Aal *et al.*, 2003).

2.1.2 Faulted Reservoir

"This reservoir is formed by shearing and offsetting of the strata as shown in Figure 2.2. The movement of the nonporous rock opposite the porous formation containing the oil/gas creates the sealing. The tilt of the petroleum-bearing rock and the faulting trap the oil/gas in the reservoir."

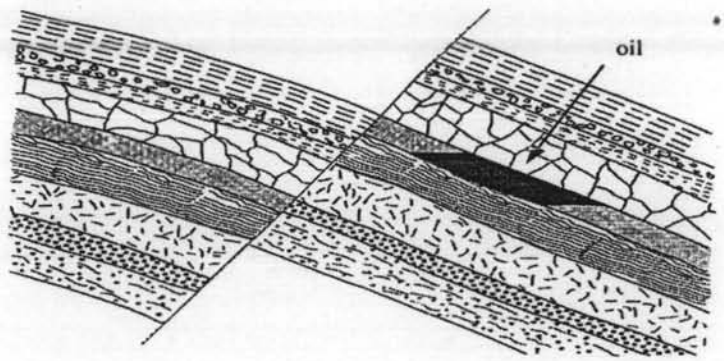

Figure 2.2 Faulted reservoir (Abdel-Aal *et al.*, 2003).

2.1.3 Salted-Dome Reservoir

"This type of reservoir structure, which takes a shape of a dome, is formed due to the upward movement of large, impermeable salt dome that deformed and lifted the overlying layers of rock. As shown in Figure 2.3, petroleum is trapped between the cap rock and an underlying impermeable rock layer, or between two impermeable layers of rock and the salt dome."

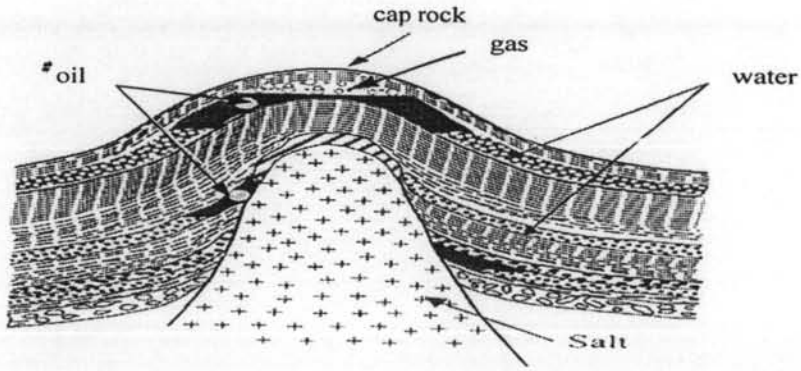

Figure 2.3 Salted-Dome reservoir (Abdel-Aal *et al.*, 2003).

2.1.4 Unconformities

"This type of reservoir structure, shown in Figure 2.4, is formed as a result of an unconformity where the impermeable cap rock is laid down across the cutoff surfaces of the lower beds."

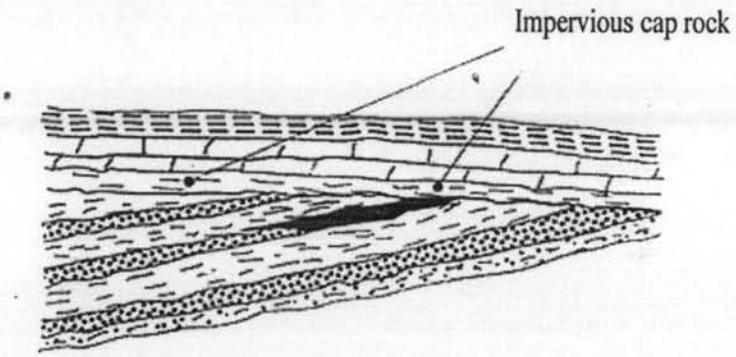

Figure 2.4 Unconformities reservoir (Abdel-Aal *et al.*, 2003).

