CHAPTER III

THEORY OF HEAT PIPE

In order for the heat pipe to operate continuously and efficiently, the maximum capillary pumping head $(\Delta P_c)_{max}$ must be greater than the total pressure drop in the pipe. This pressure drop is made of three components

(a) The pressure drop ΔP_l required to return the liquid from the condenser to the evaporation

(b) The pressure drop ΔP_v necessary to cause the vapor to flow from the evaporation to the condenser

(c) The gravitational head ΔP_g which may be zero, positive or negative

Thus

$$(\Delta P_c)_{max} = \Delta P_l + \Delta P_v + \Delta P_g \qquad (3.1)$$

In general, the pressure drop term, on the right-hand side of Eq. (3.1) increase with heat load. If a heat pipe is to operate continuously without drying out the wick, then the required capillary pressure should not exceed the maximum possible capillary pressure at any point along the heat pipe. Limitations on the heat transport capability of a heat pipe are

(1) capillary limit

(2) sonic limit

(3) entrainment limit

(4) boiling limit

3.1 Pressure Balance

The described pressure balance above can be expressed mathematically as

$$[P_v(x_{ref}) - P_v(x)] + [P_v(x)-P_l(x)] + [P_l(x)-P_l(x_{ref})] + [P_l(x_{ref}) -P_v(x_{ref})] = 0$$

If we define the followings as,

$$P_c(x) = P_v(x) - P_l(x) \tag{3.2}$$
$$P_c(x_{ref}) = P_v(x_{ref}) - P_l(x_{ref})$$
$$\Delta P_v(x-x_{ref}) = P_v(x) - P_v(x_{ref})$$
$$\Delta P_l(x_{ref}-x) = P_l(x_{ref}) - P_l(x)$$

If the reference position, x_{ref}, is chosen to be at x_{min} where the capillary pressure is minimum and equal to zero, Eq.(3.2) reduces to

$$P_c(x) = \Delta P_v(x-x_{min}) + \Delta P_l(x_{min}-x) \tag{3.3}$$

where $P_c(x)$ = capillary pressure at position ,x, along the heat pipe

$P_c(x_{ref})$ = capillary pressure at a reference position,x_{ref}

$\Delta P_v(x-x_{ref})$ = vapor pressure drop on flowing from x to x_{ref}

$\Delta P_l(x_{ref}-x)$ = liquid pressure drop on flowing from x_{ref} to x

3.2 Maximum Capillary Pressure

When a meniscus is formed at the liquid- vapor interface, can be calculated by Laplace and Young equation,

$$P_c = \sigma(1/R_1 + 1/R_2) \qquad (N/m^2) \qquad (3.4)$$

or

$$P_{cm} = 2\,\sigma/r_c \qquad (N/m) \qquad (3.5)$$

where $2/r_c = (1/R_1 + 1/R_2)$; is the surface tension coefficient of the liquid (N/m); R_1 and R_2 are the principal radii of curvature of the meniscus

Table 3.1 Expressions of Effective Capillary Radius r_c for Several Wick Structures

Wick structures	k_e Expressions[a]
Wick and liquid in series	$k_e = \dfrac{k_l k_w}{\epsilon k_w + k_l(1-\epsilon)}$
Wick and liquid in parallel	$k_e = \epsilon k_l + (1-\epsilon)k_w$
Wrapped screen	$k_e = \dfrac{k_l[(k_l + k_w) - (1-\epsilon)(k_l - k_w)]}{[(k_l + k_w) + (1-\epsilon)(k_l - k_w)]}$
Packed spheres	$k_e = \dfrac{k_l[(2k_l + k_w) - 2(1-\epsilon)(k_l - k_w)]}{[2k_l + k_w + (1-\epsilon)(k_l - k_w)]}$
Rectangular grooves	$k_e = \dfrac{(w_f k_l k_w \delta) + w k_l(0.185 w_f k_w + \delta k_l)}{(w + w_f)(0.185 w_f k_f + \delta k_l)}$

