SECTION II

GENERAL THEORIES

OF

LINEAR ANTENNAS

CHAPTER IV

THE ELECTRIC DIPOLE AND THIN LINEAR ANTENNAS

4-1. The Short Electric Dipole

Since any linear antenna may be considered as consisting of a large number of very short conductors connected in series, it is of interest to examine first the radiation properties of short conductors. From a knowledge of the properties of short conductors, we can then proceed to a study of long linear conductors such as are commonly employed in practice.

A short linear conductor is often called a short dipole. In the following discussion, a short dipole is always of finite length even though it may be very short. If the dipole is vanishingly short, it is an infinitesimal dipole.

Let us consider a short dipole such as shown in Fig. 4-1. The length L is very short compared to the wave-length ($L \ll \lambda$). Plates at the ends of the dipole provide capacitance loading. The short length and the presence of these plates result in a uniform current I along the entire length L of the dipole. The dipole may be energized by a balanced transmission line, as shown. It is assumed

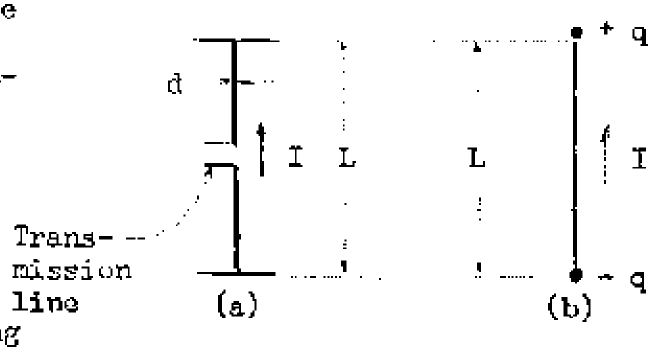

Fig. 4-1. A short dipole antenna (a) and its equivalent (b).

that the transmission line does not radiate and, therefore, its presence will be disregarded. Radiation from the end plates is also considered to be negligible. The diameter d of the dipole is small compared to its length ($d \ll L$). Thus, for purposes of analysis we may consider that the short dipole appears as in Fig. 4-1b. Here it consists simply of a thin conductor of length L with a uniform current I and point charges q at the ends. The current and charge are related by

$$\frac{dq}{dt} = I \qquad (4\text{-}1)$$

4-2. The Field of a Short Dipole[15]

The fields everywhere around a short dipole of length L placed coincident with the Z axis and with its center at the origin as in

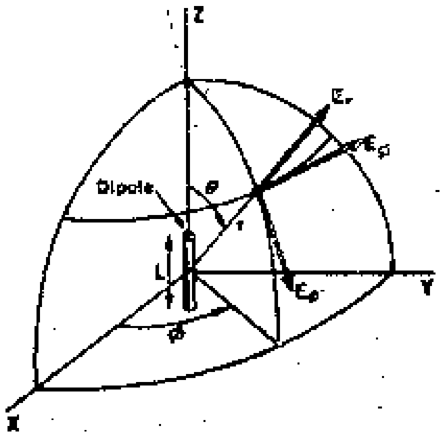

Fig. 4-2. Relation of dipole to coordinates.

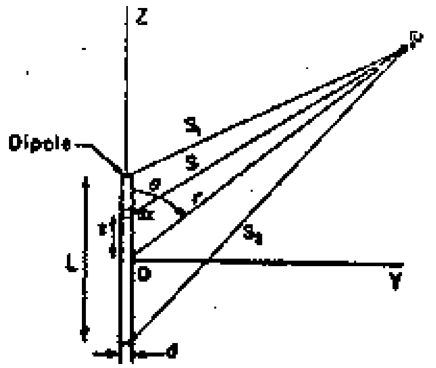

Fig. 4-3. Geometry for short dipole.