2.1.5 Lens-Type Reservoir

"This type of reservoir, the petroleum-bearing porous formation is sealed by the surrounding, *ie.*, non porous formation. Irregular deposition of sediments and shale at the time the formation was laid down is the probably cause for this abrupt change in formation porosity. An example of this reservoir is shown in Figure 2.5."

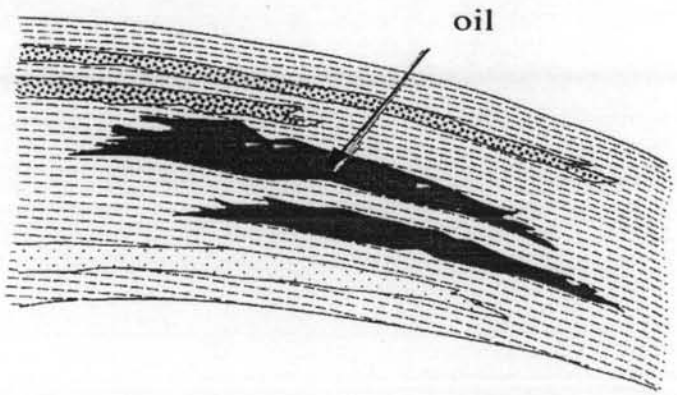

Figure 2.5 Lens-Type reservoir (Abdel-Aal *et al.*, 2003).

2.1.6 Combination Reservoir

"This type of reservoir is formed by, combinations of folding, faulted, abrupt changes in porosity, other conditions that create the trap containing oil and/or natural gas form."

2.2 Reservoir Simulation (Dawe, 2000)

"The exploration for natural gas typically begins with geologists examining the surface structure of the earth, and determining areas where it is geologically likely that petroleum or gas deposits might exist. By surveying and mapping the surface and sub-surface characteristics of a certain area, the geologist can extrapolate which areas are most likely to contain a petroleum or natural gas reservoir.

The geologist has many information attained from the rock cuttings and samples obtained from the digging of irrigation ditches, water wells, and other oil and gas wells. This information is all combined to allow the geologist to make inferences as to the fluid content, porosity, permeability, age, and formation sequence of the rocks underneath the surface of a particular area.

Geochemists will analyse possible source rocks with a view to determining the degree to which the organic material has been altered by time, heat and pressure, identifying possible petroleum source rocks, and assessing the burial history and petroleum potential of the sediments. Petrophysics comprises the study of physical properties and the fluids contained of rocks. At this stage geologists must work closely with the well log analyst, the reservoir engineers and production engineers.

Reservoir simulation implementing various numerical techniques is constructed for solving such a problem. During simulation steps require inputting of many parameters that are related to the reservoir rocks and fluids as discuss before. This integrated approach ensures that the geological model and the simulation model use the same data interpreted with the best technical competence.

Reservoir simulation is used for evaluating remaining oil and recovery factors under different operational (natural depletion, water/gas injection and production rate), comparison of development schemes for required offtake, evaluating effects of an aquifer on natural water drive, determining the effects of

uncertainties in reservoir description of complex reservoirs on development planning and studying the effects of platform location and the spacing of the wells, and, the effects of continuity of pore space and fluids, particularly with horizontal well placements.

The reservoir simulation process can be divided into following steps:

1. Obtain reservoir data from geophysicist and geologist for gridding; reservoir maps, net to gross maps.

2. Compile the input data for the initialization: porosity, permeability, saturation distribution maps, fluid distribution.

3. Define reservoir regions and aquifer; continuity, faults, zonation and layering, for coarse/fine grids; set out x-y grid on top surface map; define Δx and Δy dimensions and assign layers and the Δz values, depth of each block in layer.

