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APPENDICES

APPENDIX A

A.1 The Matrices $\mathbf{R}(\xi,z,s)$ and $\mathbf{S}(\xi,z,s)$

The matrices R and S in eqns (2.15) and (2.16), respectively, are given by (Senjuntichai and Rajapakse , 1995)

$$\mathbf{R} = [\ \mathbf{R}_1\ :\ \mathbf{R}_2\ ] \tag{A1}$$

$$\mathbf{S} = [\ \mathbf{S}_1\ :\ \mathbf{S}_2\ ] \tag{A2}$$

where

$$\mathbf{R}_1 = \begin{bmatrix} -2\mu a_3\eta\delta_2 e^{\gamma z} & -2\mu a_3\eta\delta_2 e^{-\gamma z} & a_1 z e^{\xi z} & -a_1 z e^{-\xi z} \\ 0 & 0 & 0 & 0 \\ 2\mu a_3\eta\delta_1 e^{\gamma z} & -2\mu a_3\eta\delta_1 e^{-\gamma z} & -(a_1 z - \frac{a_2}{\xi})e^{\xi z} & -(a_1 z + \frac{a_2}{\xi})e^{-\xi z} \\ 2\mu a_3\eta e^{\gamma z} & 2\mu a_3\eta e^{-\gamma z} & -2\mu a_4\eta e^{\xi z} & -2\mu a_4\eta e^{-\xi z} \end{bmatrix} \tag{A3}$$

$$\mathbf{R}_2 = \frac{1}{2}\begin{bmatrix} e^{\xi z} & e^{-\xi z} & -e^{\xi z} & -e^{-\xi z} \\ e^{\xi z} & e^{-\xi z} & e^{\xi z} & e^{-\xi z} \\ -e^{\xi z} & e^{-\xi z} & e^{\xi z} & -e^{-\xi z} \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{A4}$$

$$\mathbf{S}_1 = \mu\begin{bmatrix} -4\mu a_3\xi\eta\delta_1 e^{\gamma z} & 4\mu a_3\xi\eta\delta_1 e^{-\gamma z} & (2a_1\xi z - 1)e^{\xi z} & (2a_1\xi z + 1)e^{-\xi z} \\ 0 & 0 & 0 & 0 \\ 4\mu a_3\xi\eta\delta_2 e^{\gamma z} & 4\mu a_3\xi\eta\delta_2 e^{-\gamma z} & -2(a_1\xi z - a_4)e^{\xi z} & 2(a_1\xi z + a_4)e^{-\xi z} \\ -2a_3\delta_1 e^{\gamma z} & 2a_3\delta_1 e^{-\gamma z} & 2a_4\delta_2 e^{\xi z} & -2a_4\delta_2 e^{-\xi z} \end{bmatrix}$$

(A5)

$$\mathbf{S}_2 = \frac{\mu}{2}\begin{bmatrix} 2\xi e^{\xi z} & -2\xi e^{-\xi z} & -2\xi e^{\xi z} & 2\xi e^{-\xi z} \\ \xi e^{\xi z} & -\xi e^{-\xi z} & \xi e^{\xi z} & -\xi e^{-\xi z} \\ -2\xi e^{\xi z} & -2\xi e^{-\xi z} & 2\xi e^{\xi z} & 2\xi e^{-\xi z} \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{(A6)}$$

and

$$a_1 = \frac{1}{2(1-2\nu_u)} \quad \text{(A7)}$$

$$a_2 = \frac{(3-4\nu_u)}{2(1-2\nu_u)} \quad \text{(A8)}$$

$$a_3 = \frac{B(1+\nu_u)(1-\nu)}{3(\nu_u-\nu)} \quad \text{(A9)}$$

$$a_4 = \frac{(1-\nu_u)}{(1-2\nu_u)} \quad \text{(A10)}$$

$$c = 2\mu a_3 \kappa \eta \quad \text{(A11)}$$

$$\eta = \frac{B(1+\nu_u)}{3(1-\nu_u)} \quad \text{(A12)}$$

$$\delta_1 = \frac{\kappa\gamma\eta}{s} \quad \text{(A13)}$$

$$\delta_2 = \frac{\kappa\xi\eta}{s} \quad \text{(A14)}$$

APPENDIX B

B.1 Formulation of Bending Moment and Shear Force of Axisymmetric Circular Plate

The bending moment of the plate per unit length (Timoshenko and Woinowsky-Krieger,1959)

