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APPLICATION OF LINEAR MULTI-AGENT SYSTEM APPROACH
TO DESIGN BUILDING TEMPERATURE CONTROL SYSTEM

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A Thesis Submitted in Partial Fulfillment of the Requirements
for the Degree of Master of Engineering Program in Electrical Engineering
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ระบบควบคุมอุณหภูมิอาคารเป็นระบบเชื่อมโยงระหว่างกันขนาดใหญ่ที่มีความต้องการใช้พลังงานสูง โดยทั่วไป ระบบควบคุมอุณหภูมิอาคารอธิบายด้วยระบบเชิงเส้นเวลายืนยงโดยอาศัยอุปกรณ์ไฟฟ้าเสมือน ได้แก่ ตัวต้านทาน และตัวเก็บประจุ ในมุมมองของทฤษฎีกราฟ ระบบควบคุมอุณหภูมิอาคารมีแบบจำลองเป็นระบบเชิงเส้นหลายตัวแทน ภายใต้ทอพอโลยีการสื่อสารแบบไม่กำกับ ระบบควบคุมอุณหภูมิอาคารมีเป้าหมายหลัก เพื่อตามรอยแนววิถีของอุณหภูมิอ้างอิง ซึ่งสมมูลกับการจัดการให้ความคลาดเคลื่อนการตามรอยเป็นศูนย์ในเวลาจำกัด ดังนั้น ระบบควบคุมอุณหภูมิอาคารจึงเหมาะกับกรอบงานของการควบคุมแบบกระจายตัว การควบคุมแบบแยกศูนย์ และการควบคุมแบบรวมศูนย์ ในวิทยานิพนธ์นี้ เราจะมุ่งความสนใจเพื่อเปรียบเทียบการออกแบบ 3 วิธี ได้แก่ การควบคุมแบบเอกฉันท์กระจายตัว การควบคุมแบบเรียนรู้วนซ้ำแยกศูนย์ และการควบคุมแบบเรียนรู้วนซ้ำรวมศูนย์ เริ่มแรก เราประยุกต์ใช้การควบคุมแบบเอกฉันท์กระจายตัวกับระบบควบคุมอุณหภูมิอาคาร โดยแก้ปัญหาการจัดสรรทรัพยากรที่มีเงื่อนไขบังคับแบบยากของสัญญาณควบคุม ต่อไป เราประยุกต์ใช้การควบคุมแบบเรียนรู้วนซ้ำแยกศูนย์โดยใช้อนุพันธ์ของสัญญาณความคลาดเคลื่อนตามรอย สุดท้าย เราจัดรูปแบบการควบคุมแบบเรียนรู้วนซ้ำรวมศูนย์ เป็นปัญหาการหาค่าต่ำสุดของฟังก์ชันต้นทุนกำลังสอง ภายใต้เงื่อนไขบังคับสัญญาณควบคุมขาเข้า ปัญหานี้มีชื่อเรียกอีกว่า การควบคุมแบบเรียนรู้วนซ้ำกำลังสอง เราประยุกต์แนวทางวิธีสลับทิศทางของตัวคูณกับการออกแบบการควบคุมแบบเรียนรู้วนซ้ำกำลังสอง และหาคำตอบเชิงวิเคราะห์ของสัญญาณควบคุมในแต่ละรอบของการวนซ้ำ เราเสนอสมบัติการลู่เข้าของวิธีสลับทิศทางของตัวคูณ และระเบียบวิธีของการออกแบบ ผลลัพธ์เชิงตัวเลข แสดงให้เห็นประสิทธิผลของระเบียบวิธีเหล่านี้

ภาควิชา วิศวกรรมไฟฟ้า	ลายมือชื่อนิสิต ....
สาขาวิชา วิศวกรรมไฟฟ้า	ลายมือชื่อ อ.ที่ปรึกษาหลัก ....
ปีการศึกษา 2559	ลายมือชื่อ อ.ที่ปรึกษาร่วม ....

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KEYWORDS: BUILDING TEMPERATURE CONTROL SYSTEM; MULTI-AGENT SYSTEM; DISTRIBUTED CONSENSUS CONTROLLER; DECENTRALIZED ITERATIVE LEARNING CONTROL; QUADRATIC ITERATIVE LEARNING CONTROL; ALTERNATING DIRECTION METHOD OF MULTIPLIERS.

TUYNH VAN PHAM: APPLICATION OF LINEAR MULTI-AGENT SYSTEM APPROACH TO DESIGN BUILDING TEMPERATURE CONTROL SYSTEM. ADVISOR: PROF. DAVID BANJERDPONGCHAI, Ph.D., CO-ADVISOR: ASST. PROF. DINH HOA NGUYEN, Ph.D., 52 pp.

Building temperature control system (BTCS) is a large-scale interconnected system with a high demand of energy consumption in building. BTCS is generally described by linear time-invariant system using electric-analogous devices such as resistors and capacitors. In view of graph theory, it is modeled as linear multi-agent system (MAS) subjected to undirected communication topology. The main goal of BTCS is to track the reference temperature trajectory which is equivalent to make the tracking error go to zero in finite time. Hence, BTCS properly suits with framework of distributed control, decentralized control, and centralized control. In this thesis, we focus on comparison of three design methods, namely, distributed consensus controllers (DCC), decentralized iterative learning control (ILC), and centralized ILC. First, the DCC is applied to BTCS by solving resource allocation problem with hard control input constraints. Next, we apply decentralized ILC using the derivative of tracking error (D-type). Lastly, centralized ILC design is formulated as minimization problem with a quadratic cost function subject to control input constraints. This problem is also called Q-ILC. We apply the Alternating Direction Method of Multipliers (ADMM) approach to Q-ILC design and derive analytical solution of control input in each iteration. The convergence property of ADMM and Q-ILC algorithm are given. Numerical results are provided to illustrate the effectiveness of these algorithms.

Department:	Electrical Engineering	Student's Signature	....
Field of Study:	Electrical Engineering	Advisor's Signature	....
Academic Year:	2016	Co-Advisor's Signature	....

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LIST OF FIGURES

NOTATIONS

Indices

i, j	Index of room (agent)
k, m	Index of ILC and ADMM approach
t	Index of time slot

Parameters

P	Total power supply for all rooms in building (kW)
N	Number of room in building
T_N	Number of samples

Symbols

$\mathbb{R}$	The set of real number
$\mathbf{1}_{T_N N}$	The special $T_N N \times 1$ vector consisting of all elements equal to 1
$\mathbf{0}_{T_N N}$	The special $T_N N \times 1$ vector consisting of all elements equal to 0
I_N	The identity matrix
$\lVert H \rVert$	Euclidean norm
$\lambda_{\max}(.)$	The maximum eigenvalue

Variables

$\mathbf{x}_{i,k+1}$	State of room i at $k+1$ iteration in supper vector form
$\mathbf{u}_{i,k+1}$	The control input of room i at $k+1$ iteration in supper vector form
$\mathbf{y}_{i,k+1}$	The output system of room i at $k+1$ iteration in supper vector form
$\mathbf{e}_{i,k+1}$	The error between the desired reference trajectory with real output of room i at $k+1$ iteration in supper vector form

Acronyms

ILC	Iterative Learning Control
ADMM	Alternating Direction Method of Multipliers
MAS	Multi-Agent System
DCC	Distributed Consensus Controller

CHAPTER I

INTRODUCTION

1.1 Literature Review and Motivation

Today, energy consumption in buildings where the thermal heating ventilation and air conditioning (HVAC) systems account for the majority of total energy consumption is about 40% of total energy usage in United States whereas the building in Thailand consumes 42% total electricity usage [1]. Therefore, the modeling and design of HVAC systems with energy-saving target by binding consumption capacity, the heat source heating/cooling input are very important and urgent. Hitherto, many approaches have been proposed as reported in [2]. We will study one part of HVAC, e.g., building temperature control systems (BTCSs) since BTCS is a typical case of a large-scale system in which the thermal dynamics of individual rooms or zones are interconnected, the typical problem is the interconnection term among rooms, and the elements of BTCS always consume a large quantity of energy in a building. This is the first main reason that problems of BTCS have attracted many researchers so far. And the second one is many control approaches could be developed for large-scale interconnected dynamics of BTCS. For instance [3], the main contribution of the paper is the application of complex distributed control heat source for multi-room building in such a way that the room temperature satisfies the requirements of the occupants. To control this plant, first the authors tried to model a multi-agent system by applying thermal dynamics of room and wall. And to make it easier for design techniques and make it effective, the obtained controller should be distributed. Meaning that each room has a temperature controller, the controller can communicate with one another to achieve the common goal, i.e. the temperature of the room is the same and track the setpoint temperature signal. This is the meaning of the MAS system. Furthermore, to be suitable for practical implementation the system should be considered in discrete time domain. With the setting and resolve the issue, this paper launched three major contributions [3] as follows. First, using distributed consensus algorithms to control "heating energy coordination" to get the temperature track setpoint, and assure the constraint of "heat energy generation". Second, distributed consensus controller is designed to regulate "heat flow". The purpose of this work is to make the coordination error go to zero. Last, study "thermal behavior of the wall thermal capacitor": the other existing works discussed only the temperature of room as the main controlled variable. Therefore, the authors want to focus on the uncontrolled variable: the wall's temperature with the purpose is to use the variance between wall's temperature and the equilibrium point temperature equals to zero.

In papers [2], [4] by German Obando *et al.*, the authors use a distributed control system based on Multi-agent to control the room temperature of the building. In this paper, the authors use the graph as a tool to describe the link and interactive agents (the room's temperature) on the MAS (the building temperature). Moreover, the author has used the passivity theory to demonstrate the effectiveness of this method for heating power distribution to the rooms in the building even in the absence of a reference temperature reached by the worst load condition. By using the variance between steady state temperature errors compared to the target set criteria, so we want to reformulate to master this method completely, thereafter we hope to find a week points to improve and expand the results of this approach.

In paper [5] by John T. Wen *et al*., the authors was referring to the use of ILC for building temperature control. Results showed that the system outputs are satisfactory, sticking to the set values, but with the added problem of increase in energy consumption. Although the authors have proposed solutions to reduce energy consumption, how we choose the coefficient correction factor to save energy is still difficult. Therefore there must be specific solutions to solve these problems. We can extend this problem by varying desired temperature profiles and using time varying cost functions. The reason is that the electricity price during peak hours is higher than that used in the other hours of the day. Therefore ILC can be used to control the system in off-peak hours to minimize the energy consumption during peak hours, but the aim is to ensure the temperature of the building to stick the desired values of the user.

In paper [6] by Yi Zheng *et al*., with the proposed algorithm of Impact-Region Optimization based on DMPC applied to the control the temperature of the 4-room building, this method has improved the output quality of the entire closed-loop system without increasing the complexity of the agents in the network connection. However, the analysis and how to ensure the stability of the system when using this control method the authors have not mentioned. This problem can be expanded in the future.

From summary and discussion above, we decide to research the topic: "Application of Linear Multi-Agent System Approach to Design Building Temperature Control System". The scheme of our work is described in the following figure.

Hitherto, there have been several approaches to design distributed consensus controllers for BTC systems, e.g. [2, 7]. Those approaches can be mainly categorized into two sub-classes, namely, distributed linear (distributed Model Predictive Control, distributed consensus control) and distributed nonlinear methods (Replicator Dynamics). However, their common control purposes are to satisfy the users' requirement (comfort criterion) and to save the power consumption. More specifically, when the classical PID controller is used, [7] pointed out that it is implemented at the point quite far from optimal resource allocation because of the inefficiency of allocating energy. Usually, when non-conventional strategies are employed, the control objectives are not satisfied due to the lack of theoretical analysis. In addition, most of existing strategies contain saturation conditions resulting in strong transient responses which may harm the system actuators. To overcome the aforementioned drawbacks when applying non-conventional control strategies, [7] utilized the replicator dynamics controller (RDC). Due to the reason that distributed framework is the most suitable for large-scale interconnected system, e.g., BTCS is a typical case of this class system, in our first main work, we decide to design DCC for BTCSs described by linear time-invariant large-scale interconnected dynamics.

The BTCS is modeled by linear multi-agent systems subjected to undirected communication topology using graph theory. The main advantages of DCC is that first, it is fully distributed, then we do not need to divide the whole system into smaller subsystems. Second, it can satisfy some certain input constraints including limited power supply and user comfort. In this research, we formulate the controller design of a multi-agent system and apply DCC to a four-connected room model. In particular, we design a local controller for each room in the building that communicates with other local controllers to achieve some global goals. The main task of local controllers is to achieve objective that each room temperature can track its own desired reference temperature.

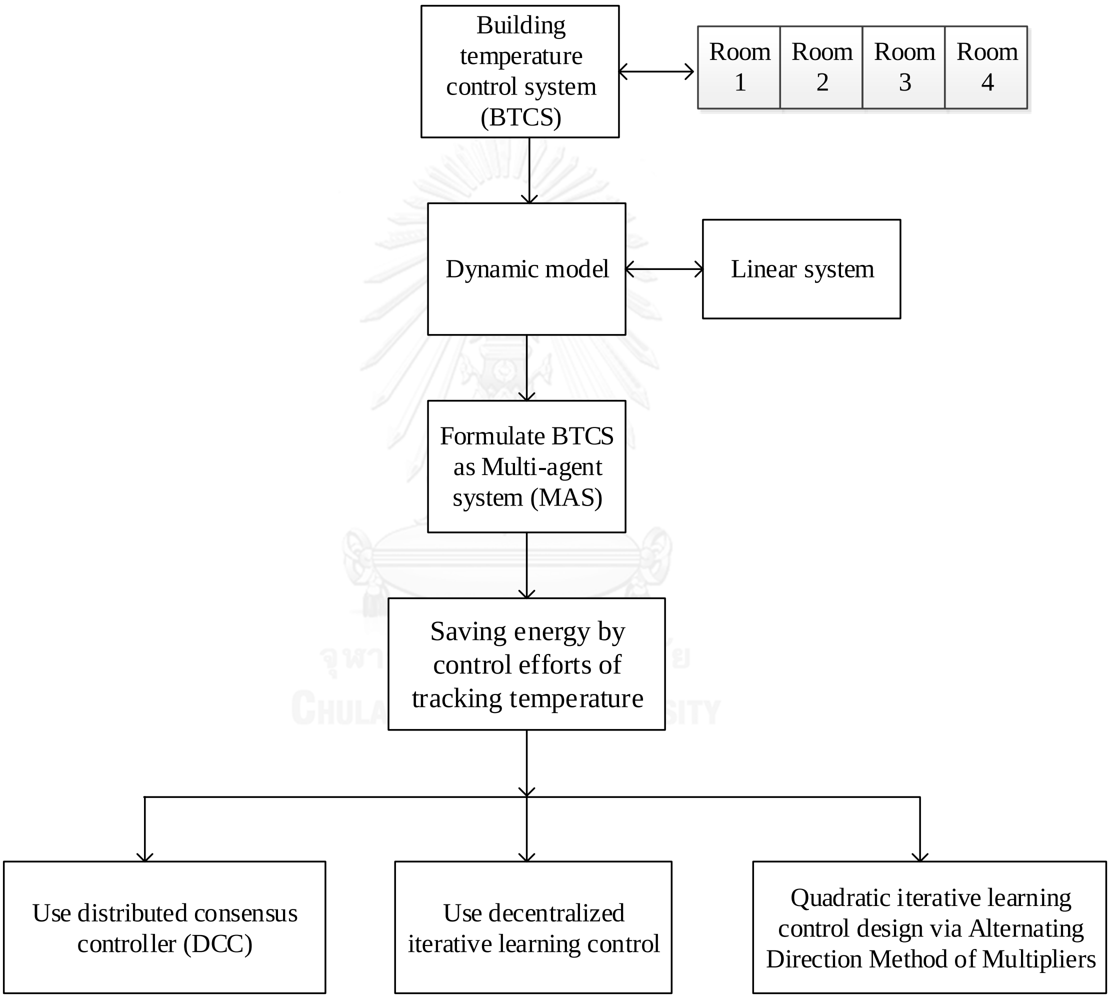

Figure 1. 1. Scheme of proposed research work.

On the other hand, when the daily desired temperature profiles of individual rooms or zones in a building are specified, ILC seems to be the most suitable control strategy. It has been well-known that ILC works well for processes with repetitive dynamics and finite-time operations [8, 9] where traditional control methods are often ineffective. The key idea of the ILC is that implementation information of previous iterations as a control input and error between the desired output trajectory and the system output is used to improve the performance quality of the system in the next iteration. The advantages of the ILC are the simple structure and the ability to track the reference input without needing an accurate model of the object. Therefore, in second of our main work, we consider the design of a decentralized ILC for the BTC systems described by linear time-invariant large-scale interconnected dynamics, which is an effective strategy treating the interconnection problem. In particular, we will design a local ILC, which means that the local input and local output of each subsystem are used to design local ILC [10] for each room/zone in the building. Local ILC does not have any connection with others. The main task of local controllers is to achieve local control objectives that each room/zone temperature tracks its own desired reference by controlling each room with power supply independently while the interconnected terms, i.e., the heat flow among rooms, are moved. By applying the methodology developed in Wu, et al. [11], and [12], and extended in Li, et al. [13], we utilize the λ-norm to analyze the convergence condition and derive the learning gain Γ_i in each subsystem.

On the third of our main task, we solve an optimization problem with quadratic criteria performance subject to hard control inputs constraints based on Alternating Direction Method of Multipliers (ADMM). The main advantage of the proposed scheme is that it gives good performance despite existing the interconnection among rooms. In this task, the ILC design is formulated as computing control input updating. Moreover, we apply ILC to a four-connected room model. It aims that the temperature of each room in building is controlled by a central controller. In particular, we will design ILC, which means that the local inputs and local outputs of subsystems are used to design control strategy [10] for whole rooms/zones in the building. ILC requires a central coordinator which collects, processes, saves, and sends data to heater located in each room [14]. The main task of ILC is to achieve local control objective that each room/zone temperature tracks its own desired reference with dependent power supply while the interconnected terms, i.e., the local states/outputs affected by the heat flow among rooms, are used to model the influence with the adjacent rooms/zones.