[a]Where k_e = effective thermal conductivity
k_l = liquid thermal conductivity
k_w = thermal conductivity of wick material
ϵ = wick porosity
w_f = groove fin thickness
w = groove thickness

3.3 Liquid Pressure Drop

The pressure drop of the liquid in wick structures can be obtained by integrating the liquid pressure gradient:

$$\Delta P_l(x_{min}-x) = P_l(x_{min}) - P_l(x)$$

$$= - \int_{X_{min}}^{X} dP_l \, dx \qquad (3.6)$$

Since liquid velocity in heat pipe wicks is generally very low, the dynamic pressure can be neglected. Then

$$\frac{dP_l}{dx} = \frac{-2\,\tau_l}{r_{h,l}} + \rho_l g \sin \qquad (3.7)$$

and

$$\frac{dP_l}{dx} = - F_l Q \pm \rho_l g \sin(\phi)$$

where

$$Re_l = 2r_{h,l}\,\rho_l v_l / \mu_l \qquad \text{(Reynolds number)} \qquad (3.8)$$

$$f_l = 2\tau_l / (\rho_l v_l^2) \qquad \text{(Drag coefficient)} \qquad (3.9)$$

$$v_l = Q / (\varepsilon A_w \rho_l \lambda) \qquad \text{(Liquid velocity-m/sec)} \qquad (3.10)$$

$$F_l = \mu_l / (K A_w \lambda \rho_l) \qquad \text{(Frictional coefficient for the liquid flow-}(N/m^2)(W\text{-}m)) \qquad (3.11)$$

K is calculated from the equation

$$K = 2\varepsilon\, r_{h,l}^2 / (f_l Re_l) \qquad \text{(Wick permeability-}m^2) \qquad (3.12)$$

F_l can be determined from K or $f_l Re_l$ by using Table 3.2 and Fig. 3.1-3.2

Table 3.2 Expressions of Wick Permeability K for Several Wick Structure

Wick structures	K Expressions
Circular artery	$K = \frac{r^2}{8}$
Open rectangular grooves	$K = \frac{2\epsilon r_{h,l}^2}{(f_l \mathrm{Re}_l)}$ $r_{h,l} = \frac{2w\delta}{w + 2\delta}$ ϵ = porosity = $\frac{w}{s}$ s = groove pitch w = groove width δ = groove depth $(f_l \mathrm{Re}_l)$ from Fig. 2-4
Circular annular wick	$K = \frac{2r_{h,l}^2}{(f_l \mathrm{Re}_l)}$ $r_{h,l} = r_1 - r_2$ $(f_l \mathrm{Re}_l)$ from Fig. 2-5
Wrapped screen wick	$K = \frac{d^2\epsilon^3}{122(1-\epsilon)^2}$ $\epsilon = 1 - \frac{1.05\pi N d}{4}$ d = wire diameter N = mesh number
Packed sphere	$K = \frac{r_s^2\epsilon^3}{37.5(1-\epsilon)^2}$ r_s = sphere radius ϵ = porosity (value depends on packing mode)

f_l is the frictional stress at the liquid-solid interface (N/m)

$r_{h,l}$ is the wick hydraulic radius (m)

ρ_l is the liquid demsity (kg/m^3)

g is the gravitational acceleration (m/sec^2)

λ is the latent heat of vaporization (J/kg)

ε is the wick porousity

ψ is the angle of inclination of the heat pipe measured from the horizontal direction