$$M_r = -D\left(\frac{d^2w}{dr^2} + \frac{v_p}{r}\frac{dw}{dr}\right) \qquad \text{(B1)}$$

$$Q = -D\left(\frac{d^3w}{dr^3} + \frac{1}{r}\frac{d^2w}{dr^2} - \frac{1}{r^2}\frac{dw}{dr}\right) \qquad \text{(B2)}$$

From eqn(2.43) ; $w(r,t) = a_0(t)r^2 \ln r + \alpha_0 + \sum_{n=1}^{N} \alpha_n(t) r^{2n} \quad , \quad 0 \le r \le 1$

$$\frac{dw}{dr} = a_0(r + 2r\ln r) + \sum_{n=1}^{N} 2n\alpha_n r^{2n-1} \qquad \text{(B3)}$$

$$\frac{d^2w}{dr^2} = a_0(3 + 2\ln r) + \sum_{n=1}^{N} 2n(2n-1)\alpha_n r^{2n-2} \qquad \text{(B4)}$$

$$\frac{d^3w}{dr^3} = a_0(3 + \frac{2}{r}) + \sum_{n=2}^{N} 2n(2n-1)(2n-2)\alpha_n r^{2n-3} \qquad \text{(B5)}$$

Substitute eqns(B3) - (B5) into eqns(B1) and (B2) we have

$$M_r(r,t) = -D\left\{a_0(3+2\ln r+\nu_p+2\nu_p\ln r)+\sum_{n=1}^{N}\alpha_n[2n(2n-1)+2n\nu_p]r^{2n-2}\right\}$$

(B6)

$$Q(r,t) = -D\left[\frac{4a_0}{r}+\sum_{n=2}^{N}4n^2(2n-2)\alpha_n r^{2n-3}\right] \qquad \text{(B7)}$$

B.2 Formulation of Strain Energy of Plate

From $$U_p = \int_0 \pi D\left[\left(\frac{d^2w}{dr^2}+\frac{1}{r}\frac{dw}{dr}\right)^2 - \frac{2(1-\nu_p)}{r}\frac{dw}{dr}\frac{d^2w}{dr^2}\right]rdr \qquad \text{(B8)}$$

and from $$\frac{1}{r}\frac{dw}{dr} = a_0(1+2\ln r)+\sum_{n=1}^{N}2n\alpha_n r^{2n-2} \qquad \text{(B9)}$$

$$\frac{d^2w}{dr^2}+\frac{1}{r}\frac{dw}{dr} = a_0(4+4\ln r)+\sum_{n=1}^{N}4n^2\alpha_n r^{2n-2} \qquad \text{(B10)}$$

$$\left(\frac{d^2w}{dr^2}+\frac{1}{r}\frac{dw}{dr}\right)^2 = a_0^2(4+4\ln r)^2+2a_0(4+4\ln r).\sum_{n=1}^{N}4n^2\alpha_n r^{2n-2}$$
$$+\sum_{n=1}^{N}4n^2\alpha_n r^{2n-2}.\sum_{n=1}^{N}4n^2\alpha_n r^{2n-2} \qquad \text{(B11)}$$

$$\frac{dw}{dr}\frac{d^2w}{dr^2} = \left[a_0(r+2r\ln r)+\sum_{n=1}^{N}2n\alpha_n r^{2n-1}\right]\left[a_0(3+2\ln r)+\sum_{n=1}^{N}2n(2n-1)\alpha_n r^{2n-2}\right]$$

(B12)

$$= a_0^2(3r + 6r\ln r + 2r\ln r + 4r(\ln r)^2 + \sum_{n=1}^{N} 2a_0(3 + 2\ln r) n\alpha_n r^{2n-1}$$

$$+\sum_{n=1}^{N} a_0(r + 2r\ln r).2n(2n-1).\alpha_n r^{2n-2} + \sum_{n=1}^{N} 2n\alpha_n r^{2n-1}.\sum_{n=1}^{N} 2n(2n-1)\alpha_n r^{2n-2}$$

