CHAPTER V

MULTIPHASE FLOW CORRELATIONS

Several multiphase flow correlations that can predict pressure loss or bottomhole flowing pressure have been published. In this study, the following existing multiphase flow correlations were used to compare with the proposed Guo's four-phase flow model. These existing multiphase flow correlations are

1) Hagedorn and Brown correlation

2) Duns and Ros correlation (original and modified)

3) Beggs and Brill correlation

4) Orkiszewski correlation

5) Fancher and Brown correlation

5.1 Hagedorn and Brown correlation

The Hagedorn-Brown correlation[6] was developed from data obtained from a 1,500 feet deep instrumented well using fluids with a wide range of viscosities. This correlation was measured for flow in tubing sizes ranging from 1 ¼ to 2 ⅞ inch O.D and included a wide range of liquid rates and gas-liquid ratio. The effects of liquid viscosity were studied by using water and oil as the liquid phase in this correlation.

Neither liquid holdup nor flow pattern were measured, but the liquid holdup was back–calculated to satisfy the measured pressure gradient after the pressure drops due to friction and acceleration were accounted for. The liquid holdup is not a true indicator of the proportion of the pipe occupied by fluid; it is merely a correlation parameter.

Hagedorn and Brown found that the liquid holdup could be correlated to four dimensionless numbers. These numbers are (1) liquid velocity number, (2) gas velocity number, (3) pipe diameter number, and (4) liquid viscosity number. They can be expressed as follows:

Liquid velocity number $$N_{LV} = 1.938 v_{sL} \left(\frac{\rho_L}{\sigma} \right)^{1/4} \tag{5.1}$$

Gas velocity number $$N_{gv} = 1.938 v_{sg} \left(\frac{\rho_L}{\sigma} \right)^{1/4} \tag{5.2}$$

Pipe diameter number $N_d = 120.872d\left(\frac{\rho_L}{\sigma}\right)^{1/2}$ (5.3)

Liquid viscosity number $N_L = 0.15726\mu_L\left(\frac{1}{\rho_L\sigma^3}\right)^{1/4}$ (5.4)

The above expressions are dimensionless when the parameters are expressed in the appropriate field units as shown below:

v_{sL} = liquid superficial velocity, ft/sec

v_{sg} = gas superficial velocity, ft/sec

ρ = liquid density, lbm/ft^3

σ = interfacial tension, dynes/cm

μ_L = liquid viscosity, centipoises (cp)

d = pipe diameter, feet

The correlating function is entered with a value of CN_L. The corrected liquid number is read from a plot of CN_L versus N_L. The secondary holdup correction factor is determined from a correlation of N_{gv}, N_L, and N_d. Once the correction factors have been determined, the holdup can be calculated. The pressure gradient due to elevation change (gravity term) is calculated from

$$\left(\frac{dp}{dL}\right)_{Gravity} = \frac{g}{g_c}\rho_L H_L + \rho_g(1 - H_L) \quad (5.5)$$

The pressure gradient due to friction is given by

$$\left(\frac{dp}{dL}\right)_{Friction} = f\frac{\rho_f v_m^2}{2g_c d} \quad (5.6)$$

where

$$\rho_f = \frac{\rho_n^2}{\rho_m}$$

$$\rho_m = \rho_L H_L + \rho_g(1 - H_L)$$

$$\rho_n = \rho_L \lambda_L + \rho_g(1 - \lambda_L)$$

$$v_m = v_{sL} + v_{sg}$$

$$\lambda_L = v_{sL} / v_m$$

The friction gradient can be written in terms of the mass flow rate (w) as

$$\left(\frac{dp}{dL}\right)_{Friction} = f\frac{w^2}{2g_c dA^2 \rho_m} \tag{5.7}$$