Finally, a numerical example of building temperature control will be provided to illustrate the effectiveness of above control schemes that we are considering and to compare it with other control schemes' in term of power consumption and time response.

1.2 Objectives

1. To improve performance and energy efficiency of heating system for Building Temperature Control System (BTCS).
2. To apply smart controllers that minimum power and still give comfortable temperature to the occupant.

1.3 Scope of thesis

1. Studying distributed framework, a most suitable control technique for large-scale interconnected system, and its possibility to combine with iterative learning control (ILC) to derive novel control design methods.

2. Designing efficient distributed consensus controller (DCC) for large-scale linear interconnected system.
3. Designing applicable decentralized ILC scheme for linear large-scale interconnected system.
4. Designing applicable optimization ILC via Alternating Direction Method of Multipliers (ADMM) for linear large-scale interconnected system.
5. Applying the aforementioned algorithm to real model of BTCS.

1.4 Methodology

1. A review and survey of large-scale interconnected system, e.g. one typical case is BTCS, using distributed framework as well as ILC.
2. Model BTCS as a linear large scale interconnected time-invarying system by using electric-analogous.
3. Formulate the controller design problem of a multi-agent system using graph theory and employing distributed resource allocation algorithm when considering the power supply constraints.
4. Separate the whole system into a number of subsystems, then formulate, and apply decentralized D-type ILC algorithm for each subsystem.
5. Formulate the ILC design problem as computing control input updating via Alternating Direction Method of Multipliers (ADMM).
6. Apply designed algorithms to dynamical model of BTCS.
7. Analyze the output response of the feedback system and control input.
8. Choose good parameters, setup these parameters to the system and compare performance of designed controllers with the existing method.
9. Write conference paper for publication and thesis.

1.5 Expected Results

1. Reduce power consumption and improve output performance when comparing with existing approaches.
2. Provide a systematic way to design efficient algorithms for large-scale interconnected systems.

1.6 Achievements

This thesis have achieved the results as follows:

1. We have proposed the formulation with the main design objectives by using three typical control techniques, e.g., DCC, decentralized ILC, centralized ILC via ADMM, in large-scale setting.
2. We provide the implementations when using these control techniques that give the good performance for BTCS.

1.7 Summary

In chapter 1, we briefly discussed overview of BTCS approaches and the existing research related to this topic. Then, we provided the scope of this thesis and the methodology to fulfill the expected outcomes, and the achievements are listed.

CHAPTER II

BACKGROUND

This chapter presents the definitions, basic background, and properties of mathematic tool related to design problem utilized in this thesis. First, Section 2.1 presents the building temperature control system. The definition and properties of multi-agent system are presented in Section 2.2.

2.1 Building Temperature Control System

According to [2, 4], how to describe the mathematical expression of the thermal dynamics for a building correctly is quite difficult because the temperatures of the rooms in the building are not homogeneous. However, for simplicity we split the system into several major subsystems which are corresponding to the rooms in the building and assuming their temperatures is uniform and the heat flow occurs only from one room to the other. When considering ways to control any system, we classify the input-output variables as the uncontrolled variables and controlled variables. Specifically, in thermal dynamics of a building, the temperature is the controlled variable, and heat fluxes can be used in every room of the building. The heat fluxes among rooms depend on the temperature differences between them and these disparities are characterized by physical variables as conduction, convection, and radiation. The thermal dynamics of a building depends on its own topology, which includes the number of the rooms and layout space of each room. This is the reason for the differences between thermal dynamic building models. The two main components that build up a building thermal dynamics are rooms and walls. Hence, let us investigate a multi-room building, of which each room is surrounded by a limited number of walls and arranged into various types. Consider a simple and popular case in construction, all the rooms of a multi-room building are arranged in a row.

There are many ways to model the building thermal model. One typical way is using the conservation of energy and mass to control the volume of a thermal multi-zone building (e.g., 4 rooms). For simplifying, we use the resistor to model the wall's thermal model and the capacitor to model the room's thermal model as follows. According to [15], the physical meaning of thermal resistance is to model the heat flow among room because of the deviation temperature $Q = \frac{\Delta T}{R}$ where Q (W), ΔT ($^{\mathrm{O}}\mathrm{C}$) and R $\left(\frac{^{\mathrm{O}}\mathrm{C}}{\mathrm{W}}\right)$ denote the value of heat pass through resistance, the deviation temperature, and thermal resistance. The potentiality to store the heat of wall is presented by thermal capacitance and expressed by $Q = C\dfrac{d\Delta T}{dt}$, where C $\left(\dfrac{\text{Wh}}{^{\text{o}}\text{C}}\right)$ is the thermal capacitance.

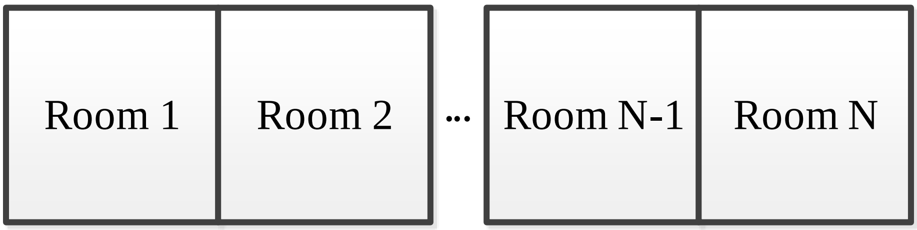

Figure 2. 1. An example of a building with N rooms in series [10].

Applying the "energy balance" principle, we have the thermal dynamic equation for each room in building as follows

$$C_i\dot{T}_i = \sum_{j\in N_i} R_{ij}^{-1}(T_j(t) - T_i(t)) + R_{ia}^{-1}(T_i^a(t) - T_i(t)) + u_i(t), \tag{2.1}$$

where $T_i(t)$, and $T_i^{ref}(t)$ are the room's temperature and its temperature reference of the i^{th} room, $u_i(t)$ is the heating power that is supplied by heater, C_i is the thermal capacitance of i^{th} room, R_{ij} is the thermal resistance of the wall to connect i^{th} room with j^{th} room, R_{ia} is the thermal resistance of the wall to connect i^{th} room with environment, $\alpha_{ii} = (R_{ii-1}^{-1} + R_{ii+1}^{-1} + R_{ia}^{-1})$ is the total of each inverse-thermal resistance of i^{th} room, $T_i^a(t)$ is the ambient temperature that i^{th} room connects directly, N_i is the set that consists of all the rooms which can contact to the i^{th} room directly.

Note that the thermal dynamic of room that is represented by analogy circuit is often changing over time interval due to the characteristic of these devices and heat flows among rooms. Hence, to simplify we assume that a thermal dynamic system is a time-invariant system, and all the parameters are constant in the time domain. The linear state-space model can be described as follows:

$$\dot{x}(t) = Ax(t) + B_u u(t) + B_a T^a(t), \tag{2.2}$$

where $x(t) = [T_1(t), T_2(t), ..., T_N(t)]^{\text{T}}$ is the state vector,

$u(t) = [u_1(t), u_2(t), ..., u_N(t)]^{\text{T}}$ is the control input vector,

$T^a(t) = [T_1^a(t), T_2^a(t), ..., T_N^a(t)]^T$, and

$x^{ref}(t) = [T_1^{ref}(t), T_2^{ref}(t), ..., T_N^{ref}(t)]^T$

are the vector of ambient temperature contacted directly with rooms, and the reference temperature, respectively. Let $e_i(t) = x_i(t) - x_i^{ref}(t)$ denote the tracking error. The BTCS in our consideration is a class of large-scale interconnected systems consisting of some physically divided subsystems where each subsystem connects with other subsystems by their states. The system matrices[1] are

$$A = \begin{bmatrix} \frac{-\alpha_{11}}{C_1} & \frac{R_{12}^{-1}}{C_1} & 0 & 0 & \cdots & 0 \\ \frac{R_{21}^{-1}}{C_2} & \frac{-\alpha_{22}}{C_2} & \frac{R_{23}^{-1}}{C_2} & 0 & \cdots & 0 \\ 0 & \frac{R_{32}^{-1}}{C_3} & \frac{-\alpha_{33}}{C_3} & \frac{R_{34}^{-1}}{C_3} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & \cdots & \cdots & \frac{R_{NN-1}^{-1}}{C_N} & \frac{-\alpha_{NN}}{C_N} \end{bmatrix};$$

$$B_u = \begin{bmatrix} \frac{1}{C_1} & 0 & 0 & 0 & \cdots & 0 \\ 0 & \frac{1}{C_2} & 0 & 0 & \cdots & 0 \\ 0 & 0 & \frac{1}{C_3} & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & \frac{1}{C_N} \end{bmatrix}; B_a = \begin{bmatrix} \frac{R_{1a}^{-1}}{C_1} & 0 & 0 & 0 & \cdots & 0 \\ 0 & \frac{R_{2a}^{-1}}{C_2} & 0 & 0 & \cdots & 0 \\ 0 & 0 & \frac{R_{3a}^{-1}}{C_3} & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & \frac{R_{Na}^{-1}}{C_N} \end{bmatrix}.$$

2.2 Multi-Agent System

"MAS is a multi-agent system in which each agent is an entity that senses its environment and acts upon it, and all the agents in this system are interacted to exchange information with each other" [16].

According to [17], MASs have been attracted by a lot of researchers by its practical applications in many fields such as system biology, power grids, and energy management systems. They can be analyzed and designed under the context of MASs. The main characteristic of MASs is that we can achieve the global targets by executing the local measurement and control at each agent and combining of agents using that local information. One of the most important issue in MASs is the

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[1]The parameters of BTCS can be found in [4]

consensus problem due to its attraction in both the theoretical and practical issues. The reference [2] says that the consensus control for MASs has been mightily interested, and there are a lot of results on this issue. The main purpose of the consensus algorithm is to design the distributed formalities so that a finite amount of state variables corresponding to each agent in MASs can achieve the correspondence.

According to [17-19], the graph theory is a useful tool to describe the interconnection among agents in MASs. Hence, the introduction of some basic concepts of the graph theory, e.g., a weighted graph, set of node, set of edges, adjacency matrix, etc., is necessary and mentioned in this section. Define $G = \{V, E, A\}$ be a weighted of a directed graph to represent the information structure in Multi-Agent Systems including *N* agents. In which, each node and each edge in G represent an agent and the link to connect with other agents, respectively. $V = \{V_1, V_2, ...V_N\}$ and $E = \{(V_i, V_j) : V_i, V_j \in V\} \subseteq V \times V$ are the vertices set and the edges set of *G*, respectively. There exists an edge if i^{th} agent can receive information from j^{th} agent, it is represented by an edge $e_{ij} \in E$. We use the symbol $N_i \triangleq \{j : e_{ij} \in E\}$ to represent the neighbours of vertex set of i^{th} agent. Let a_{ij} is an element of a non-negative symmetric adjacency matrix A of graph G, the calculation of a_{ij}, i.e., $a_{ij} > 0$ if $e_{ij} \in E$ and $a_{ij} = 0$ if $e_{ij} \notin E$. Moreover, there does not exist an edge connecting to a node with itself, i.e., $a_{ij} = 0$. The in-degree of vertex i is denoted by $\deg_i^{in} \triangleq \sum_{j \in N_i} a_{ij}$ thus $D = diag\{\deg_i^{in}\}_{i=1,\ldots,\, N}$ is the in-degree matrix of G. Continually, Laplacian matrix L of G is introduced. This can be calculated by $L = D - A$. The Laplacian matrix is singular, i.e., $L\mathbf{1}_N = 0$. We consider the balanced condition of graph G by using the concept of out-degree $\deg_i^{out} \triangleq \sum_{j \in N_i} a_{ji}$. G becomes balanced graph if $\deg_i^{in} = \deg_i^{out}$, $\forall\ i = 1, \ldots N$. A set of edges that has a tail at i^{th} agent and head at j^{th} agent is called a directed path connecting vertices i and j in graph G. The graph G is proved to contain a spanning tree if there is a node root leading to a driven path to all other nodes. Remember that an undirected graph ($a_{ij} = a_{ji}$) is a special case of a directed graph. An undirected graph G is connected if and only if $rank(L) = N - 1$ [19].

2.3 Summary

This chapter briefly discusses basic concept of mathematical notation and background relevant to building thermal dynamic model and multi-agent system. First, we provide some mathematical notations and their definitions used this thesis. Next, the description of BTCS are introduced. BTCS is modeled as a linear large-scale interconnected system by using electric-analogous devices such as resistors and capacitors. Finally, we discuss the principle and typical properties of MAS by employing graph theory.

CHAPTER III

DISTRIBUTED CONSENSUS CONTROLLER DESIGN

This chapter presents the principle of DCC design and its application for BTCS. This chapter starts with the problem formulation of DCC design that links directly with application for BTCS in Section 3.1. Then, Section 3.2 present the detail concepts of consensus algorithm and its design procedure. Lastly, in Section 3.3, the numerical example when applying DCC for BTCS and comparison with existing method, namely, replicator dynamic control (RDC) are given to illustrate the effectiveness of proposed control techniques.

3.1 Problem Formulation

In this section, we use all ideas in reference [2] to apply for our model. However, it is noted that the main differences between the idea of this chapter and [2] are on the model of BTCS and on the inverse-barrier functions to guarantee the input constraints. The purpose of BTCS is to keep the temperature of the room close to the desired value. Thus, to control all the room temperatures to be exactly the same is not necessarily needed in this case. To achieve control objectives, assuming each room has one actuator (heating/cooling), and they are bound to physical and mechanical properties depending on the type of heating system installed for the building. The actuator is provided by one total heating power *P* located in one particular location in the building. From which, *P* is distributed to the heater located in each room. Consumed heating power $u_i(t)$ at the i^{th} room, and the remaining power $u_{N+1}(t)$, i.e., the difference between *P* and $u_i(t)$ in all rooms, satisfies

$$\sum_{i=1}^{N} u_i(t) + u_{N+1}(t) = P\,. \tag{3.1}$$

In which the thermal storage capacity of the actuator located at each room is within the following limits

$$u_{i\_min} \le u_i \le u_{i\_max}, \forall i = 1,...,N\,, \tag{3.2}$$

$$u_{N+1}(t) \ge 0\,. \tag{3.3}$$

If *P* is large enough, this is a necessary condition for the room temperature to stick setpoint temperature using the individual controller.

In fact, when the BTCS is operating the worst case may happen, i.e., *P* is not enough making the control strategy fail to work well because the power allocation does not work at the optimal point, particularly in the worst case load. Therefore, a good alternative to solve the above problem is to use the "dynamic resource allocation strategy". The goal of this solution is to provide $u_i(t)$ among rooms with the assumption that each of the building's occupants received an equal heat source. References [2], [20] and [21] have proposed distributed resource allocation (RA) which is formulated as optimization problem

$$\min \sum_{i=1}^{N+1} \left(\frac{1}{2} e_i^2(t) + \varepsilon_i \beta_i(u_i(t))\right) \tag{3.4}$$

subject to (2.2), (3.1), and (3.3).

where

$$\beta_i(u_i(t)) = \left[\frac{1}{(u_{i\_max} - u_i(t))} + \frac{1}{(u_i(t) - u_{i\_min})}\right]$$

is called inverse-barrier function. DCC is designed to track the reference temperatures while satisfying given constraints. It is the solution of optimization problem (6).

3.2 Consensus Algorithm

Quality indicators proposed in [2] show that this is a good strategy to solve the problems mentioned above, which leads to an agreement between $e_i(t), \forall i = 1,...,N$ in the rooms. The main purpose of [2] is to use DCC based on a MAS, where each agent has a linear dynamic and performs a consensus protocol to coordinate decisions. To perform one consensus protocol, the authors in [2] designed the lead equalization, with the assumption that the i^{th} agent manages the function $f_i(e_i(t), u_i(t))$ and uses only local information

$$f_i(e_i(t), u_i(t)) = e_i(t) + \varepsilon_i \varphi_i(u_i(t)), \tag{3.5}$$

where

$$\varphi_i(u_i(t)) = \left[\frac{1}{(u_{i\_max} - u_i(t))^2} + \frac{1}{(u_i(t) - u_{i\_min})^2}\right] \tag{3.6}$$

denote the derivative of a inverse-convex barrier function $\beta_i(u_i(t))$, in which ε_i is positive real number for tuning $\varphi_i(u_i(t))$. The task of the second term in (3.5) is to keep the value of $u_i(t)$ within the allowed value range. The characteristic of $\varphi_i(u_i(t))$ is that it is a strictly increasing continuous function when (3.2) is satisfied; $\varphi_i(u_i(t)) \to -\infty$ as $u_i(t) \to u_{i\_min}$; $\varphi_i(u_i(t)) \to +\infty$ as $u_i(t) \to u_{i\_max}$. ε_i is chosen in such a way that the effect of $\varphi_i(u_i(t))$ is minimized when $u_i(t)$ is located far from the boundary in the feasible domain. The value of ε_i converges to zero, provided that it satisfies criterion of $\varphi_i(u_i(t))$ We can think that $u_{N+1}(t)$ is a virtual agent and represented by a vertex $N+1$ in communication topology. Hence, the coordination of agents is represented by the graph $G_c = \{V \cup (N+1),\ E_c,\ A_c\}$, where E_c and A_c correspond to the set of edges and weighted of the adjacency matrix of G_c, respectively. We notice the fact that G_c has the same all the same properties of G as mentioned earlier.

