

# Chapter I

## Introduction

### 1.1 Motivation

At present, there are two main optimization approaches, the derivative-based and derivative-free methods. The derivative-based schemes, such as Newton's and conjugate gradient (CG) methods, have long been used in engineering applications. On the other hand, the derivative-free optimization, such as genetic algorithm (GA) and simulated annealing (SA), are much less employed in mechanical engineering applications, mostly confined to the field of control, machine intelligence and CAD/CAM [1], [2]. Compared to the derivative-based schemes, the derivative-free methods do not need functional derivative of a given objective function. They, instead, rely on repeated evaluation of the objective function and obtain the search direction under nature-inspired heuristic guidelines. Although the derivative-free schemes are generally slower than derivative-based methods, they are much more effective for complicated objective functions and combinatorial problems as the methods do not require differentiable objective functions.

GA is a derivative-free population-based optimization method of which search mechanisms are based on the Darwinian concept of survival of the fittest. Originally, GA is established to solve single-objective optimization problems (SOOPs) [3], [4]; subsequently it is adapted to solve multi-objective optimization problems (MOOPs) which have a number of objective functions to be minimized or maximized. In the thesis, the GA for a multi-objective optimization problem (MOOP) is called the multi-objective evolutionary algorithm (MOEA).

Most real-world optimization problems are MOOPs; for example, the decision to buy a computer by its performance and price is an MOOP with two objectives – the performance maximization and price minimization. As it is almost

improbable that only one solution is optimum in all objectives for a given MOOP, the multiple optimum solutions of MOOPs – the Pareto optimal solutions – are used. It is easy to compare solutions of SOOP, while solutions of MOOPs are compared by the Pareto domination [5], which is firstly defined by Vilfredo Pareto and shall be described in Chapter 2. If a given solution dominates other solutions, it is better than the rest. Thus, for a given solution set, the non-dominated solutions are the best solutions of the set.

A multi-objective evolutionary algorithm (MOEA) embeds the Pareto domination concept into a genetic algorithm (GA). In a GA, an objective of a solution  $i$  is directly used to evaluate the fitness of the solution. On the other hand, an MOEA employs Pareto domination concept to assign fitness or rank of a solution from objectives of the solution. The purposes of the MOEA are to find good non-dominated solutions which must possess 2 qualities – they should be close to the Pareto-optimal front and they are diverse along their front [5].

The structural design optimization may be divided into 3 main categories – the size, shape and topology design [6], [7]. In sizing optimization, the topology and shape are held constant while specific dimensions of the structure are modified. Examples include designs of plate thickness [8] and cross-sectional areas of truss and frame elements [9]. The shape optimization maintains a constant topology while the shape is modified and design variables that produce optimum component shape are determined. The prime example is the shape optimization by varying shape parameterisation and local curve fitting [10], [11]. It should be noted that the sizing optimization typically occurs as a consequence of the shape optimization process.

Meanwhile, the topology optimization finds the optimal lay-out of structure within a specified region and may be divided into 3 main categories – the discrete truss design [12], unit cell properties, e.g. orientation and porosity in composite materials [13], and continuum topology optimization. The purpose of the

continuum topology optimization is to allow the creation of new boundaries. The space that contains the structure is specified and divided into rectangular grids. By selective filling these grids or leaving the space empty, different configurations are obtained. Many structural topology optimization problems have been solved by derivative-based optimizers such as [14]-[24]. These optimizers require an explicit mathematical formulation of decision variables of a topology optimization problem, even though it is very difficult to identify the objective formulations of topology optimization problems [25].

Therefore a genetic algorithm (GA), a derivative-free optimizer, is more suitable to the topology optimization problems; in this thesis, the GA and computational mechanics have been combined for continuum topology optimization. The GA randomly generates a population within the solution space set which is then evolved repeatedly toward structures that can perform superlatively. Most of structural topology optimization problems are focused on single-objective optimization problems. If there is more than one objective to be designed, they can be considered as single-objective problems by optimizing only one selected objective and treating other objectives as problem restrictions. However, it is generally better if all objectives are optimized; hence the multi-objective optimization is needed for the problems.

In this thesis, MOEA is chosen to solve the multi-objective continuum topology optimization problems whilst the finite volume method (FVM) is selected for the objective calculation. The advantages of this FVM scheme are direct representation of conservative laws, straight forward physical interpretation and simple discretization. Moreover the FVM does not require the connectivity analysis as in finite element method (FEM) [6] due to its superlative physical representation as control volumes with only one shared corner vertex are not physically attached [26].