This expression simplifies to

$$\left(\frac{dp}{dL}\right)_{Friction} = f\frac{w^2}{2.9652\times10^{11}\,\rho_m d^5} \tag{5.8}$$

where

w = mass flow rate, lbm/day

ρ_m = density based on liquid holdup, lbm/ft^3

d = pipe inside diameter, feet

f = two-phase friction factor, dimensionless

The two-phase friction factor is correlated with a two-phase Reynolds number using the standard Moody diagram. The two-phase Reynolds number is defined as

$$N_{Re} = \frac{\rho_n v_m d}{\mu_m} \tag{5.9}$$

where

$$\mu_m = \mu_L^{H_L} \times \mu_g^{(1-H_L)}$$

The acceleration due to acceleration is given by

$$\left(\frac{dp}{dL}\right)_{Acceleration} = \frac{\rho_m \Delta\left(v_m^2\right)}{2g_c dL} \tag{5.10}$$

where v_m is the difference in mixture velocity between the inlet and outlet ends of a pipe element. The acceleration gradient is applied as a correction (E_k) to the sum of the gravity and friction gradients as follows:

$$E_k = \frac{dL}{dP}\left(\frac{dP}{dL}\right) = \frac{\rho_m \Delta\left(v_m^2\right)}{2g_c dP} \tag{5.11}$$

The total pressure drop can be calculated from

$$\left(\frac{dP}{dL}\right)_{Total} = \frac{\left(\frac{dp}{dL}\right)_{Gravity} + \left(\frac{dP}{dL}\right)_{Friction}}{1 - E_k} \tag{5.12}$$

The refinements suggested by Brill and Hagedorn have been implemented in PROSPER. These refinements are

1) Griffith correlation for bubble flow, and

2) Limit on liquid holdup to be always greater than the no-slip holdup.

Some additional refinements have been added to the basic Hagedorn-Brown correlation. The additional refinements are

1) Beggs and Brill deviation correction for liquid holdup, and

2) Explicit calculation of acceleration term instead of using the E_k correction method.

The Hagedorn-brown correlation is probably the most widely used for all oil wells over a wide range of well conditions. It works well for bubble and slug flow regimes in a wide range of applications. At low flow rates, it underpredicts flowing pressures. This can result in an optimistic value for minimum stable flow rates. Errors are greatest for large bore deviated wells in the 35-70° range with moderate water cuts where water/oil slip may be significant.

5.2 Duns and Ros correlation (original and modified)

Duns and Ros correlation[4] was based on some 4,000 runs and 20,000 data points. The experiments were conducted in a laboratory facility at low pressure using air, oil, and water in 10 m test section and pipe diameters ranged from 3.2 to 8.02 cm. The liquid holdup was measured by radioactive tracers and the flow pattern was observed by transparent test section.

Correlations were developed for slip velocity (from which the holdup can be calculated) and friction factor for three distinct flow regimes. The flow regimes are defined as functions of the dimensionless quantities N_{gv}, N_{LV}, L_a, L_s, L_m, and N_d where

Gas velocity number $$N_{gv} = v_{sg}\left(\frac{\rho_L}{g\sigma}\right)^{1/4} \tag{5.13}$$

Liquid velocity number $$N_{LV} = v_{sL}\left(\frac{\rho_L}{g\sigma}\right)^{1/4} \tag{5.14}$$

Diameter number $N_d = d\left(\frac{\rho_L g}{\sigma}\right)^{1/2}$ (5.15)

Liquid viscosity number $N_L = \mu_L\left(\frac{g}{\rho_L \sigma^3}\right)^{1/4}$ (5.16)

$$L_s = 50 + 36N_{LV}$$

$$L_m = 75 + 84N_{LV}^{0.75}$$

Flow regimes are determined using the following relations:

Region I (Bubble flow) $0 \le N_{gv} \le (L_1 + L_2 N_{LV})$

Region II (Slug flow) $(L_1 + L_2 N_{LV}) < N_{gv} < L_s$

Region III (Mist flow) $N_{gv} > L_m$

Duns and Ros Flow Map

Duns and Ros developed a dimensionless slip velocity correlation from which the actual slip velocity and liquid holdup can be calculated using the relation

$$S = v_s\left(\frac{\rho_L}{\sigma_L g}\right)^{1/4} \tag{5.17}$$

where

$$v_s = v_g - v_L = \frac{v_{sg}}{(1 - H_L)} - \frac{v_{sL}}{H_L}$$

Solving for liquid holdup yields

$$H_L = \frac{v_s - v_m + \left[(v_m - v_s)^2 + 4v_s v_{sL}\right]^{1/2}}{2v_s} \tag{5.18}$$

The procedure for calculating the gravity pressure gradient is as follows:

1) Calculate the dimensionless slip velocity S using the appropriate correlation. The correlation for S is different for each flow regime.