Fig. 4-2 can be expressed in terms of vector and scalar potentials. It is assumed that the medium surrounding the dipole is air or vacuum. According to Kraus, the retarded vector potential of the electric current has only one component, namely, $\vec{A}_z$. Its value is

$$\vec{A}_z = \frac{\mu L I_o e^{jw(t-\frac{r}{c})}}{4\pi r} \qquad (4-2)$$

The retarded scalar potential V of a charge distribution is

$$V = \frac{I_o L\cos\theta\, e^{jw(t-\frac{r}{c})}}{4\pi\epsilon c}\left(\frac{1}{r} + \frac{c}{jw}\cdot\frac{1}{r^2}\right) \qquad (4-3)$$

Equations (4-2) and (4-3) express the vector and scalar potentials everywhere due to a short dipole. The only restrictions are that $r \gg L$ and $\lambda \gg L$. These equation give the vector and scalar potentials at a point P in Fig. 4 - 3 in terms of the distance r to the point from the center of the dipole, the angle θ, the length of the dipole L, the current on the dipole, and some constants.

Knowing the vector potential $\vec{A}$ and the scalar potential V, the electric and magnetic fields may then be obtained from the relations

$$\vec{E} = -jw\vec{A} - \nabla V \qquad (4-4)$$

$$\vec{H} = \frac{1}{\mu}\nabla \times \vec{A} \qquad (4-5)$$

For the case of the far field, we have effectively only two field components, E_θ and H_ϕ, given by

$$E_\theta = \frac{jwI_oL\sin\theta\, e^{jw(t-\frac{r}{c})}}{4\pi\epsilon c^2 r} \qquad (4-6)$$

and

$$H_\phi = \frac{jw\, I_oL\sin\ e^{jw(t-\frac{r}{c})}}{4\pi c r} \qquad (4-7)$$

Taking the ratio of E_θ to H_ϕ as given by (4-6) and (4-7), we obtain

$$\frac{E_\theta}{H_\phi} = \frac{1}{\epsilon c} = \sqrt{\frac{\mu}{\epsilon}} = 377 \text{ ohms} \qquad (4\text{-}8)$$

This is the intrinsic impedance of free space.

Comparing (4-6) and (4-7) we note that E_θ and H_ϕ are in time phase in the far field. We note also that the field patterns of both are proportional to $\sin\theta$. The pattern is independent of ϕ, so that the space pattern is doughnut-shaped, being a figure of revolution of the pattern in Fig. 4-4 about the axis of the dipole.

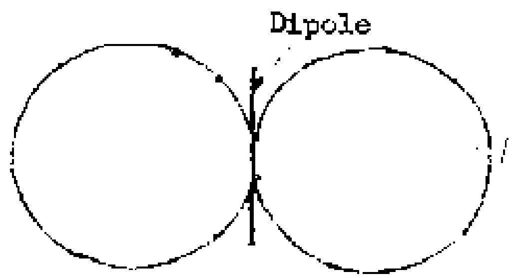

Fig. 4-4 Near - and far - field patterns of E_θ and H_ϕ components for short dipole.

4-3. Radiation Resistance of Short Electric Dipole [16]

The radiation resistance of the short dipole of Fig. 4-1. can be found as follows. The Poynting vector of the far field is integrated over a large sphere to obtain the total power radiated. This power is then equated to I^2R where I is the rms. current on the dipole and R is the resistance, called the radiation resistance of the dipole.

The average Poynting vector is given by

$$\vec{P} = \frac{1}{2} \mathrm{Re} (\vec{E} \times \vec{H}^{*}) \qquad (4\text{-}9)$$

The far - field components are E_θ and H_ϕ so that the radial component of the Poynting vector is

$$P_r = \frac{1}{2} \mathrm{Re}\, E_\theta H_\phi^{*} \qquad (4\text{-}10)$$

where E_θ and H_ϕ^{*} are complex

The far - field components are related by the intrinsic impedance of the medium. Hence,

$$E_\theta = H_\phi Z = H_\phi \sqrt{\frac{\mu}{\epsilon}} \qquad (4\text{-}11)$$