To measure the success of the allocation process, [2] uses the variance of the difference between x_i^{ref} and temperature at steady state $x_{i,ss}$ denoted by Var and expressed by (3.7). Let $e_{i,ss} = x_i^{ref} - x_{i,ss}$ denote the tracking error at steady state.

$$Var = \frac{1}{N}\sum_{i=1}^{N}\left(e_{i,ss} - \overline{e_{i,ss}}\right)^2 , \tag{3.7}$$

where $\overline{e_{i,ss}}$ is the mean value of $e_{i,ss}$, the mean is over tracking errors of rooms, it is not over the tracking error of one room in time domain. The control strategy tries to make *Var* small as much as possible. The faster the speed of convergence of f_i and variance is, the better performance of DCC will be. Reference [2] have executed resource allocation (RA) for satisfying RA constraints from (2.2) to (3.3), acting of control signals $u_i(t)$, driving the system to reach output consensus, i.e., $f_i(e_i(t), u_i(t)) = 0$. The authors of [2] have proposed the consensus protocol for each agent as follows:

$$\dot{u}_i = \gamma \sum_{j \in N_i} a_{ij}[f_j(e_j(t), u_j(t)) - f_i(e_i(t), u_i(t))], \tag{3.8}$$

which means that each agent uses only the local information and receives its neighbours' information, γ is the tuning parameter ($\gamma > 0$). We rewrite Eq. (3.8) in the following compact form

$$\dot{u}_e = -\gamma L_c f , \tag{3.9}$$

where $u_e = [u_1(t), \ldots, \; u_{N+1}(t)]^T$, L_c is the Laplacian matrix of G_c, $f = [f_1(e_1(t), u_1(t)), \ldots, f_N(e_N(t), u_N(t)), \; f_{N+1}(u_{N+1}(t))]^T$. The changing of the control signals $u_i(t), \; \forall i = 1, \ldots, N$ will be affected by the choice of γ. If γ is very large, $u_i(t)$ will change abruptly, and the output response will have a large overshoot. Based on analysis of the output response, we may choose a suitable range for γ. Let

$$f_{N+1}(u_{N+1}(t)) = -\varepsilon_{N+1}\frac{1}{(u_{N+1}(t) - u_{N+1\_min})^2} \tag{3.10}$$

denote the derivative strictly increasing continuous function when $u_{N+1}(t) \in [u_{N+1\_min}; +\infty)$; $f_{N+1}(u_{N+1}(t)) \to -\infty$ as $u_{N+1}(t) \to u_{N+1\_min}$, $f_{N+1}(u_{N+1}(t)) \to +\infty$ as $u_{N+1}(t) \to +\infty$.

Under some constraints from (3.1) to (3.3), it is shown by [2] that if $\sum_{i=1}^{N+1} u_i(0) = P$ then $\sum_{i=1}^{N+1} u_i(t) = P,\ \forall t \geq 0$ (see proof of proposition 4.1 in [2]).

MAS with the kinematic of agents described by expression (3.9) reaches the output consensus if the following condition is fulfilled.

$$\lim_{t\to\infty} \left| f_i(e_i(t), u_i(t)) - f_j(e_j, u_j(t)) \right| = 0,\ \forall i, j{=}1,\ldots,\ N, \tag{3.11}$$

which means that $f_i(e_i(t), u_i(t))$ will converge to be the same value at steady-state, thus the two following terms will be consensus, i.e.,

$$e_i(t) = e_j(t) \text{ and } \varepsilon_i \varphi_i(u_i(t)) = \varepsilon_j \varphi_j(u_j(t)).$$

Therefore, the closed-loop system will be stable. Reference [2] has proved that control system is asymptotically stable at equilibrium point $x_i^*(t),\ u_N^*(t),\ u_{N+1}^*(t)$ if graph G_c is connected and $\sum_{i=1}^{N+1} u_i(0) = P$, by using a suitable Lyapunov function candidate *V* satisfying $\dot{V} < 0$ (see proof of proposition 4.3 in [2]). If the controller operates well and *P* is adequate, then $e_i(t) = e_j(t) = 0$. This means that $x_i(t)$ converges to $x_i^{ref}(t)$ for all $i = 1,\ldots,N$.

The main task is to design DCC for coordination network. It means to find $u_i(t)$ for each agent described by (3.9) in such a way that $e_i(t),\ \forall i = 1,\ldots,N$ reaches consensus while they exchange information through graph G_c.

Next, we propose the design procedure for DCC.

Step 1: Model

Given the position of u_{N+1} in G_c, compute L_c.

Step 2: Set the initial conditions

Given *P*, u_{i_min}, and u_{i_max}, set the initial value of control inputs $u_i(0) = \dfrac{P}{N+1}$, and choose ε, γ.

Step 3: Consensus controller

Set the consensus controller, which is described by (3.9).

To find the control law u_e, we consider the coordination state-space linear dynamic system described by

$$\begin{cases} \dot{u}_e = -\gamma L_c f \\ y_e = C_e u_e \end{cases}. \tag{3.12}$$

On the other hand, we can solve the problem of the closed-loop control system in order to achieve the output in time domain when combining the thermal dynamic model (2.2) with DCC (3.9) by solving the ordinary differential equation (ODE) with initial conditions $x_i(0)$ and $u_i(0)$.

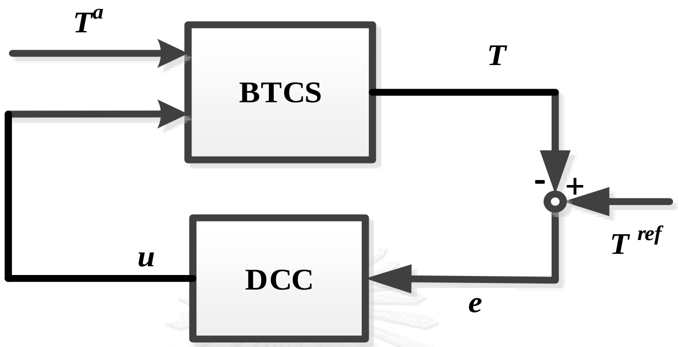

Figure 3. 1. Block diagram of a closed-loop BTCS [22].

3.3 Numerical Example

3.3.1 System Descriptions

To illustrate the output response when applying DCC, we consider four rooms (N = 4) temperature control with almost all of the parameters and description are taken from [4, 22], and shown in Figure 2. 1.

Table 3. 1. The parameters of BTCS [4].

Parameter	**value**	**Parameter**	**value**
Room 1		**Room 4**	
C_1 $(\frac{\text{Wh}}{^{\circ}\text{C}})$	7.476×10^4	C_4 $(\frac{\text{Wh}}{^{\circ}\text{C}})$	7.476×10^4
R_{12}^{-1} $(\frac{^{\circ}\text{C}}{\text{W}})$	35.392	R_{43}^{-1} $(\frac{^{\circ}\text{C}}{\text{W}})$	35.392
R_{1a}^{-1} $(\frac{^{\circ}\text{C}}{\text{W}})$	159.067	R_{4a}^{-1} $(\frac{^{\circ}\text{C}}{\text{W}})$	159.067
Room 2		**Room 3**	
C_2 $(\frac{\text{Wh}}{^{\circ}\text{C}})$	7.476×10^4	C_3 $(\frac{\text{Wh}}{^{\circ}\text{C}})$	7.476×10^4

R_{21}^{-1} $(\frac{^{\circ}\text{C}}{\text{W}})$	35.392	R_{32}^{-1} $(\frac{^{\circ}\text{C}}{\text{W}})$	35.392
R_{23}^{-1} $(\frac{^{\circ}\text{C}}{\text{W}})$	35.392	R_{33}^{-1} $(\frac{^{\circ}\text{C}}{\text{W}})$	35.392
R_{2a}^{-1} $(\frac{^{\circ}\text{C}}{\text{W}})$	123.675	R_{3a}^{-1} $(\frac{^{\circ}\text{C}}{\text{W}})$	123.675

When adding u_5 on communication topology next to room 4, we obtain the following G_c as follows.

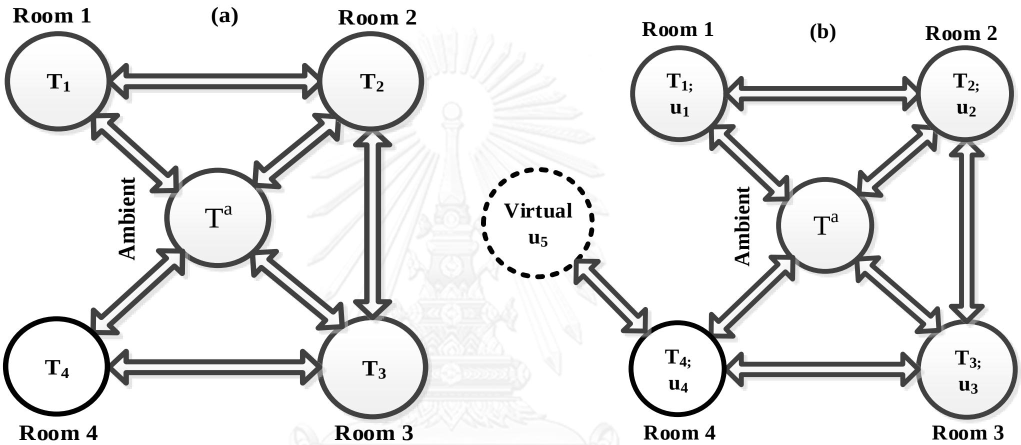

Figure 3. 2. a) Four connected rooms with associated graph represent thermal interconnections in BTCS and b) Communication topology when adding a virtual agent $u_{N+1}(u_5)$.

The Laplacian matrix corresponding to Figure 3. 2b is as follows:

$$L_c = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix}$$

3.3.2 Numerical Results

We assume that the ambient temperature equals to 10°C. The desired reference temperature at room 1, room 2, room 3, and room 4 is 23°C, 22°C, 21°C, and 20°C, respectively. The maximum of *P* is equal to 6.6 kW, which will supply to heaters. We choose $u_{i\_min} = 0$ kW; $u_{i\_max} = 3$ kW; $\gamma = 1.1$, and $\varepsilon = 100$. With these conditions, the numerical results are shown in Figure 3. 3-Figure 3. 5.

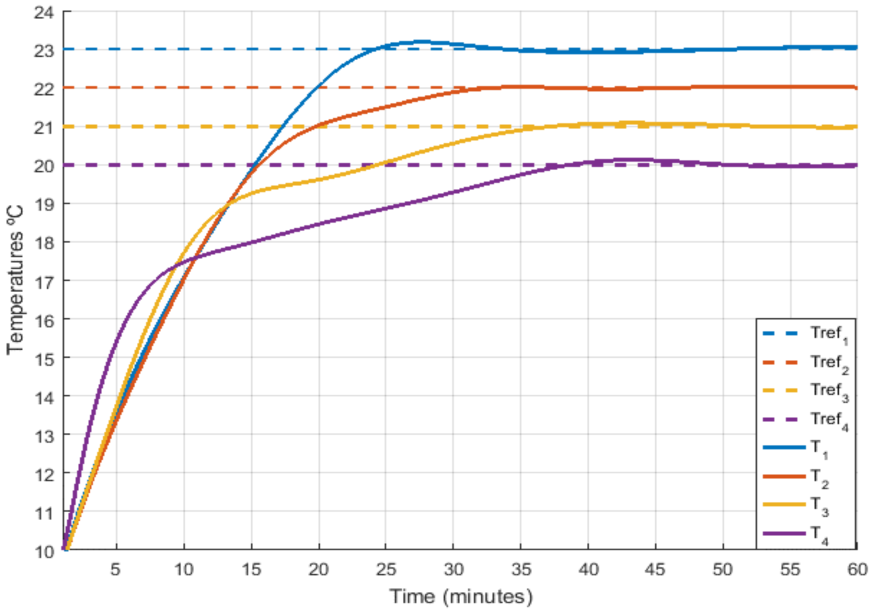

Figure 3. 3. Room's temperature when applying Distributed Consensus Controller.

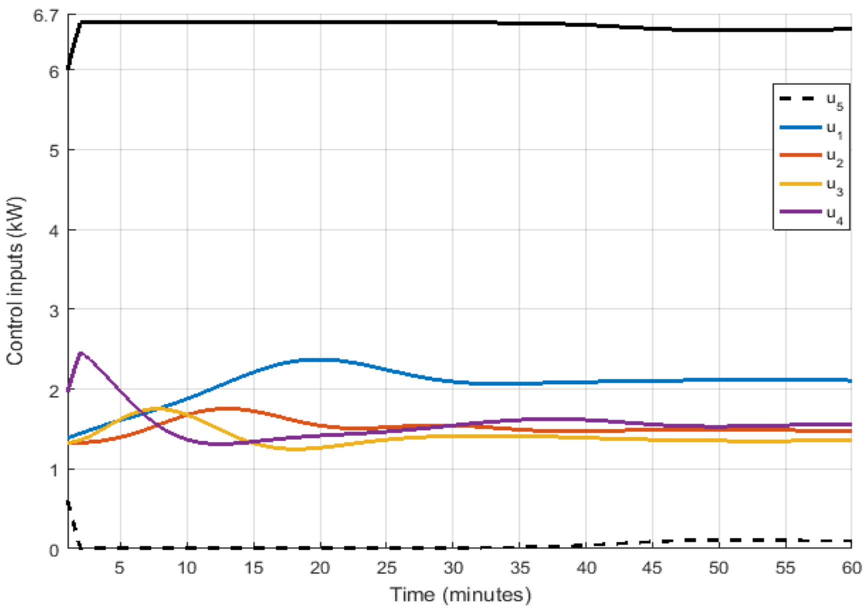

Figure 3. 4. Control signals when applying Distributed Consensus Controller.

Figure 3. 3 shows that the room temperatures track their setpoints without overshoot. Figure 3. 4 provides the control input, i.e., the power supply for each room and the total consumed power. It also depicts that control inputs satisfy constraints (3.1)(3.2), and (3.3). Figure 3. 5a shows the value of $f_i(e_i(t), u_i(t))$, which converges to zero at steady-state. It implies that the closed-loop system is stable. Figure 3. 5b shows the value of tracking error which converges to zero because P supply is enough. Figure 3. 5c depicts the value of $\varphi_i(u_i(t))$ to penalize control inputs. Figure

3. 5d demonstrates the effectiveness of DCC which makes variance go to zero at steady-state.

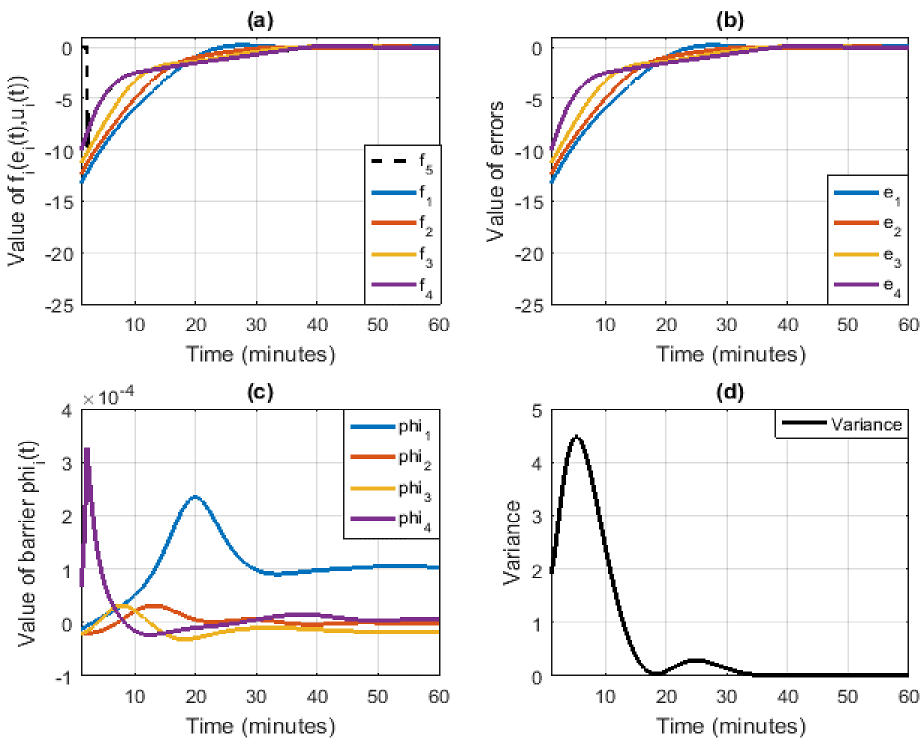

Figure 3. 5. Value of $f_i(e_i(t),u_i(t))$, $e_i(t)$, $\varphi_i(u_i(t))$ and variance.

Table 3. 2. Comparison of performance between RDC and DCC.

Controller / Criteria		RDC	DCC	Change (%)
Settling time (min)	Room 1	58.228	44.777	-23.10
	Room 2	66.456	55.443	-16.57
	Room 3	70.886	64.827	-8.55
	Room 4	72.215	70.642	-2.18
Peak of power supply (kW)		6.8	6.6	-2.94

We compare the results of RDC and DCC. Numerical results show that RDC gives longer settling time and its control input fluctuates with maximum magnitude of approximately 3.6 kW (see Fig. 6 in [7]). This saturated control signal may seriously harm the actuator. Meanwhile, DCC gives a better response with a smaller settling time. It is observed that with a DCC the settling time of room 1 output decreases by about 23% and for room 4, about 2% (compare Fig. 6 of [7] and Figure 3. 3 of this chapter). Moreover, the DCC needs less consumed power than that of RDC. The DCC's total power has a negligible fluctuation (only 0.2 kW). It is good not only for response of room temperature, but also for actuators. Performance of both control techniques are compared in Table 3. 2. It ensures the advantages of distributed consensus algorithm.

Next, we show the experimental results when using derivative of inverse-barrier function and varying γ in Table 3. 3. We observe that with small and large γ the stability of the system can be maintained. More specifically, for BTCS model, $\gamma \in [0.01;\ 1.1]$ would give better results, namely, less settling time and reduction of peak of power supply. For convenience, we omit the output response and control input. All the room temperatures reach their references, respectively, with longer settling time. In addition, in these cases, the control inputs oscillate with large magnitude, and the output response have large overshoots. Moreover, if γ is very large, the output responses take much longer settling time. Finally, we choose $\gamma = 1.1$ and the tracking error of system can be well maintained. The output responses have a small overshoot and less settling time. It reflects the best performance of the system.

Table 3. 3. Comparison of performance when varying γ.

Criteria \ DCC		$\gamma = 0.01$	$\gamma = 0.1$	$\gamma = 1.1$	$\gamma = 10$
Settling time (min)	Room 1	127.908	159.05	44.777	200.25
	Room 2	61.963	126.065	55.443	200.35
	Room 3	60.82	97.073	64.827	200.183
	Room 4	86.285	115.332	70.642	200.3
Peak of power supply (kW)		6.6	6.6	6.6	6.6

3.4 Summary

This chapter presents the design of DCC for BTCS. DCC is designed based on MAS framework and we employ resource allocation algorithm. Comparing between DCC and RDC [7], the performance of DCC is better in terms of less power consumption and faster time response.

