CHAPTER 2

CONCEPTUAL FRAMEWORK

Economists and financial market participants often hold quite different points of view about the pricing of assets[3]. Given the assumption of rational behavior and of rational expectations, economists usually believe that the price of an asset must simply reflect market fundamentals, that is to say, can only depend on information about current and future returns from this asset. Deviations from this market fundamental value are taken as evidence of irrationality. On the other hand, financial market participants believe that fundamentals are only part of what determines the price of assets. Extraneous events may well influence the price, is believed by other participants to do so; "crowd psychology" becomes an important determinant of prices.

Oliver J. Blanchard and Mark W. Watson (1982) mentioned that "the economists have overstated their case. Rationality of both behavior and of expectations often does not imply that the price of an asset be equal to its fundamental value. In other words, there can be rational deviations of the prices from fundamental value, rational bubbles".

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[3] Oliver J. Blanchard and Mark W. Watson, "Bubbles, Rational Expectations and Financial Markets," NBER Working Paper series. No. 945 (July 1982) : 1-28.

2.1) Rational Bubbles

Grant McQueen and Steven Thorley (1994) stated that *"Rational speculative bubbles allow stock prices to deviate from their fundamental value without assuming irrational investors*[4]*"*. Investors realize that prices of assets exceed fundamental values, but they believe there is a good probability that the bubble will continue to expand and yield a high return. The probability of a crash is exactly compensated by the probability of a high return; thus, the model shows the rationality of staying in the market despite the overvaluation. They also concluded, "the empirical characteristics of bubbles include a long-run up in price or a long run of positive abnormal returns followed by a crash".

2.1.1) Bubble Model

A simple efficient market condition is that the expected return of an asset is equal to the required return

$$E_t\,[R_{t+1}] \quad = \quad r_{t+1} \qquad (1)$$

where E_t denotes the mathematical expectation given the information set at time t, $R_{t+1} \equiv (p_{t+1} - p_t + d_{t+1})/p_t$, and r_{t+1} is the time-varying required rate of return. In terms of

4 Grant McQueen and Steven Thorley, "Bubbles, Stock Returns and Duration Dependence," Journal of Financial and Quantitative Analysis 29, No.3 (September 1994) : 379-401.

prices, the competitive equilibrium condition in Equation (1) states that the current price equals the expected future price and dividend discounted at the tax-adjusted return required by investors,

(2) $$p_t = \frac{E_t[p_{t+1} + d_{t+1}]}{(1 + r_{t+1})}$$

Solving Equation (2) recursively yields one solution to the equilibrium condition: the *fundamental value* of the asset,

(3) $$p_t^* = \sum_{i=1}^{\infty} \frac{E_t[d_{t+1}]}{\prod_{j=1}^{i} (1 + r_{t+1})}$$

However, **Shiller(1978), Blanchard and Watson(1982), and West(1987),** among others, note that any price of the form,

(4) $$p_t = p_t^* + b_t \qquad \text{where} \qquad E_t[b_{t+1}] = (1 + r_{t+1})b_t$$

is a solution to the equilibrium condition as well. Thus, the market price can deviate from the fundamental value by a rational speculative bubble factor, b_t, if, on average, the factor grows at the required rate of return.

Diba and Grossman (1987), (1988) show that the bubble model in equation (4) rules out negative bubbles since they have to grow more negative over time, yet total stock prices will never be negative. The theoretic potential for positive, but not negative, bubbles suggests that bubble tests should allow for nonlinearity. In contrast to traditional variance bound and time series tests, the duration dependence test developed in the thesis specifically allows for nonlinearity by estimating separate hazard rates for runs of positive and negative returns.

The following rational bubble process, based on **Blanchard and Watson (1982),** allows for bubbles to grow and burst,

$$(5) \qquad b_{t+1} = \frac{(1 + r_{t+1})b_t}{\pi} - \frac{(1 - \pi)a_0}{\pi} \quad \text{with probability } \pi,$$

$$= a_0, \qquad \text{with probability } (1-\pi).$$

In this process, the bubble factor grows by the exact amount needed to compensate investors for the probability, 1- π, that the bubble will crash and the price will revert to the small initial bubble value, $a_0 > 0$. In Blanchard and Watson's original model, the bubble crashes to zero, $a_0 = 0$. The additional of the a_0 term, similar to **West (1988) and Bollerslev and Hodrick (1992)**, facilitates the initial bubble and allows for multiple crashes since bubble cannot restart after they crash completely (see **Diba and Grossman (1987), (1988)**). In order for the Blanchard and Watson model to be consistent with the two traditional characteristics of bubbles, a long run-up in price followed by a crash, the probability of the bubble continuing, π, must be greater than ½. **Blanchard and Watson (1982)**, show that the expected duration of the bubbles is $(1-\pi)^{-1}$; therefore, the restriction that π>1/2 result in the average bubble lasting more than 2 periods.

