CHAPTER IV

APPLICATIONS OF CERTAINTY EQUIVALENT AND RISK TOLERANCE CONCEPT

This chapter presents the applications of the risk tolerance and certainty equivalent concept that we have discussed from the previous chapter. The hypothetical example of drilling investment prospects are set up to illustrate the applications of this concept. The dominant application is to incorporate the risk attitude of decision makers to the financial risk into the decision analysis process hence assist decision makers selecting the appropriate investment project that more represent their truly risk attitude.

4.1 Using certainty equivalent indicting the best share among prospects.

Instead of making a decision based on the EMV concept which fails to take risk attitude of a decision maker into account by maximizing the expected value, the decision maker should make a decision by maximizing the certainty equivalent which is more truly and adequately incorporates risk preference of a decision maker into account. The applications are adapted from Cozzolino (1978) exponential risk aversion. The following 5 simple drilling prospects are set up as an example of illustrating the applications of certainty equivalent approach to show how to combine the risk preference concept into the investment decision making. The certainty equivalent valuation provides guidance to the firm in terms of value of diversification and risk sharing by guiding the appropriate participation level among various investment projects that consistent with firm's risk propensity. Let's consider the following example. Suppose that the company is considering investing in the 5 drilling risky prospects with the information shown in Table 4.1.

Table 4.1: The five drilling risky prospects.

Prospect	Producer NPV (MM$)	Dry Hole Cost (MM$)	Probability of Producer Occurrence	EMV (MM$)
1	100	15.0	0.55	48.3
2	70	30.0	0.65	35.0
3	110	25.0	0.45	35.8
4	90	40.0	0.65	44.5
5	85	15.0	0.35	20.0

Assume that the company agrees on 28 million dollar as their risk tolerance of the firm (In real world problem, the risk tolerance of the firm has much more complication in methods to determine. Therefore, in this section the risk tolerance is set up in order to illustrate the applications of the concept.) Of those five drilling projects present to the firm, the certainty equivalent at different working interest of the projects is calculated from equation 3.3 as stated before. Table 4.2 shows the computations of certainty equivalent value at different working interest for the firm with a risk tolerance of 28 million dollar.

Table 4.2: Calculations of certainty equivalent at various working interest of five drilling projects.

Wi	Certainty Equivalent of the Project				
	1	2	3	4	5
0%	0.0000	0.0000	0.0000	0.0000	0.0000
5%	2.2656	1.6467	1.5882	2.0497	0.9003
10%	4.2366	3.0810	2.7896	3.7366	1.6097
15%	5.9160	4.2961	3.6288	5.0494	2.1416
20%	7.3131	5.2886	4.1373	5.9863	2.5109
25%	8.4421	6.0587	4.3511	6.5557	2.7328
30%	9.3212	6.6101	4.3075	6.7749	2.8225
35%	9.9717	6.9495	4.0433	6.6685	2.7948
40%	10.4163	7.0867	3.5931	6.2665	2.6636
45%	10.6782	7.0337	2.9879	5.6016	2.4419
50%	10.7802	6.8043	2.2553	4.7073	2.1416
55%	10.7438	6.4133	1.4187	3.6161	1.7733
60%	10.5886	5.8759	0.4978	2.3582	1.3467
65%	10.3324	5.2073	-0.4908	0.9609	0.8701
70%	9.9912	4.4223	-1.5337	-0.5520	0.3509
75%	9.5785	3.5346	-2.6199	-2.1596	-0.2046
80%	9.1062	2.5573	-3.7405	-3.8445	-0.7909
85%	8.5845	1.5021	-4.8883	-5.5922	-1.4032
90%	8.0217	0.3797	-6.0578	-7.3905	-2.0375
95%	7.4251	-0.8004	-7.2444	-9.2296	-2.6903
100%	6.8007	-2.0301	-8.4445	-11.1013	-3.3588

Then the certainty equivalent of those five drilling projects is plotted versus working interests as shown in Figure 4.1. It is obvious from the graph that for each prospect, there is a best working interest for a given the company's risk tolerance of 28 million dollar. That best working interest is the %WI which yields the greatest certainty equivalent.

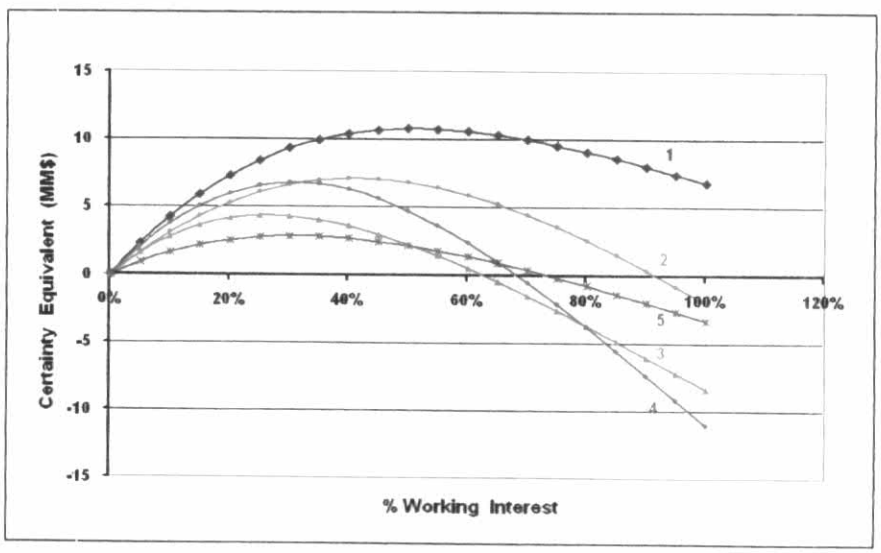

Figure 4.1: Plot of CE at various working interest of five drilling prospects.