CHAPTER IV

DECENTRALIZED ITERATIVE LEARNING CONTROL DESIGN

Iterative Learning Control (ILC) is a control algorithm whose key feature is that the information of the previous iterations such as the control input and the error between the desired output trajectory and the system output is used to improve the performance quality of the system in the next iteration [10, 23]. According to [24], decentralized iterative learning control is one type of non-centralized controller. The main idea of designing this scheme is to separate the whole system into a number of subsystems. Once above works are completed, we can divide the big problem into several sub-problems and solve them individually and easily. Hence, the separated calculation would be more efficient, and the optimal solution can be achieved with faster convergence rate than dealing with the whole main problem.

The main difference between the distributed configuration and the decentralized configuration. In distributed scheme, some local controllers can share information together while in decentralized scheme, there is no shared information among local controllers. Consequently, the distributed scheme has better performance than the decentralized one in time response but its costs of communication and computation are larger [24]. In the second work, we aim to design decentralized ILC and investigate the power consumption of the decentralized scheme. The design and implement in decentralized ILC will be considered next.

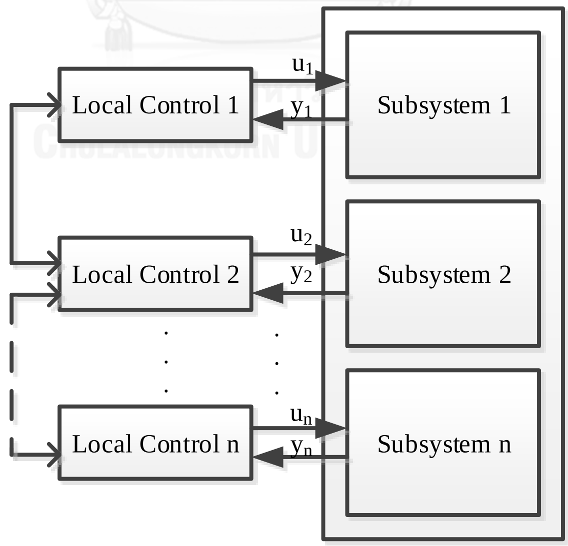

Figure 4. 1. Distributed scheme [20].

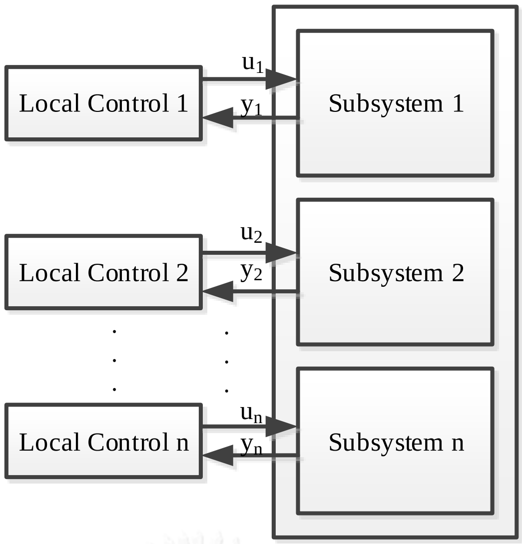

Figure 4. 2. Decentralized scheme [20].

4.1 Problem Formulation

Consider a linear large-scale interconnected system S including N interconnected-subsystem $S_i,\ i=1,\ldots,N$ expressed by the following equation

$$\begin{cases} \dfrac{dx_i(t)}{dt} = A_i x_i(t) + B_i u_i(t) + \sum\limits_{j\in N_i} A_j x_j(t) \\ y_i(t) = C_i x_i(t) \end{cases}, \tag{4.1}$$

where $x_i(t)\in\mathbb{R}^{n_i},\ u_i(t)\in\mathbb{R}^{m_i},\ y_i(t)\in\mathbb{R}^{l_i}$ correspond to the state vector, control input vector, output vector of i^{th} agent, respectively. A_i is called the state matrix, B_i is the input matrix, C_i is the output matrix of i^{th} agent. These matrices are constant matrices. A_j, which is an unknown constant matrix of appropriate dimension is represented interconnection between i^{th} agent and j^{th} agent.

For each subsystem $S_i,\ i=1,\ldots,\ N$, we assume that there exists a local desired output trajectory $y_i^{ref}(t)\in\mathbb{R}^{l_i}$ is defined in a finite- time interval $t\in[t_0,T]$. We define the local tracking error between the local desired output trajectory and the real output error of each subsystem as follows $e_i(t)\triangleq y_i^{ref}(t)-y_i(t)$, where $e_i(t)\in\mathbb{R}^{l_i}$ .The main goal is to determine the decentralized ILC rules for subsystems with unknown desired initial state $x_i^{ref}(t_0)$ such that the local tracking error goes to zero when the iteration number approaches infinity, i.e.,

$$\lim_{k\to\infty}\left\|e_{i,k}(t)\right\| = \lim_{k\to\infty}\left\|y_{i,k}^{ref}(t)-y_{i,k}(t)\right\| = 0,\ \forall t\in[t_0,T_N].$$

It means that we can find a series of control input $u_{i,k}(t)$ and initial state value $x_{i,k}(t_0)$ in the next iteration satisfying $\lim_{k\to\infty} \| e_{i,k}(t) \| = 0$ and $\lim_{k\to\infty} \| x_{i,k}^{ref}(t_0) - x_{i,k}(t_0) \| = 0$ by using an arbitrary initial control input values $u_{i,0}(t)$ and an arbitrary initial state $x_{i,0}(t_0)$ at the first iteration $k = 0$.

4.2 Assumptions

Assumption 1. For each subsystems $S_i,\ i = 1,\ldots,N$ the desired local output trajectory $y_i^{ref}(t)$ is continuous differentiable vector function on $t \in [t_0, T]$ [11].

Assumption 2. The matrix $C_i B_i$, $(i = 1,\ldots,\ N)$ is full row rank [11].

The meaning of Assumption 1 is to ensure that the output trajectory of each subsystem can track a continuously desired output trajectory. This assumption makes sure that we can apply the D-type ILC to control each subsystem. The decentralized ILC rules for each subsystem would be feasible by using Assumption 2. This assumption is a criterion to derive the learning gain to guarantee the closed-loop system to be asymptotically stable. We will further discuss in the next section.

4.3 Algorithm

There are many kinds of ILC such as D-type ILC, PD-type ILC, and PID-type ILC. In this work, we investigate D-type ILC for each local controller with the reason that it is possible to implement decentralized ILC using the local control input and output data. In [25], with D-type ILC, local tracking objective in each interconnected subsystem can be easily solved. That is to say, we can overcome the issue caused by the unknown interconnected terms and insufficiency state information from other subsystems.

The control objective is to make the output system tracking a desired output reference from any arbitrary initial input and initial state, and the kinetic behavior can treat the various parameter of the system [25]. To achieve this objective, a D-type ILC algorithm is chosen to derive control inputs in each iteration. The mathematical updating input algorithm proposed by Arimoto *et al.* [25] can be expressed as follows.

$$u_{i,k+1}(t) = u_{i,k}(t) + \Gamma_i \frac{d}{dt} e_{i,k}(t), \tag{4.2}$$

where $t = 1,\ldots,T_N$ denote a sampling time, Γ_i is the learning gain matrix of i^{th} local controller, $u_{i,k}(t)$, and $\frac{d}{dt} e_{i,k}(t)$ are the control input and the derivative output error, i.e., $e_{i,k}(\text{t}) = y_{i,k}^{ref}(t) - y_{i,k}(t)$ of i^{th} subsystem at iteration k, respectively.

To implement this algorithm, we assume an initial control input at the first iteration

$k = 0$, i.e., $u_{i,0}(t)$. It means that at the initial iteration, $u_{i,0}(t)$ has the same value for all sampling time t. The main idea of this algorithm is to find the control input $u_{i,k}(t)$ at each sampling time t of each iteration k, so after the initial iteration condition, control input $u_{i,k}(t)$ will be computed by (4.2). The main advantage of D-type ILC algorithm is that it does not require accurate model while ensuring the tracking when the algorithm is repetitively applied. In other words, the good tracking can be satisfied by applying a simple algorithm. Moreover, the D-type ILC is the most suitable updating rule for reference tracking among decentralized systems [26]. Most of the existing approaches are assumed that initial states are the same at all iterations. Recently, some research investigates a way to alleviate the above hypothesis by proposing the initial state learning algorithm described as follows

$$x_{i,k+1}(t_0) = x_{i,k}(t_0) + B_i \Gamma_i e_{i,k}(t_0), \tag{4.3}$$

where $x_{i,0}(t_0)$ is an initial state at the first sampling time of the initial iteration.

We can see that the decentralized ILC (4.2) and (4.3) use only available actions of local measurement $y_{i,k}(t)$. Then only information of input matrix B_i and output matrix C_i is utilized to design the learning gain. It does not require information of dynamic matrix A_i then ILC can work well for uncertainty model [10], [27].

The stopping criteria for iterative learning procedure are given as follows.

$$\|e_k\| \le \varepsilon_{\mathrm{dilc}}, \tag{4.4}$$

$$k = iter\_\max\ , \tag{4.5}$$

where $\varepsilon_{\mathrm{dilc}}$ denotes a error tolerance selected by the designer, and $iter\_\max$ is the maximum number of iteration.

4.4 Convergence Analysis

In this section, we analyze convergence condition using the ILC algorithm as discussed earlier, and then propose a way to design learning gain Γ_i for each controller. Main theorem is as follows Wu [11, 12], and Li *et. al* [13].

Consider large scale interconnected systems (4.1) which satisfies assumption 2. Given desired local output trajectory $y_i^{ref}(t),$ which satisfies assumption 1, over the finite time interval $t \in [t_0, T]$, by employing the decentralized ILC law described by (4.2) and the initial state learning control law described by (4.3), then the local output error $e_i(t)$ of each subsystem can be guaranteed to asymptotically converge to zero, i.e., for each $i = 1,\ldots,N$ and any $t \in [t_0, T_N]$,

$$\lim_{k\to\infty} \| e_{i,k}(t) \| = \lim_{k\to\infty} \| y_{i,k}^{ref}(t) - y_{i,k}(t) \| = 0,\ \forall t \in [t_0, T], \tag{4.6}$$

if there exists an iterative learning gain control matrix Γ_i such that

$$|| I_i - C_i B_i \Gamma_i || < 1,\ i = 1,\ldots,N. \tag{4.7}$$

The existence of such ILC matrix gain is ensured by the following theorem.

Theorem [27]: Assume the existence of the product of matrix $C_i B_i$, there always exists a learning gain Γ_i such a way that (4.7) holds.

Proof [27]: From assumption 2 that $C_i B_i$ is a full column rank, then employing Moore-Penrose pseudo-inverse, $C_i B_i$ is invertible, and there exist a pseudo-inverse

$(C_i B_i)^{\dagger} = (C_i B_i)^T [(C_i B_i)(C_i B_i)^T]^{-1}$, which makes

$(C_i B_i)(C_i B_i)^{\dagger} = I$. Then, if we choose $\Gamma_i = q_i (C_i B_i)^{\dagger}$, where q_i is a positive real number, then $|| I_i - C_i B_i \Gamma_i || = 0$ when $q_i = 1$.

Relating the effectiveness of q_i, the value q_i affects the value of Γ_i, then it affects the magnitude of $|| I_i - C_i B_i \Gamma_i ||$. The larger the value of q_i is, the smaller the value of $|| I_i - C_i B_i \Gamma_i ||$ will be. Hence, if the value of q_i is very close to 1, the value of the corresponding norm $|| I_i - C_i B_i \Gamma_i ||$ is small. When the above occurs, the convergence speed of tracking error is very fast. If q_i is close to 0, the opposite occurrence will happen.

The convergence of decentralized ILC rule (4.2) is not affected by the physically interconnected term. However, the transient response is affected by the interconnected term [25].

Next, we will describe how to choose the learning gain to satisfy the convergence condition and to make the system asymptotically stable. Design procedure of decentralized ILC scheme consists of 4 steps. Employing this design procedure, we obtain ILC which fulfills convergence requirement. We observe that when (4.7) holds, it is possible to design local controller using the local information of a subsystem model and ignoring the interconnected terms [26]. We will propose the design procedure for D-type decentralized ILC.

Step 1: Learning gains Γ_i.

By solving the inequality (4.7), we can obtain the feasible learning gain Γ_i [18] expressed by

$$\Gamma_i = q_i (C_i B_i)^{\mathrm{T}} [(C_i B_i)(C_i B_i)^{\mathrm{T}}]^{-1}, \tag{4.8}$$

where q_i is a positive number and $0 < q_i < 1$.

After choosing diagonal learning gains Γ_i, verify whether the inequality (4.7) is satisfied or not.

Step 2: Set the initial conditions

Set $k := 0$, given the initial value of control input $u_{i,k}(t) = u_0$, and the initial states $x_{i,k}(0) = x_0$.

Implement the 1st iteration, measure output $y_{i,0}(t)$ and calculate errors $e_{i,0}(t)$.

Step 3: Solve iterative learning problem

Update control signal $u_{i,k+1}(t)$ according to (4.2), and compute initial states $x_{i,k+1}(0)$ using (4.3).

Implement the 2nd iteration, apply $u_{i,k+1}(t)$ and $x_{i,k+1}(0)$ to the system and measure the outputs $y_{i,k+1}(t)$ and compute the errors $e_{i,k+1}(t)$. Then, reset the initial conditions.

Step 4: Check the stoping criteria

If (4.4) or (4.5) is true, then, stop the iteration, else, set $k := k+1$ and return to step 3.

4.5 Numerical Example

4.5.1 Numerical Conditions

To illustrate the output response when applying decentralized ILC, we consider the building temperature control consisting of four-room ($N = 4$) temperature control with four independent electrical heaters and almost all of the parameters and descriptions taken from [7], and shown in [10]. We assume that the ambient temperature equals to 10°C. The total power *P* is located in one place in building, which will supply for each of the heaters placed in each room.

We choose the initial condition of state vector and the initial control input as follows:

$$u_{1,0}(\mathrm{t}) = u_{2,0}(\mathrm{t}) = u_{3,0}(\mathrm{t}) = u_{4,0}(\mathrm{t}) = 0;$$
$$x_{1,0}(0) = x_{2,0}(0) = x_{3,0}(0) = x_{4,0}(0) = 10.$$

Let the sample time be 1 minute, the number of samples is 501. The desired reference output is

$$y_i^{ref} = \begin{cases} (10^o C, 10^o C, 10^o C, 10^o C)^T, & t \in [0,100) \cup [400,500] \\ (0.13t - 3,\ 0.12t - 2,\ 0.11t - 1,\ 0.1t)^T, & t \in [100,200) \\ (23^o C, 22^o C, 21^o C, 20^o C)^T, & t \in [200,300) \\ (-0.13t + 62, -0.12t + 58, -0.11t + 54, -0.1t + 50)^T, & t \in [300,400) \end{cases} \quad (4.9)$$

For stopping criteria, we choose

$$\varepsilon_{\mathrm{dilc}} = 10^{-2} \text{ and } iter\_\max = 16000.$$

For the decentralized ILC given in (4.2), and (4.3) with the convergence condition (4.7)we choose the learning gains Γ_i, i = 1, 2, 3, 4, and then check the convergence condition described by (4.7) as follows.

$$q_i = 0.9, \text{ then } \Gamma_i = 1210.5, \left\| I_i - C_i B_i \Gamma_i \right\| = 0.2 < 1.$$

It is noted that all convergence conditions are satisfied.

4.5.2 Numerical Results

The simulation run with maximum 16000 iterations, and the tracking errors of output system converge to zero at iteration $k = 5000$ demonstrated in the following Figures.

Applying D-type decentralized ILC control law (4.2) and (4.3) to BTCS, we get the output responses of a closed-loop system gradually track the desired reference temperatures shown in Figure 4. 3.

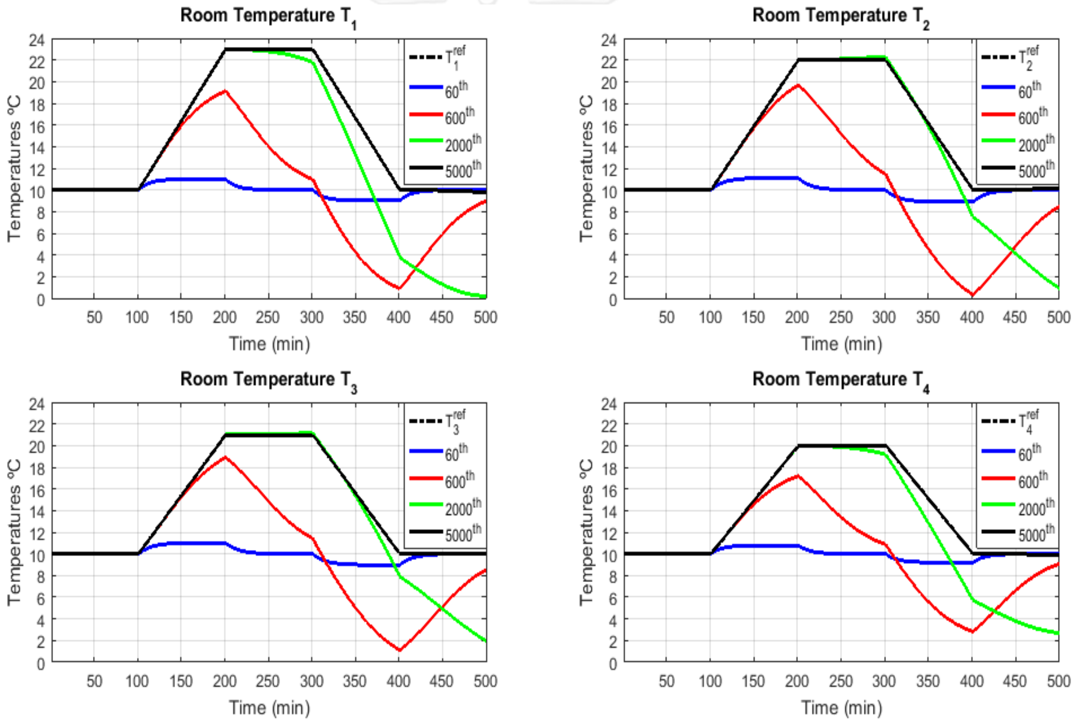

Figure 4. 3. Room temperature when using $q_i = 0.9$.