Thus, (4-10) becomes

$$P_r = \frac{1}{2} \mathrm{Re}\, Z H_\phi H_\phi^{*} = \frac{1}{2} |H_\phi|^2 \mathrm{Re}\, Z = \frac{1}{2} |H_\phi|^2 \sqrt{\frac{\mu}{\epsilon}} \qquad (4\text{-}12)$$

The total power radiated W is then

$$W = \iint P_r \, ds = \frac{1}{2} \sqrt{\frac{\mu}{\epsilon}} \int_0^{2\pi} \int_0^{\pi} |H_\phi|^2 r^2 \sin\theta \, d\theta \, d\phi \qquad (4\text{-}13)$$

where the angles are as shown in Fig. 4-2. and $|H_\phi|$ is the absolute value of the magnetic field, which from (4-7) is

$$|H_\phi| = \frac{w I_0 L \sin\theta}{4 \pi c r} \qquad (4\text{-}14)$$

Substituting this into (4-13) we have

$$W = \frac{1}{32} \sqrt{\frac{\mu}{\epsilon}} \frac{\beta^2 I_0^2 L^2}{\pi^2} \int_0^{2\pi} \int_0^{\pi} \sin^3\theta \, d\theta \, d\phi \qquad (4\text{-}15)$$

Upon integrating, (4-15) becomes

$$W = \sqrt{\frac{\mu}{\epsilon}} \frac{\beta^2 I_0^2 L^2}{12\pi} \qquad (4\text{-}16)$$

This is the average power or rate at which energy is streaming out of a sphere surrounding the dipole. Hence, it is equal to the power radiated. Assuming no losses, it is also equal to the power delivered to the dipole. Therefore, W must be equal to the square of the rms current I flowing on the dipole times a resistance R called the radiation resistance of the dipole. Thus

$$\sqrt{\frac{\mu}{\epsilon}}\ \frac{\beta^2 I_0^2 L^2}{12\pi} = \left(\frac{I_0}{\sqrt{2}}\right)^2 R \qquad (4\text{-}17)$$

Solving for R,

$$R = \sqrt{\frac{\mu}{\epsilon}}\ \frac{\beta^2 L^2}{6\pi} \qquad (4\text{-}18)$$

For air or vacuum $\sqrt{\frac{\mu}{\epsilon}} = 377 = 120\,\pi$ ohms so that (4-18) becomes

$$R = 80\pi^2 \left(\frac{L}{\lambda}\right)^2 = 80\pi^2 L_\lambda^2 \qquad (4\text{-}19)$$

The only restriction is that $\lambda \gg L$

4-4. The Thin Linear Antenna[17]

In this section, the far - field patterns of thin linear antennas is expressed. It is assumed that the antennas are symmetrically fed at the center by a balanced two wire transmission line. The antennas may be of any length, but it is assumed that the current distribution is sinusoidal. Current - distribution measurements indicate that this is a good assumption provided that the antenna is thin, that is, when the conductor diameter is less than, say, $\lambda / 100$. Thus, the sinusoidal current distribution approximates the natural distribution on thin antennas. Examples of the approximate natural - current distributions on a number of thin, linear center - fed antennas of different length are illustrated in Fig. 4-5. The current are in phase over each $\frac{1}{2}$ - wavelength

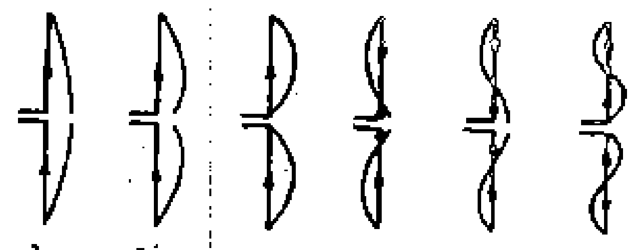

Fig. 4-5. Approximate natural current distribution for thin, linear center - fed antennas of various lengths.

section and in opposite phase over the next.

