CHAPTER III
MATHEMATICAL MODELING

This chapter describes the model geometry, model assumption, model formulation which performs the relationship between transient mass transport and Darcy's law and model algorithm in finite difference and finite element methods.

3.1 Model Description

Naturally, shapes of petroleum reservoirs have different geometries. For example, anticline shape has been found where the impermeable rock traps the natural gas under the ground, like an umbrella. Natural gas can be recovered by drilling a well through the impermeable rock. Gas in these reservoirs is typically under extremely high pressure, thereby releasing from the reservoir itself.

In this work, the reservoir model initially assumed for the petroleum reservoir as a simple rectangular shape. The model was used to investigate the reservoir behaviours such as pressure distribution, wellbore pressure, bottom well pressure, and production time after gas withdrawal or injection. Afterwards, the interior islands and curved edge were added into the regular shape to simulate more realistic the reservoir behaviours.

The geometries of reservoir are illustrated in Figures 3.1(a), (b) and (c). The white and gray regions represent the permeable and impermeable rock, respectively. The reservoir was surrounded by impermeable rock or water that no mass transfer along these boundaries. Figure 3.1(c) illustrates the actual reservoir geometry which contains 12 withdrawal wells (white dots) and is surrounded by impermeable rock. The size of this reservoir performs in SI unit.