Many MOEAs had been developed in last decade; most of these MOEAs [27]–[33] are tested only on well-established benchmark problems and focus on two-objective optimization problems. It has not been studied whether these developed MOEAs are suitable for multi-objective topology optimization, which are real-world problems and may not have only two optimized objectives [34]. This thesis presents improved MOEAs for multi-objective continuum topology optimization, in which the improved MOEAs are tested not only on well-established benchmark problems but also on continuum topology optimization problems. Hopefully, it shall be helpful for researches in structural design optimization.

There are two main parts in thesis – the improvement of multi-objective evolutionary algorithms (MOEAs) and implementations of MOEAs for multi-objective continuum topology optimization problems. Three proposed MOEAs are introduced in the first part. The co-operative co-evolution multi-objective algorithm (CCMOA) is the first presented MOEA. It introduces the co-operative co-evolution strategy, which is originally developed for single-objective optimization problems, into a multi-objective optimization problem (MOOP). The co-operative co-evolution is successfully employed for multi-objective continuum topology optimization problems [35], [36]. The second proposed MOEA, the compressed-objective genetic algorithm (COGA-I) [37] and, more importantly, its improved version, the improved compressed-objective genetic algorithm (COGA-II), is proposed for optimization problems with three-or-more objectives. The third proposed MOEA is the co-operative co-evolutionary improved compressed-objective genetic algorithm (CCCOGA-II) which results from the integration of co-operative co-evolution into improved compressed-objective genetic algorithm (COGA-II). It is also proposed for an optimization problem with three-or-more objectives as COGA-II.

The effectivenesses of proposed MOEAs are evaluated by comparing them with well-established advanced MOEAs – fast elitist non-dominated sorting

genetic algorithm (NSGA-II) [28], and improved strength Pareto evolutionary algorithm (SPEA-II) [30]. All MOEAs under consideration – CCMOA, COGA-II, CCCOGA-II, NSGA-II, and SPEA-II, are tested on well-known unconstrained benchmark problems ZDT1-6 [38] and DZTL1-7 [39] for optimization problems with two objectives and three-or-more objectives respectively. In the second part, these MOEAs are used to solve multi-objective continuum topology optimization problems, which are the heat conduction, linear-elastic and multi-disciplinary thermo-elastic problems.

## **1.2 Research Objectives**

This thesis is roughly divided into two parts – the improvement of multi-objective evolutionary algorithms (MOEAs) and implementations of MOEAs to multi-objective continuum topology optimization problems. The objectives of this thesis are as follows:

- 1.2.1 To study the well-established multi-objective evolutionary algorithms (MOEAs).
- 1.2.2 To introduce the co-operative co-evolution for multi-objective optimization problem (MOOP), resulting in the so-called co-operative co-evolutionary multi-objective algorithm (CCMOA).
- 1.2.3 To introduce the new multi-objective evolutionary algorithm (MOEA), improved compressed-objective genetic algorithm (COGA-II), for MOOPs with three-or-more objectives.
- 1.2.4 To integrate the co-operative co-evolution into the compressed-objective genetic algorithm (COGA-II). The resulting algorithm is the co-operative co-evolutionary compressed-objective genetic algorithm (CCCOGA-II).
- 1.2.5 To study the performance improvement due to the purposed multi-objective evolutionary algorithms – CCMOA, COGA-II, and CCCOGA-II.



- 1.2.6 To study the finite volume method (FVM) for heat conduction, elastic solids and thermo-elastic solid simulations.
- 1.2.7 To implement MOEAs and FVM for multi-objective continuum topology optimization problems.
- 1.2.8 To study the effectiveness of multi-objective evolutionary algorithms (MOEAs) – CCMOA, COGA-II, and CCCOGA-II – for solving the multi-objective continuum topology optimization problems.

### **1.3 Research Scopes**

This thesis studies the improvement of MOEA and implementation of MOEAs to optimize multi-objective continuum topology problems. The scopes of this thesis are as follows:

- 1.3.1 The co-operative co-evolutionary multi-objective algorithm (CCMOA), which employs the co-operative co-evolution for multi-objective optimization problems (MOOPs), is presented and tested on selected optimization problems with 2-6 objectives.
- 1.3.2 The improved compressed-objective genetic algorithm (COGA-II) for an optimization problem with three-or-more objectives is purposed and tested on selected optimization problems with 3-6 objectives.
- 1.3.3 The co-operative co-evolutionary improved compressed-objective genetic algorithm (CCCOGA-II), the resulting algorithm from the integration of co-operative co-evolution into improved compressed-objective genetic algorithm (COGA-II), is introduced and tested on selected optimization problems with 3-6 objectives.
- 1.3.4 The introduced MOEAs – co-operative co-evolutionary multi-objective algorithm (CCMOA), improved compressed-objective genetic algorithm (COGA-II), and co-operative co-evolutionary improved compressed-

objective genetic algorithm (CCCOGA-II) – are compared with two well-established MOEAs – fast elitist non-dominated sorting genetic algorithm (NSGA-II), and improved strength Pareto evolutionary algorithm (SPEA-II).

