CHAPTER V

CONCLUSION AND OPEN PROBLEMS

5.1 Conclusion

We have investigated values and bounds of the clique covering numbers and the clique partition numbers of the k-power of graphs. There are results as follows :

Clique parameters of the k-power of paths :

1. For $n, k \in \mathbf{N}$,

$$cc(P_n^k) = \begin{cases} 1 & \text{if } \ k \geq n-1, \\ n-k & \text{if } \ 1 \leq k < n-1. \end{cases}$$

2. For $n \in \mathbf{N}$,

$$cp(P_n^2) = \begin{cases} 1 & \text{if } \ n = 1, 2, 3, \\ n-1 & \text{if } \ n = 2r \ , \ r \geq 2, \\ n-2 & \text{if } \ n = 2r+1 \ , \ r \geq 2. \end{cases}$$

Clique parameters of the k-power of cycles :

1. For $n, k \in \mathbf{N}$, $cc(C_n^k) \leq n$.
2. For $n, k \in \mathbf{N}$ where $k < \frac{n}{3}$, $cc(C_n^k) = n$.
3. For $n \in \mathbf{N}$,

$$cc(C_n^2) = \begin{cases} 1 & \text{if } \ 1 \leq n \leq 5, \\ 4 & \text{if } \ n = 6, \\ n & \text{if } \ n \geq 7. \end{cases}$$

4. For $n \in \mathbf{N}$,

$$cp(C_n^2) = \begin{cases} 1 & \text{if } 1 \leq n \leq 5, \\ 4 & \text{if } n = 6, \\ n+1 & \text{if } n = 2r+1, r \geq 3, \\ n & \text{if } n = 2r, r \geq 4. \end{cases}$$

Clique parameters of the k-power of pyramids :

1. For $n, k \in \mathbf{N}$,

$$cc(PG_n^k) = \begin{cases} 1 & \text{if } k \geq n-1, \\ \frac{(n-k)(n-k+1)}{2} & \text{if } 1 \leq k < n-1. \end{cases}$$

2. For $n \in \mathbf{N}$.

   (i) If $n = 1, 2$ *or* 3, then $cp(PG_n^2) = 1$.

   (ii) If $n = 2r+1$ where $r \geq 2$, then

$$\frac{(n-2)(n-1)}{2} \leq cp(PG_n^2) \leq \frac{(n-1)(7n-19)}{4}.$$

   (iii) If $n = 2r$ where $r \geq 2$, then

$$\frac{(n-2)(n-1)}{2} \leq cp(PG_n^2) \leq \frac{7n^2}{4} - 5n + 4.$$

Clique parameters of the k-power of ladders :

1. For $n, k \in \mathbf{N}$,

$$cc((P_2 \times P_n)^k) = \begin{cases} 1 & \text{if } k \geq n, \\ 2(n-k)+2 & \text{if } 2 \leq k < n, \\ 3n-2 & \text{if } k = 1. \end{cases}$$

2. For $n \in \mathbf{N}$.

   (i) If $n = 1$ *or* 2, then $cp((P_2 \times P_n)^2) = 1$.

   (ii) If $n = 2r + 1$ where $r \geq 1$, then

$$2n - 2 \leq cp((P_2 \times P_n)^2) \leq \frac{5n-3}{2}.$$

   (iii) If $n = 2r$ where $r \geq 2$, then

$$2n - 2 \leq cp((P_2 \times P_n)^2) \leq \frac{5n-4}{2}.$$

Clique parameters of the k-power of grids :

1. For $m, n, k \in \mathbf{N}$ where $k < min\{m, n\}$ and k is odd,

$$cc((P_m \times P_n)^k) \geq 2mn - k(m+n).$$

2. For $m, n, k \in \mathbf{N}$ where $k < min\{m, n\}$ and k is even,

$$cc((P_m \times P_n)^k) \geq mn - k^2.$$

3. For $m, n \in \mathbf{N}$ where $m, n > 2$,

$$cc((P_m \times P_n)^2) = mn - 4.$$

4. For $m, n \in \mathbf{N}$ where $m, n > 2$.

   (i) If $m = 2r_1 + 1$ and $n = 2r_2 + 1$ where $r_1, r_2 \geq 1$, then

$$mn - 4 \leq cp((P_m \times P_n)^2) \leq 2mn - \frac{5m}{2} - 2n + \frac{9}{2}.$$

   (ii) If $m = 2r_1 + 1$ and $n = 2r_2$ where $r_1 \geq 1$ and $r_2 \geq 2$, then

$$mn - 4 \leq cp((P_m \times P_n)^2) \leq 2mn - \frac{3m}{2} - 2n + \frac{5}{2}.$$

   (iii) If $m = 2r_1$ and $n = 2r_2 + 1$ where $r_1 \geq 2$ and $r_2 \geq 1$, then

$$mn - 4 \leq cp((P_m \times P_n)^2) \leq 2mn - \frac{3n}{2} - 2m + \frac{5}{2}.$$

   (iv) If $m = 2r_1$ and $n = 2r_2$ where $r_1, r_2 \geq 2$, then

$$mn - 4 \leq cp((P_m \times P_n)^2) \leq 2mn - \frac{3m}{2} - \frac{n}{2} - 3.$$

5.2 Open Problems

This thesis brings some open problems for future work as follows :

Find values or improve bounds of

1. The clique covering numbers of the k-power of cycles where $\frac{n}{3} \leq k < \lfloor \frac{n}{2} \rfloor$.

2. The clique covering numbers of the k-power of grids where $k \geq 3$.

3. The clique partition numbers of the square of pyramids, ladders and grids.

4. The clique partition numbers of the k-power of paths, cycles, pyramids, ladders and grids where $k \geq 3$.

5. The clique covering numbers and the clique partition numbers of the k-power of other graphs, for example, bipartite graphs, $P_m \times P_n \times P_r$ etc.

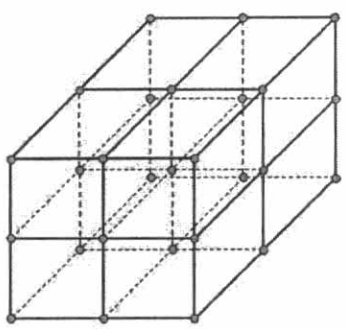

Figure 5.1: $P_3 \times P_3 \times P_3$