CHAPTER III

RESERVOIR SIMULATION

Webster's dictionary defines *simulate* as *to assume the appearance of without the reality*. Simulation of petroleum reservoir performance refers to the construction and operation of a model whose behavior assumes the appearance of actual reservoir behavior. The model itself is either physical or mathematical. A mathematical model is simply a set of equations that, subject to certain assumptions, describes the physical processes active in the reservoir. Although the model itself obviously lacks the reality of the oil or gas field, the behavior of a valid model simulates that of the field.

In the development of a reservoir simulator there are some major formulation steps those are shown in the Figure 3.1. In this figure, the formulation process outlines the basic assumptions in the simulator, states those assumptions in precise mathematical terms and applies them to the reservoir. The result of the formulation process is a set of non linear partial differential equations (PDE) that describes the flow through the porous media. Then the non linear PDE's are converted to linear algebraic equations by discretization. Once the simulator equations are linearize, then these linear equations are solved numerically.

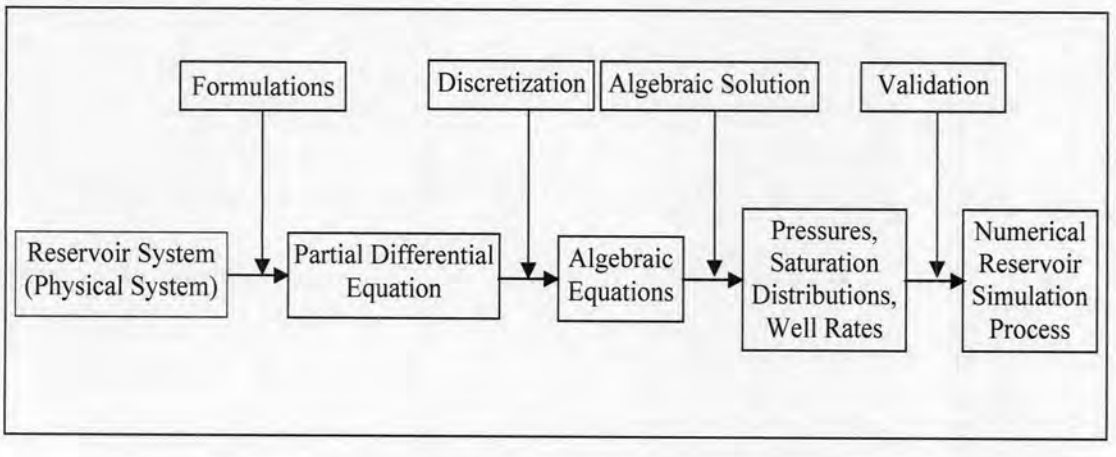

Figure 3.1: Major Steps used to Develop Reservoir Simulator.

The purpose of simulation is estimation of field performance under one or more operating schemes. Whereas the field can operate only once, at considerable

expense; a model can be operated or run many times at low expense over a short period of time. Observation of model performance under different operating conditions aids selection of an optimal set of conditions for the reservoir.

3.1 Formulation of Simulation Equation

To understand the fluid flow in the reservoir it is must to express some system of equations that govern the behavior of these fluids. After having development such a system of equations then the effect of the varying conditions can be analyzed. The basic equations of reservoir simulation are obtained by combining Conservation of Mass, Conservation of Momentum and Rate Equation. These governing equations together with the necessary boundary conditions and initial conditions can be solved numerically to predict the performance of the reservoir under different conditions.

3.1.1 Conservation of Mass

The law of conservation of mass is a material balance equation for a control volume of the system to be modeled. It can be achieved by equating the accumulation of mass within the control volume with the difference between the mass in and out of the control volume.