4. Check the initialization data for errors.

5. Compare gross rock volume, porosity values and compute pore volume.

6. Calculate initial pressure and saturation distributions in reservoir; when the initial data are put into the model. The model is initialized so that saturations and pressure computed throughout the model; check on previous material balance equation for oil in place, gas in place, free gas in place, water in place and pressure. If the static calculations agree, the model is validated; if not, then the input and calculations must be re-checked.

7. Read well data for simulation. For each cell block in the simulator grid system, there must be a value of dimensions for cell and grid blocks, thickness, elevation from fluid contacts, porosity, absolute/effective permeability, rock compressibility, capillary and relative permeability data, pressure-dependent data, thermodynamic data, viscosity.

8. Initial reservoir pressure, initial phase saturations.

9. Calculate the size of the next time step.

10. Modify computer-calculated transmissibilities.

11. Calculate the pressure and saturation distributions at the next time step; solving method and solver are important.

12. Graphical output in colour with fluid movements, saturations, pressure.
13. Compare results with what might be reasonable anticipated."

2.3 Finite Difference Method (FDM) (Chapra and Canale, 2002)

"A simple and efficient method for solving ordinary differential equations (ODEs) in problem regions with simple boundaries. The method requires the construction of a mesh defining local coordinate surfaces. For each node of this mesh, the unknown function values are found."

From the definition of the derivative is given as

$$\left.\frac{df(x)}{dx}\right|_{x_0} = f'(x_0) = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0} \tag{2-1}$$

Let :

$$h = x - x_0 \tag{2-2}$$

then, the derivative is approximated by

$$f'(x_0) = \frac{f(x_0 + h) - f(x_0)}{h} \tag{2-3}$$

A function f(x), which is continuous and differentiable in the interval $[x_0, x]$, can be represented by a Taylor series.

$$f(x) = f(x_0) + (x - x_0) f'(x_0) + \frac{(x - x_0)^2 f''(x_0)}{2!} + \frac{(x - x_0)^3 f'''(x_0)}{3!} + \ldots + \frac{(x - x_0)^n f^{(n)}(x_0)}{n!} + R_n(x) \tag{2-4}$$

where $R_n(x)$ is called remainder. This term lumps together the remaining terms in the infinite series from (n+1) to infinity; therefore represents in "truncation error", and the remainder term is given by

$$R_n(x) = \frac{(x - x_0)^{n+1} f^{(n+1)}(\xi)}{(n+1)!} \tag{2-5}$$

The value of ξ is an unknown function of x; therefore, it is impossible to evaluate the remainder, or truncation error term exactly.

"The advantage of FDM is this method takes less memory, which stores neighbor information and is especially needed at the boundaries because FDM handle the periodic boundary conditions in a different way. However, FDM also has many drawbacks, *i.e.*, more complication to set up a program, high computation time per time step and restriction to handle rectangular shapes and simple alterations."

2.3.1 Partial Differential Equations

"Partial differential equations are used to characterize systems of engineering problems where the behavior of a physical quantity is represented in terms of rate of change with respect to two or more independent variables.

A given function u is depending on both x and y, and the partial derivative of u with respect to x at an arbitrary point (x, y) is defined as

$$\frac{\partial u}{\partial x} = \lim_{\Delta x \to 0} \frac{u(x + \Delta x, y) - u(x, y)}{\Delta x} \tag{2-6}$$

The partial derivative with respect to y is defined as

$$\frac{\partial u}{\partial y} = \lim_{\Delta y \to 0} \frac{u(x, y + \Delta y) - u(x, y)}{\Delta y} \tag{2-7}$$

An equation involving partial derivative of an unknown function of two or more independent variables is called partial differential equations, or PDEs. For example,

$$\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial y^2} + u = 1 \qquad \text{.....2}^{\text{nd}} \text{ order}$$

$$\frac{\partial^3 u}{\partial x^2 \partial y} + x\frac{\partial^2 u}{\partial y^2} + 8u = 5y \qquad \text{.....3}^{\text{rd}} \text{ order}$$