(B13)

Substitute (B11) and (B13) into (B8) we have

$$U_p = \int_0^1 \pi D \left\{ \begin{array}{l} \left[\begin{array}{l} a_0^2 r(4 + 4\ln r)^2 + 2a_0(4 + 4\ln r)\sum_{n=1}^{N} 4n^2\alpha_n r^{2n-1} \\ + \sum_{n=1}^{N} 4n^2\alpha_n r^{2n-1} \sum_{n=1}^{N} 4n^2\alpha_n r^{2n-1} \end{array} \right] \\ -2(1-v_p) \left\{ \begin{array}{l} a_0^2(3r + 8r\ln r + 4r(\ln r)^2 + \sum_{n=1}^{N} 2a_0 n\alpha_n r^{2n-1}(2 + 2n + 4n\ln r) + \\ \sum_{n=1}^{N} 2n\alpha_n r^{2n-1} \sum_{n=1}^{N} 2n(2n-1)\alpha_n r^{2n-2} \end{array} \right] \end{array} \right\} dr$$

(B14)

After Integration and rewritten eqn.(B14) in term of i, j where i, j = 2 to N+1

$$U_p = \pi D\left[4a_0^2 - 2(1-v_p)\frac{a_0^2}{2} \right] + \pi D\left[\{8a_0(2i-3)\}^T\{\alpha\} - 2(1-v_p)\{2a_0(i-1)\}^T\{\alpha\} \right]$$

$$+\pi D\left[\{\alpha\}^T \left\{ \frac{4(i-1)^2.4(j-1)^2}{2i+2j-6} \right\} \{\alpha\} - 2(1-v_p)\{\alpha\}^T \left\{ \frac{4(i-1)(j-1)(2j-3)}{2i+2j-6} \right\} \{\alpha\} \right]$$

(B15)

$$U_p = \pi D\left[(3+v_p)a_0^2\right] + \{\alpha\}^T \left[\frac{4(i-1)(j-1)\pi D}{2i+2j-6} \right] \left[4(i-1)(j-1) - 2(1-v_p)(2i-3)\right]\{\alpha\}$$

$$+\pi D.4a_0\left[(3+v_p)i - v_p - 5\right]\{\alpha\} \qquad , \qquad 2 \le i,j \le N+1$$

(B16)

The strain energy of the plate corresponding to the assumed displacement function can be expressed in the Laplace domain as

$$U_p = \pi D(3+\nu_p)\bar{a}_0^2 + < Q^p > \{\bar{\alpha}\} + \{\bar{\alpha}\}^T [K^p]\{\bar{\alpha}\} \tag{B17}$$

where

$$Q_1^p = 0 \tag{B18}$$

$$Q_i^p = \pi D\left[4\bar{a}_0\left(4(i-1)-(1-\nu_p)i\right) - 4\bar{a}_0(1+\nu_p)\right] \quad ; \quad 2 \le i \le (N+1) \tag{B19}$$

$$K_{1j}^p = K_{j1}^p = 0 \tag{B20}$$

$$K_{ij}^p = \frac{4(i-1)(j-1)\pi D}{(2i+2j-6)}\left[4(i-1)(j-1) - 2(1-\nu_p)(2i-3)\right] \quad ; \quad 2 \le i,j \le (N+1) \tag{B21}$$

B.3 Formulation of Strain Energy of Half-Space

$$U_h = \frac{1}{2}\int_V \sigma_{ij}\varepsilon_{ij} dV \tag{B22}$$

$$U_h = \frac{1}{2}\int_0^1 \left[2\pi r \overline{T}_z(r,H,s)\overline{w}(r,s)\right] dr \tag{B23}$$

where

$$\overline{T}_z(r,s) = \sum_{n=0}^{N} \overline{\alpha}_n(s)\breve{T}_{nz}(r) + \overline{a}_0(s).T_z^*(r) \tag{B24}$$