We clearly see that the output responses tend to track the desired reference trajectory when the number of iteration increases. After 5000 iterations, the output responses track their desired reference with no overshoot. Moreover, Figure 4. 4 and Figure 4. 5 illustrate the control input converges with small fluctuation. This control makes the tracking error of temperature output asymptotically converge to zero. In Figure 4. 6, we vary q_i and observe that the smaller value of q_i is, the slower convergence speed of output response will be. When q_i is small $(q_i = 0.3)$, it needs at least 15000 iterations to track the desired references. The control objective is fulfilled when using the decentralized ILC despite the complexity of the interconnected term. In addition, we can easily set up and implement the algorithm since the D-type decentralized ILC is formulated as the first-order derivative of tracking errors and this algorithm can reduce the complexity of the system by treating the interconnected term.

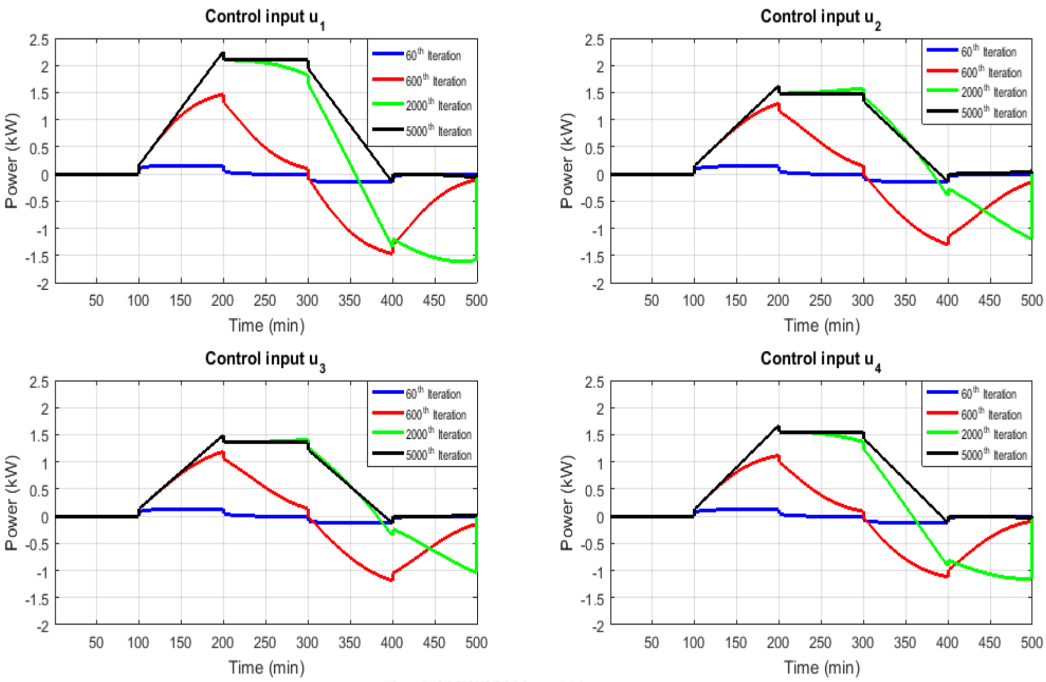

Figure 4. 4. Power supply when using $q_i = 0.9$.

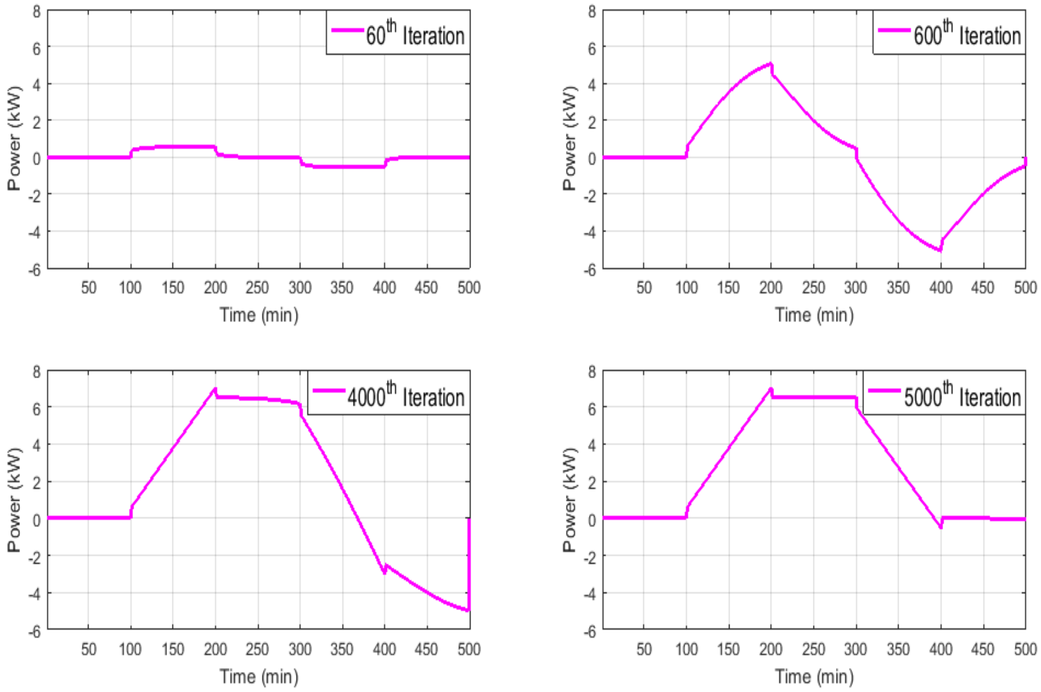

Figure 4. 5. Total power supply when using $q_i = 0.9$.

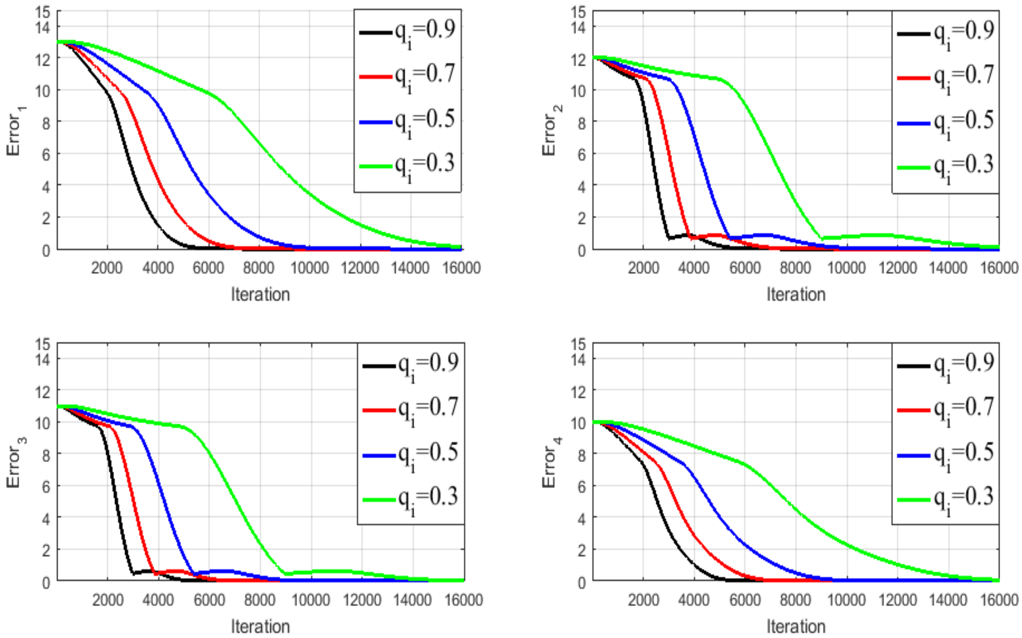

Figure 4. 6. Errors between the desired and output temperature when varying q_i .

Next, we will simulate the trapezoidal response and show the comparison of output responses and control inputs between two controllers. The decentralized ILC uses $q_i = 0.9$ and a maximum of 5000 iterations while DCC uses the best tuning parameters $\gamma = 1.1$ and $\varepsilon = 100$ referred in [2, 22]. The comparison is specified by the magnitude of steady-state error of temperature output subject to a ramp-input. This error is the difference between the desired reference temperature and the output response at steady-state. Furthermore, we will compare the peak of the control signal at steady state and transition state.

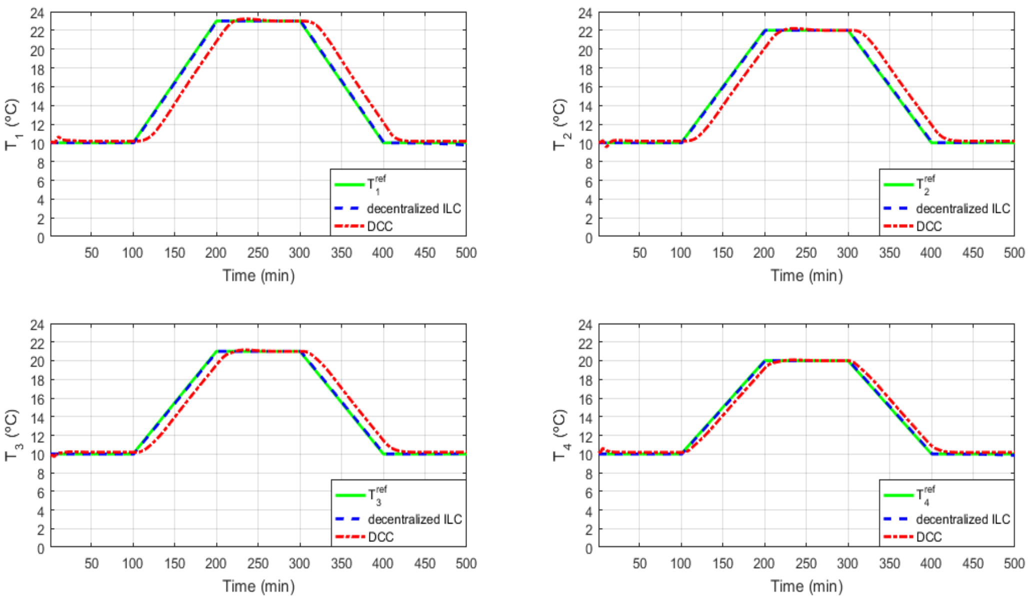

Figure 4. 7. Comparison of output responses between decentralized ILC and DCC.

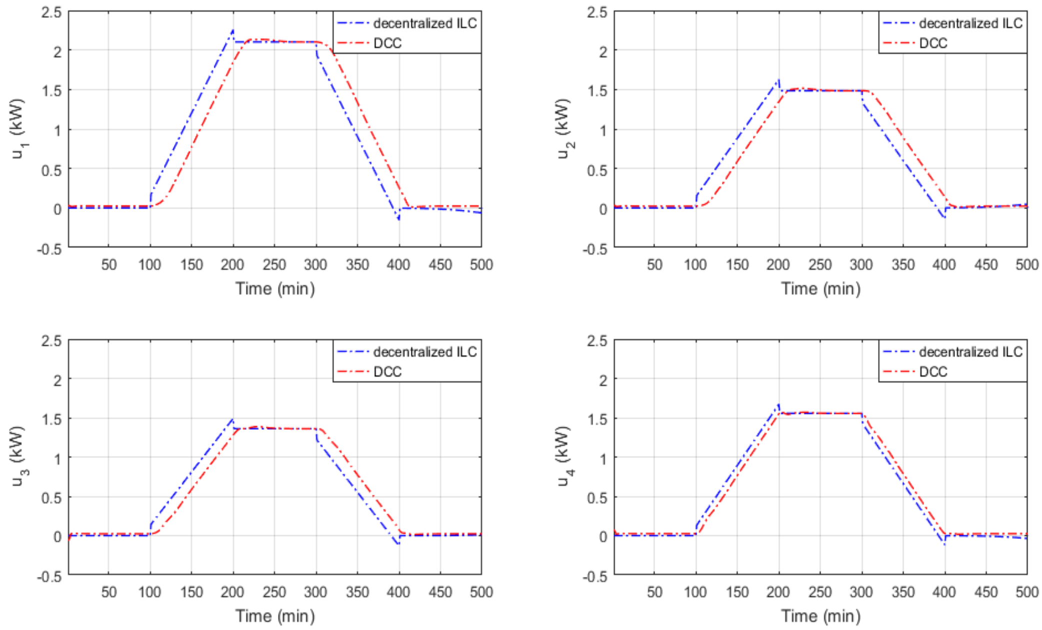

Figure 4. 8. Comparison of control inputs between decentralized ILC and DCC.

Figure 4. 7 shows that the output responses of both control techniques track their desired reference trajectories. When reference-temperature is constant, the steady-state error goes to zero. However, when the reference temperature is a ramp input, the output of DCC has error ranging from 0.5-2 degrees whereas the output of decentralized ILC has no error. On the other hand, when reference temperature is constant, the steady-state error goes to zero for both controllers. Figure 4. 8 illustrates the control input or the power supply of each room.

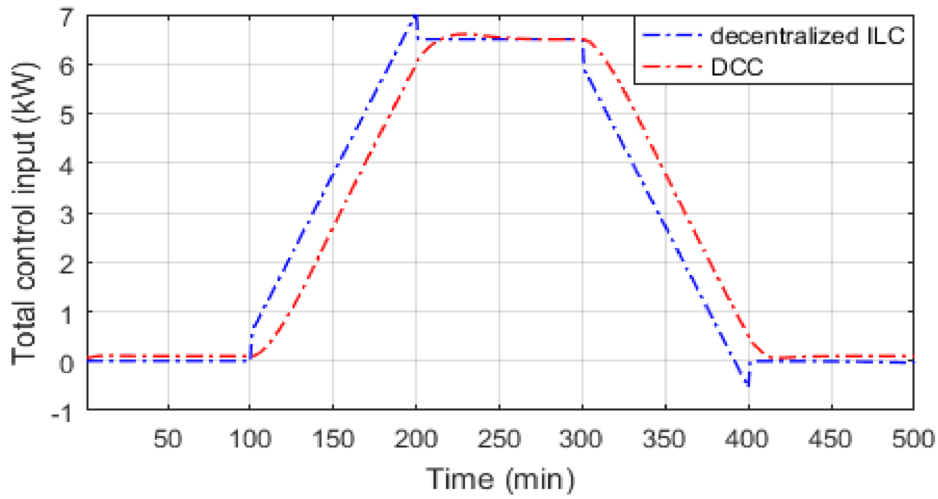

Figure 4. 9. Comparison of total control input between decentralized ILC and DCC.

Figure 4. 9 shows the total power supply for 4 rooms of both cases. When considering the ramp-up inputs, the decentralized ILC spends more total control input than that of DCC. On the other hand, during the ramp-down inputs, decentralized ILC employs less total control input than that of DCC. When the reference temperature is

constant, decentralized ILC uses the same magnitude of total control input as that of DCC.

Table 4. 1. Comparison of output performance when reference is ramp input.

Output	Steady-state error (^oC)	
	Decentralized ILC	DCC
T_1	0	2
T_2	0	2
T_3	0	1
T_4	0	0.5

Table 4. 2. Comparison of control input between decentralized ILC and DCC.

Peak magnitude of control input	Ramp-up input		Ramp-down input	
	DILC	DCC	DILC	DCC
u_1 (kW)	2.25	1.84	1.93	2.10
u_2 (kW)	1.63	1.34	1.33	1.48
u_3 (kW)	1.49	1.28	1.22	1.36
u_4 (kW)	1.67	1.55	1.42	1.56
Total control input at transition state (kW)	7.04	6.01	5.90	6.5

4.6 Summary

This chapter utilized a decentralized ILC algorithm for BTCS. First, we employ the D-type algorithm to design the local ILC for each subsystem in a decentralized scheme. Moreover, we show how to properly choose the ILC matrix gain. The control strategy is applied to a four-room model and we compare the output performance and control input between decentralized ILC and DCC. It reveals that decentralized ILC outperforms DCC for tracking trapezoidal reference input and uses less total control input at the transition state.

CHAPTER V

QUADRATIC ITERATIVE LEARNING CONTROL DESIGN

This chapter aims to design of Iterative Learning Control (ILC) based on Alternating Direction Method of Multipliers (ADMM) with application to BTCS. The organization of this chapter is as follows. Section 5.1 presents formulation of Q-ILC design. Section 5.2 gives the main results of ADMM approach to Q-ILC design consisting of update variables. The convergence property are analyzed in Section 5.3. The implement algorithm is shown in Section 5.4. Lastly, Section 5.5 gives numerical results of ILC design for BTCS to illustrate effectiveness of proposed approach.

5.1 Formulation of Q-ILC Design

Consider the discrete-time linear ILC system described by linear time-invariant state-space model [9, 23, 28].

$$\begin{cases} x_k(t+1) = Ax_k(t) + B_u u_k(t) + B_a T_k^a(t) \\ y_k(t) = Cx_k(t) \end{cases}, \tag{5.1}$$

where $x \in \mathbb{R}^N$, $u \in \mathbb{R}^{q_u}$, $T_a(t) \in \mathbb{R}^{q_a}$, $y \in \mathbb{R}^p$, are state vector, control input, ambient temperature vector, and output of system (5.1); A, B_u, B_a, C are the system matrices with appropriate dimensions.