The equations of the far - field for a symmetrical, thin, linear, center - fed antenna of length L as referred to Fig. 4-6. are

$$H_\phi = \frac{j(I_o)}{2\pi r}\left[\frac{\cos(\beta L\cos\theta/2) - \cos(\beta L/2)}{\sin\theta}\right] \qquad (4-20)$$

Multiplying H_ϕ by $Z = 120\pi$ gives E_θ as

$$E_\theta = \frac{j\,60(I_o)}{r}\left[\frac{\cos(\beta L\cos\theta/2) - \cos(\beta L/2)}{\sin\theta}\right] \qquad (4-21)$$

where $(I_o) = I_o e^{jw(t - \frac{r}{c})}$, and $\beta = 2\pi/\lambda$

The shape of the far-field pattern is given by the factor in the brackets. The factors preceding the brackets in (4-20) and (4-21) give the instantaneous magnitude of the fields as functions of the antenna current and the distance r. To obtain the rms value of the field, we let (I_0) equal the rms current at the location of the current maximum. There is no factor involving phase in (4-20) or (4-21), since the center of the antenna is taken as the phase center. Hence any phase change of the fields as a function of θ will be a jump of 180° when the pattern factor changes sign.

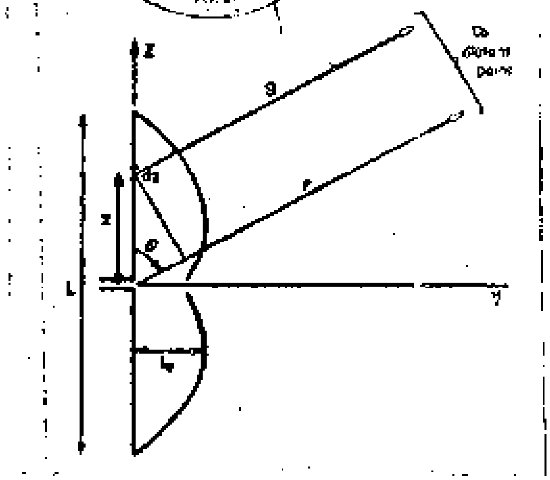

Fig. 4-6 Relations for symmetrical, thin, linear, center - fed antenna of length L.

4-4a. Half - wavelength Antenna. When $L = \lambda/2$, the pattern factor becomes

$$E = \frac{\cos(\pi/2 \cos\theta)}{\sin\theta} \qquad (4\text{-}22)$$

This pattern is shown in Fig. 4-7. It is only slightly more directional than the pattern of an infinitesimal of short dipole which is given by $\sin\theta$. The beam width between half - power points of the 1/2 wavelength antenna is 78° as compared to 90° for the short dipole.

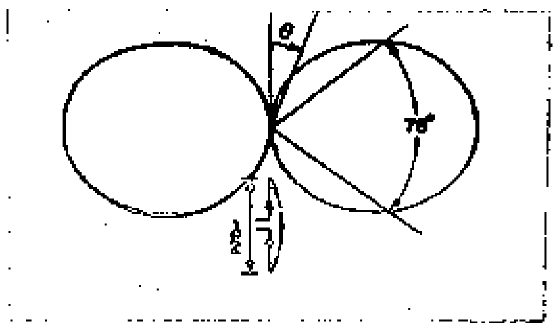

Fig. 4-7. Far - field pattern of 1/2 - wavelength antenna. The antenna is center - fed, and the current distribution is assumed to be sinusoidal.

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4-5. Radiation Resistance of 1/2 - Wavelength Antenna.