For one dimension,

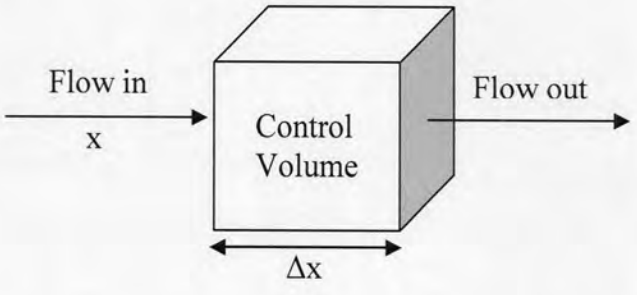

Figure 3.2: Fluid flow through porous media.

Mass Balance Equation

The material balance equation over time Δt is

[Mass in] – [Mass out] = [Mass accumulation] + [Mass injection]

$$[m_i] - [m_o] = [m_a] - [m_s] \tag{3.1}$$

where, Mass in, $m_i = m_x|_x \, A \, \Delta t$ (3.2)

Mass out, $m_o = m_x|_{x+\Delta x} \, A \, \Delta t$ (3.3)

Mass accumulation, m_a = [Mass in at time t+Δt] – [Mass in at time t]

Thus,

Mass accumulation, $m_a = [\, m_v A \Delta x \,]_{t+\Delta t} - [\, m_v A \Delta x \,]_t$ (3.4)

Mass injection, $m_s = (-\, q_m) \; A \Delta x \Delta t$ (3.5)

where, m_x= x component of mass flux, mass/(area-time)

m_v= mass contained in unit volume of reservoir, mass/volume

q_m= mass injection (–), mass injected/(area-time)

Then,

$$[m_x|_x - m_x|_{x+\Delta x}] \, A \, \Delta t = [\, m_v A \Delta x \,]_{t+\Delta t} - [\, m_v A \Delta x \,]_t + q_m \, A \Delta x \Delta t$$

$$\left[\frac{[m_x|_x - m_x|_{x+\Delta x}]}{\Delta x} \right] = \frac{[m_v|_{t+\Delta t} - m_v|_t]}{\Delta t} + q_m$$

The limits are $\Delta x \to 0$, $\Delta t \to 0$, then

$$-\frac{\partial m_x}{\partial x} = \frac{\partial}{\partial t}(m_v) + q_m \tag{3.6}$$

3.1.2 Conservation of Momentum

The conservation of momentum is modeled using Darcy's law. The mass flux can be expressed as the product of the density and velocity

$$\text{Mass Flux} = m_x = \rho u_x \tag{3.7}$$

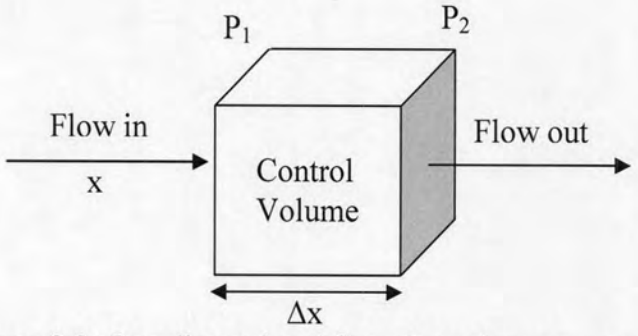

Figure 3.3: One dimensional flow through porous media.

For single phase, one-dimensional flow, Darcy's law is described as

$$u_x = -\frac{k}{\mu}\frac{\partial p}{\partial x} \tag{3.8}$$

where, k = permeability

For three dimensional single phase flow,

$$\vec{u} = -\frac{k}{\mu}\left[\nabla p + \rho\frac{\vec{g}}{g_c}\right] \tag{3.9}$$

where, $\vec{g}$ = gravity force

g_c = conversion constant

$$\rho\frac{\vec{g}}{g_c} = \rho\frac{g}{g_c}\nabla z = \gamma\nabla z \tag{3.10}$$

where, z is positive in the downward vertical direction.

