INTRODUCTION

The multiplicative structure of any ring is by definition a semigroup with zero. Then it is valid to ask whether a given semigroup S has S^0 isomorphic to the multiplicative structure of some ring. If it does, S is said to *admit a ring structure*. Let $\mathcal{R}$ be the class of all semigroups admitting ring structure. Semigroups admitting ring structure have long been studied. In 1961, Kogalovski [7] showed that the class $\mathcal{R}$ is not axiomatizable. If S is a member of $\mathcal{R}$, then there is an isomorphism φ from S^0 onto $(R, \cdot)$ for some ring $(R, +, \cdot)$. If we define an addition $\oplus$ on S^0 by

$$x \oplus y = \varphi^{-1}(\varphi(x) + \varphi(y)) \quad \text{for all } x, y \in S,$$

then $(S^0, \oplus, \cdot)$ is a ring which is isomorphic to $(R, +, \cdot)$ through the mapping φ. Hence a semigroup $S \in \mathcal{R}$ if and only if there is an operation $+$ on S^0 such that $(S^0, +, \cdot)$ is a ring where $\cdot$ is the operation on S^0. A brief survey of results obtained in this area was given by Peinado [9]. For various studies in this area, see [4], [8], [11], [12], [1], [13], [14] and [5].

Additively commutative semirings with zero are a generalization of rings. In [10], the author defined semigroups admitting the structure of an additively commutative ring with zero analogously. He determined when some semigroups admit such a structure.

We know that nearrings generalize rings and many examples and important results of nearrings can be found in [2], the book written by Clay. Some important right nearrings which are not rings are $(M(\mathbb{R}), +, \circ)$, $(C(\mathbb{R}), +, \circ)$ and $(D(\mathbb{R}), +, \circ)$ where $M(\mathbb{R})$ is the set of all functions $f : \mathbb{R} \to \mathbb{R}$, $C(\mathbb{R}) = \{f \in M(\mathbb{R}) \mid f \text{ is continuous}\}$, $D(\mathbb{R}) = \{f \in M(\mathbb{R}) \mid f \text{ is differentiable}\}$ and $+$ and $\circ$ are the usual addition and composition of functions. Observe that these nearrings are additively commutative. In this research, left nearrings and right nearrings are assumed to be additively commutative. The three above nearrings have no multiplicative zero. However, every right nearring has a multiplicative left zero and every left nearring has a multiplicative right zero. By definition, a zero-symmetric left [right] nearring has a multiplicative zero. It follows that if S has no left [right] zero, then S is not isomorphic to the multiplicative structure of a right [left] nearring. Then the most reasonable definition of a semigroup admitting a right [left] nearring structure is as follows : A semigroup S is said to *admit a right [left] nearring structure* if S or S^0 is isomorphic to the multiplicative structure of some right [left] nearring, or equivalently

(1) there is an operation $+$ on S such that $(S, +, \cdot)$ is a right [left] nearring where $\cdot$ is the operation on S or

(2) there is an operation $+$ on S^0 such that $(S^0, +, \cdot)$ is a right [left] nearring where $\cdot$ is the operation on S^0.

In this research, we characterize when semigroups of our interest admit a right nearring structure and a left nearring structure.

Chapter I contains basic definitions and quoted results which are needed for our study. See [3] and [4] for more details.

Chapter II deals with standard transformation semigroups. We provide necessary and sufficient conditions for each of these transformation semigroups to admit a right nearring structure and a left nearring structure. Important results of this chapter are that every full transformation semigroup on a nonempty set X always admit a right nearring structure but it admit a left nearring structure only the case that $|X| = 1$ and this is also true for the partial transformation semigroup on X. One of the main tools is that for any nonempty set X, there is an operation $+$ on X such that $(X, +)$ is an abelian group. Some results in [14] are useful for this chapter.

In Chapter III, we give characterizations determining when certain matrix groups admit a right nearring structure and a left nearring structure. Some techniques in [6] are important for this study.

In the last chapter, some other semigroups are studied in the same manner.

Semigroups we consider in this chapter are the infinite cyclic semigroup, left zero semigroups, right zero semigroups, Kronecker semigroups, dihedral groups and alternating groups.