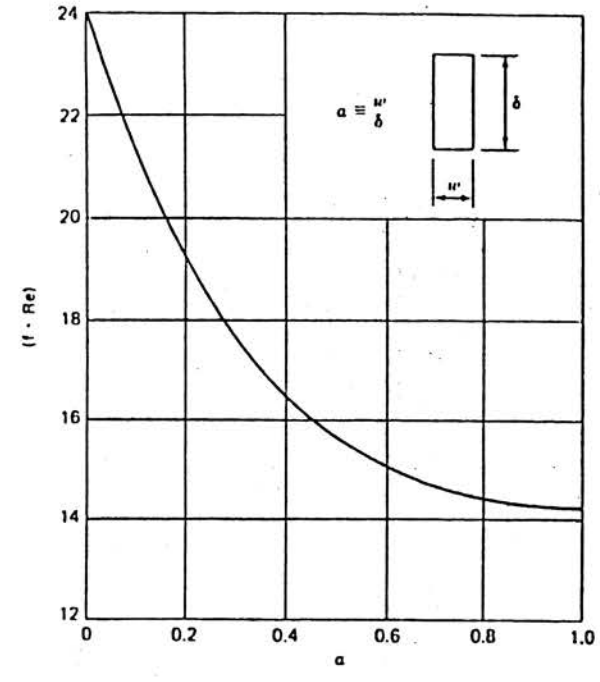

Fig 3.1 Frictional coefficients for laminar flow in rectangular tubes

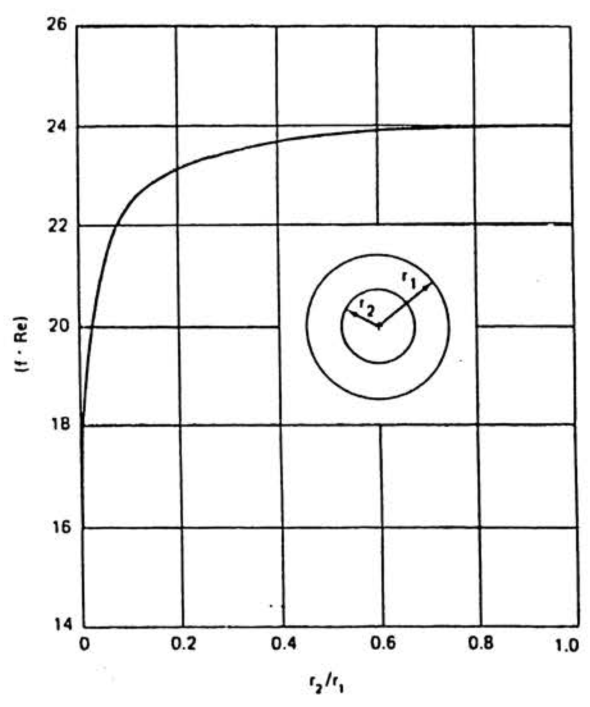

Fig 3.2 Frictional coefficients for laminar flow in circular annuli

3.4 Vapor Pressure Drop

The vapor pressure drop in the heat pipe vapor flow passage is calculated by integrating the vapor pressure gradient:

$$\Delta P_v(x-x_{min}) = P_v(x) - P_v(x_{min}) = \int_{x_{min}}^{x} \frac{dP_v}{dx}\,dx \qquad (3.13)$$

In steady state, the mass flow rate for the vapor is equal to that for the liquid at the same axial position. However, because of the low density for the vapor in comparison with that for the liquid, the vapor velocity can be considered much larger than the liquid velocity. Under these circumstances, the vapor pressure gradient results not only from the frictional drag but also from the dynamic effort, and the flow of vapor may be laminar or turbulent. The compressibility of the vapor may also become important.

The principle of conservation of axial momentum (neglecting gravitational force because of the low vapor density) requires that

$$\frac{dP_v}{dx} = -F_v Q - D_v \frac{dQ^2}{dx} \qquad (3.14)$$

where the frictional pressure coefficient for the vapor flow,

$$F_v = \frac{(f_v Re_v)\mu_v}{2r_{h,v}^2 A_v \rho_v \lambda} \qquad (3.15)$$

$$R_{e_v} = \frac{2r_{h,v}Q}{A_v \mu_v \lambda} \qquad (3.16)$$

and the dynamic pressure coefficients for the vapor flow is

$$D_v = \frac{\beta}{A_v^2 \rho_v \lambda^2} \tag{3.17}$$

Value of coefficients F_v and D_v for the presssure gradient are dependent upon flow conditions. For vapor flow in a circular duct, Mach number and Reynolds number can be used to determined F_v and D_v in Table 3.3

