# CHAPTER IV

## NUMERICAL RESULTS AND CONCLUSIONS

The formulation which have been derived in Chapter 3, have been used to develop a computer program in FORTRAN 77 language, which is included in Appendix C for practical applications of this study. The program computes deflection, normal and transverse bending moment and twisting moment at any interior point $(\rho,\theta)$ of circular and annular plates with arbitrary combination of boundary conditions and interior columns subject to concentrated loads and uniformly distributed load. Two circular and two annular plates of different boundary conditions have been tested and compared with the results by other investigators and those from a finite element method. In each case Poisson's ratio is taken to be 0.3.

Computation has been performed on a PRIME 9750 computer using double precision arithmetic but this program can run also on an IBM personal computer with little modification on input/output unit specifier.

In the first example, a clamped circular plate under a singular load, P , acting at the center and at location (0.5,0.0), as shown in Fig. 5a, is computed by subdividing the boundary into 36 intervals. The deflections, normal and transverse bending moments of this study, which are plotted against the argument of $\rho$ in Fig. 5b and 5c, are in very close agreement with the analytical results which proposed by Timoshenko and Woinowsky-Krieger [13].

The second example is a uniformly loaded circular plate clamped over a section of the boundary subtending an angle $2\epsilon$ at the plate center, and simply supported over the remainder of the edge as shown in Fig. 6a. The problem is computed by subdividing the boundary into 36 intervals and the results in terms of deflections and normal bending moments are plotted in Figs. 6b and 6c along the diameter of symmetry, for $\epsilon = 45^\circ, 90^\circ$, and $135^\circ$. Corresponding results of deflections and the normal bending moments by Conway and Farnham [2] are also shown for comparison.

A uniformly loaded annular plate, $b/a = 0.25$, with mixed boundary conditions (Fig. 7a) is calculated in the third example by subdividing the outer and inner boundary into 38 and 18 intervals respectively. The deflections and normal bending moments which are plotted against the argument of $\rho$ in Fig. 7b, 7c, 7d and 7e show good agreement with those of Sriswasdi [4].

The last example is purposely presented to show that arbitrary combination of boundary conditions and interior supports such as a uniformly loaded annular plate, $b/a = 0.25$, with three interior columns, as shown in Fig. 8a, may be examined. The results in Figs. 8b, 8c and 8d are computed by subdividing the outer and inner boundary into 36 and 18 intervals respectively. The results when compared with those from a finite element program, SAP IV, in which the symmetrical half of the plate is modeled by 230 trapezoidal elements, are in good harmony. However, while the present method used only 1 min. 34 sec. of PRIME 9750 computer time, the SAP IV consumed 6 min. 8 sec. Moreover, less data preparation is obviously achieved by this type of boundary technique.

In this present study, the boundary element technique has been applied to circular and annular plate with aribitrary combination of boundary conditions and interior columns subjected to concentrated loads and uniformly distributed load. By approximating each unknown function on the boundary by a set of discrete constant values over each interval, the boundary integral equations can be replaced by a set of algebraic equations. Numerical results obtained by the present study are in good agreement with those from other method of approach.

The unknown discrete values of normal slope, normal bending moment and Kirchhoff's shear along the mixed boundary conditions of the plate problems in Example 2 to 4 are plotted in Figs. 9 to 11 respectively. Normal bending moments show a square root singularity at the brink of the clamped supports [3,4], but Kirchhoff's shear cannot be plotted in smooth curves. However, this variation has no appreciable effects on the result at points of the sufficiently large distance away from the edge as discussed in Saint-Venant Principle [14]. To improve this variation, a suitable shape function other than rectangular may be used in the discrete intervals.