PDEs can also focus on linear and second-order equations. For two independent variables, PDEs can be expressed in the following general form :

$$A\frac{\partial^2 u}{\partial x^2} + B\frac{\partial^2 u}{\partial x \partial y} + C\frac{\partial^2 u}{\partial y^2} + D = 0 \qquad \text{(2-8)}$$

where, A, B and C are functions of x and y, and D is a function of x, y, u, $\partial u/\partial x$ and $\partial u/\partial y$. However, the partial differential equations can be classified into three forms as follow,

$B^2 - 4AC < 0$ .....elliptic

$B^2 - 4AC = 0$ .....parabolic

$B^2 - 4AC > 0$ .....hyperbolic"

2.3.2 The Alternating-Direction Implicit (ADI) Scheme

"The alternating-direction implicit or ADI, scheme is one of the finite difference method. This method provides means of solving parabolic equations in two spatial dimensions using tridiagonal matrices. The principle of this scheme is to employ two different equations which are used in turnover successive time-steps at

each of duration Δt/2. The first equation is implicit only in the y-direction and the second is implicit in the x-direction. The detailed calculation of this scheme can be explained by solving a heat-conducting equation in two dimensions as follow,

$$\frac{\partial T}{\partial t} = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \tag{2-9}$$

Each time increment is executed in two steps, as shown in Figure 2.6.

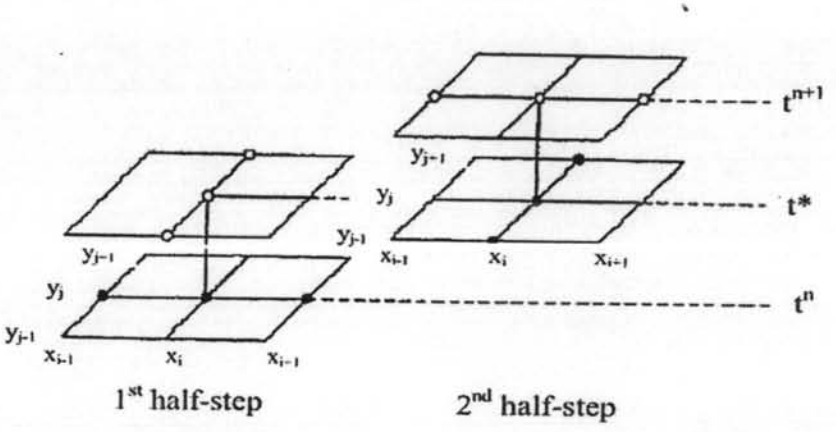

Figure 2.6 The two half-steps used in implementing the ADI scheme for solving parabolic equations in two spatial dimensions (Chapra and Canale, 2002).

For the first step, Eq. (2-9) is approximated by

$$\frac{T_{i,j}^{l+1/2} - T_{i,j}^{l}}{\Delta t/2} = k\left[\frac{T_{i+1,j}^{l} - 2T_{i,j}^{l} + T_{i-1,j}^{l}}{(\Delta x)^2} + \frac{T_{i,j+1}^{l+1/2} - 2T_{i,j}^{l+1/2} + T_{i,j-1}^{l+1/2}}{(\Delta y)^2}\right] \tag{2-10}$$

where, l, l+1/2 and l+1 are the time step, i and j location in x and y axis, respectively.