Thus the Darcy's law can be written in the vector form as

$$\vec{u} = -\frac{k}{\mu}\left[\nabla p - \gamma\nabla z\right] \tag{3.11}$$

3.1.3 Mass Balance and Darcy's Law Combination

Substituting m_x and $m_v = \rho\phi$, then we get from Equation 3.6,

$$-\frac{\partial(\rho u_x)}{\partial x} = \frac{\partial}{\partial t}(\rho\phi) + q_m \tag{3.12}$$

In three dimensional form,

$$-\nabla(\rho\vec{u}) = \frac{\partial}{\partial t}(\rho\phi) + q_m \tag{3.13}$$

Combining of Equations 3.11 and 3.13, we get the following

$$-\nabla\left[\frac{\rho k}{\mu}(\nabla p - \gamma\nabla z)\right] = \frac{\partial}{\partial t}(\rho\phi) + q_m \tag{3.14}$$

3.1.4 Simulation Flow Equation

Dividing Equation 3.14 by ρ_{sc} and using the definition of formation volume factor (FVF), $B = \frac{\rho}{\rho_{sc}}$ yields the following form

$$-\nabla\left[\frac{k}{B\mu}(\nabla p - \gamma\nabla z)\right] = \frac{\partial}{\partial t}\left(\frac{\phi}{B}\right) + q$$

Finally,
$$-\nabla[\lambda(\nabla p - \gamma\nabla z)] = \frac{\partial}{\partial t}\left(\frac{\phi}{B}\right) + q \qquad (3.15)$$

where, $\lambda = \frac{k}{B\mu}$ and $q_m = \rho q$

q = volumetric flow rate.

This Equation is obtained by combination of the appropriate form of Darcy's law and mass conservation equation. Here fluid density is expressed in the implicit form of the FVF as a function of pressure.

Now, two phase simulation equations can be developed from Equation 3.15 as follows:

Water Phase:

$$-\nabla[\lambda_w(\nabla p_w - \gamma_w\nabla z)] = \frac{\partial}{\partial t}\left(\phi\frac{S_w}{B_w}\right) + q_w \qquad (3.16)$$

where, S_w = water saturation

B_w = water FVF

q_w = volumetric flow rate of water

Gas Phase:

$$-\nabla\left[R_{sw}\lambda_w(\nabla p_w - \gamma_w\nabla z) + \lambda_g(\nabla p_g - \gamma_g\nabla z)\right] = \frac{\partial}{\partial t}\left(\phi\left(R_{sw}\frac{S_w}{B_w} + \frac{S_g}{B_g}\right)\right) + R_{sw}q_w + q_g \qquad (3.17)$$

where, S_g = gas saturation

B_g = gas FVF

q_g = volumetric flow rate of gas

R_{sw} = solution gas-water ratio

The transmissibility for phase *i* is

$$\lambda_i = \frac{kk_{ri}}{\mu_i B_i}$$

For water and gas two phase simulation, there are four unknowns which are p_w, S_w, p_g and S_g. Already there are two equations, Equation 3.16 and 3.17, and another two equations can be get from the following relations:

The saturation can be expressed as

$$S_w + S_g = 1 \qquad (3.18)$$

The capillary pressure can be expressed as

$$P_{cwg} = P_g - P_w \qquad (3.19)$$

where, P_{cwg} = gas-water capillary pressure

After knowing the petrophysical properties, fluid properties, initial reservoir conditions, grid block sizes and time steps, these four unknowns p_w, S_w, p_g and S_g can be solved from Equation 3.16 to 3.19. These equations are discretized and solved numerically. A computer program which solves these numerical equations is called reservoir simulator.

3.2 Simulation Solution Procedure

The simulator itself computes fluid flow throughout the reservoir. The fluid flow equations are expressed in non linear partial differential form for each fluid present. Next, these PDE's are discretize, a process of converting the PDE's into algebraic equations, in finite difference method. The discretization process results in a system of non linear algebraic equations which can be solved more easily. The major steps are followed for the formulation of simulator is shown in Figure 3.1. Then the unknown parameters can be calculated for each grid block and for each time step.