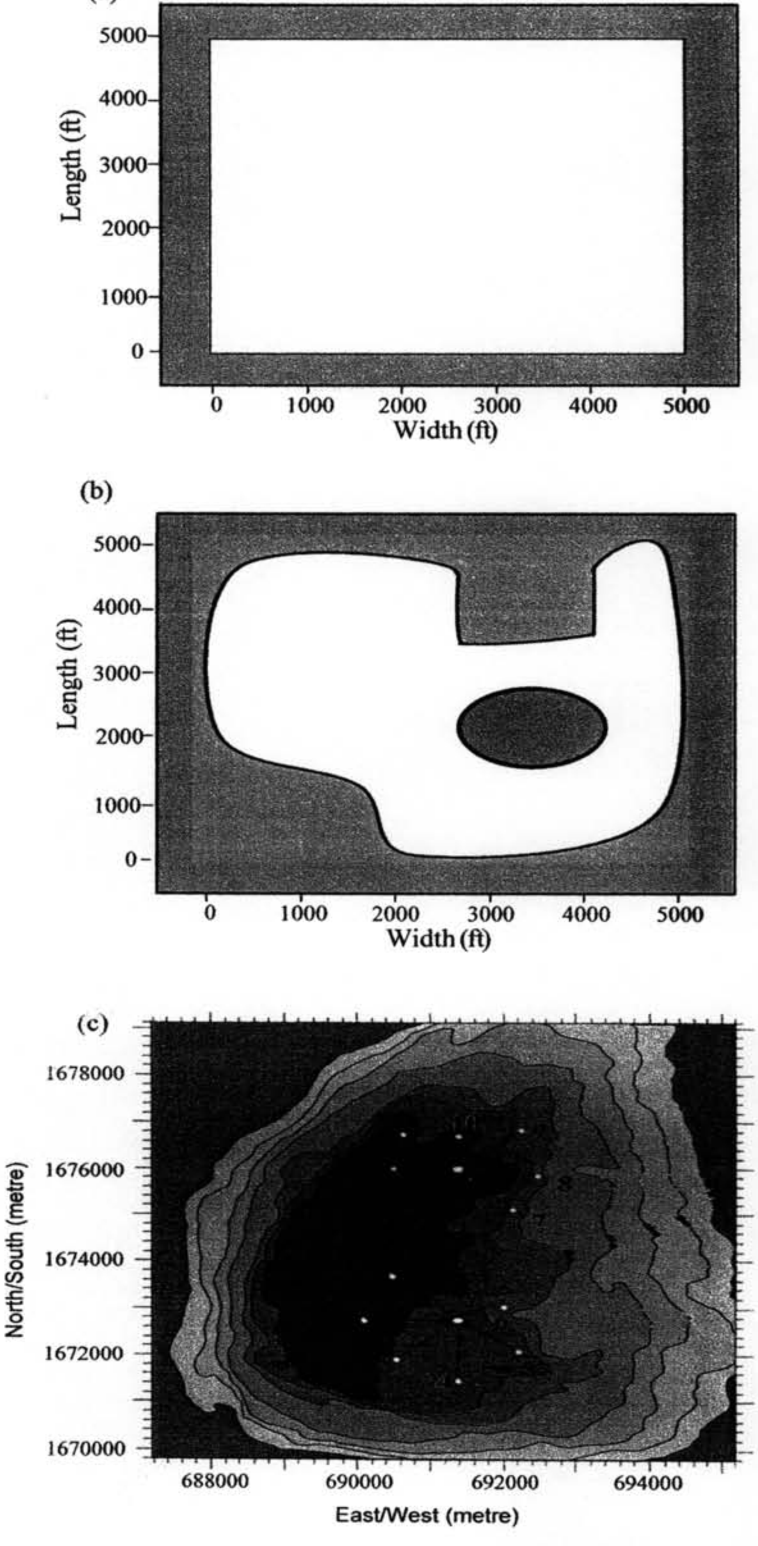

Figure 3.1 Geometries of modeling reservoir, (a) regular shape, (b) irregular shape, (c) carbonate reservoir.

3.2 Model Assumptions

In this work, some specifications of reservoir properties are assumed as follows otherwise stated:

1. The permeability, k, was assumed constant along the reservoir.
2. The gas (predominantly methane) behaves ideally, and so has a compressibility factor $Z = 1$.
3. The porosity and reservoir temperature are uniform throughout the reservoir.

3.3 Model Formulation

In the natural gas production, flow of natural gas into the well depends on the pressure drop in the reservoir, $p_r - p_w$, where p_r is average reservoir pressure and p_w is wellbore pressure. The relationship between flow rate and pressure drop occurring in the porous medium is very complex and depends on many parameters such as rock properties, fluid properties and flow regime.

For single phase flow in porous medium, the fluid velocity ($\overline{V}$) throughout the porous medium can be determined using the Darcy's law (Wilkes, 1999).

$$\overline{V} = -\frac{k}{\mu}\nabla p \tag{3-1}$$

where, k is the rock permeability, μ the gas viscosity, ∇p the pressure gradient.

The three dimensional mass balance equation (combining with Darcy's law) as a function of pressure in the rectangular coordinate can be written as (Wilkes, 1999),

$$\frac{\partial}{\partial x}\left(\frac{p}{ZT}\frac{k}{\mu}\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{p}{ZT}\frac{k}{\mu}\frac{\partial p}{\partial y}\right)+\frac{\partial}{\partial z}\left(\frac{p}{ZT}\frac{k}{\mu}\frac{\partial p}{\partial z}\right)-\frac{ZMR}{MW}=\varepsilon\frac{\partial(p/ZT)}{\partial t} \tag{3-2}$$

where, ε is porosity; Δz, the reservoir thickness; M, the mass withdrawal rate per unit volume; MW, the molecular weight of gas; p, the reservoir pressure; R, the gas constant, T, the reservoir temperature; and Z, the compressibility factor.

The volumetric flow rate per volume (q_s) at standard conditions is given by,

$$q_s = \frac{M}{\rho_s} = \frac{ZMRT_s}{p_s MW} \tag{3-3}$$

Place q_s into Eq. (3-2).

$$\frac{\partial}{\partial x}\left(\frac{p}{ZT}\frac{k}{\mu}\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{p}{ZT}\frac{k}{\mu}\frac{\partial p}{\partial y}\right)+\frac{\partial}{\partial z}\left(\frac{p}{ZT}\frac{k}{\mu}\frac{\partial p}{\partial z}\right)-\frac{q_s p_s}{T_s} = \varepsilon\frac{\partial(p/ZT)}{\partial t} \tag{3-4}$$