Thus, the approximation of $\partial^2 T/\partial x^2$ is written at the based point t^l where values of temperature are known. Consequently, only three temperature terms in the approximation of $\partial^2 T/\partial y^2$ are unknown. For the case of square grids ($\Delta x = \Delta y$), Eq. (2-10) can be expressed as

$$-\lambda T_{i,j-1}^{l+1/2} + 2(1+\lambda)T_{i,j}^{l+1/2} - \lambda T_{i,j+1}^{l+1/2} = \lambda T_{i-1,j}^{l} + 2(1-\lambda)T_{i,j}^{l} + \lambda T_{i+1,j}^{l} \quad (2\text{-}11)$$

where, $\lambda = k\,\Delta t/(\Delta x)^2$, $T_{i,j}^{l+1/2}$ is intermediate value at the end of each time step. For the second step from $t^{l+1/2}$ to t^{l+1}, Eq. (2-10) is approximated by

$$\frac{T_{i,j}^{l+1} - T_{i,j}^{l+1/2}}{\Delta t/2} = k\left[\frac{T_{i+1,j}^{l+1} - 2T_{i,j}^{l+1} + T_{i-1,j}^{l+1}}{(\Delta x)^2} + \frac{T_{i,j+1}^{l+1/2} - 2T_{i,j}^{l+1/2} + T_{i,j-1}^{l+1/2}}{(\Delta y)^2}\right] \quad (2\text{-}12)$$

In contrast to first step, the approximation of $\partial^2 T/\partial y^2$ is now implicit, For square grid, the equation can be written as

$$-\lambda T_{i-1,j}^{l+1} + 2(1+\lambda)T_{i,j}^{l+1} - \lambda T_{i+1,j}^{l+1} = \lambda T_{i,j-1}^{l+1/2} + 2(1-\lambda)T_{i,j}^{l+1/2} + \lambda T_{i,j+1}^{l+1/2} \quad (2\text{-}13)$$

Therefore, a temperature at each time step can be determined by solving Eqs. (2-11) and (2-13) together."

2.4 Finite Element Method (FEM) (Chapra and Canale, 2002)

"Finite Element method is developed to solve the partial differential equations (PDEs) in some systems that FDM is not suitable. FEM can be applied to solve the system of governing equations with irregular shape, unusual boundary conditions and heterogeneous composition, as shown in Figure 2.7."

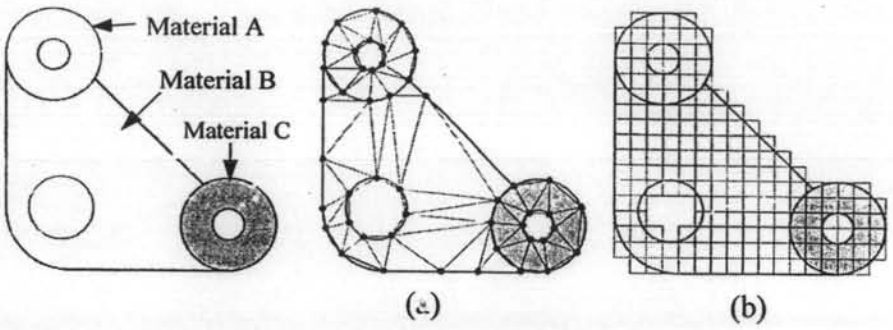

Figure 2.7 The difference between (a) FEM and (b) FDM (Chapra and Canale, 2002).

"The basic concept is that a mathematical model is subdivided into simple shape regions called "element". The geometry of the element is employed in these conditions, *i.e.*, 1D geometry uses the straight line or curved segment. For 2D geometry, the triangular and/or quadrilateral shapes are used. For 3D geometry, the tetrahedral, pentahedral and/or hexahedral shape are applied. As shown in Figure 2.7a, the triangular shape elements in FEM fit with the irregular geometry quite well rather than rectangular shape elements in FDM (Figure 2.7b)."

"An approximate solution for the PDE can be developed for each of these elements. Several methods are available for solving the equation in each element separately. Generally, the conventional methods used the direct approach, the method of weighted residuals, and the variable approach (Galerkin method). Then, the total solution is generated by linking all elements together called "assembly"."

"The advantage of FEM is highly accurate method for the problem of arbitrary shaped and dipping interfaces, higher quality of FEM approximation than in the corresponding FDM approach, but FEM requires large demand on computer memory and high computation costs."