Taylor series expansion is a useful tool in numerical analysis which provides a means for converting most well-behaved functions into simple polynomials and can be written as:

$$P(x+\Delta x) = P(x) + \frac{\Delta x}{1!}\frac{\partial P}{\partial x} + \frac{(\Delta x)^2}{2!}\frac{\partial^2 P}{\partial x^2} + \frac{(\Delta x)^3}{3!}\frac{\partial^3 P}{\partial x^3} + \cdots \quad \cdots + \frac{(\Delta x)^N}{N!}\frac{\partial^N P}{\partial x^N}$$

Consider the following first derivative finite difference forms of Taylor series expansion:

Forward Difference $$\frac{\partial P}{\partial x} = \frac{P(x+\Delta x) - P(x)}{\Delta x} \tag{3.20}$$

Backward Difference $$\frac{\partial P}{\partial x} = \frac{P(x) - P(x-\Delta x)}{\Delta x} \tag{3.21}$$

Central Difference can be obtained by subtracting Equation 3.20 and 3.21:

Central Difference $$\frac{\partial P}{\partial x} = \frac{P(x+\Delta x) - P(x-\Delta x)}{2\Delta x} \tag{3.22}$$

The example of finite difference simulation equation for the water phase in the x-direction is shown as follows:

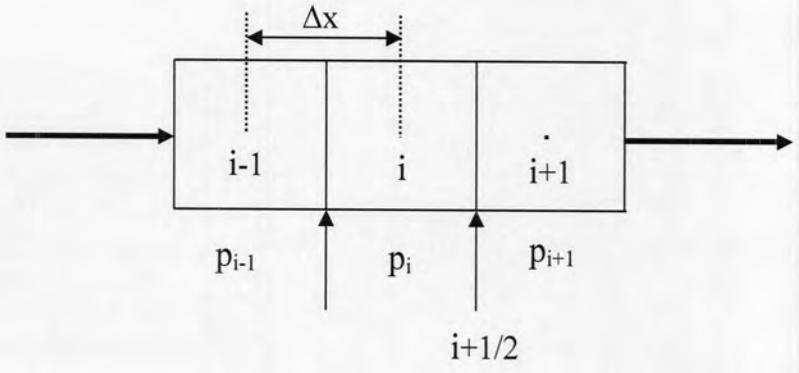

Figure 3.4: One Dimensional Simulation Grid

Neglecting the gravity term from Equation 3.16, the flow equation for water in the x-direction at grid block *i* becomes

$$-\frac{\partial}{\partial x}\left[\lambda_{w,i}\left(\frac{\partial p_w}{\partial x}\right)_i\right] = \frac{\partial}{\partial t}\left(\phi\frac{S_w}{B_w}\right)_i + q_w \tag{3.23}$$

where,
$$\left(\frac{\partial p_w}{\partial x}\right)_i = \left(\frac{\partial p_w}{\partial x}\right)_{i+\frac{1}{2}} + \left(\frac{\partial p_w}{\partial x}\right)_{i-\frac{1}{2}} \quad (3.24)$$

Then,
$$-\frac{1}{\Delta x}\left[\lambda_{w,i}\left(\left(\frac{\partial p_w}{\partial x}\right)_{i+\frac{1}{2}} - \left(\frac{\partial p_w}{\partial x}\right)_{i-\frac{1}{2}}\right)\right] = \frac{\phi_i}{B_{w,i}}\left(\frac{\partial S_w}{\partial t}\right)_i + q_w \quad (3.25)$$

Finally the finite difference form of the water flow equation in the x-direction at grid block *i* becomes

$$-\frac{\lambda_{w,i}}{(\Delta x)^2}\left[\left(\left(p_{w,i+1} - p_{w,i}\right)_{i+\frac{1}{2}} - \left(p_{w,i} - p_{w,i-1}\right)_{i-\frac{1}{2}}\right)\right] = \frac{\phi_i}{B_{w,i}\Delta t}\left(S_{w,i}^{n+1} - S_{w,i}^{n}\right)_i + q_w \quad (3.26)$$

where, n = current time level

n+1 = next time level

Δt = time step

There are two following basic means of solving the simulator equations as shown in Figure 3.5.