Let we define

$$\begin{aligned} \mathbf{x}_k &= [x_k(1)^T \, x_k(2)^T \ldots \, x_k(T_N)^T]^T \\ \mathbf{u}_k &= [u_k(0)^T \, u_k(1)^T \ldots \, u_k(T_N-1)^T]^T \\ \mathbf{T}_k^a &= [T_k^a(0)^T \, T_k^a(1)^T \ldots \, T_k^a(T_N-1)^T]^T \\ \mathbf{y}_k &= [y_k(1)^T \, y_k(2)^T \ldots \, y_k(T_N)^T]^T \end{aligned}, \tag{5.2}$$

where $\mathbf{y}_k$, $\mathbf{u}_k$, $\mathbf{T}_k^a$, $\mathbf{x}_k$ are said to be the equivalent super vectors consisting of the state vector, control input, ambient temperature, and output of system at all sample times in the time interval $[0, T_N]$, at the k^{th} iteration. We rewrite the system (5.1) in the super vector framework in order to express the input-output relationship as follows

$$\mathbf{y}_k = G_u \mathbf{u}_k + G_a \mathbf{T}_k^a + F\mathbf{x}_k(0), \tag{5.3}$$

where $G_u \in \mathbb{R}^{T_N q_u \times T_N p}$ is called a Markov matrix, which are the impulse responses of the discrete-time system (5.1) to present the relation between the control input $\mathbf{u}_k \in \mathbb{R}^{T_N q_u}$ and output system $\mathbf{y}_k \in \mathbb{R}^{T_N p}$ at different samples; $G_a \in \mathbb{R}^{T_N q_a \times T_N p}$

is the matrix to express relation between ambient temperature $\mathbf{T}_k^a \in \mathbb{R}^{T_N q_a}$ and output system $\mathbf{y}_k \in \mathbb{R}^{T_N p}$; in addition, $F \in \mathbb{R}^{T_N N \times T_N p}$ is the matrix relating initial state $\mathbf{x}_k(0) \in \mathbb{R}^{T_N N}$ and output $\mathbf{y}_k \in \mathbb{R}^{T_N p}$.

Without loss of generality, assuming that the relative degree of system (5.1) is unity, i.e., the condition $CB \neq 0$ is fulfilled. Then, the three following matrices can be expressed [9] in general forms

$$
\begin{aligned}
G_u &= \begin{bmatrix} CB_u & 0 & 0 & \cdots & 0 \\ CAB_u & CB_u & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ CA^{T_N-1}B_u & CA^{T_N-2}B_u & \cdots & \cdots & CB_u \end{bmatrix}, \\
G_a &= \begin{bmatrix} CB_a & 0 & 0 & \cdots & 0 \\ CAB_a & CB_a & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ CA^{T_N-1}B_a & CA^{T_N-2}B_a & \cdots & \cdots & CB_a \end{bmatrix}, \\
F &= \begin{bmatrix} CA & CA^2 \cdots & CA^{T_N} \end{bmatrix}^T.
\end{aligned}
\tag{5.4}
$$

Let e is the tracking error between desired reference trajectory r and the output system y expressed [9] as

$$e(t) = r(t) - y(t). \tag{5.5}$$

Then, we have the tracking error in the super vector framework [9]

$$\mathbf{e}_k = \mathbf{r} - \mathbf{y}_k, \tag{5.6}$$

where

$$\mathbf{e}_k = [e_k(1)^T \, e_k(2)^T \ldots \, e_k(T_N)^T]^T,$$

$$\mathbf{r} = [r(1)^T \, r(2)^T \ldots \, r(T_N)^T]^T.$$

Note that the reference inputs are iteration-invariant, so the iteration index k is ignored in the super-vector $\mathbf{r}$; the initial condition $\mathbf{x}_k(0)$ and ambient temperature $\mathbf{T}_k^a$ are identical. Then, the tracking error update model of system (5.3), which is computed by minimizing the ILC optimization problem are defined by

$$
\begin{aligned}
\mathbf{e}_{k+1} &= \mathbf{r} - \mathbf{y}_{k+1} = \mathbf{r} - \mathbf{y}_k - (\mathbf{y}_{k+1} - \mathbf{y}_k) \\
\mathbf{e}_{k+1} &= \mathbf{e}_k - G_u \Delta\mathbf{u}_{k+1}
\end{aligned},
\tag{5.7}
$$

where $\Delta\mathbf{u}_{k+1} = \mathbf{u}_{k+1} - \mathbf{u}_k$ denote the difference of control input between two continuous iterations, namely, control input update of the system.

To design ILC input, we use the quadratic performance criterion expressed as follows:

$$\mathbf{J}_{k+1}(\Delta\mathbf{u}_{k+1}) = \mathbf{e}_{k+1}^{T} Q \mathbf{e}_{k+1} + \Delta\mathbf{u}_{k+1}^{T} R \Delta\mathbf{u}_{k+1}, \tag{5.8}$$

where *Q*, *R* are positive definite matrices.

According to [9, 29], to design controller, we often use the linearization model around the working point with small deviation. Then, the constraint on control inputs $u_{i,k+1}$ is considered to make sure that the system will not diverge far from equilibrium point. In addition, the constraint on the deviation input between continuous iterations $\Delta\mathbf{u}_{i,k+1}$ is added to guarantee the smooth and safe running of the control system.

Moreover, the total control input $\sum_{i=1}^{N} \mathbf{u}_{i,k+1}$ should be bounded to saving energy. Hence, we consider the following constraints of the control inputs.

C1. Bounded magnitude: $\mathbf{u}_{i,l} \le \mathbf{u}_{i,k+1} \le \mathbf{u}_{i,h}$.

C2. Bounded rate w.r.t. iteration index:

$$\Delta\mathbf{u}_{i,l} \le \Delta\mathbf{u}_{i,k+1} \le \Delta\mathbf{u}_{i,h},\ \Delta\mathbf{u}_{i,l} \le 0,\ \Delta\mathbf{u}_{i,h} \ge 0.$$

C3. Bounded magnitude of total input: $\sum_{i=1}^{N} \mathbf{u}_{i,k+1} \le P$.

The constraints C1 and C2 can be combined to be a constraint [9]

$$\begin{bmatrix} -I \\ I \\ -I \\ I \end{bmatrix} \Delta\mathbf{u}_{i,k+1} \le \begin{bmatrix} -\mathbf{u}_{i,l} + \mathbf{u}_{i,k} \\ \mathbf{u}_{i,h} - \mathbf{u}_{i,k} \\ -\Delta\mathbf{u}_{i,l} \\ \Delta\mathbf{u}_{i,h} \end{bmatrix}$$

We can write constraints in a form of box constraints for vectors as

$$\prod \Delta\mathbf{u}_{i,k+1} \le \phi, \tag{5.9}$$

where $\prod = \begin{bmatrix} -I \\ I \end{bmatrix};\ \phi = \begin{bmatrix} -\underline{\Delta}\mathbf{u}_i \\ \bar{\Delta}\mathbf{u}_i \end{bmatrix};$

$\underline{\Delta}\mathbf{u}_i = \max\{\mathbf{u}_{i,l} - \mathbf{u}_{i,k}, \Delta\mathbf{u}_{i,l}\}$; $\bar{\Delta}\mathbf{u}_i = \min\{\mathbf{u}_{i,h} - \mathbf{u}_{i,k}, \Delta\mathbf{u}_{i,h}\}$.

Let X, $\underline{X}$, and $\bar{X}$ contain the sequence, the lower bounded rate, and the upper bounded rate of control input change in iteration as follows:

$X = [\Delta\mathbf{u}_{1,k+1}^{T}, \Delta\mathbf{u}_{2,k+1}^{T}, \ldots, \Delta\mathbf{u}_{N,k+1}^{T}]^{T}$,

$\underline{X} = [\underline{\Delta}\mathbf{u}_1^T, \underline{\Delta}\mathbf{u}_2^T, ..., \underline{\Delta}\mathbf{u}_N^T]^T$,

$\bar{X} = [\bar{\Delta}\mathbf{u}_1^T, \bar{\Delta}\mathbf{u}_2^T, ..., \bar{\Delta}\mathbf{u}_N^T]^T$.

Then, the constraint (5.9) is in the form as follows:

$$\underline{X} \leq X \leq \bar{X} \tag{5.10}$$

On the other hand, constraint C3 can now be reformulated as follows:

$$\mathbf{1}_{T_N N}^T X \leq \hat{P}, \tag{5.11}$$

Substitute (5.7) into (5.8), and rewrite this performance index in terms of X, we obtain

$$\mathbf{J}_{k+1}(X) = X^T(R + G_u^T Q G_u)X - 2\mathbf{e}_k^T Q G_u X + \mathbf{e}_k^T Q \mathbf{e}_k$$

We observe that there is a variable X. Since $\mathbf{e}_k$ is known from previous iteration, in the design, $\mathbf{e}_k^T Q \mathbf{e}_k$ can be omitted from the cost function.

$$\mathbf{J}_{k+1}(X) = X^T(R + G_u^T Q G_u)X - 2\mathbf{e}_k^T Q G_u X$$

Then, the design of control input is formulated as min problem

$$\min_{X \in \Xi} X^T \tilde{R} X + \tilde{Q} X,$$

where $\tilde{R} \in \mathbb{R}^{T_N N \times T_N N}$, $\tilde{Q} \in \mathbb{R}^{1 \times T_N N}$,

$$\tilde{R} = (R + G_u^T Q G_u), \tag{5.12}$$

$$\tilde{Q} \triangleq -2\mathbf{e}_k^T Q G_u, \tag{5.13}$$

Ξ is the convex set defined by (5.10), and (5.11).

In this work, we aim to solve this optimization by using ADMM. In order to employ ADMM for solving the following minimizing optimization problem

$$\min X^T \tilde{R} X + \tilde{Q} X \tag{5.14}$$

s.t. (5.10), and (5.11),

we need to reformulate it to a suitable form. We define an auxiliary variable *Z*, which is the one algorithm state in ADMM [14, 30], and two convex sets, namely Ω_1, Ω_2 corresponding to (5.11) and (5.10), respectively where (5.11) now is defined in terms of Z.

$$\Omega_1(X) = \{X \in \mathbb{R}^{T_N N} : 1^T X_i \leq \hat{P} \ \forall i = 1, ..., N\}, \tag{5.15}$$

$$\Omega_2(Z) = \{Z \in \mathbb{R}^{T_N N} : \underline{X}_i \leq Z_i \leq \bar{X}_i \ \forall i = 1, ..., N\}. \tag{5.16}$$

Consequently, the following two indicator functions are defined to incorporate these sets Ω_1, Ω_2.

$$I_1(X) = \begin{cases} 0 & : X \in \Omega_1, \\ \infty & : X \notin \Omega_1, \end{cases}; \quad I_2(Z) = \begin{cases} 0 & : Z \in \Omega_2, \\ \infty & : Z \notin \Omega_2. \end{cases}$$

Finally, we get the formulation for the considering ILC problem with equality constraint $X = Z$.

$$\begin{aligned} &\min X^T \tilde{R} X + \tilde{Q} X + I_1(X) + I_2(Z) \\ &\text{s.t.} \;\; X - Z = 0 \end{aligned}, \tag{5.17}$$

5.2 The Main Result

Because the indicator function of a convex set is proper, closed, and convex, the cost function with respect to *X* and *Z* in (5.17) is also proper, closed, and convex [14, 30]. This formulation is actually fitted into ADMM framework. The augmented Lagrangian associated with (5.17) is defined as follows:

$$\begin{aligned} L_\rho(X,Z,W) \triangleq\; & X^T \tilde{R} X + \tilde{Q} X + I_1(X) + I_2(Z) \\ & + \eta^T (X - Z) + \frac{\rho}{2} \left\| X - Z \right\|_2^2 \end{aligned}, \tag{5.18}$$

where $\eta^T \in \mathbb{R}^{T_N N}$ is a Lagrange multiplier, and ρ is scalar penalty parameter. The augmented Lagrangian (5.18) can be written as follows.

$$\begin{aligned} L_\rho(X,Z,W) \triangleq\; & X^T \tilde{R} X + \tilde{Q} X + I_1(X) + I_2(Z) \\ & + \frac{\rho}{2} \left\| X - Z + W \right\|_2^2 \end{aligned}, \tag{5.19}$$

where $W = \frac{1}{\rho} \eta \in \mathbb{R}^{T_N N}$ is called a scaled Lagrange multiplier. Accordingly, the optimization problem (5.17) is iteratively solved by the updating variables in scalar form as follows [14, 30].

$$X^{m+1} := \arg\min_X L_\rho(X, Z^m, W^m) \tag{5.20}$$

$$Z^{m+1} := \arg\min_Z L_\rho(X^{m+1}, Z, W^m) \tag{5.21}$$

$$W^{m+1} := W^m + X^{m+1} - Z^{m+1} \tag{5.22}$$

If the two following criteria are satisfied, the iterative process (5.20)-(5.22) is completed [14, 30],

$$\left\| r^m \right\|_2 \leq \varepsilon^{\text{pri}} \;, \left\| s^m \right\|_2 \leq \varepsilon^{\text{dual}} \;, \tag{5.23}$$

where $r^m \triangleq X^m - Z^m$ and $s^m \triangleq -\rho(Z^m - Z^{m-1})$ are the primal and dual residuals at iteration m [30]; $\varepsilon^{\text{pri}} > 0$ and $\varepsilon^{\text{dual}} > 0$ are primal and dual feasibility tolerances chosen as follows [30].

$$\begin{aligned} \varepsilon^{\text{pri}} &= \sqrt{T_N N}\varepsilon^{\text{abs}} + \varepsilon^{\text{rel}} \max\{\left\| X^m \right\|_2, \left\| -Z^m \right\|_2\}, \\ \varepsilon^{\text{dual}} &= \sqrt{T_N N}\varepsilon^{\text{abs}} + \varepsilon^{\text{rel}} \left\| \rho W^m \right\|_2, \end{aligned} \tag{5.24}$$

where $\varepsilon^{\text{abs}} > 0$ and $\varepsilon^{\text{rel}} > 0$ are absolute and relative tolerance that depend on the scale of typical values. The value of ε^{rel} should be selected to be 10^{-3} or 10^{-4} [14, 30].

The updating variables X, Z, W can be separated into individual updates at each time slot and can be implemented in a centralized manner. When the variables X in (5.20) and Z in (5.21) have updated, respectively, the update of W in (5.22) is decentralized [14]. Hence, we only focus on the updates of X , and Z .

5.2.1 X-update Step

In this section, we would like to show the update of variable X in (5.20). Regarding the definition of indicator function [14], it is clear to see that X^{m+1} is the solution of the following optimization problem,

$$\begin{aligned} &\min \quad f(X) \\ &\text{s.t. (5.11)}, \end{aligned} \tag{5.25}$$

where $f(X) = X^T \tilde{R} X + \tilde{Q} X + \frac{\rho}{2} \left\| X - Z + W \right\|_2^2$.

This is the convex optimization problem since the cost function $f(X)$ is in quadratic form and the inequality constraint (5.11) is affine [14]. Therefore, the strong duality holds for (5.25) and (5.11), i.e., the optimal solution X^{m+1} are found by employing the Karush-Kuhn-Tucker (KKT) conditions.

Applying KKT condition [31], we have

$$\frac{\partial f(X)}{\partial X} + \mu^{m+1} \frac{\partial}{\partial X}(1^T X - \hat{P}) = 0, \tag{5.26}$$

$$\mu^{m+1}(1^T X - \hat{P}) = 0, \tag{5.27}$$

$$\mu^{m+1} \geq 0, \tag{5.28}$$

where $\mu^{m+1} \in \mathbb{R}$ is Lagrange multiplier in the iteration $m+1$ of ADMM approach.

To find the solution, we consider the active characteristic of constraint, after that, we check the sign of resulting Lagrange multiplier. Here, we set constraint to be active or inactive.

By assuming the constraint (5.11) is active ($\mu^{m+1} > 0$), we obtain

$$\mathbf{1}_{T_N N}^T X = \hat{P}$$

From (5.26), we obtain the solution X

$$2X^T \tilde{R} + \tilde{Q} + \rho(X - Z + W)^T + \mu \mathbf{1}_{T_N N}^T = 0$$

$$X = [-\rho(-Z + W) - \tilde{Q}^T - \mu \mathbf{1}_{T_N N}](2\tilde{R} + \rho I_{T_N N})^{-1}. \qquad (5.29)$$

Multiplying both side of (5.29) with $\mathbf{1}_{T_N N}^T$, we obtain

$$\mathbf{1}_{T_N N}^T X = \mathbf{1}_{T_N N}^T [-\rho(-Z + W) - \tilde{Q}^T - \mu \mathbf{1}_{T_N N}](2\tilde{R} + \rho I_{T_N N})^{-1}. \qquad (5.30)$$

Then, comparing this solution (5.30) with $\hat{P}$.

By assuming the constraint (5.11) is inactive ($\mu^{m+1} = 0$), we obtain

$$\mathbf{1}_{T_N N}^T X < \hat{P}$$

From (5.26), we obtain the solution X

$$2X^T \tilde{R} + \tilde{Q} + \rho(X - Z + W)^T = 0$$

$$X = [-\rho(-Z + W) - \tilde{Q}^T](2\tilde{R} + \rho I_{T_N N})^{-1} \qquad (5.31)$$

Multiplying both side of (5.31) with $\mathbf{1}_{T_N N}^T$, we acquire

$$\mathbf{1}_{T_N N}^T X = \mathbf{1}_{T_N N}^T [-\rho(-Z + W) - \tilde{Q}^T](2\tilde{R} + \rho I_{T_N N})^{-1} \qquad (5.32)$$

Then, comparing this solution (5.32) with $\hat{P}$.

Without loss of generality, we assume that the optimal solution X can be expressed by the following form

$$X^{m+1} = [-\rho(-Z + W) - \tilde{Q}^T - \bar{\mu} \mathbf{1}_{T_N N}](2\tilde{R} + \rho I_{T_N N})^{-1}, \qquad (5.33)$$

where

$$\bar{\mu} = \frac{\widehat{P} + \mathbf{1}_{T_N N}^T [\rho(-Z + W) + \tilde{Q}^T](2\tilde{R} + \rho I_{T_N N})^{-1}}{-\mathbf{1}_{T_N N}^T \mathbf{1}_{T_N N}(2\tilde{R} + \rho I_{T_N N})^{-1}}. \qquad (5.34)$$

5.2.2 Z-update Step

The update of Z in (5.21) is shown more detail in this section. We can easily see that Z^{m+1} is the solution of the following optimization problem,

$$\min \quad \left\| X^{m+1} - Z + W^{m} \right\|_2^2 \tag{5.35}$$

$$\text{s.t.} \quad \underline{X} \le Z \le \bar{X}, \tag{5.36}$$

which is convex optimization problem, because the cost function (5.35) is quadratic function and the constraints (5.36) are polyhedral [14].