2) Solve for the slip velocity, v_s

3) Solve for the liquid holdup, H_L

4) Calculate the mixture density, $\rho_m = \rho_L H_L + \rho_g (1 - H_L)$

5) Calculate the gravity pressure gradient

$$\left(\frac{dP}{dL}\right)_{Gravity} = \frac{g}{g_c}\rho_m \tag{5.19}$$

Bubble Flow Regime

The bubble flow regime is defined by a gas velocity number falling between zero and an upper limit $0 \le N_{gv} \le (L_1 + L_2 N_{LV})$.

For bubble flow, the parameters are calculated as follows:

Dimensionless slip velocity:

$$S = F_1 + F_2 N_{Lv} + F_3'\left[\frac{N_{gv}}{1+N_{Lv}}\right]^2 \tag{5.20}$$

The bubble slip velocity numbers F_1, F_2, F_3, and F_4 are correlated with the liquid viscosity number N_L.

F_3' is obtained from

$$F_3' = F_3 - \frac{F_4}{N_d} \tag{5.21}$$

The Duns and Ros friction loss for bubble flow is given by

$$\left(\frac{dP}{dL}\right)_{Friction} = f_w \frac{\rho_L v_{sL} v_m}{2 g_c d} \tag{5.22}$$

From experimental data, Duns and Ros obtained the following expression for f_w:

$$f_w = f_1\, f_2 / f_3 \tag{5.23}$$

f_1 is obtained from the Moody diagram as a function of the liquid Reynolds number

$$N_{\mathrm{Re}L} = \frac{\rho_L v_{SL} d}{\mu_L} \tag{5.24}$$

Note that for low values of Reynolds number corresponding to laminar flow conditions, the friction factor becomes independent of pipe roughness.

The factor f_2 is a correction for the in-situ gas–liquid ratio.

The factor f_3 is an additional correction for both liquid viscosity and in–situ gas–liquid ratio. It becomes important for viscosities greater than approximately 50 centistokes.

$$f_3 = 1 + f_1\left(\frac{v_{sg}}{50v_{sL}}\right)^{1/2} \tag{5.25}$$

The acceleration term is insignificant for the bubble flow regime and is therefore not calculated.

Slug Flow Regime

For the slug flow regime [$(L_1 + L_2N_{LV}) < N_{gv} < L_s$], the dimensionless slip velocity is calculated as follows:

$$S = (1 + F_5)\frac{(N_{gv})^{0.982} + F_6'}{(1 + F_7N_{Lv})^2} \tag{5.26}$$

The slug slip velocity numbers F_5, F_6, and F_7 are found from a plot as a function of liquid viscosity number N_L. The friction pressure gradient is calculated using the same procedure as for bubble flow. The acceleration term is considered to be negligible in the slug flow regime.

Mist Flow Regime

For the mist flow regime ($N_{gv} > L_m$), the slip velocity is taken as zero. This is because with high gas flow rates, the liquid and gas travel with essentially the same velocity. With no slip, the mixture density can be calculated directly from

$$\rho_n = \rho_L\lambda_L + \rho_g\lambda_g = \rho_L\left(\frac{v_{sL}}{v_m}\right) + \rho_g\left(\frac{v_{sg}}{v_m}\right) \tag{5.27}$$

In the mist flow regime, the friction term is based on the gas phase only.

$$\left(\frac{dP}{dL}\right)_{Friction} = f\frac{\rho_g v_{sg}^2}{2g_c d} \tag{5.28}$$

The friction factor f is read from the Moody diagram as a function of the gas Reynolds number.