Table 3.3 Expression of Vapor Frictional Coefficient F_v and Dynamic Coefficient D_v

Flow conditions	F_v [a]	D_v [a]
$Re_v < 2300$, $M_v < 0.2$	$\frac{8\mu_v}{r_{h,v}^2 A_v \rho_v \lambda}$	$\frac{1.33}{A_v^2 \rho_v \lambda^2}$
$Re_v < 2300$, $M_v > 0.2$	$\left(\frac{8\mu_v}{r_{h,v}^2 A_v \rho_v \lambda}\right)\left(1 + \frac{\gamma_v - 1}{2} M_v^2\right)^{-1/2}$	$\frac{1.33}{A_v^2 \rho_v \lambda^2}$
$Re_v > 2300$, $M_v < 0.2$	$\left(\frac{0.019\mu_v}{A_v r_{h,v}^2 \rho_v \lambda}\right)\left(\frac{2r_{h,v}Q}{A_v \lambda \mu_v}\right)^{3/4}$	$\frac{1}{A_v^2 \rho_v \lambda^2}$
$Re_v > 2300$, $M_v > 0.2$	$\left(\frac{0.019\mu_v}{A_v r_{h,v}^2 \rho_v \lambda}\right)\left(\frac{2r_{h,v}Q}{A_v \lambda \mu_v}\right)^{3/4}\left(1 + \frac{\gamma_v - 1}{2} M_v^2\right)^{-3/4}$	$\frac{1}{A_v^2 \rho_v \lambda^2}$

3.5 Effective Thermal Conductivity of Wick Structures

The primary heat transfer mechanisms for heat pipes are.

(i) heat conduction across the container wall and the liquid saturated wick at the evaporator section with subsequent evaporation at the liquid-vapor interface of that section

(ii) convective transport of latent heat by vapor from the evaporator to the condenser

(iii) heat conduction across the liquid saturated wick and the container wall at the condenser section with subsequent condensation at the liquid-vapor interface of that section

Heat transfer in the liquid-saturated wick can be considered a

1. Solid and liquid in series
2. Solid and liquid in parallel

The thermal conductivity that occured is the effective thermal conductivity as shown in Table 3.4

Table 3.4 Expressions of Effective Thermal Conductivity k_e for Liquid Saturated Wicks

Wick structures	k_e Expressions[a]
Wick and liquid in series	$k_e = \frac{k_l k_w}{\epsilon k_w + k_l(1-\epsilon)}$
Wick and liquid in parallel	$k_e = \epsilon k_l + (1-\epsilon)k_w$
Wrapped screen	$k_e = \frac{k_l[(k_l + k_w) - (1-\epsilon)(k_l - k_w)]}{[(k_l + k_w) + (1-\epsilon)(k_l - k_w)]}$
Packed spheres	$k_e = \frac{k_l[(2k_l + k_w) - 2(1-\epsilon)(k_l - k_w)]}{[2k_l + k_w + (1-\epsilon)(k_l - k_w)]}$
Rectangular grooves	$k_e = \frac{(w_f k_l k_w \delta) + w k_l(0.185 w_f k_w + \delta k_l)}{(w + w_f)(0.185 w_f k_f + \delta k_l)}$

3.6 Limit to Heat Transport

Heat pipe will operate efficiently depends on continuously flow of the working fluid. The maximum heat transfer rate of a heat pipe is confined by the following limitations

1. Capillary limit
2. Sonic limit
3. Entrainment limit
4. Boiling limit

These limitations on maximum heat transport are shown in Fig. 3.3.