The rational speculative bubbles model allows for unexpected price changes, $\epsilon_{t+1} \equiv (R_{t+1} - r_{t+1})\, p_t$, from two unobservable sources: unexpected changes in the fundamental value,

(6) $$\mu_{t+1} = p^*_{t+1} + d_{t+1} - (1 + r_{t+1})p^*_t ,$$

and unexpected changes in bubble,

(7) $$\eta_{t+1} = b_{t+1} - (1 + r_{t+1})b_t$$

The observable unexpected price change $\epsilon_{t+1} = \mu_{t+1} + \eta_{t+1}$ equals the sum of the fundamental and bubble changes,

(8) $$\epsilon_{t+1} = \mu_{t+1} + \frac{(1-\pi)}{\pi}((1 + r_{t+1})b_t - a_0),$$ with probability π,

$$= \mu_{t+1} - (1 + r_{t+1})b_t + a_0 ,$$ with probability $(1-\pi)$

As required by the efficient markets condition, the expected value of the total price innovation is zero. However, the probability of the positive innovation or abnormal return can be greater than ½ even if the fundamental innovations are symmetric around zero. This is due to the inherent skewness of the bubble innovations. If the bubble continues, its innovation is positive and small relative to an infrequent but large negative innovation if the bubble bursts. The asymmetry of bubble innovations results in observed excess returns that tend to be positive while the bubble continues, causing autocorrelation and longer runs of positive excess return than expected from a temporally independent series.

For illustrative purposes, assume that μ_{t+1} is unimodal and symmetrically distributed with mean zero. From equation (8), the probability of a negative innovation, is $\lambda_{t+1} \equiv prob\ [\in_{t+1} < 0]$

$$(9)\quad \lambda_{t+1} = \pi F\left[-\frac{(1-\pi)}{\pi}((1+r_{t+1})b_t - a_0)\right] + (1-\pi)F\left[(1+r_{t+1})b_t - a_0\right]$$

where $F(.)$ is the cumulative density function for μ_{t+1}. For value of a_0, and consequently b_t, greater than zero, the observed excess return distributed is negatively skewed so that λ_{t+1} is less than ½. Specially, the probability of a negative innovation decreases with the bubble factor,

$$(10)\quad \frac{\partial \lambda_{t+1}}{\partial b_t} = -(1-\pi)(1+r_{t+1})\left[f\left(-\frac{(1-\pi)}{\pi}((1+r_{t+1})b_t - a_0)\right) - f((1+r_{t+1})b_t - a_0)\right] < 0.$$

With π >1/2, the absolute value of the argument of the second density, *f(.)*, is greater than the first , making the term in square brackets positive. This illustration assumes fundamental innovations are symmetrically distributed. From many numerical analysis shows that the larger the bubble, the smaller the probability of an observed negative price change.

In addition to autocorrelation and skewness, bubble also induce kurtosis by mixing low variance return distributions associated with small bubbles with higher variances as the bubbles grow. Unfortunately, diagnostic tests for the bubbles based on positive autocorrelation, skewness, or kurtosis are inconclusive, even if significant, because fundamental price movements can also be associated with these attributes.

Equation (5) and (10) suggest a more discriminating testable implication about price innovations without directly observable fundamental values. Equation (5) requires that the bubble is explosive, growing in each period that it survives. Equation (10) shows that larger bubble factors result in a lower probability of a negative excess return. As the bubble component grows, it begins to dominate the fundamental component; consequently, negative abnormal returns become less likely, occurring primarily when the bubble crashes. Thus, a log run of positive excess returns suggests the presence of the bubble, and a bubble decreases the probability of a negative abnormal return. Together, Equation (5) and (10) imply that if price contain bubbles, then run of observed positive abnormal returns will exhibit duration dependence with an inverse relationship between the probability of a run ending and the length of the run. Formally, the probability of a negative observed innovation conditional in a sequence of i prior positive innovations, $h_i = \Pr ob(\in_t < 0 | \in_{t-1} > 0, \in_{t-1} > 0, ..., \in_{t-i} > 0, \in_{t-i-1} < 0)$, decreases with i. That is, Equations (5) and (10) imply that $h_{i+1} < h_i$ for all i if bubble are present. This is true even if the bubble crashes probability, (1-π), is constant. Since bubbles cannot be negative, a similar inequality does not hold for runs of negative abnormal returns. Consequently, bubbles generate duration dependence in runs of positive, but not negative abnormal returns.

