CHAPTER III

PROXIMITY EFFECTS

If a normal metal N is deposited on top of a superconducting metal S, and if the electrical contact between the two is good, Cooper pairs can leak from S to N. This "proximity effect" is of comparatively long range. The thickness of the leakage region on the N side is typically 10^3 Å (and can be made even longer if desired).

The first experiments suggesting the existence of a proximity effect were performed very early by Meissner (51). He measured supercurrents between two superconducting wires separated by a thin gold layer (deposited on one of the wires).

Cooper (52) has suggested an intuitive argument for the possibility that the superconducting properties of thin metallic films may be strongly affected by direct contact with other metals. He emphasized that in the BCS theory one must clearly distinguish between the range of the attractive interaction between electrons, and the distance over which as a result of this interaction the electrons are correlated into Cooper pairs. The range of the interaction is very short (10^{-8} cm); the "size" of the wave packets of the pairs, on the other hand, is of the order of the coherence length, that is, 10^{-4} cm. Because of this long coherence length the Cooper pairs can extend a considerable distance into a region in which the interaction between electrons is not attractive. Thus when a thin layer of superconducting material is in contact with a layer of normal metal, the zero-momentum pairs formed because of the attractive interaction in the superconductor extend into both layers. As a result the ground state energy of this thin bimetallic layer is characterized by some average over both metals of the parameter N(O)V, which in turn determines the energy gap of the layer and its transition temperature, according to equations (1.32)and(1.45). The form of this average of course depends on the nature of the boundary between the two metals; the better the contact, the more effective is a superimposed layer in changing the properties of the substrate. Regardless of how one accounts for this, one would expect the average to depend also in some manner on the relative thickness of the two layers. The thicker the normal layer, the smaller the average interaction, and the more the energy gap width and transition temperature are decreased from the values they would have if only the superconductor were present.

The literature dealing with the superconducting proximity effect is extensive for both experiment and theory. The major theoretical difficulty is that the superconducting order parameter in a normal-superconducting (NS) sandwich is spatially dependent. It is therefore only possible to perform a detailed calculation based on Gor'kov's equation (55) at temperature T near the superconducting critical temperature T_c of the sandwich. The deGennes-Werthamer theory(53,54)calculates T_c for "dirty" NS sandwiches from Gor'kov's equations by imposing certain boundary conditions. This calculation explicitly involves the effective coherence length ξ in each film, and it is assumed that in each film the electronic mean free path ℓ is much smaller than ξ and the film thickness. The excitation spectrum in dirty NS sandwiches has been computed only near T_c.

However, using a tunneling model for the proximity effect, McMillan (4) has been able to calculate for all temperature $T < T_c$ the tunneling density of state in each film of "clean" NS sandwiches, for which $\ell \sim$ film thickness. In this model the elctrical contact is replaced by potential barrier and tunneling through this barrier is described by the transfer Hamiltonian of Cohen et al (56). For the model to be applicable the thickness of each film must be smaller than the corresponding coherence length, so that the properties of each film may be considered constant across its thickness. Hence calculations for the McMillan model do not involve the coherence length.

3.1 The McMillan's Model (7)

The physical system under consideration is represented in Fig. 3.1. The superconducting film is usually a Pb film (because of the high transition temperature), of thickness ~ 1000 Å. The film in contact with the Pb film is taken to be a normal metal, say, Ag, with a thichkness of the same order of magnitude. Both films must be clean, i.e., the mean free path ℓ is of the same order as the film thickness, and the coherence length ξ is larger than the film thickness.

The Hamiltonian for such a sandwich in the McMillan tunneling model (7) is the sum of the Hamiltonian for the N and S slabs and the tunneling Hamiltonian

$$H = H_N + H_S + H_T , \tag{3.1}$$

$$H_T = T_m \sum_{nn'} (a^\dagger_{n\uparrow} b_{n'\uparrow} + b^\dagger_{-n'\downarrow} a_{-n\downarrow}) + \text{H.c.}, \tag{3.2}$$

where $a^\dagger_{ns}$ creates an electron in a one electron state (labeled by n)

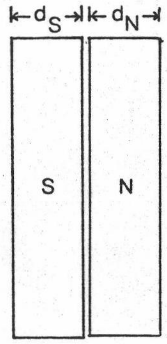

Fig. 3.1. Geometry of the NS films. Film S is a BCS superconductor, film N is a normal metal. The films have thicknesses d_S and d_N, respectively, and both films have the same area A.

with spin s in the normal film, $b^{\dagger}_{ns}$ creates an electron in state n and spin s in the superconducting film. T_m, the transfer matrix element is assumed to be independent of n and n'.