In the absence of capital limitation, the best share of each prospect is that which maximizes certainty equivalent. So, from the certainty equivalent valuations chart in Figure 4.1, the firm should invest in the prospect 1, 2, 3, 4 and 5 with the working interest of 50%, 40%, 25%, 30% and 30% respectively in all prospects.

4.2 Applying certainty equivalent in portfolio balancing.

Recall that the above example is applicable for the case that the company has no limit on the availability of fund by investing with the best share which gives the greatest certainty equivalent in the prospects. But generally most firms usually work under a certain budget. In this case, the technique for determination of the optimal share under capital limitation is presented. If there are N prospects available for the firm, the firm should participate in each prospect with the fraction that does not make the total cost of all N prospects exceeds the allocated budget B. This technique is used to optimize the total risk adjusted value (RAV) from N opportunities without exceeding the applicable constraint on available resources. The technique used here is

applied with the prospect with two outcomes which are gain and loss outcome. The example is applied from Mian (2002). The following two equations are used to calculate the optimum working interest of each prospect so a balanced portfolio within a constrained budget is achieved.

$$B = RT \sum_{i=1}^{N} \frac{C_i}{NPV_i + C_i} \ln \left[\frac{\left(\frac{p_i}{1-p_i} \right) \left(\frac{NPV_i}{C_i} - H \right)}{1+H} \right] \tag{4.1}$$

$$WI_i = \frac{RT}{NPV_i + C_i} \ln \left[\frac{\left(\frac{p_i}{1-p_i} \right) \left(\frac{NPV_i}{C_i} - H \right)}{1+H} \right] \tag{4.2}$$

where

RT	=	risk tolerance value (currency unit)
p_i	=	probability of each outcome i (fraction)
B	=	budget of the corporate (currency unit)
C_i	=	investment cost of the project (currency unit)
NPV_i	=	Net Present Value of the project (currency unit)
H	=	the first determined value using in the SOLVER function

In order to calculate the working interest of the N prospects within the budget constraint B, the SOLVER add-ins program in EXCEL is used by using a linear optimization scheme. The H in equation 4.1 is the first determined by SOLVER and then used in equation 4.2 to calculate the optimum working interest for each opportunity. Let's consider the previous five drilling prospects in Table 4.1 again for the company with a risk tolerance of 28 million dollars and an *assumed* available budget of 35 million dollars. If the company has limited capital of 35 million dollars what the optimum working interest of each prospect that the firm should participate in order to prevent the firm's budget exceed an available budget. The calculations of an optimum working interest for those five projects with a constrained budget are shown in Table 4.3.

Table 4.3: The calculations of portfolio balancing of five drilling prospects in EXCEL.

	A	B	C	D	E	F	G
1							
2		Prospect	Producer NPV (MM$)	Dry Hole Cost (MM$)	Probability of Producer Occurrence	EMV (MM$)	RT MM$
3							
4		1	100	15.0	0.55	48.3	28
5		2	70	30.0	0.65	35.0	
6		3	110	25.0	0.45	35.8	
7		4	90	40.0	0.65	44.5	
8		5	85	15.0	0.35	20.0	
9							
10							
11		WI_{best}	B	WI_i	WI_i*Cost (MM$)	RAV (MM$)	
12		0.50	0.2420	0.45	6.7749	10.6841	
13		0.40	0.3464	0.32	9.6996	6.7942	
14		0.25	0.1888	0.21	5.2865	4.2110	
15		0.30	0.3429	0.24	9.5999	6.4705	
16		0.30	0.1300	0.24	3.6391	2.7087	
17		42.2500	35.0000	1.4690	35.0000	30.8685	
18							
19		H	0.230652				

1. **Calculations method**

   1.1 Equation 4.1 and 4.2 are used in Cells C12:C16 and D12:D16 respectively.

   1.2 Cell C19 inputs initial guess value of *H*.

   1.3 SOLVER part: Set solver parameters as follow

      1.3.1 Set Target Cell: **C17**

      1.3.2 Equal to: Value of: **35**

      1.3.3 By Changing Cell: **C19**

      1.3.4 Click on **Solve** to perform calculations.

2. **Results**

The WI_{best} values in Cell B12:B16 is the best participation level that we have calculated from the previous section. If the company participates at the best share for all five drilling prospects, the total cost in cell B17 (42.25 $MM) would exceed the available budget (35 $MM). Therefore, in each available project, the firm should participate in the % working interest as shown in Table 4.4.

Table 4.4: Results of the portfolio balancing.

Prospect	Producer NPV (MM$)	Dry Hole Cost (MM$)	Probability of Success	Working Interest	Cost at each WI
1	100	15	0.55	45.17%	6.77
2	70	30	0.65	32.33%	9.70
3	110	25	0.45	21.15%	5.29
4	90	40	0.65	24.00%	9.60
5	85	15	0.35	24.26%	3.64
					35.00

Therefore, if the company has limited capital of 35 million dollars, the firm should invest in the prospect 1, 2, 3, 4, and 5 with optimum working interest of 45%, 32%, 21%, 24% and 24% respectively.

In short, the certainty equivalent can assist the firm in selecting the appropriate working interest among the various investment options. By utilizing the certainty equivalent concept, the risk preference of a decision maker is already combined into the decision making model, hence this is more realistic measure of value than the EMV approach. Next chapter presents the methodology of assessment of risk tolerance and consistency of the E&P firm that we have discussed so far.