$$N_{\mathrm{Re}} = \frac{\rho_g v_{sg} d_{hy}}{\mu_g} \tag{5.29}$$

where

d_{hy} = the hydraulic diameter of the flow string

In mist flow, there is a film of liquid on the pipe wall. The ripples of the wall film cause a drag on the gas. This process is governed by a form of the Weber number

$$N_{We} = \frac{\rho_g v_{sg}^2 \varepsilon}{\sigma} \tag{5.30}$$

Liquid viscosity also has an influence which is accounted for by making N_{we} a function of a dimensionless number containing the liquid viscosity

$$N_\mu = \frac{\mu_L^2}{\rho_L \sigma\ \varepsilon} \tag{5.31}$$

The value of pipe roughness may be very small, but ε/d never becomes smaller than the value for the pipe itself. At the transition to slug flow, ε/d approaches 0.5. Between these limits, ε/d can be calculated from the following equations:

$$N_{We} N_\mu < 0.005 : \frac{\varepsilon}{d} = \frac{0.0749 \sigma_L}{\rho_g V_{sg}^2 d} \tag{5.32}$$

Values of f for the mist flow regime can be found for ε/d > 0.05 from

$$f = \left[\frac{1}{\left[4 Log_{10} \left(0.27 \varepsilon / d \right)^{1.73} \right]} \right] \times 4 \tag{5.33}$$

As the wave height on the walls increases, the actual area available for flow of gas is reduced to (d- ε). Duns and Ros suggested that the prediction of friction loss could be refined by substitution of (d- ε) for d and

$$\frac{v_{sg} d^2}{(d - \varepsilon)^2} \tag{5.34}$$

for v_{sg} throughout the calculation of friction gradient. In this case, the determination of roughness is iterative.

In mist flow, the acceleration term can be written as

$$\left(\frac{dP}{dL} \right)_{Acceleration} = \frac{v_m v_{sg} \rho_n}{g_c p dL} \tag{5.35}$$

Let's define an acceleration parameter E_k as follows:

$$E_k = \frac{V_m V_{sg} \rho_n}{g_c p} \tag{5.36}$$

then, the total pressure gradient can be calculated from

$$\frac{dP}{dL} = \frac{\left(\frac{dP}{dL}\right)_{Gravity} + \left(\frac{dP}{dL}\right)_{Friction}}{1 - E_k} \tag{5.37}$$

Transition (Slug-Mist) Flow Regime

For the region ($L_s < N_{gv} < L_m$), linear interpolation of the total pressure gradients is used to determine the total pressure gradient. This means that when N_{gv} falls between L_s and L_m, pressure gradients must be calculated using both slug flow and mist flow correlations as follows:

$$\frac{dP}{dL} = A\left(\frac{dP}{dL}\right)_{Slug} + B\left(\frac{dP}{dL}\right)_{Mist} \tag{5.38}$$

where

$$A = \frac{L_m - N_{gv}}{L_m - L_s}, B = \frac{N_{gv} - L_s}{L_m - L_s} = 1 - A$$

Duns and Ros correlation has been found to perform better in mist flow than most others. It is particularly useful for condensate wells. Although the accuracy of pressure gradient predictions in slug flow is generally inferior to Hagedorn and Brown, prediction of minimum stable flow rates using the minimum value of the Duns and Ros flowing bottomhole pressure is generally accurate.

In PROSPER, additional refinements have been made to the basic Duns and Ros method as follows:

1) Beggs and Brill[9] deviation correction for holdup is implemented
2) Gould et al flow map[18] which more accurately predicts the onset of mist flow for some conditions is used, and
3) Explicit calculation of the acceleration term.

5.3 Beggs and Brill correlation

The Beggs and Brill correlation[9] can be used in both horizontal and inclined flow. This correlation was developed from experimental data obtained in a small scale test facility. The parameters studied and their ranges of variation were

(1) gas flow rate (0 to 300 Mscf/D)

(2) liquid flow rate (0 to 30 gal/min)

(3) average system pressure (35 to 95 psia)

(4) pipe diameter (1 and 1.5 in)

(5) liquid holdup (0 to 0.870)

(6) pressure gradient (0 to 0.8 psi/ft)

(7) inclination angle (-90 to +90)

(8) horizontal flow pattern.

Air and water were the fluids used. Liquid and gas rates were varied to enable all flow patterns to be observed with the pipe horizontal.