Defining the gas potential (compared with reference pressure, p_r) as

$$\phi = \int_{pr}^{p} \frac{p}{Z\mu} dp \tag{3-5}$$

Assuming that reference pressure (p_r) is equal 0, Eq. (3-5) becomes

$$\phi = p^2/2\mu \tag{3-6}$$

or

$$p = (2\mu\phi)^{1/2} \tag{3-7}$$

Eq. (3-4) is rearranged to

$$\frac{\partial}{\partial x}k\frac{\partial \phi}{\partial x}+\frac{\partial}{\partial y}k\frac{\partial \phi}{\partial y}+\frac{\partial}{\partial z}k\frac{\partial \phi}{\partial z}-\frac{q_s T p_s}{T_s} = \varepsilon\gamma\frac{\partial \phi}{\partial t} \tag{3-8}$$

where, $\gamma = \frac{\mu}{p} = \frac{\mu}{\sqrt{2\mu\phi}}$, $\quad \beta = \varepsilon * \sqrt{\frac{\mu}{2}}$

Using assumptions indicated previously, Eq. (3-8) for gas withdrawal from a reservoir becomes,

$$k\left(\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2}\right)-\alpha q_s=\frac{\beta}{\sqrt{\phi}}\frac{\partial\phi}{\partial t} \qquad (3\text{-}9)$$

where, $\alpha=\dfrac{Tp_s}{T_s}$

In case of gas injection into reservoir, the equation becomes

$$k\left(\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2}\right)+\alpha q_s=\frac{\beta}{\sqrt{\phi}}\frac{\partial\phi}{\partial t} \qquad (3\text{-}10)$$

In case of flow from the reservoir into the i^{th} well, (Figure 3.2)

$$Q_i=\pm\frac{\pi\Delta zk}{\alpha\mu}\frac{(p_{wi}^2-p_{bi}^2)}{\ln\frac{r_e}{r_w}-0.5} \qquad (3\text{-}11)$$

where, plus and minus signs represent injection and withdrawal, respectively. And the flow in the well,

$$Q_i=\pm\frac{\pi}{\sqrt{32}}\left(\frac{MW}{RT\rho_s}\right)\sqrt{\frac{\pm gD^5(e^{\alpha_1 L}p_t^2-p_{wi}^2)}{f_F(e^{\alpha_1 L}-1)}} \qquad (3\text{-}12)$$

where, $\alpha_1=2MWg/RT$, plus and minus signs represent injection and withdrawal, respectively.

The volumetric flow rate per volume (q_s) is related to volume flow rate (Q_i) by

$$Q_i=q_s(\Delta x\Delta y\Delta z) \qquad (3\text{-}13)$$

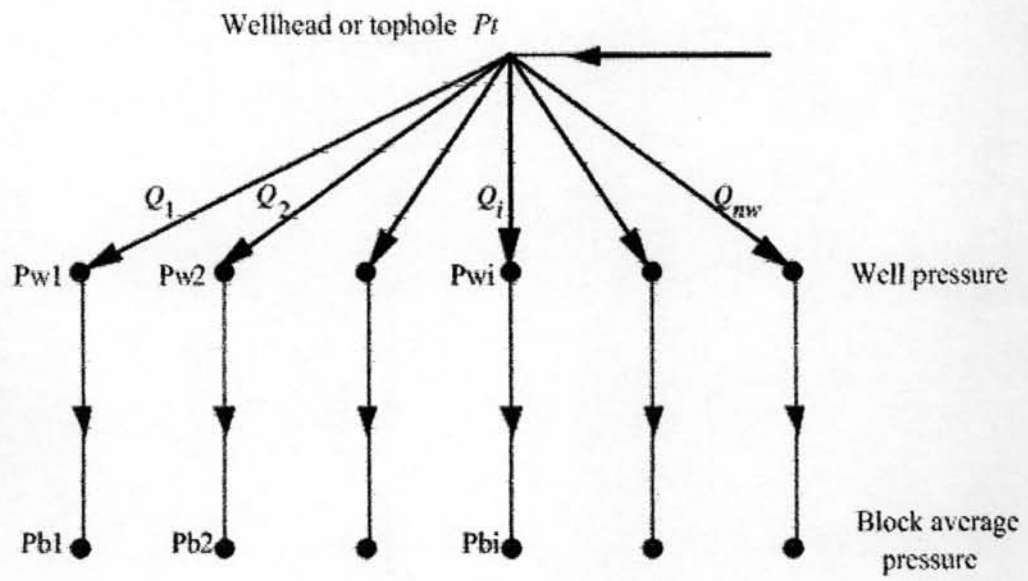

Figure 3.2 Flows in the block / well system (Wilkes, 1999).