2.4.1 The Galerkin Method

"The Galerkin method is the most commonly used in finite element analysis. Galerkin method used to solve both ordinary and partial differential equations as well as the various boundary conditions."

The Galerkin method used for approximating the solution of differential equation in each element.

$$Lu + f = 0 \tag{2-14}$$

where, L is differential operator and f is a source term. The trial functions can perform below

$$u \cong v = c_0 + \sum_{j=1}^{n} \gamma_j c_j \tag{2-15}$$

From Galerkin statement,

$$\int_V \gamma_i (Lu + f) dV = 0 \quad , \text{i} = 1, 2, \ldots, \text{n} \tag{2-16}$$

Substitution variable 'u' by the trial function, v, Eq. (2-16) becomes

$$\int_V \gamma_i (L(c_0 + \sum_{j=1}^{n} \gamma_j c_j) + f) dV = 0 \tag{2-17}$$

Then, Eq. (2-17) is rearranged into a matrix form, as indicated below

$$\text{Ac} = \text{b} \tag{2-18}$$

which, the vector of coefficients, $c = [c_1, c_2, \ldots, c_n]$ can be determined. The a and b are defined as,

$$a_{ij} = \int_V \gamma_i L \gamma_j dV \tag{2-19}$$

$$b_i = \int_V \gamma_i (f + Lc) dV \tag{2-20}$$

2.5 Literature Review

Khuzhayorov and Burnashev (2001) studied the model of the multiphase flow of an oil-gas-condensate system in porous media. The "Black oil" model is extended to describe the four-phase flow of the oil-gas-condensate systems in porous media by considering the solubility of gas in the condensate, water, and oil phases and to condensate evaporation into the gas phase. The three-dimensional transient-state model equations are reduced to one-dimensional transient-state forms by the partial integration method. The finite-difference method is used for the model solution. The calculations of the hydrodynamical parameters of an oil-gas-condensate reservoir element are carried out for several cases. This study provides a comparison of the effect of the different exploitation methods of the reservoir fluids production.

William and Roland (2002) studied three-dimensional finite element simulation of three-phase flow in a deforming fissured reservoir. The development of a capacity for predicting the exploitation of structurally complicated and fractured oil reservoirs. The mathematical formulation of a three-phase, three-dimensional fluid flow and rock deformation in fractured reservoirs is presented. The presented formulation consists of both the equilibrium and multiphase mass conservation equations of coupling between the fluid flow and solid deformation, which were usually ignored in the reservoir simulation literature. A Galerkin-based finite element method is applied to discretize the governing equations in space and a finite difference scheme is used to match the solution in time.

Baoyan *et al.* (2003) studied the sequential method for the black-oil reservoir simulation on unstructured grids. This paper presents new results for applying the sequential solution method to the black-oil reservoir simulation with

unstructured grids. The fully implicit solution method was successfully applied to reservoir simulation with unstructured grids. However, the complexity of the fully implicit method and the irregularity of the grids are a very complicated structure of linear equation systems (LESs) and in high computational cost to solve them. The sequential method is applied to reduce the size of the LESs by the low implicit degree of this method. These techniques are applied to field-scale models of both saturated and undersaturated reservoirs.

Suarez-Rivera *et al.* (2004) studied numerical analysis of open-hole multilateral completions minimizes the risk of costly junction failures. This model analyzes 3D compaction in an oil reservoir after an open-hole horizontal multilateral well is emplaced and is being pumped. The model couples fluid flow with Darcy's law from the Earth Science Module to displacements from the Stress and Strain applications from the FEMLAB core package. It focuses on elastic deformation. Related analyses can also be conducted for elasto-plastic materials, which are automated in the structural mechanics module.

