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**MULTI-OBJECTIVE EVOLUTIONARY ALGORITHMS
FOR CONTINUUM TOPOLOGY OPTIMIZATION**

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A Dissertation Submitted in Partial Fulfillment of the Requirements
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This thesis employs multi-objective evolutionary algorithms (MOEAs) to solve continuum topology optimization problems – heat conduction, linear-elastic, and thermo-elastic problems with 3-6, 2-5, and 2-5 design objectives, respectively. Three MOEAs – co-operative co-evolutionary multi-objective algorithm (CCMOA), improved compressed-objective genetic algorithm (COGA-II), and co-operative co-evolutionary improved compressed-objective genetic algorithm (CCCOGA-II) – are proposed.

These MOEAs are tested against 2 well-established MOEAs – fast non-dominated sorting genetic algorithm (NSGA-II), and improved strength Pareto evolutionary algorithm (SPEA-II) – using benchmark problems with 2-6 objectives – ZDT1-6, DTLZ1-7, linked DTLZ2, and linked DTLZ6 – as well as the continuum topology optimization problems. After simulations, the proposed MOEAs outperform NSGA-II and SPEA-II in both types of problems.

With the proposed MOEAs, the progressive refinement run with increasingly refined grid for problems with 2-3 design objectives, and the objective increasing run which starts with only few objectives and keeps adding others until all design objectives are considered for problems with many design objectives, are used to solve the continuum topology optimization problems. The searching runs with the proposed MOEAs can obtain reliable solutions for the problems. Optimized topological solutions by the proposed MOEAs can be very useful for further detailed design such as shape and sizing optimization.

From simulation results, it can be concluded that MOEAs are more suited to solve continuum topology optimization problems than derivative-based optimizers. Moreover, they can solve complex or non-linear continuum topology optimization problems, which are very difficult for the derivative-based optimizers. In addition, the proposed MOEAs outperform the existing algorithms in both benchmark and continuum topology optimization problems.

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Abbreviations

Avg	Average
CAD	Computer Aided Design
CAM	Computer Aided Mechanics
CCCOGA-II	Co-operative co-evolutionary improved compressed-objective genetic algorithm
CCGA	Co-operative co-evolutionary genetic algorithm
CCMOA	Co-operative co-evolution multi-objective algorithm
CFD	Computational fluid dynamics
CG	Conjugate gradient
CNSGA	Controlled elitist non-dominated sorting genetic algorithm
COGA-I	Compressed-objective genetic algorithm
COGA-II	Improved compressed-objective genetic algorithm
DSA	Design sensitivity analysis
DTZL1-7	Deb, Thiele, Zitzler, and Laumanns 's DTZL problems
FEM	Finite element method
FON	Fonseca 's FON problem
FVM	Finite volume method
GA	Genetic algorithm
KUR	Kursawe 's KUR problem
L-DTLZ	Linked Deb, Thiele, Zitzler, and Laumanns 's DTZL problems
M_1	Average distance of non-dominated solutions to the true Pareto optimal front
MOCCGA	Multi-objective co-operative co-evolutionary genetic algorithm
MODCGA	Multi-objective diversity control oriented genetic algorithm
MOEA	Multi-objective evolutionary algorithm
MOGA	Multi-objective genetic algorithm
MOOP	Multi-objective optimization problem
MRH_{pa}	Maximum value of ratios of hole perimeter to hole area

Abbreviations (continue)

NPGA	Niched Pareto genetic algorithm
NSCCGA	Non-dominated sorting co-operative co-evolution genetic algorithm
NSGA	Non-dominated sorting genetic algorithm
NSGA-II	Fast elitist non-dominated sorting genetic algorithm
POL	Poloni 's POL problem
PAES	Pareto-archive evolution strategy
R_{pa}	Ratio of structural perimeter to structural area
SA	Simulated annealing
SBX	Simulated binary crossover
SCH1-2	Schaffer 's SCH problems
SD	Standard deviation
<i>SDT</i>	summation of distances
SGA	Standard genetic algorithm
SOOP	Single-objective optimization problem
SPEA	Strength Pareto evolutionary algorithm
SPEA-II	Improved strength Pareto evolutionary algorithm
VEGA	Vector evaluated genetic algorithm
VNT1-2	Viennet 's VNT problems
WS	Winning score
ZDT1-6	Zitzler, Deb, and Thiele 's ZDT problems