CHAPTER III

CALCULATION OF THE MODE SHAPE AND FREQUENCIES

We use inverse interpolation and linear interpolation to find the mode shapes and frequencies. Suppose we have a function $y = f(x)$ from which for a given value of x_i a value y_i is assigned. To find x when some y_j is given ($i \neq j$) we use inverse interpolation. If x_j is given ($i \neq j$)then we use direct interpolation to find y.

The calculation is done in the following steps:

1. Fix F, and determine the value of y_1 that makes $y'_{10} = 0$, by using inverse interpolation.
2. Repeat the calculation for different frequencies F. The results obtained are in Table 15 - 22.
3. From the Table 15 - 22 calculate the corresponding value of y'''_{10} by using direct interpolation. This gives y'''_{10} as a function of F, when $y'_{10} = 0$. Results are shown in Table 23 - 30.
4. Determine the frequency F for which $y'''_{10} = 0$, using inverse interpolation. This is the vibration frequency, and the corresponding values of $y_0, y_1, y_2 \cdots\cdots y_{10}$ give the shape of the mode. The result obtained is F = 1289 H_z (Hertz), see table 31.

Table 15

Calculation the value of y_1 that makes $y''_{10} = 0$, where $F = 1000$

$F = 1000$, $DF = 100$, $y_1 = -1.0$, $Dy_1 = 0.1$

y'_{10}	y_1	1	2	3	4	5
-0.31640	-1.0					
-0.13930	-0.9	-0.8213439				
0.03765	-0.8	-0.8212851	-0.8212976			
0.21470	-0.7	-0.8212766	-0.8213174	-0.8212771		
0.39170	-0.6	-0.8212618	-0.8213224	-0.8212799	-0.8212737	
0.56870	-0.5	-0.8212631	-0.8213280	-0.8212810	-0.8212747	-0.8212716

Since $y'_{10} = 0$ the left-hand column contains the "parts" used in the process.

We obtain $y_1 = -0.8212716$.

Table 16

$F = 1100$, $DF = 100$, $y_1 = -1.0$, $Dy_1 = 0.1$

y'_{10}	y_1	1	2	3	4	5
-0.38450	-1.0					
-0.19110	-0.9	-0.8011892				
0.002268	-0.8	-0.8011728	-0.8011730			
0.19560	-0.7	-0.8011550	-0.8011723	-0.8011731		
0.3890	-0.6	-0.8011635	-0.8011807	-0.8011730	-0.8011732	
0.5825	-0.5	-0.8011892	-0.8011892	-0.8011729	-0.8011732	-0.8011732

$$y'_{10} = 0$$

$$y_1 = -0.8011732$$

Table 18

$F = 1300$, $DF = 100$, $y_1 = -1.0$, $Dy_1 = 0.1$

y'_{10}	y_1	1	2	3	4	5
-0.512l0	-1.0					
-0.31130	-0.9	-0.7652964				
-0.08031	-0.8	-0.7652405	-0.7652211			
0.15070	-0.7	-0.7652490	-0.7652685	-0.7652362		
0.38180	-0.6	-0.7652456	-0.7652736	-0.7652202	-0.7652401	
0.61280	-0.5	-0.7652355	-0.7652759	-0.7652274	-0.7652392	-0.7652418

$y'_{10} = 0$

$y_1 = -0.7652418$

Table 17

$F = 1200,\ DF = 100,\ y_1 = -1.0,\ Dy_1 = 0.1$

y'_{10}	y_1	1	2	3	4	5
-0.4598	-1.0					
-0.2404	-0.9	-0.7824976				
-0.0370	-0.8	-0.7824976	-0.7824976			
0.1714	-0.7	-0.7824976	-0.7824976	-0.7824976		
0.3858	-0.6	-0.7824976	-0.7824976	-0.7824976	-0.7824976	
0.5972	-0.5	-0.7824976	-0.7824976	-0.7824976	-0.7824976	-0.7824976

$y'_{10} = 0$

$y_1 = -0.7824976$

Table 19

$P = 1400,\ DF = 100,\ y_1 = 1.0,\ Dy_1 = 0.1$

y'_{10}	y_1	1	2	3	4	5
-0.6326	-1.0					
-0.3802	-0.9	-0.7493661				
-0.1278	-0.8	-0.7493661	-0.7493661			
0.1245	-0.7	-0.7493330	-0.7493661	-0.7493661		
0.3769	-0.6	-0.7493413	-0.7493536	-0.7493629	-0.7493677	
0.6293	-0.5	-0.7493462	-0.7493586	-0.7493648	-0.7493684	-0.7493696