With a flow rate set up, the pipe inclination was varied so that the effect of angle on holdup could be observed. Holdup correlations were developed for each of the three horizontal flow regimes. The liquid holdup is first calculated as if the pipe were horizontal and then corrected for pipe inclination. Beggs and Brill found that the holdup was a maximum at approximately +50°.

Beggs and Brill modified their flow map from that originally published to include a transition zone between the segregated and intermittent flow regimes (see below). The following dimensionless parameters are used to identify the flow regime that would exist if the pipe were horizontal:

$$N_{FR} = \frac{v_m^2}{gd} \tag{5.39}$$

$$\lambda_L = \frac{v_{sL}}{v_m} \tag{5.40}$$

The Beggs and Brill flow regime numbers are

$$L_1 = 316\lambda_L^{0.302} \tag{5.41}$$

$$L_2 = 0.0009252\lambda_L^{-2.4684} \tag{5.42}$$

$$L_3 = 0.5\lambda_L^{-6.738} \tag{5.43}$$

The horizontal flow regimes are determined as follows:

Segregated flow

Limits: $\lambda_L < 0.01$ and $N_{FR} < L_1$

or $\lambda_L \geq 0.01$ and $N_{FR} < L_2$

Transition flow

Limits: $\lambda_L \geq 0.01$ and $L_2 < N_{FR} \leq L_3$

Intermittent flow

Limits: $0.01 \leq \lambda_L < 0.4$ and $L_3 < N_{FR} \leq L_1$

or $\lambda_L \geq 0.4$ and $L_3 < N_{FR} \leq L_4$

Distributed flow

Limits: $\lambda_L < 0.4$ and $N_{FR} \geq L_1$

or $\lambda_L \geq 0.4$ $N_{FR} > L_4$

When the flow falls in the transition regime, the liquid holdup is calculated using both the segregated and intermittent expressions and linearly interpolated using the following weighting factors:

$$A = \frac{L_3 - N_{FR}}{L_3 - L_2} \tag{5.44}$$

$$\mathrm{B} = 1 - \mathrm{A} \tag{5.45}$$

The same equations are used to calculate liquid holdup for all flow regimes. The coefficients and exponents used in the equations are changed for each flow regime. Liquid holdup is given by

$$H_{L(\phi)} = H_{L(0)}\psi \tag{5.46}$$

where $H_{L(0)}$ is the holdup which would exist for the same flow conditions in a horizontal pipe. The equivalent horizontal holdup is given by

$$H_{L(0)} = \frac{a\lambda_L^b}{N_{FR}^c} \tag{5.47}$$

where *a*, *b,* and *c* are taken from Table 5.1 according to flow regime.

Table 5.1: The values of *a*, *b*, and *c* for different flow patterns.

Flow pattern	*a*	*b*	*c*
Segregated	0.98	0.4846	0.0868
Intermittent	0.845	0.5351	0.0173
Distributed	1.065	0.5824	0.0609

with the constraint that $H_{L(0)} \geq \lambda_L$.

The effect of pipe inclination is accounted for using the liquid holdup inclination correction factor which is defined as follows:

$$\psi = 1 + C(\sin(1.8\phi) - 0.333\sin^3(1.8\phi)) \tag{5.48}$$

where ϕ is the actual angle of the pipe from horizontal. For vertical upward flow, $\phi = 90°$ and ψ becomes

$$\psi = 1 + 0.3\ \mathrm{C} \tag{5.49}$$

where

$$C = (1-\lambda_L) In(d\lambda_L^e N_{LV}^f N_{FR}^g) \tag{5.50}$$

where *d*, *e*, *f*, and *g* are determined for each flow regime from Table 5.2.

Table 5.2: The values of *d, e, f,* and *g* for different flow patterns.

Flow pattern	*d*	*e*	*f*	*g*
Segregated uphill	0.011	-3.768	3.539	-1.614
Intermittent uphill	2.96	0.305	-0.4473	0.0978
Distributed uphill	No correction		C=0, ψ=1, $H_L \neq f(\phi)$	
All flow pattern downhill	4.7	-0.3692	0.1244	-0.5056

with the restriction that $C \geq 0$.

