CHAPTER I

INTRODUCTION

The purpose of this thesis is to continue the studies by Wanida Israngkul Na Ayudhya (1957) of a generalization of the binomial coefficients and the binomial theorem. The following is a summary of Wanida's results.

The binomial coefficients can be written in the form

$$^{n}C_{r} = \frac{n!}{r!\,(n-r)!} \quad \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

This defines a function on the lattice points in the first quadrant of the r,n - plane. The values of the function may be extended to the lattice points of the third and fourth quadrants by expanding the binomial series.

$$(1+a)^{n} = {}^{n}C_{0}\,1 + {}^{n}C_{1}a + {}^{n}C_{2}a^{2} + \ldots\ldots\ldots\ldots ,$$

either with $|a| < 1$ or $|a| > 1$, where n is a negative integer.

By substituting gamma functions for the factorials in (1) we obtain

$$^{n}C_{r} = \frac{\Gamma(n+1)}{\Gamma(r+1)\Gamma(n-r+1)} \quad \ldots\ldots\ldots\ldots\ldots(2)$$

which has values on every point in the r,n -plane, except on the singular lines, n = -1, -2, -3, ... and so on. Mrs. Wanida removed the singularities on the lattice points of these lines in two ways.

1. By taking the limit along the line $r = r_1$ to the lattice point (r_1, n_1). From either direction one obtains the coefficients of the expansion of $(1+a)^n$ that converges when $|a| < 1$.

2. By taking the limit along the line $n = r + (n_2 - r_2)$ to the lattice point (r_2, n_2). From either direction one obtains the coefficients of the expansion of $(1+a)^n$ that converges when $|a| > 1$.

In the present thesis, we shall remove the singularities on the lattice points of the singular lines by taking the limit along every line passing through that point, not only for the two special lines mentioned above.