$y'_{10} = 0$

$y_1 = -0.7493696$

Table 20

$\bar{P} = 1500$, $DF = 100$, $y_1 = 1.0$, $Dy_1 = 0.1$

y'_{10}	y_1	1	2	3	4	5
-0.7305	-1.0					
-0.4550	-0.9	-0.7348457				
-0.1796	-0.8	-0.7347976	-0.7347662			
0.09576	-0.7	-0.7347687	-0.7347821	-0.7347766		
0.3711	-0.6	-0.7347495	-0.7347927	-0.7347748	-0.7347772	
0.6466	-0.5	-0.7347687	-0.7348139	-0.7347766	-0.7347766	-0.7347778

$y'_{10} = 0$

$y_1 = -0.7347778$

Table 21

$F = 1600,\ DF = 100,\ y_1 = -1.0,\ Dy_1 = 0.1$

y'_{10}	y_1	1	2	3	4	5
-0.8364	-1.0					
-0.5362	-0.9	-0.7213857				
-0.2360	-0.8	-0.7213857	-0.7213852			
-0.0614	-0.7	-0.7213671	-0.7213707	-0.7213737		
0.3643	-0.6	-0.7213625	-0.7213719	-0.7213700	-0.7213727	
0.6645	-0.5	-0.7213671	-0.7213774	-0.7213832	-0.7213726	-0.7213728

$y'_{10} = 0$

$y_1 = -0.7213728$

Table 22

$F = 1700$, $DF = 100$, $y_1 = -1.0$, $Dy_1 = 0.1$

y'_{10}	y_1	1	2	3	4	5
-0.9505	-1.0					
-0.6238	-0.9	-0.7090629				
-0.2971	-0.8	-0.7090603	-0.7090579			
0.02954	-0.7	-0.7090424	-0.7090757	-0.7090740		
0.3562	-0.6	-0.7090380	-0.7090471	-0.7090530	-0.7090758	
0.6829	-0.5	-0.7090424	-0.7090531	-0.7090564	-0.7090747	-0.7090770

$y'_{10} = 0$

$y_1 = -0.7090770$

Table 23

Calculation the value of y''_{10} corresponding to

$y_1 = -0.8212716$

$F = 1000,\ y_1 = -1.0,\ Dy_1 = 0.1,\ i = 0,1,2,\text{---}5$

y_i	$y_* - y_1$	y''_{10}	1	2	3	4	5
-1.0	0.1787284	-0.02136					
-0.9	0.0787284	-0.01166	-0.0040233				
-0.8	-0.0212716	-0.001961	-0.0040233	-0.0040233			
-0.7	-0.1212716	0.007741	-0.0040218	-0.0040230	-0.0040234		
-0.6	-0.2212716	0.01744	-0.0040211	-0.0040295	-0.0040226	-0.0040214	
-0.5	-0.3212716	0.02714	-0.0040233	-0.0040142	0.0040239	-0.0040231	-0.0040272

$y'_{10} = 0$

$y_* = y_1 = -0.8212716$

$y''_{10} = -0.0040272$

Table 24

$F = 1100,\ y_1 = -1.0,\ \Delta y_1 = 0.1,\ i = 0,1,2\text{---},5$

y_1	$y_* - y_1$	y''_{10}	1	2	3	4	5
-1.0	0.1988267	-0.00658					
-0.9	0.0988267	-0.014173	-0.0030150				
-0.8	-0.0011733	-0.002875	-0.0030131	-0.0030131			
-0.7	-0.1011733	0.00898	-0.0030144	-0.0030131	-0.0030131		
-0.6	-0.2011733	0.02083	-0.0030147	-0.0030193	-0.0030131	-0.0030131	
-0.5	-0.3011733	0.032690	-0.0030292	-0.0030104	-0.0030131	-0.0030131	-0.0030131

$$y'_{10} = 0$$

$$y_* = y_1 = -0.8011733$$

$$y''_{10} = -0.0030131$$

Table 25

$F = 1200,\ y_1 = -1.0,\ Dy_1 = 0.1,\ i = 0,1,2,\text{---}5$

y_1	$y_0 - y_1$	y''_{10}	1	2	3	4	5
-1.0	0.2175024	-0.02259					
-0.9	0.1175024	-0.01833	-0.0015741				
-0.8	0.0175024	-0.001478	-0.0015835	-0.0015851			
-0.7	-0.0824976	0.01018	-0.0015835	-0.0015828	-0.0015846		
-0.6	-0.1824976	0.02144	-0.0015842	-0.0015811	-0.0015833	-0.0015857	
-0.5	-0.2824976	0.03270	-0.0015842	-0.0015820	-0.0015814	-0.0015904	-0.0015771