Fully Implicit (Implicit Pressure Implicit Saturation) Method: This method uses saturations at the old time step to implicitly calculate pressure and saturation at the new time step. In this method, pressure and saturations at the new time level are determined simultaneously. This method is unconditionally stable. But with large time steps in the Fully Implicit method there is introduction of numerical dispersion, an error when the Taylor series approximation is used to replace derivatives with finite differences.

IMPES (Implicit Pressure Explicit Saturation) Method: This method solves for pressure at new time level using saturations at the old time level, and then uses the pressures at the new time level to explicitly calculate saturations at the new time level. The IMPES method has severe restrictions on the time step size. The maximum time step size in an IMPES simulator can be estimated by applying the rule of thumb that throughput in any block should not exceed 10% of the pore volume of the block.

Fully Implicit methods do more calculations in a time step than the IMPES method, and are stable over longer time steps. The unconditional stability of the Fully Implicit method means that a Fully Implicit simulator can solve problems faster than IMPES methods by taking significantly longer time steps.

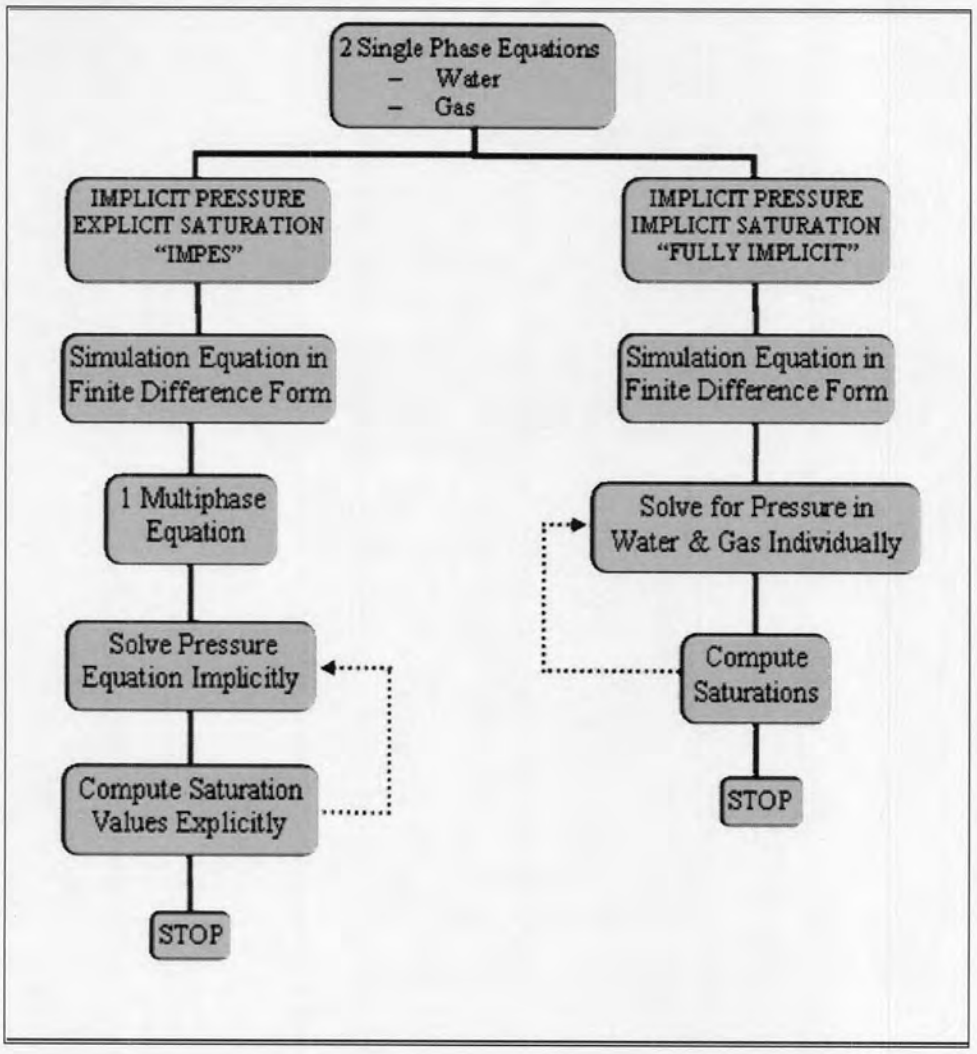

Figure 3.5: Flow chart of simulation solution process.

3.3 Reservoir Simulation Workflow

The workflow of reservoir simulation followed in this study consists of Reservoir Model Construction and Performance Predictions. Both of the steps are discussed in details as follows:

3.3.1 Reservoir Model Construction

To construct the reservoir model, the following tasks are performed:

1. **Selection of modeling approach.** This involves making decisions on several keys, fundamental aspects of the model. The fundamental aspects should be based on the physical characteristics of the reservoir, and the objectives of the simulation study.

2. **Define the reservoir.** This description includes structure of the reservoir, net to gross ratio. The structure and net to gross ratio come from depth map and log respectively.
3. **Design grids and define reservoir rock and fluids properties.** The reservoir rock and fluids properties are porosity, absolute permeability, fluid properties, water and gas saturation, water and gas relative permeability, capillary pressure, initial temperature and pressure. The sources of data are as follows:
    - Grid Dimensions: Defined by user.
    - Porosity: Logging
    - Permeability: Porosity-Permeability correlation
    - Fluid (Water and Gas) Properties such as B, C, μ, ρ: Correlations