The friction loss term is defined as

$$\left(\frac{dP}{dL}\right)_{Friction} = f_{tp}\frac{\rho_n v_m^2}{2g_c d} \tag{5.51}$$

where

$$\rho_n = \rho_L\lambda_L + \rho_g\lambda_g$$

$$f_{tp} = f_n\left[\frac{f_{tp}}{f_n}\right]$$

The no-slip friction factor is determined from the smooth pipe curve on the Moody diagram or calculated using

$$f_n = 1/[2\log[N_{\mathrm{Re}}/(4.5223\log N_{\mathrm{Re}} - 3.8215)]]^2 \tag{5.52}$$

Using the following Reynolds number

$$N_{\mathrm{Re}} = \frac{\rho_n v_m d}{\mu_n} \tag{5.53}$$

where

$$\mu_n = \mu_L \lambda_L + \mu_g \lambda_g$$

The ratio of the two-phase to no-slip friction factor is calculated from

$$\frac{f_{tp}}{f_n} = e^{s} \tag{5.54}$$

where

$$S = \left[\frac{\ln(y)}{\left\{-0.0523 + 3.182\ln(y) - 0.8725[\ln(y)]^2 + 0.01853[\ln(y)]^4\right\}} \right]$$

and

$$y = \frac{\lambda_L}{H_{L(\phi)}^2}$$

S becomes unbounded at a point in the interval $1 < y < 1.2$. In this region, S is calculated using:

$$S = ln\ (2.2\,y - 1.2) \tag{5.55}$$

Although the acceleration pressure gradient is small, it is included for increased accuracy.

$$\left(\frac{dP}{dZ}\right)_{Acceleration} = \frac{\rho_s v_m v_{sg}}{g_c p}\left[\frac{dP}{dZ}\right] \tag{5.56}$$

If the acceleration term is defined as

$$E_k = \frac{\rho_s v_m v_{sg}}{g_c p} \tag{5.57}$$

The total pressure gradient can be expressed as

$$\left(\frac{dP}{dZ}\right)_{Total} = \frac{\left(\frac{dP}{dZ}\right)_{Gravity} + \left(\frac{dP}{dZ}\right)_{Friction}}{1 - E_k} \tag{5.58}$$

where

$$\left(\frac{dP}{dZ}\right)_{Gravity} = \frac{g}{g_c}\rho_s$$

The approach of including the acceleration term as an overall correction factor to the total gradient is convenient and sufficiently accurate when the acceleration term is small. To improve accuracy when acceleration is large, PROSPER calculates the acceleration term explicitly and adds it to the gravity and friction terms to find the pressure gradient.

5.4 Orkiszewski correlation

Orkiszewski[8] used the data of Hagedorn and Brown and the field data from the 148 wells conditions to develop a new correlation to be used in the bubble and slug flow patterns. The Orkiszewski correlation combines the Griffith and Wallis method for bubble flow with a new correlation for slug flow and the Duns and Ros method for mist flow. The data of Hagedorn and Brown was used as the basis for the slug flow correlation.

The flow patterns considered by Orkiszewski are bubble flow, slug flow, transition flow and mist flow. In the slug-flow liquid distribution coefficient was used to calculate the liquid density rather than liquid holdup. A distinction was made as to which equations are used to calculate the liquid distribution coefficient depending on whether oil or water was the continuous liquid phase and if the mixture velocity was greater than 10 ft/sec.

Bubble Flow

The bubble flow regime is defined by

$$\frac{q_g}{q_t} < (L)_B$$

where

$$(L)_B = 1.071 - (0.2218 v_t 2 / d_h) \tag{5.59}$$

The value of $(L)_B$ is constrained to be grater than or equal to 0.13.

Liquid holdup in the bubble flow regime is given by

$$H_L = 1 - \frac{1}{2}\left[1 + \frac{q_t}{v_s A_p} - \left[(1 + q_t / v_s A_p)^2 - 4 q_g / v_s A_p\right]^{1/2}\right] \tag{5.60}$$

The value of the bubble slip velocity (v_s) is taken to be constant at 0.8 ft/sec.