Goodarz *et al.* (2004) studied numerical solution for natural gas production from methane hydrate dissociation. This paper describes a one-dimensional model for natural gas production from dissociation of methane hydrate in a confined reservoir by a depressurizing well. This approach accounts for the heat released by hydrate dissociation and convection–conduction heat transfer in the gas and hydrate zone. The system of governing equations is solved using a finite-difference scheme. Distributions of temperature and pressure in hydrate and gas regions and time evolutions of natural gas output are also evaluated. It is shown that rate of gas production relies on well pressure. In addition, both heat conduction and convection in the hydrate zone is important.

Chamnakyut (2004) studied a simulation of underground storage of natural gas. The numerical simulation program for predicting the pressure profile of natural gas in an underground storage reservoir was developed. The computer program was written for a two-dimensional model and a single layer to solve a governing equation that is a partial differential equation to obtain an approximate solution of the pressure distribution. Pressure profiles were predicted and the amount of gas withdrawn from a reservoir was calculated using the numerical method called the implicit alternating-

direction method (IAD) and important input parameters such as permeability, porosity, initial pressure. The program was written to predict regular and irregular shapes of gas reservoirs at constant and inconstant permeability, the pressure distribution and production rate at different time steps.

Ruben (2004) studied a variational multiscale finite element method for multiphase flow in porous media. A stabilized finite element method for numerical solution of multiphase flow in porous media was based on a multi-scale decomposition of pressures and fluid saturations into resolved (or grid) scales and unresolved (or subgrid) scales. The sub grid problem is modeled using an algebraic approximation. This model requires the definition of a matrix of intrinsic time scales. The performance of the method with simulations of a water flood in a heterogeneous oil reservoir was illustrated. The proposed method yields stable, highly accurate solutions on very coarse grids, which is compared with those obtained by the classical Galerkin method or the upstream finite difference method.

Ridha (2004) used of reservoir simulation to optimize recovery performance. A three-dimensional finite-difference reservoir simulator integrated in an EOR expert system was used to determine the reservoir management and production strategies to optimize the oil recovery from a carbonate reservoir. After screening the reservoir for an appropriate EOR process on the basis of its properties, it was determined that miscible carbon-dioxide injection is the most suitable process. The management strategies involved studying the different injection techniques to maximize the project profitability. All simulation runs were conducted using permeability fields that have been conditioned with core data taken from wells in the field. This well configuration was shown to yield the best oil recovery compared to other well configurations.

Ursin (2004) investigated on fluid flow in gas condensate reservoirs: the interplay of forces and their relative strengths. Natural production from gas condensate reservoirs is characterized by gas condensation and liquid dropout in the reservoir. The effects of liquid condensation reduced productivity and loss of production. Successful forecast of well productivity and reservoir production depends on detailed understanding of the effect of various forces acting on fluid flow in time and space. The production rate gas condensate reservoirs is thus indirectly

related to the interplay of fundamental forces, such as the viscosity, the capillary, the gravitational and their relative strengths, demonstrated by various dimensionless numbers. The material balance, the reservoir fluid flow and the well bore flow calculations are performed on a cylindrical reservoir model. The ratios between fundamental forces are calculated and dimensionless numbers are defined. The interplay of forces, demonstrated by these numbers, are calculated as function of radial dimension and reservoir pressure.

Henderson *et al.* (2005) studied supercritical fluid flow in porous media: modeling and simulation. The supercritical flow in porous materials employed in chromatography, supercritical extraction and petroleum reservoirs. The fluid is constituted of one pure substance and flow behavior is monophasic, highly compressible and isothermal. The porous media is isotropic, possibly heterogeneous with rectangular format. The heterogeneity of porous media is modeled by a simple power law, which describes the relationship between permeability and porosity. The modeling of the hydrodynamic phenomena incorporates the Darcy's law and the mass balance equation. A conservative finite-difference scheme is used to discretize differential equations. The results of the simulation for pressure and mobility of supercritical and liquid propane flowing through porous media are presented. The mobility of fluid flow in porous media depended on the permeability and fluid viscosity of fluid.