The cost function (5.35) and constraint (5.36) can be decomposed in the following problem for $i = 1,...,N$.

$$\min \ (Z_i^2 - 2(X_i^{m+1} + W_i^m)Z_i), \forall i = 1,...,N, \tag{5.37}$$

$$\text{s.t.} \quad \underline{X}_i \le Z_i \le \bar{X}_i, \forall i = 1,...,N. \tag{5.38}$$

We observe that problem (5.37) with constraint (5.38) has a scalar variable, then its optimal solution can be computed as follows:

$$Z_i^{m+1}(t) = \begin{cases} \underline{X}_i & : X_i^{m+1}(t) + W_i^m(t) < \underline{X}_i(t) \\ X_i^{m+1} + W_i^m & : \underline{X}_i(t) < X_i^{m+1}(t) + W_i^m(t) < \bar{X}_i(t) \\ \bar{X}_i & : \bar{X}_i < X_i^{m+1}(t) + W_i^m(t), \ \forall i = 1,...,N \end{cases}. \tag{5.39}$$

5.3 Convergence Property

The following theorem shows that the convergence of ADMM approach can be guaranteed.

Theorem [14]: The solutions of ADMM converge as $m \to \infty$

$$r^m \to 0, \ p(X^m) \to p^*, \ X^m \to X^*, \ Z^m \to Z^*, \tag{5.40}$$

where $p(X^m) \triangleq X^{m^T} \tilde{R} X^m + \tilde{Q} X^m + I_1(X) + I_2(Z)$; p^* is the minimum cost of (20), $X^* = Z^*$ are the minimizer of (5.17), respectively.

Proof [30, 32]: All elements of $g(X^m)$ are proper, closed, and convex function. In addition, the Lagrangian is defined by

$$L_0(X,Z,W) \triangleq X^T \tilde{R} X + \tilde{Q} X + I_1(X) + I_2(Z) + \eta^T (X - Z),$$

has a saddle point. It is because the first two elements, $X^T \tilde{R} X$ and $\tilde{Q} X$ are quadratic functions, respectively, and $I_1(X)$, $I_2(Z)$ are indicator functions [14].

Since L_0 has a saddle point (X^*, Z^*, W^*),

$$L_0(X^*, Z^*, \eta^*) \le L_0(X^{m+1}, Z^{m+1}, \eta^*) \tag{5.41}$$

$$\begin{aligned} L_0(X^{m+1}, Z^{m+1}, \eta^*) &= (X^{m+1})^T \tilde{R} X^{m+1} + \tilde{Q} X^{m+1} \\ &\quad + I_1(X) + I_2(Z) + \eta^{*T}(X^{m+1} - Z^{m+1}) \\ &= p^{m+1} + \eta^{*T} r^{m+1} \end{aligned} \tag{5.42}$$

Combining (5.41)-(5.42), it is easy to see that

$$p^* \le p^{m+1} + \eta^{*T} r^{m+1} \tag{5.43}$$

Let $h(X) = X^T \tilde{R} X + \tilde{Q} X + I_1(X)$; $g(Z) = I_2(Z)$.

Since Z^{m+1} minimizes $g(Z) - (\eta^{m+1})^T Z$, then

$$\begin{aligned} h(X^{m+1}) + (\eta^{m+1} + \rho(Z^{m+1} - Z))^T X^{m+1} &\le h(X^*) \\ &+ (\eta^{m+1} + \rho(Z^{m+1} - Z))^T X^*, \end{aligned} \tag{5.44}$$

and

$$g(Z^{m+1}) - (\eta^{m+1})^T Z^{m+1} \le g(Z^*) - (\eta^{m+1})^T Z^{m+1} \tag{5.45}$$

Adding (5.44) with (5.45) and using $X^* = Z^*$,

$$\begin{aligned} p^{m+1} - p^* &\le -(\eta^{m+1})^T r^{m+1} \\ &+ \rho(Z^{m+1} - Z^m)^T(-r^{m+1} - (Z^{m+1} - Z^*)) \end{aligned} \tag{5.46}$$

Define $V^m = \frac{1}{\rho}\left\|\eta^{m+1} - \eta^m\right\|_2^2 + \rho\left\|Z^{m+1} - Z^m\right\|_2^2$.

Adding (5.43) and (5.46), multiplying both sides by 2 and rearranging, we obtain

$$\begin{aligned} 2(\eta^{m+1} - \eta^*)^T r^{m+1} + 2\rho(Z^{m+1} - Z^m)^T r^{m+1} \\ + 2\rho(Z^{m+1} - Z^m)^T (Z^{m+1} - Z^*) \le 0 \end{aligned} \tag{5.47}$$

Using the update:

$$r^{m+1} = \frac{1}{\rho}(\eta^{m+1} - \eta^m) \tag{5.48}$$

in (5.47), we obtain

$$\begin{aligned} &\frac{1}{\rho}\left(\left\|\eta^{m+1} - \eta^*\right\|_2^2 - \left\|\eta^m - \eta^*\right\|_2^2\right) + \rho\left\|r^{m+1} + (Z^{m+1} - Z^m)\right\|_2^2\Bigg) \\ &+ 2\rho\left[(Z^{m+1} - Z^m)(Z^{m+1} - Z^*)\right] \le 0 \end{aligned}$$

$$V^{m+1} \le V^{m} - \rho \left\| r^{m+1} \right\|_2^2 - \rho \left\| Z^{m+1} - Z^{m} \right\|_2^2 \\ + 2\rho (r^{m+1})^T (Z^{m+1} - Z^{m}) \tag{5.49}$$

Since Z^{m+1} minimizes $g(Z) - (\eta^{m+1})^T Z$, and Z^{m} minimizes $g(Z) - (\eta^{m})^T Z$ after adding we have

$$g(Z^{m+1}) - (\eta^{m+1})^T Z^{m+1} \le g(Z^{m}) - (\eta^{m+1})^T Z^{m},$$

and

$$g(Z^{m}) - (\eta^{m})^T Z^{m} \le g(Z^{m+1}) - (\eta^{m})^T Z^{m+1}$$

to get

$$(\eta^{m+1} - \eta^{m})^T (Z^{m+1} - Z^{m}) \ge 0. \tag{5.50}$$

Combining (5.48) with (5.50), we have

$$2\rho (r^{m+1})^T (Z^{m+1} - Z^{m}) \ge 0 \tag{5.51}$$

From (5.49) and (5.51),

$$V^{m+1} \le V^{m} - \rho \left\| r^{m+1} \right\|_2^2 - \rho \left\| Z^{m+1} - Z^{m} \right\|_2^2 \tag{5.52}$$

Because $V^{m} \le V^{0}$, η^{m} and Z^{m} are bounded [32], thus

$$\rho \sum_{m=0}^{\infty} \left[\left\| r^{m+1} \right\|_2^2 - \left\| Z^{m+1} - Z^{m} \right\|_2^2 \right] \le V^{0},$$

which results in $\left\| r^{m+1} \right\|_2^2 \to 0$, $\left\| Z^{m+1} - Z^{m} \right\|_2^2 \to 0$ when $m \to \infty$. From (5.43) and (5.46), we have $\lim_{m \to \infty} p^{k} = p^{*}$.

Therefore, the proposed algorithm converges or Eq. (5.40) is satisfied.

5.4 Algorithm

The stopping criteria for ADMM are given by (5.23), or

$$m = m\_\max. \tag{5.53}$$

The stopping criteria for iterative learning procedure are given as follows.

$$\left\| e_k \right\| \le \varepsilon, \tag{5.54}$$

$$k = iter\_\max \tag{5.55}$$

where ε denotes a tolerance of error; $iter\_\max$, $m\_\max$ are the maximum number of iteration of ILC design, and ADMM approach, respectively. We summarize the main results by giving the Q-ILC design with following steps.

Step 1: Set the initial conditions

Set $k := 0$, given the initial value of control input $\mathbf{u}_k = \mathbf{u}_0$, and the initial states $\mathbf{x}_k = \mathbf{x}_0$. Implement the 1st iteration, measure output $\mathbf{y}_0$ and calculate errors $\mathbf{e}_0$.

Compute $\widehat{P}$, $\tilde{R}$, $\tilde{Q}$, and $\overline{\mu}$ as in (5.11), (5.12), (5.13), and (5.34).

Step 2: ADMM algorithm

for $1 \leq m \leq m\_\max$ **execute**

Compute X^{m+1} by (5.33).

Compute Z^{m+1} by (5.39).

Calculate W^{m+1} by (5.22).

if (5.23) is true or $m = m\_\max$ **then**

break;

end

end

Step 3: Update control input in original ILC design

Caculate $\Delta\mathbf{u}_{k+1} := X^*$, then find the update control input $\mathbf{u}_{k+1} = \mathbf{u}_k + \Delta\mathbf{u}_{k+1}$.

Apply $\mathbf{u}_{k+1}$ to the system and measure the output $\mathbf{y}_{k+1}$ and compute the error $\mathbf{e}_{k+1}$ Then, go to step 4.

Step 4: Check the stopping criteria

If (5.54) or (5.55) is true, then, stop the iteration, else, set $k := k+1$ and return to step 2.

It is noted that the convergence speed of ADMM problem will be affected by the value of ρ, which is appeared in the X-update step. The larger value of ρ will increase the weight on consent with variable Z, while the smaller value of ρ will lead the X-update step towards setting more importance on reducing the cost function, one constraint is satisfied by each that guaranteed. If the ADMM algorithm starts with an initial feasible point X_0, the cost can be decreased faster with a small value of ρ [33]. When dealing with a convex problem fitted into ADMM framework, it always converges for all $\rho > 0$. In addition, the solution of ADMM problem will be affected by the weighting matrices Q, R and the specification of constraints control inputs C1-C3. Hence, in design, we can empirically vary these parameters until getting a good solution and convergence speed [33].

5.5 Numerical Example

5.5.1 Numerical Conditions

In this subsection, we give the simulation conditions on the building temperature control composing of four rooms shown in [10] to illustrate the output response when applying proposed algorithm. Most of the parameters and physical descriptions are taken from [7, 10]. We assume that the ambient temperature equals to 10°C. The total power $P = 7\ kW$ and is located in the building, which will supply thermal energy to each room. We choose the initial condition of the initial control inputs and state vector as follows:

$$\mathbf{u}_{1,0} = \mathbf{u}_{2,0} = \mathbf{u}_{3,0} = \mathbf{u}_{4,0} = \mathbf{0}_{T_N};$$

$$\mathbf{x}_{1,0} = \mathbf{x}_{2,0} = \mathbf{x}_{3,0} = \mathbf{x}_{4,0} = 10 \times \mathbf{1}_{T_N}.$$

Let the sample time be 1 minute, the number of samples is 501. The desired reference output is a trapezoidal input as (4.9).

The constraints inputs are given as follows:

$$\begin{array}{ll} \mathbf{u}_l = \mathbf{0}_{T_N N}, & \mathbf{u}_h = 2500 \times \mathbf{1}_{T_N N}, \\ \Delta\mathbf{u}_l = -1000 \times \mathbf{1}_{T_N N}, & \Delta\mathbf{u}_h = 1800 \times \mathbf{1}_{T_N N} \end{array}. \tag{5.56}$$

The weighting matrices are selected as follows:

$$Q = 10^3 \times I_{T_N N},\ R = 10^{-3} \times I_{T_N N}.$$

In addition, the parameter of ADMM algorithm are chosen as follows:

$$\varepsilon^{\text{abs}} = 10^{-4},\ \varepsilon^{\text{rel}} = 10^{-3},\ \rho = 7.$$

For stopping criteria, we choose

$$\varepsilon = 10^{-2},\ m\_\max = 1000, \text{ and } iter\_\max = 10000.$$

5.5.2 Numerical Results

The numerical results are depicted by Figure 5. 1-Figure 5. 7. First, the room temperatures gradually track their target trajectories shown in Figure 5. 1. Moreover, Figure 5. 2 and Figure 5. 3 show that the control inputs converge and satisfy the constraints C1 and C3 assigned by (5.56). Additionally, Figure 5. 4 shows that the difference of control input w.r.t. iteration index $\Delta\mathbf{u}_{i,k+1}$ are satisfied as constraint C2, and their variables values goes to zero. Figure 5. 5 and Figure 5. 6 verify the monotonic convergence of the system error shown by $\|e_k\|_\infty$ and $\|e_k\|_Q$.

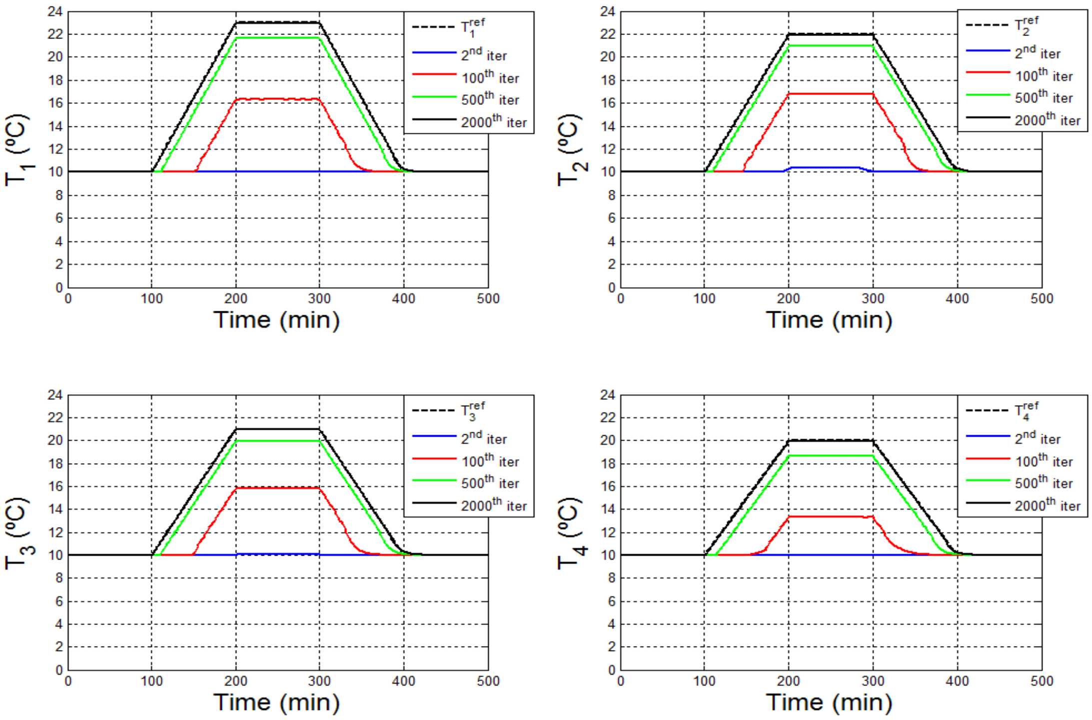

Figure 5. 1. Room temperature when using $\rho = 7$.

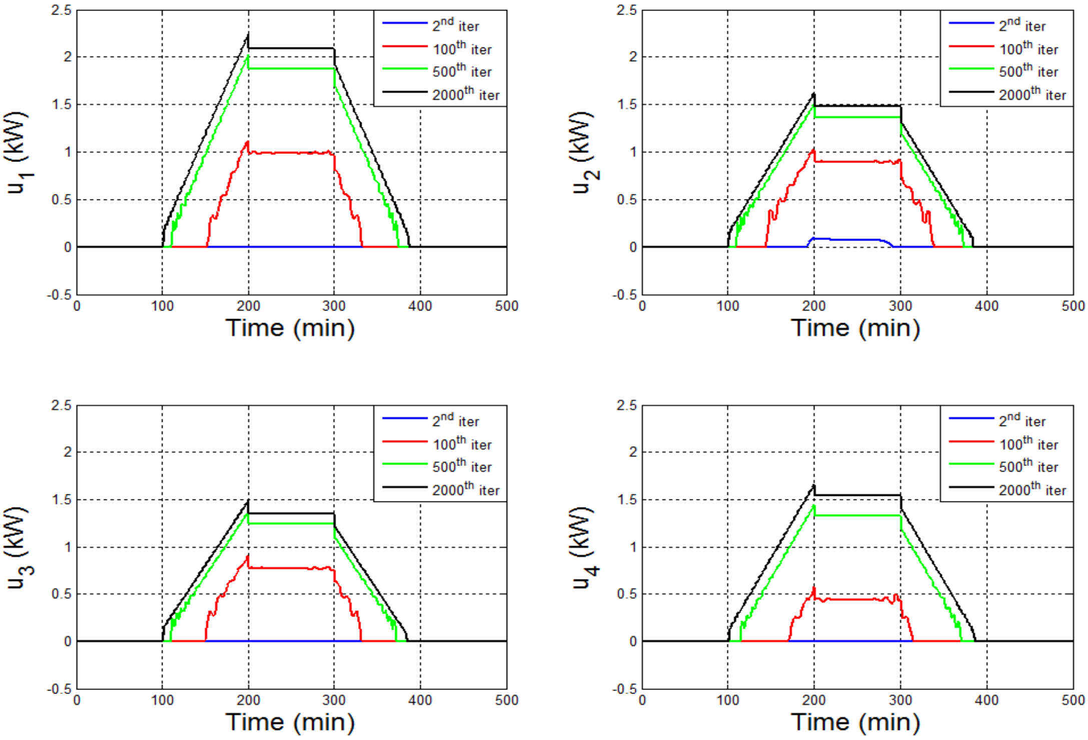

Figure 5. 2. Power supply when using $\rho = 7$.