$y'_{10} = 0$

$y_0 = y_1 = -0.7824976$

$y''_{10} = -0.0015771$

Table 26

$P = 1300,\ y_1 = -1.0,\ Dy_1 = 0.1,\ i = 0,1,2 \text{---} 5$

y_1	$y_* - y_1$	y''_{10}	1	2	3	4	5
-1.0	0.2347130	-0.03948					
-0.9	0.1347130	-0.02255	0.0002569				
-0.8	0.0347130	-0.005626	0.0002488	0.0002380			
-0.7	-0.0652870	0.01130	0.0002495	0.0002490	0.0002318		
-0.6	-0.1652870	0.02823	0.0002469	0.0002512	0.0002476	0.0002215	
-0.5	-0.2652870	0.04516	0.0002469	0.0002469	0.00002583	0.0002299	0.0002176

$$y'_{10} = 0$$

$$y_* = y_1 = -0.7652870$$

$$y''_{10} = 0.0002176$$

Table 27

$F=1400,\ y_1=-1.0,\ Dy_1=0.1,1=0,\ 1,2,\ldots.5$

y_1	y_6-y_1	y_{10}'''	1	2	3	4	5
-1.0	0.2506304	-0.04733					
-0.9	0.1506304	-0.02745	0.0024853				
-0.8	0.0506304	-0.00758	0.0024785	0.0024751			
-0.7	-0.0493696	0.01229	0.0024825	0.0024802	0.0024778		
-0.6	-0.1493696	0.03216	0.0024803	0.0024836	0.0024790	0.0024772	
-0.5	-0.2493696	0.05204	0.0024653	0.0025027	0.0024801	0.0025147	0.0024772

$$y_{10}'=0$$

$$y_6=y_1=-0.7493696$$

$$y_{10}'''=0.0024772$$

Table 28

$F=1500,\ y_1=-1.0,\ Dy_1=0.1,\ 1=0,\ 1,2,\ldots 5$

y_1	y_*-y_1	y'''_{10}	1	2	3	4	5
-1.0	-0.265222	-0.05623					
-0.9	-0.165222	-0.03312	0.0050628				
-0.8	-0.065222	-0.01000	0.0050761	0.0050842			
-0.7	0.034778	0.01310	0.0050618	0.0050619	0.0050697		
-0.6	0.134778	0.03621	0.0050628	0.0050628	0.0050772	0.0050671	
-0.5	0.234778	0.05933	0.0050681	0.0050650	0.0050800	0.0050679	0.0050660

$$y'_{10}=0$$

$$y_*=y_1=-0.7347778$$

$$y'''_{10}=0.0050660$$

Table 29

$F=1600,\ y_1 = -1.0,\ \Delta y_1 = 0.1,\ i=0,\ 1,\ 2,\ \ldots 5$

y_1	$y_0 = y_1$	y''_{10}	1	2	3	4	5
-1.0	0.2786272	-0.06630					
-0.9	0.1786272	-0.03964	0.0079820				
-0.8	0.0786272	-0.01298	0.0079820	0.0079820			
-0.7	-0.0213728	0.01367	0.0079760	0.0079770	0.0079780		
-0.6	-0.1213728	0.04033	0.0079720	0.0079770	0.0079770	0.0079780	
-0.5	-0.2213728	0.06700	0.0079600	0.0079870	0.0079830	0.0079820	0.0079732