    - Water and Gas Saturations: Logging
    - Water and Gas Relative Permeability: Core Analysis
    - Capillary Pressure: Core Analysis
    - Initial Temperature: Logging
    - Initial Pressure: Logging.
4. **Select an appropriate simulation model.** Reservoir simulator is classified based on the following characteristics:
    - Fluid Description
        - Black Oil
        - Compositional
        - Chemical
    - Temperature
        - Isothermal
        - Thermal
    - Simulation Solution Method
        - IMPES
        - Fully Implicit
    - Coordinate System
        - Cartesian
        - Radial
        - Spherical

In this study, the reservoir simulator ECILPSE 100 is used. ECLIPSE 100 is a Black Oil and isothermal simulator. The fully implicit method was selected as solution method as it is stable method irrespective of time step size. The selected grid system is Cartesian coordinate.

3.3.2 Performance Predictions

In this phase, most of the study objectives are met. According to the study objective, the variables to be varied are designed. Sensitivity analyses of defined variables are then performed and discussions are made on different simulation cases result. The best sand candidate is then selected on the basis of the cumulative injection volume of water.

In summary, reservoir simulation is a process to simulate the reservoir performance under various operating conditions by solving numerical equations, describes the physical process in the reservoir. The numerical equations are formulated by the combination of conservation of mass and conservation of momentum. Finally, numerical equations are solved for pressure and saturation of each grid blocks for each phase. There are two basic means of solving the simulation equations such as Fully Implicit and IMPES. In Fully Implicit method, pressure and saturations at the new time level are determined simultaneously where as in IMPES method, solves for pressure at new time level using saturations at the old time level, and then uses the pressures at the new time level to explicitly calculate saturations at the new time level. As Fully Implicit method is unconditionally stable, therefore, it is selected as solution method in this study.

The workflow of reservoir simulation consists of two main steps which are Reservoir Model Construction and Performance Predictions. Reservoir Model Construction comprises of (1) selection of modeling approach, (2) define the reservoir, (3) design grids and reservoir rock and fluid properties and (4) select an appropriate simulation model. In Performance Prediction phase sensitivity analysis are performed and make conclusion.