2.2) Duration Dependence

To test duration dependence, returns are transformed into series of run lengths on positive and negative observed abnormal returns. For example, a return series of four positive abnormal returns followed by three negative, two positives and finally four negative abnormal returns is transformed into two data sets: a set for runs of positive abnormal returns with values of 4 and 2 and a set for runs of negative abnormal returns with values of 3 and 4. The separation of returns into two states is similar in nature to the tests by **Blanchard and Watson (1982), Evans (1986), and Mcqueen and Thorley (1991).**

Formally, the data consist of a set, S_T, of T observations on the random run length, I. A run is defined as a sequence of abnormal returns of the same sign. Thus, I is a positive valued discrete random variable generated by some discrete density function, $f_i \equiv$ Prob$(I=i)$, and corresponding cumulative density function, $F_i \equiv$ Prob$(I<i)$. Define N_i and

P_i as the count of complete runs and partial runs, respectively, of length i in the sample. A partial run may occur in the beginning (left-censored) and at the end (right-censored) of the time period being examined. The density function version of the log likelihood is

$$(11) \qquad L(\theta \,|S_T) = \sum_{i=1}^{\infty} N_i Lnf_i + P_i Ln(1 - F_i),$$

where θ is vector of parameters. The hazard function, $h_i \equiv \text{Prob}(I{=}i \,|\, I \geq i)$, represents the probability that a run ends at i, given that it lasts until i. A hazard function specification describe data in terms of conditional probability in contrast to the density function specification, which focus on unconditional probabilities. The choices between a hazard and density specification depends on the economic question of interest. This article investigates whether the probability that a stock return run continues is conditional on the length of the run; consequently, the hazard specification is appropriate. An additional reason for using the hazard specification is the lack of closed-form multi-parameter discrete density functions. The hazard function is related to the density function by

$$(12) \qquad h_i = \frac{f_i}{(1 - F_i)} \qquad \text{and} \qquad f_i = h\prod_{j=1}^{i=1}(1 - h_i)$$

Using the relationship in Equation (12), the hazard function version of the log likelihood is

$$(13) \qquad L(\theta \,|S_T) = \sum_{i=1}^{\infty} N_i Lnh_i + M_i Ln(1 - h_i) + Q_i Ln(1 - h_i),$$

where M_i and Q_i are the count of completed and partial runs with a length greater that i, respectively. The terms containing P_i and Q_i in the log likelihood (Equation (11) and (13)) are included to incorporate information contained in partial runs and may be ignored in large samples.

To perform tests of duration dependence, a functional form must be chosen for the hazard function. Similar to **Mcdonald, McQueen, and Thorley (1992)**, the tests of duration dependence in this thesis are based on the logistic transformation of the log of i

(14) $$h_i = \frac{1}{1 + e^{-(\alpha + \beta Lni)}}$$

The log-logistic function transforms the unbounded range of $\alpha + \beta \text{Ln}(i)$ into the (0,1) space of h_i, the conditional probability of ending a run. The null hypothesis of no bubbles implies that the probability of a run ending is independent of the prior returns or that positive and negative abnormal returns are random. In term of the model, the null hypothesis of no duration dependence is that $\beta = 0$ (constant hazard rate or geometric density function). The bubble alternative suggests the probability of a positive run ending should decrease with thee run length or that the value of the slope parameter, β, is negative (decreasing hazard rate).

Tests are performed by substituting Equation (14) into (13) and maximizing the log likelihood function with respect to α and β. Under a suitable description of the data set, hazard function can be estimated as logit regression. In the logit transformation, the independent variable is the log of the length of the run and the dependent variable is 1 (0) if the run ends (does not end) in the next period. The Likelihood Ratio Test (LRT) of $\beta = 0$ is asymptotically distributed χ^2 with one degree of freedom.