$y'_{10} = 0$

$y_0 = y_1 = -0.7213728$

$y''_{10} = 0.00797316$

Table 30

$F=1700,\ y_1 = -1.0,\ \Delta y_1 = 0.1,\ i = 0, 1, 2, \ldots 5$

y_1	$y_0 - y_1$	y''_{10}	1	2	3	4	5
-1.0	0.2909230	-0.07765					
-0.9	0.1909230	-0.04711	0.0111979				
-0.8	0.0909230	-0.01658	0.0111788	0.0111614			
-0.7	-0.0090770	0.01395	0.0111788	0.0111788	0.0111772		
-0.6	-0.1090770	0.04448	0.0111788	0.0111788	0.0111788	0.0111771	
-0.5	-0.2090770	0.07502	0.0111804	0.0111895	0.0111699	0.0111775	0.0111770

$$y'_{10} = 0$$

$$y_0 = y_1 = -0.7090770$$

$$y''_{10} = 0.0111770$$

Table 31

Calculation the value of F that makes $y''_{10} = 0$

y''_{10}	F	1	2	3	4	5	6	7
0.0111770	1700	1351						
0.0079732	1600	1351						
0.0050660	1500	1334	1304					
0.0024772	1400	1315	1299	1294				
0.0002176	1300	1292	1290	1289	1289			
-0.0015771	1200	1262	1277	1283	1287	1289		
-0.0030131	1100	1227	1261	1277	1286	1289	1289	
-0.0040272	1000	1185	1241	1269	1284	1289	1289	1289

We obtain F = 1289

5. Then we find $y_1 = -0.7671044$, by using inverse interpolation, for F = 1289 H_z which is shown in the Table 32.

6. This frequency F = 1289 H_z is the vibration frequency, for which $y''_{10} = 0$, and the corresponding values of $y_0, y_1, y_2 \cdots y_{10}$ can be obtained as follows:

From the equations (8) and (9)

$$y_1 = -y_{-1} = -0.7671044$$

$$z_1 = \frac{CF^2}{2} a_0 y_0 \text{ , for the iron } C = 0.185 \times 10^{-9}$$

$$\frac{C}{2} = 0.925 \times 10^{-10}$$

$$CF^2 = 0.0003078$$

$$\frac{CF^2}{2} = 0.0001537$$

Then $Z_1 = 0.0001537\ (1).\ (-1.0)$

$= -0.0001537$

By using the relations(6) and (7) we obtain

$$y_2 = \frac{z_1}{(a_1)^2} + 2y_1 - y_0$$

$$= \frac{-0.0001537}{(1.0)^2} + 2(-0.7671044) - (-1)$$

$$= -0.5343625$$

$$z_2 = CF^2\, a_1 y_1 + 2z_1 - z_0$$

$$= -0.0005435$$

and so on. The table of these results is shown below, table 33.

Table 32

Calculation the value of y_1 corresponds to F = 1289

F_1	$F - F_1$	y_1	1	2	3	4	5
1000	289	-0.8212715					
1100	189	-0.8011732	-0.7631874				
1200	89	-0.7824976	-0.7652432	-0.7670728			
1300	-11	-0.7652418	-0.7673296	-0.7671018	-0.7670986		
1400	-111	-0.7493696	-0.7693224	-0.7670525	-0.7670618	-0.7671026	
1500	-211	-0.7347778	-0.7712781	-0.7670103	-0.7670543	-0.7671010	-0.7671044