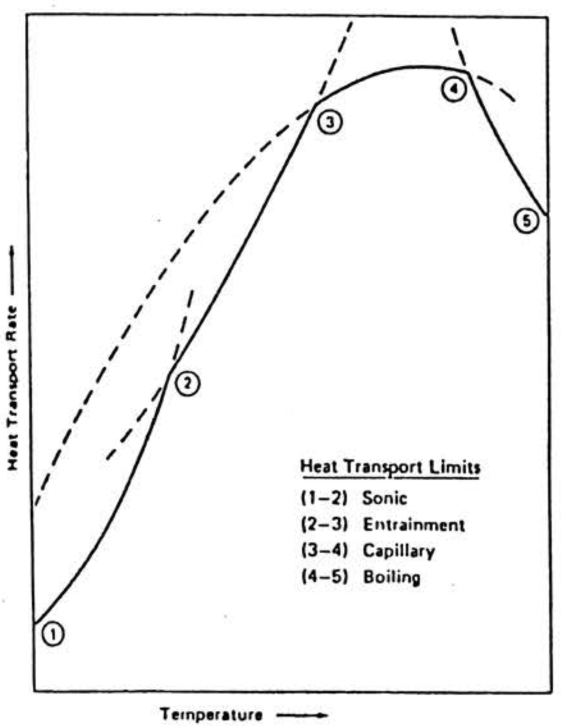

Fig. 3.3 Limitation to Heat Transport in the Heat Pipe

It is necessary for the operating point to be chosen in the area below these curves.

1. Sonic limit

This limitation occurs with the high temperature heat pipe with liquid metal as working fluid. Liquid metal will vaporate quickly at the initial stage which forces vapor from evaporation section to the condensation section. But the vapor velocity can not exceed the sonic limit. We can determined this limitation from Levy's equation

$$Q_{s,max} = A_v \rho_v \lambda \left[\frac{\gamma_v R_v T_v}{2\ (\gamma_v + 1)}\right]^{\frac{1}{2}} \tag{3.18}$$

2. Entrainment limit

This limitation occurs when shear force from vapor-liquid interface is equal or greater than liquid surface tension which causes liquid to entrain with vapor. This limitation can be determined form

$$Q_{e,max} = A_v \lambda \left[\frac{\sigma \rho v}{2r_{h,s}}\right]^{\frac{1}{2}} \tag{3.19}$$

where $r_{h,s}$ is hydraulic radius of wick

3. Boiling limit

This limitation occurs because the radial heat flux is so high that causes boiling in wick and vapor from boiling hinders the flow of working fluid. This limitation can be determined from

$$Q_{b,max} = \frac{2 \pi L_e K_e T_v}{\lambda \rho_v \ln(n/r_v)} \left[\frac{2\sigma}{r_n} - P_c \right] \tag{3.20}$$

where r_n is the radius of vapor bubbles

4. Capillary limit

This limitation occurs when vapor flow rate in heat pipe is equal to the maximum flow of the working fluid in wick. That is when maximum capillary pressure balances with the total pressure drop from eqns. (3.3),(3.6),(3.7),(3.13),(3.14). This results is

$$P_{cm} = \int_{X_{min}}^{X_{max}} [-F_v Q - D_v \frac{dQ^2}{dx} + F_l Q + \rho_l g \sin(\psi)] \, dx \qquad (3.21)$$

where ψ is the antigravity tilt angle

$$x_{min} = 0, \; x_{max} = L_t, \; P_{cm} = 2/r_c - \Delta P_\perp$$

At steady state $\frac{dQ^2}{dx} = 0$ then (3.21) becomes

$$(QL)_{c,max} = \frac{2\sigma}{r_c} - \frac{\Delta P_\perp - \rho_l g \sin(\psi)}{F_e + F_v} \qquad (3.22)$$

$$Q_{c,max} = (QL)_{c,max} \,/\, [\, \tfrac{1}{2} L_c + L_a + \tfrac{1}{2} L_c \,] \qquad (3.23)$$