In second-order self-consistent perturbation theory, the electron self energies in films S and N in Nambu matrix notation (57) are

$$\Sigma_S(\omega) = \Sigma_S^{ph} + T_m^2 \sum_n G_{Nn}(\omega) \quad (3.3a)$$

$$\Sigma_N(\omega) = \Sigma_N^{ph} + T_m^2 \sum_{n'} G_{Sn'}(\omega) \quad (3.3b)$$

where $G_{Nn}(\omega)$ and $G_{Sn'}(\omega)$ are the matrix Green's functions in film N and S, respectively. In the usual way, we write

$$G_{Sn}(\omega) = \left[Z_S(\omega)\omega \mathbf{1} - \varepsilon_n \tau_3 - \phi_S(\omega) \tau_1\right]^{-1} \quad (3.4a)$$

$$G_{Nn'}(\omega) = \left[Z_N(\omega)\omega \mathbf{1} - \varepsilon_{n'} \tau_3 - \phi_N(\omega) \tau_1\right]^{-1} \quad (3.4b)$$

where $\mathbf{1}$ is the 2 X 2 unit matrix, τ_1 and τ_3 are Pauli matrices, $Z(\omega)$ is the renormalization function, and $\phi(\omega)$ is the unrenormalized gap function in terms of the frequency ω. The quantity ε_n is the energy of the n<u>th</u> one-electron state. Substitution of Eq. (3.4) into Eq. (3.3) gives, after averaging over impurity sites, the following self-consistent equations for ϕ_S, ϕ_N, Z_S and Z_N:

$$\phi_S(\omega) = \Delta_S^{ph} + \frac{\Gamma_S \, \phi_N(\omega)}{\left[\phi^2(\omega) - Z_N(\omega)\omega^2\right]^{-\frac{1}{2}}} \quad (3.5a)$$

$$Z_S(\omega) = 1 + \frac{\Gamma_S \, Z_N(\omega)}{\left[\phi^2(\omega) - Z_N^2(\omega)\,\omega^2\right]^{-\frac{1}{2}}} \quad (3.5b)$$

$$\phi_N(\omega) = \Delta_S^{ph} + \frac{\Gamma_N \, \phi_S(\omega)}{\left[\phi_S^2(\omega) - Z_S^2(\omega)\,\omega^2\right]^{-\frac{1}{2}}} \quad (3.6a)$$

$$Z_N(\omega) = 1 + \frac{\Gamma_N \, Z_S(\omega)}{\left[\phi_S^2(\omega) - Z_S^2(\omega)\,\omega^2\right]^{-\frac{1}{2}}} \quad (3.6b)$$

where Δ_S^{ph} is the order parameter of the superconducting film given self-consistently by

$$\Delta_S^{ph} = \lambda_S \int_0^{\omega_D} d\omega \; \mathrm{Re}\left\{ \frac{\phi_S(\omega)}{\left[\phi_S^2(\omega) - Z_S^2(\omega)\,\omega^2\right]^{-\frac{1}{2}}} \right\} \quad (2.7)$$

where ω_D is the Debye cutoff frequency for the superconducting film, λ_S is the BCS coupling constant in film S.

In these equations, Γ_N and Γ_S are defined in terms of the tunneling matrix element T_m as follows:

$$\Gamma_N = h/2\,\tau_N = \pi T_m^2 \, A d_S N_S(0) \quad (3.8a)$$

$$\Gamma_S = h/2\,\tau_S = \pi T_m^2 \, A d_N N_N(0) \quad (3.8b)$$

where $N_N(0)$ and τ_N are the density of states (per unit volume for one spin orientation, at the Fermi level) and the relaxation time in film N of thickness d_N and area A, and similarly for film S. Hence,

$$\Gamma_N / \Gamma_S = d_S N_S(0) / d_N N_N(0) \quad (3.9)$$

This relation implies that the numbers of electrons crossing the barrier in opposite directions are equal, as required.

The relaxation time is given by $\tau_N = 2Bd_N/(V_{FN}\sigma)$ so that

$$\Gamma_N = hV_{FN}\,\sigma \, / \, 4Bd_N \quad (3.10)$$

where V_{FN} is the Fermi velocity, σ the probability that an electron incident on the barrier from film N will be transmitted, and $2Bd_N$ the length traveled in the film N between successive collisions with the barrier. McMillan suggests that for clean films B is constant with value $\sim$ 2.

According to McMillan's theory, the transition temperature T_c of the proximity sandwich is given by

$$\ln(T_c^B/T_c) = \frac{\Gamma_N}{\Gamma_S + \Gamma_N}\left[\psi\left(-\tfrac{1}{2} + \frac{\Gamma_S + \Gamma_N}{2\pi T_c}\right) - \psi(-\tfrac{1}{2})\right] \qquad (3.11)$$

where ψ is the digamma function (25) and T_c^B is the transition temperature of the bulk superconductor.

Experimental measurements made on clean NS sandwiches have yield at least qualitative agreement with the McMillan model. Adkins and Kington (58) measured the tunneling densities of states for both film in Pb-Cu sandwiches. They found that the McMillan model accounts for the main features of their observations. In addition, reasonable agreement with theory is obtained for the dependence of the induced energy gap on normal film thickness. Kaiser and Zuckermann (1) showed that the McMillan model gives a very good fit to Minnigerode's measurement (59) of the dependence of T_c for Pb-Cu sandwiches on thickness of the Pb and Cu films.