The friction term is given by

$$\left(\frac{dP}{dh}\right)_{Friction} = f\frac{\rho_L v_L^2}{2g_c d_{hy}} \tag{5.61}$$

The friction factor f is read from the Moody diagram using a Reynolds number defined as

$$N_{Re} = \frac{\rho_L d v_L}{\mu_L} \tag{5.62}$$

The acceleration term is considered to be negligible in the bubble flow regime.

Slug Flow

The slug flow regime is defined by $\frac{q_g}{q_t} > (L)_B, \bar{\bar{v}}_g (L)_s$. For slug flow, the average density is given by

$$\bar{\rho} = \frac{W_t + \rho_L v_s A_p}{q_t + v_s A_p} + \delta\rho_L \tag{5.63}$$

where δ = the liquid distribution coefficient, and

$v_s = C_1 C_2 \sqrt{g d_{hy}}$

C_1 can be determined from the plot of C_1 versus the bubble Reynolds number (N_b).
C_2 can also be determined from the plot of C_2 versus the bubble Reynolds number (N_b) and Reynolds number (N_{Re}).
The bubble Reynolds number (N_b) and Reynolds number (N_{Re}) are given by

$$N_b = \frac{1488 \rho_L v_s d_{hy}}{\mu_L} \tag{5.64}$$

$$N_{Re} = \frac{1488 \rho_L v_t d_{hy}}{\mu_L} \tag{5.65}$$

v_s can be calculated using the following expressions:

For $N_{Re} \le 3{,}000$

$$v_s = (0.546 + 8.74 \times 10^{-6} N_{Re})\sqrt{g d_{hy}} \tag{5.66}$$

For $N_{Re} \ge 8{,}000$

$$v_s = (0.35 + 8.74 \times 10^{-6} N_{Re})\sqrt{g d_{hy}} \tag{5.67}$$

For $3{,}000 < N_{\mathrm{Re}} < 8{,}000$

$$v_s = \frac{1}{2}\left[v_{si} + \sqrt{v_{si}^2 + \frac{13.59\mu_L}{\rho_L \sqrt{d_{hy}}}} \right] \tag{5.68}$$

where

$$v_{si} = (0.251 + 8.74 \times 10^{-6} N_{\mathrm{Re}})\sqrt{g d_{hy}}$$

The value of δ is calculated using different expressions depending on the mixture velocity and the continuous liquid phase as shown in Table 5.3.

Table 5.3: The calculation of δ using the different mixture velocity and the different continuous liquid phase.

Continuous liquid phase	Mixture velocity	Equations used for δ calculation
Water	<10	$\delta = \frac{0.013 \log \mu_L}{d_{hy}^{1.38}} - 0.681 + 0.232 \log v_t - 0.428 \log d_{hy}$
Water	>10	$\delta = \frac{0.045 \log \mu_L}{d_{hy}^{0.799}} - 0.709 - 0.162 \log v_t - 0.888 \log d_{hy}$
Oil	<10	$\delta = \frac{0.0127 \log(\mu_L + 1)}{d_{hy}^{1.415}} - 0.284 + 0.167 \log v_t + 0.113 \log d_{hy}$
Oil	>10	$\delta = \frac{0.0274 \log(\mu_L + 1)}{d_{hy}^{1.371}} + 0.161 + 0.569 \log d_{hy} + X$ where $X = -\log v_t \{[0.01 \log(\mu_L + 1) / d_{hy}^{1.371}] + 0.397 + 0.63 \log d_{hy}\}$

Orkizewski did not define criteria for determining whether oil or water is the continuous phase. In a water-oil emulsion, water will generally be the continuous phase above a water cut of approximately 75%.

The value of δ is constrained by the following limits:

For $v_t < 10$,

$$\delta \geq -0.065 v_t$$

For $v_t > 10$,

$$\delta \geq \frac{-v_s A_p}{q_t + v_s A_p}(1 - \frac{\bar{\rho}}{\rho_L})$$

These constraints are designed to eliminate pressure discontinuities between flow regimes. However, significant discontinuities still occur at v_t of 10 ft/sec. This can cause significant problems, especially in large diameter pipes. Although Orkizewski can give excellent results in many wells, the use of Orkizewski is discouraged due to the danger of encountering a pressure discontinuity during pressure matching and VLP calculations.