So
$$U_h = \int_0^1 \pi r\left[\sum_{n=0}^{N} \overline{\alpha}_n \breve{T}_{nz}(r) + \overline{a}_0 T_z^*(r)\right].\overline{w}(r,s) dr \tag{B25}$$

where
$$\overline{w}(r,s) = \overline{a}_0(s) r^2 \ln r + \sum_{n=0}^{N} \overline{\alpha}_n(s) r^{2n} \quad , \quad 0 \le r \le 1 \tag{B26}$$

$$U_h = \int_0^1 \left[\begin{array}{l} \pi r \sum_{n=0}^{N} \overline{\alpha}_n \breve{T}_{nz}(r).\overline{a}_0 r^2 \ln r + \pi r \overline{a}_0^2 T_z^*(r).r^2 \ln r + \pi r \sum_{n=0}^{N} \overline{\alpha}_n \breve{T}_{nz}(r).\sum_{n=0}^{N} \overline{\alpha}_n r^{2n} \\ + \pi r \overline{a}_0 T_z^*(r).\sum_{n=0}^{N} \overline{\alpha}_n r^{2n} \end{array}\right] dr$$

(B27)

From the discretization of circular surface into M ring elements. So the above expression can be expressed in form of summation of M terms.

$$\begin{aligned} U_h = &\sum_{l=1}^{M}\left(\pi r_l.\sum_{n=0}^{N} \overline{\alpha}_n \breve{T}_{nz}(r_l).\overline{a}_0 r_l^2 \ln r_l.\Delta r_l\right) + \sum_{l=1}^{M}\left(\pi r_l \overline{a}_0^2.T_z^*(r_l).r_l^2 \ln r_l.\Delta r_l\right) \\ &+ \sum_{l=1}^{M}\left(\pi r_l.\sum_{n=0}^{N} \overline{\alpha}_n \breve{T}_{nz}(r_l).\sum_{n=0}^{N} \overline{\alpha}_n r_l^{2n}.\Delta r_l\right) + \sum_{l=1}^{M}\left(\pi r_l \overline{a}_0.T_z^*(r_l).\sum_{n=0}^{N} \overline{\alpha}_n r_l^{2n}.\Delta r_l\right) \end{aligned}$$

$$= \sum_{l=1}^{M} \left\langle \pi r_l^3 \overline{a}_0 \breve{T}_{z(i-1)}(r_l) \ln r_l \,.\, \Delta r_l \right\rangle \langle \overline{\alpha} \rangle + \sum_{l=1}^{M} \left(\pi r_l^3 \overline{a}_0^2 T_z^*(r_l) \ln r_l \,.\, \Delta r_l \right)$$
$$+ \sum_{l=1}^{M} \langle \overline{\alpha} \rangle^T \left\langle \pi r^{2j-1} \,.\, \breve{T}_{z(i-1)}(r_l) \right\rangle \langle \overline{\alpha} \rangle + \sum_{l=1}^{M} \left\langle \pi r_l^{2i-1} \overline{a}_0 T_z^*(r_l) .\, \Delta r_l \right\rangle \langle \overline{\alpha} \rangle$$

$$U_h = \{\overline{\alpha}\}^T \left[K^h \right] \{\overline{\alpha}\} + < Q^h > \{\overline{\alpha}\} + \sum_{j=1}^{M} \pi r_j T_{zj}^* \overline{a}_0^2 r_j^2 \ln r_j \Delta r_j \qquad \text{(B28)}$$

where

$$K_{ij}^h = \sum_{l=1}^{M} \breve{T}_{z(i-1)}(r_l) .\, \pi r_l^{2j-1} \Delta r_l \quad , \; 1 \le i, j \le (N+1) \qquad \text{(B29)}$$

$$Q_i^h = \sum_{j=1}^{M} \left[\pi r^{2i-1} T_{zj}^* \Delta r_j + \pi r_j^3 \ln r_j \,.\, \breve{T}_{z(i-1)}(r_j) .\, \Delta r_j \right] \overline{a}_0 \quad , \; 1 \le i \le (N+1)$$

(B30)

BIOGRAPHY

Mr. Sukkasem Lalitkulthorn was born in Bangkok 1971. He graduated from Faculty of Engineering , Kasetsart University in 1992. He continued his study for Master Degree in Civil Engineering at Chulalongkorn University in 1994.