To find the radiation resistance, the Poynting vector is integrated over a large sphere yielding the power radiated, and this power is then equated to $(I_o / \sqrt{2})^2 R_o$, where R_o is the radiation resistance at a current maximum point and I_o is the peak value in time of the current at this point. The total power radiated W was given in (4-13) in terms of H_ϕ for a short dipole. In (4-13) H_ϕ is the absolute value. Hence, the corresponding value of H_ϕ for a linear antenna is obtained from (4-20) by putting $\; j\,(I_o) \; = \; I_o$.
Substituting this into (2-13), we obtain

$$W \;=\; 30\, I_o^2 \int_0^{\pi} \frac{\left[\cos\left(\frac{\beta L}{2}\cos\theta\right) - \cos\frac{\beta L}{2}\right]^2}{\sin\theta}\, d\theta \qquad (\,4\text{-}23\,)$$

Equating the radiated power as given by (4-23) to $I_o^2 R_o /2$ we have

$$W \;=\; \frac{I_o^2 R_o}{2} \qquad (\,4\text{-}24\,)$$

and for $L = \lambda/2$, we obtained

$$R_o \;=\; 30\,\mathrm{Cin}(\,2\pi\,) \;=\; 30 \times 2.44 \;=\; 73 \text{ ohms} \qquad (\,4\text{-}25\,)$$

which is the value at the center of the antenna or at the terminals of the transmission line. The terminal impedance also includes some inductive reactance in series with R_o (see Chapt. 5). To make the reactance zero requires that the antenna be a few percent less than 1/2 wavelength. This shortening also results in a reduction in the value of the radiation resistance.

4-6. Radiation Resistance at a Point Which Is Not a Current Maximum.

If we calculate, for example, the radiation resistance of a 3/4 wavelength antenna (see Fig. 4-5) by the above method, we obtain its value at a current maximum. This is not the point at which the transmission line is connected. Neglecting antenna losses, the value of radiation resistance so obtained is the resistance R_o which would appear at the terminals of a transmission line connected at a current maximum in the antenna, provided that the current distribution on the antenna is the same as when it is center - fed as in Fig. 4-5. Since a change of the feed point from the center of the antenna may change the current distribution, the radiation resistance R_o is not the value which would be measured on a 3/4 wavelength antenna or on any symmetrical antenna whose length is not an odd number of ½ wavelengths. However, R_o can be easily transformed to the value which would appear across the terminals of the transmission line connected at the center of the antenna.

This may be done by equating (4-24) to the power supplied by the transmission line, given by $\frac{I_1^2 R_1}{2}$, where I_1 is the current amplitude at the terminals and R_1 is the radiation resistance at this point. See Fig. 4-7. Thus,

$$\frac{I_1^2}{2} R_1 = \frac{I_o^2}{2} R_o \qquad (4\text{-}26)$$

Where R_o is the radiation resistance calculated at the current maximum. Thus, the radiation resistance appearing at the terminals is

$$R_1 = \left(\frac{I_o}{I_1}\right)^2 R_o \qquad (4\text{-}27)$$

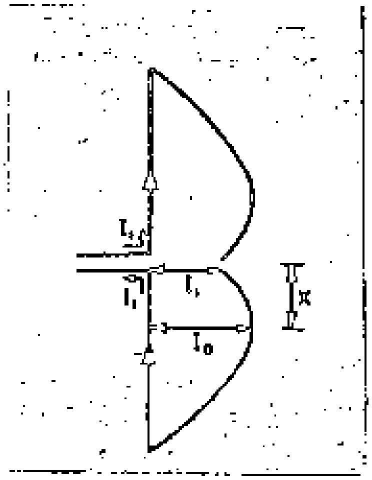

Fig. 4-8. Relation of current I_1 at transmission - line terminals to current I_o at current maximum.

The current I_1 at a distance x from the nearest current maximum, as shown in Fig. 4-8, is given by

$$I_1 = I_o \cos \beta x \qquad (4\text{-}28)$$

Therefore, (4-27) can be expressed

$$R_1 = \frac{R_o}{\cos^2 \beta x} \qquad (4\text{-}29)$$

When $x = 0$, $R_1 = R_o$; but when $x = \lambda/4$, $R_1 = \infty$ if $R_o \neq 0$. However, the radiation resistance measured at a current minimum ($x = \lambda/4$) is not infinite as would be calculated from (4-29), since an actual antenna is not infinitesimally thin and the current at a minimum point is not zero. Nevertheless, the radiation resistance at a current minimum may in practice be very large, that is, thousands of ohms.