The friction term for slug flow is given by

$$\left(\frac{dp}{dh}\right)_{Friction} = \frac{fv_t^2}{2g_c d_{hy}}\left[\left[\frac{q_L + v_s}{q_t + v_s A_p}\right] + \delta\right] \tag{5.69}$$

where f is taken from the Moody diagram using the Reynolds number

$$N_{\text{Re}} = \frac{\rho_L d v_t}{\mu_L} \tag{5.70}$$

The acceleration term is considered to be negligible in the slug flow regime.

Transition Flow

The transition flow regime is defined by $(L)_M > \overline{\overline{v_g}} > (L)_S$. The total pressure gradient is found by linear interpolation between the slug and mist flow boundaries using the interpolation scheme of Duns and Ros.

Mist Flow

The mist flow regime is defined by $\overline{\overline{v_g}} > (L)_M$. The method of Duns and Ros is used for mist flow.

5.5 Fancher and Brown correlation

Fancher and Brown correlation[7] was obtained from an 8,000 feet experimental field well to conduct flowing pressure gradient tests under conditions of continuous, multiphase flow through 2 ⅜ -in OD tubing. These tests were conducted for flow rates ranging from 75 to 936 B/D at various gas-liquid ratios from 105 to 9,433 scf/bbl. The test well was capable of making 1,000 B/D of total liquid (95 percent salt water).

These tests were compared with Poettmann and Carpenter correlation and indicated that deviations occur for certain ranges of flow rates and gas-liquid ratio. Because of these deviations, an empirical correlation based on their method was developed to fit the field data for the particular ranges of flow rates and gas-liquid ratios involved.

Viscous shear was also found to be negligible in these tests due to the high degree of turbulence of both phases. However, the original Poettmann and Carpenter correlation was extended to cover the lower density ranges (in particular less than 10 lb/cu ft for 2" tubing), which Poettmann stated was outside the range of their original data. This lower density range was further correlated by using the producing gas-liquid ratio as an additional parameter.

The irreversible-energy-loss term was evaluated by back-calculation using field data to determine the pressure gradient from the following equation:

$$f = 7.413\times10^{10}(\rho)(d)^5 \times 100\frac{\left[\left(\frac{dp}{dH}\right)-\rho\right]}{q^2}\times M^2 \qquad (5.71)$$

where f = Fanning-type friction factor

ρ = flowing density, lb/ft^3

dp/dH = pressure gradient, psi/ft

q = oil flow rate, B/D

d = diameter of pipe, ft

The Fanning-type friction was plotted against the numerator of the Reynolds number. The scattering of points in this correlation suggests that an important parameter (or parameters) was neglected. By employing the gas-liquid ratio as an additional parameter, a correction was developed between the Fanning friction factor and the numerator of the Reynolds number for three ranges of gas-liquid ratios when comparing this correlation to that of Poettmann and Carpenter, only a small deviation is found for gas-liquid ratios below 1.5 Mscf/bbl and *qM/d' s* between 15 and 50. This is to be expected because Poettmann and Carpenter correlation was based on data in these ranges. Also, it should be noted that, as the gas-liquid ratio increases, the wall friction decreases. Although the friction factor is decreasing, the pressure gradient increases at high gas-liquid ratios due to a decrease in the flowing density as shown in the following equation:

$$\frac{dp}{dH}=\frac{1}{144}\left[\rho+f\left(\frac{q^2M^2}{7.413\times10^{10}\rho d^5}\right)\right] \qquad (5.72)$$

Therefore, as the gas-liquid ratio increases and the liquid rate decreases, pressure gradients calculated from Equation 5.72 are influenced more and more by the second term of Equation 5.72.

Corrections were developed for the producing gas-liquid ratio. Corrections have been previously developed for in-situ gas-liquid ratios. However, it was found that excellent agreement was obtained by using the producing gas-liquid ratio as a parameter.