1.3.5 All five MOEAs – co-operative co-evolutionary multi-objective algorithm (CCMOA), compressed-objective genetic algorithm (COGA-II), co-operative co-evolutionary improved compressed-objective genetic algorithm (CCCOGA-II), fast elitist non-dominated sorting genetic algorithm (NSGA-II), and improved strength Pareto evolutionary algorithm (SPEA-II) – are tested by the benchmark problems ZDT1-6 for two-objective optimization problems and DTLZ1-7 for optimization problems with 3-6 objectives.

1.3.6 There are two disciplines of computational mechanics in this thesis – heat conduction and linear elastic solids problems, which are combined into a thermo-elastic solids problem, representing a multidisciplinary scenario.

1.3.7 All five employed MOEAs – co-operative co-evolutionary multi-objective algorithm (CCMOA), improved compressed-objective genetic algorithm (COGA-II), co-operative co-evolutionary compressed-objective genetic algorithm (CCCOGA), fast elitist non-dominated sorting genetic algorithm (NSGA-II), and improved strength Pareto evolutionary algorithm (SPEA-II) – are used to solve simple multi-objective continuum topology optimization problems. The introduced MOEAs are only tested with simple test cases to ensure that the MOEAs are suitable for this type of problems. No complex real-life problems are considered.

1.3.8 C programs are developed for all previous topics – 1.3.1 to 1.3.7.

#### **1.4 Research Benefits**

The benefits from this research are as follows.

- 1.4.1 The introduction of MOEAs – improved compressed-objective genetic algorithm (COGA-II), co-operative co-evolutionary multi-objective algorithm (CCMOA), and co-operative co-evolutionary improved compressed-objective genetic algorithm (CCCOGA-II) – are empirically shown to be superior to the well-established multi-objective evolutionary algorithms (MOEAs) – fast elitist non-dominated sorting genetic algorithm (NSGA-II) and improved strength Pareto evolutionary algorithm (SPEA-II) in multi-objective optimization problems (MOOPs), regardless of number of objectives. This study is particularly useful in theoretical MOEA research, especially in that concerning multi-objective optimization problems (MOOPs) with three-or-more objectives.
- 1.4.2 The multi-objective continuum topology optimization problems can be efficiently solved by MOEAs. Subsequently, researchers in mechanical engineering applications may be motivated to employ more MOEAs for mechanical engineering optimization problems. The purposed MOEAs in this thesis may be used in the future to solve the more complicated multi-objective continuum topology optimization problems as the basis for components designs, which is the first stage of the design process before the shape and sizing optimization.

## **1.5 Research Methodologies**

The research proceeds by following these steps.

- 1.5.1 To study the well-established multi-objective evolutionary algorithms (MOEAs).
- 1.5.2 To study the co-operative co-evolution for evolutionary computation and present the co-operative co-evolutionary multi-objective algorithm (CCMOA).



- 1.5.3 To formulate and present the improved compressed-objective genetic algorithm (COGA-II) for MOOPs with three-or-more objectives.
- 1.5.4 To integrate the co-operative co-evolution into the improved compressed-objective genetic algorithm (COGA-II), the resulting algorithm is referred to as the co-operative co-evolutionary improved compressed-objective genetic algorithm (CCCOGA-II).
- 1.5.5 To study the well-known unconstrained benchmark problems – ZDT1-6 for optimization problems with two-objective and DZTL1-6 for optimization problems with three-or-more objectives.
- 1.5.6 To study the unstructured, cell-centered finite volume method.
- 1.5.7 To study the MOEA performances for solving multi-objective continuum topology optimization problems.
- 1.5.8 To develop the computer program in C language for all previously stated MOEAs, benchmark problems and proposed topology optimization problems.
- 1.5.9 To write papers for publications in international conferences and journals.
- 1.5.10 To analyze and conclude the results, and write the complete thesis.