$y_1 = -0.7671044$

Table 33

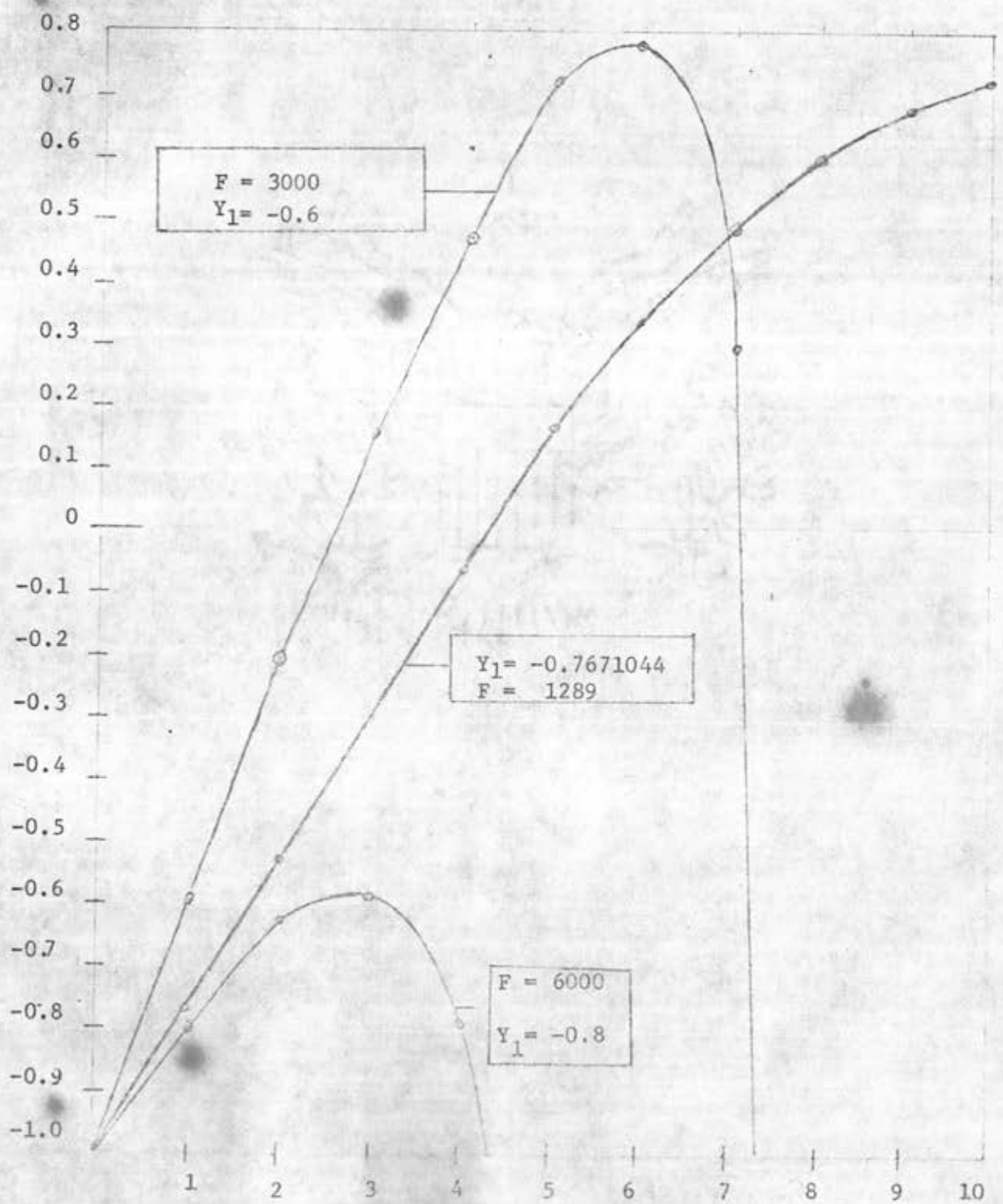
Values of y_i and z_i, i=0, 1, 2, 3 .....10, corresponding to F=1289, $y_1 = -0.7671044$

i	$z_{i+1}=CF^2 a_1 y_1 + 2z_1 - z_{i-1}$	$y_{i+1}=\frac{z_i}{(a_1)3} + 2y_1 - y_{i-1}$
0	$z_0 = 0.0000000$	$y_0 = -1.0000000$
1	$z_1 = -0.0001537$	$y_1 = -0.7671044$
2	$z_2 = -0.0005435$	$y_2 = -0.5343625$
3	$z_3 = -0.0010978$	$y_3 = -0.3021641$
4	$z_4 = -0.0017451$	$y_4 = -0.0710635$
5	$z_5 = -0.0024143$	$y_5 = 0.1582920$
6	$z_6 = -0.0031200$	$y_6 = 0.3304196$
7	$z_7 = -0.0037748$	$y_7 = 0.4775872$
8	$z_8 = -0.0043561$	$y_8 = 0.5945564$
9	$z_9 = -0.0048459$	$y_9 = 0.6766768$
10	$z_{10} = -0.0091521$	$y_{10} = 0.7200300$

They are represented in graphical form in the Figure (1)

Figure 1

Mode Shape of the actually frequency, F=1289



CONCLUSION

For the non-uniform bar case we can find the mode shape by using the difference equation and the given parameters varying along the bar length. From the derived table one can obtain the mode shape by using the interpolation and inverse interpolation method.

Thus by using the numerical method with the aid of a computer and a desk calculator we can obtain the mode shape and its natural frequencies. These results are useful in finding the overtones of musical instruments for which the cross-section is non-uniform.