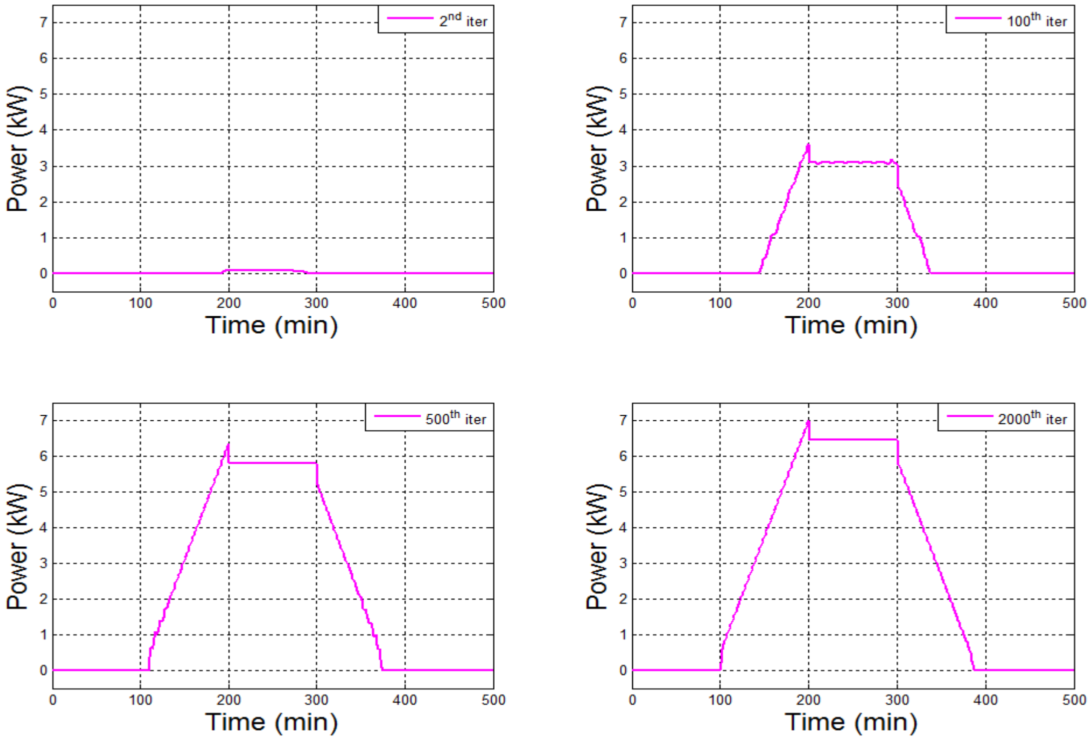

Figure 5. 3. Total power supply when using $\rho = 7$.

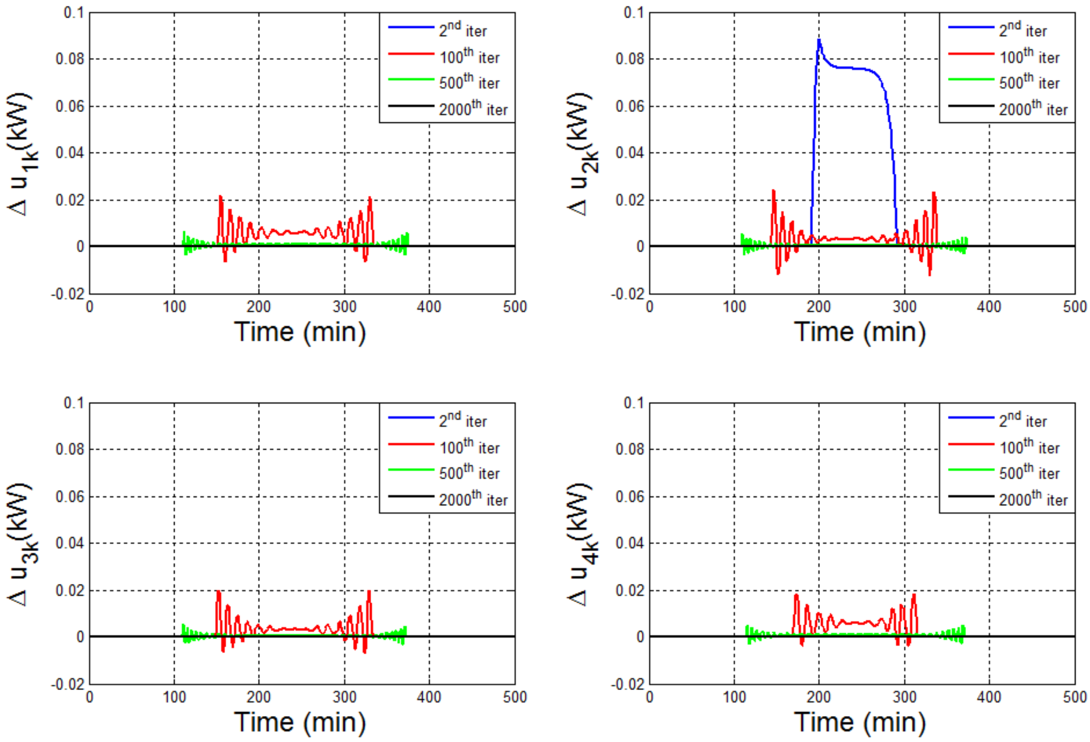

Figure 5. 4. The difference of control input w.r.t. iteration index.

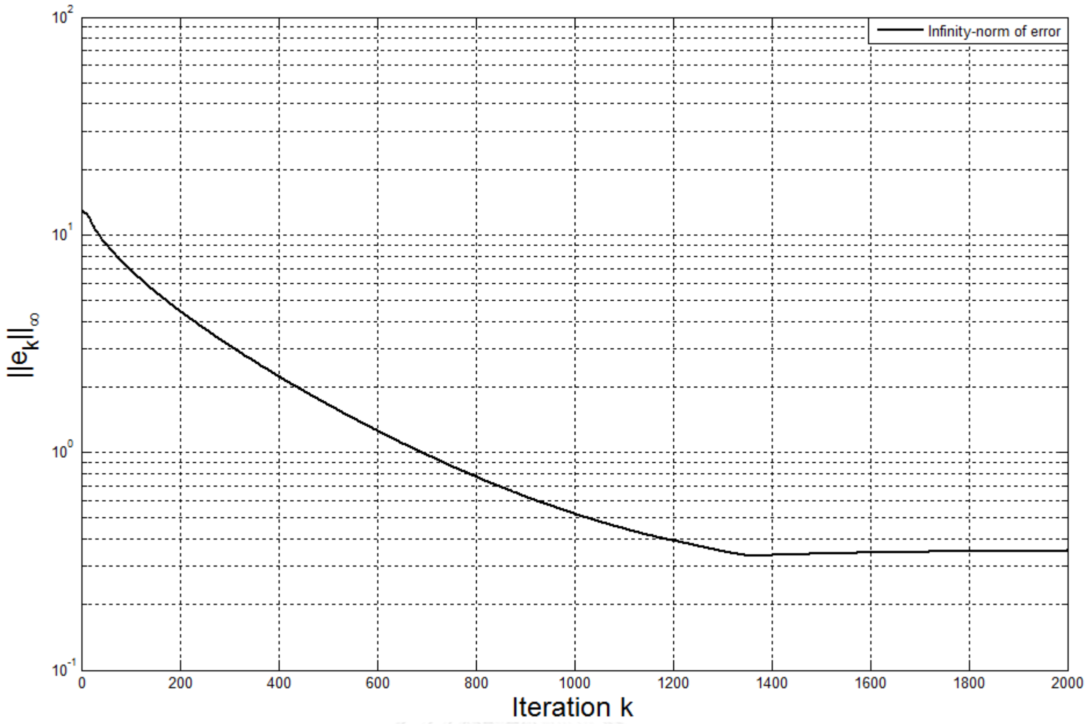

Figure 5. 5. Infinity-norm of error vs. the number of iteration.

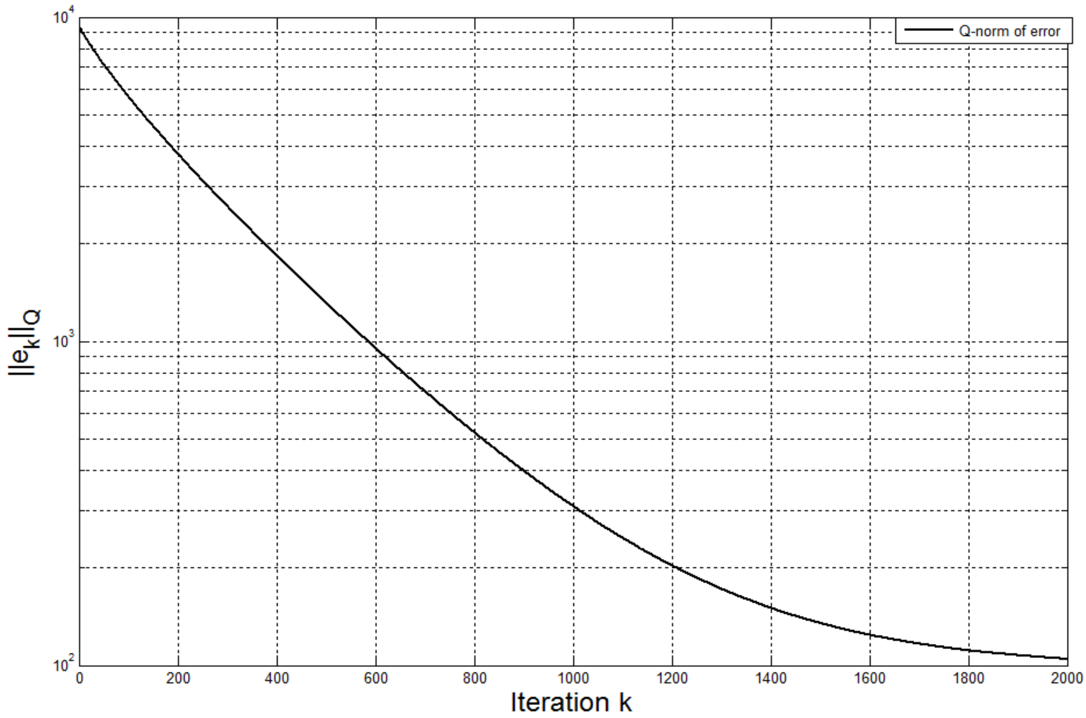

Figure 5. 6. Q-norm of error vs. the number of iteration.

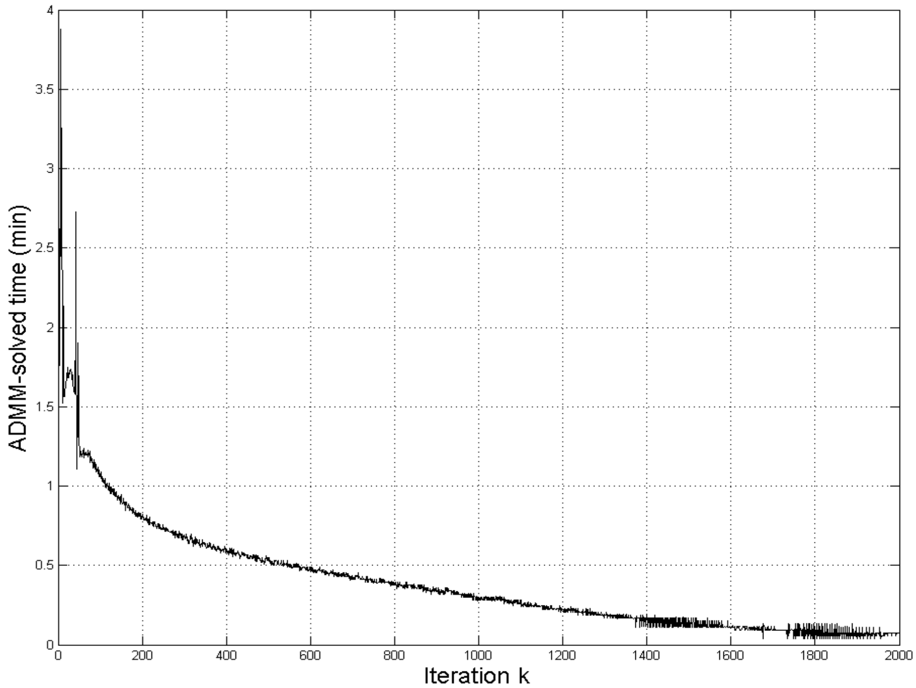

Figure 5. 7. Computational time of ADMM.

In this work, we use the computer with CPU Intel Core i5-6400 2.70 GHz, RAM 16 GB, MATLAB 8.1 to simulate the proposed algorithm. Figure 5. 7 reveals the computational time to solve ADMM problem in step 2. It decreases when the number of iterations of ILC loop increases. When running the more iterations of ILC loop, the better tracking performance is, hence the smaller of the control update $\Delta\mathbf{u}_{i,k+1}$ and the tracking error $\mathbf{e}_{i,k+1}$ are. It is supposed that the solution of ADMM problem acquires farther to the boundary of the constraints, then the computational time spends shorter [9].

5.6 Summary

This chapter presents the design of ILC based ADMM method for BTCS. ILC is designed based on ADMM approach to achieve the updating control law by analytical solution. The proposed algorithm has clarified to work well with a linear system with hard control inputs constraints.

CHAPTER VI

CONCLUSION AND FUTURE WORK

6.1 Conclusions

This thesis gives a formulation of BTCS as MAS and proposes the design of three controllers for BTCS. In particular, we show how to tune parameters used in the design utilized by distributed consensus control, decentralized ILC and centralized ILC scheme. Numerical results illustrate the effectiveness of proposed methods and then compare with the existing method. To summary the main results in this thesis, we point out as follows.

Chapter 1 briefly presents the research motivation and literature review to cover an overview of BTCS approaches. Next, the objective, scope, achievements of this thesis are given.

Chapter 2 focuses on introducing the background relevant to BTCS and a useful tool, i.e., MAS used in distributed scheme.

In Chapter 3, the detailed design procedure of DCC is proposed. By employing MAS theory in distributed framework and the inverse-barrier function, the control inputs can be forced to lie in the boundary whereas the outputs system can still track their desired trajectory in case of enough power supply. On the other hand, this method optimally apportions the available power supply for all agents so that all agent can be received the same welfare comfort. The simulation results show the good performance of DCC with criteria such as DCC gives faster convergence and consumes less power than that of RDC.

Chapter 4 gives the design of decentralized ILC controller for BTCS. We employ D-type ILC algorithm in the decentralized scheme and then conduct the influence of learning gain onto convergence speed of output performance. Finally, numerical results illustrate the effectiveness of decentralized ILC and are compared with that of distributed consensus controller (DCC) designed in Chapter 3. The results show that output responses of both controllers can track trapezoidal reference and consume the same amount of total power at steady state. However, decentralized ILC gives output response without overshoot, and its convergence is faster than that of DCC. It shows that decentralized ILC outperforms DCC for tracking trapezoidal reference. Convergence analysis reveals that tracking speed depends on the choice of learning gain.

Chapter 5 gives the design of centralized ILC for BTCS based on the ADMM. The proposed method is the main contribution of this thesis. First, the formulation of constrained Q-ILC design is new. Second, we develop a new approach to solve the design problem. In particular, we employ ADMM approach to find an optimal control input update. Actually, this approach is applicable to a set of the optimization problem in the form similar to that of the constrained Q-ILC. The novelty lies in the way we define the new convex sets to reformulate the constrained Q-ILC to the form of ADMM, by which we derive an analytical solution for the update of each iteration. This method can also guarantee the hard input constraints. Lastly, numerical examples illustrate the effectiveness of this method. The results exhibit that temperature outputs track the desired trapezoidal reference without overshoot and control inputs satisfies the hard constraints as expected. It reveals that ADMM approach is suited with ILC design problem.

6.2 Future Works

There are some restrictions in this thesis which could be further investigated in future.

Firstly, the dynamic models of BTCS should be more complex than that used in this thesis. Typically, thermal dynamics of rooms and walls are modeled using electric-analogous elements such as capacitors (C) and resistors (R). Unless the walls are made of thermal-isolated materials, the aforementioned model is enough to describe the thermal dynamics for the whole building. Each analog electronic circuit has its own advantages and disadvantages. When the wall thermal dynamics are modeled by 3R2C and 3R4C and when comparing with the performance of 1R, the analogy circuit using 3R2C and 3R4C give a better spectral analysis, step change, and frequency analysis term (c.f. Section II.A of [3]). Moreover, when using 3R2C circuit, the performance of wall thermal dynamic is as good as 3R4C circuit, and 3R2C circuit can reduce much complexity than that of 3R4C circuit. Thus, in the future work, we will use 3R2C model as it is more accurate than the others. After having the complex BTCS model, our idea of saving energy is equivalent to minimizing both thermal losses and control effort for tracking a given temperature profile. Instead of only considering the tracking problem with certain preset temperatures, we consider taking into account other constraints such as users' comfort requirements (usually the range of required temperatures set by rooms' occupants), or limited heating/cooling power supplies.

Secondly, in DCC design of Chapter 3, we just choose the tuning parameter in distributed consensus algorithm and penalize control limit using in inverse-barrier function based on observing the output and input performance. The consensus-based algorithm is referred to the existing work, which considers solving static optimization resource allocation problem. It is not fully useful to utilize for dynamic resource allocation problem. Then the future work will focus on finding systematic ways to design distributed consensus controller for solving dynamic resource allocation problem.

Thirdly, in Chapter 4, although decentralized ILC algorithm gives the better performance than that of DCC in Chapter 3 with the almost same conditions. All tuning parameters affecting to Γ_i are chosen based on the experience of designers. It should be extended to answer the question what would an optimal learning gain Γ_i be.

Lastly, when we consider the large-scale system, distributed framework is most suitable control strategy due to it requires the coordination among controllers and allows the agents to share the information on the state/input of neighboring agents [20]. Our future work will develop systematic ways to design distributed ILC via ADMM approach for this class of large-scale interconnected system and then to compare with the results of decentralized ILC as well as with the existing solver in terms of computational cost and convergence speed. In years to come, this research should be tested with our proposed algorithm on the real test-bed of BTCS.

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VITA

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LIST OF PUBLICATIONS

1. T. V. Pham, D. H. Nguyen, and D. Banjerdpongchai, "Design of Distributed Consensus Controller for Building Temperature Control System," in Proc. of the 9th AUN/SEED-Net Regional Conference on Electrical and Electronics Engineering, pp. 185-190, Hanoi, Vietnam, 17-18 November 2016.

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