Lagrangian Relaxation Method for Integrated Vehicles and Drones Routing Model

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การขนส่งสินค้าปัจจุบันนี้มีความต้องการเพิ่มขึ้นเป็นอย่างมากเนื่องจากการมาของระบบการซื้อขายผ่านอินเตอร์เน็ตทำให้หลายบริษัทได้พัฒนาการขนส่งรูปแบบต่าง ๆ เพื่อตอบสนองตลาดและลดต้นทุน หนึ่งในนั้นคือการใช้โดรนไร้ผู้บังคับร่วมกับยานพาหนะอื่นเช่นรถบรรทุก โดยโดรนสามารถออกจากทั้งคลังสินค้าหรือจากรถบรรทุกเพื่อไปส่งสินค้าแล้วกลับมาที่ฐานพัก เพื่อรอรถบรรทุกมารับกลับหรือบินกลับคลังสินค้าเอง ปัญหาการจัดเส้นทางของยานพาหนะร่วมกับโดรน (Vehicle Routing Problem with Drone) ที่ศึกษาเป็นปัญหาการจัดเส้นทางการส่งสินค้าของรถบรรทุกและโดรนไปพร้อม ๆ กัน ซึ่งเป็นปัญหาที่มีความซับซ้อนมากจึงไม่สามารถใช้วิธีการแก้ปัญหาแบบทั่วไปได้ งานวิจัยนี้ได้พัฒนาเทคนิคการผ่อนปรนปัญหาแบบลากรานจ์ (Lagrangian Relaxation) เพื่อนำมาใช้ควบคู่กับเทคนิค Branch-and-Price ในการคำนวนขอบเขตล่างของคำตอบในแต่ละขั้นของ Branch-and-Price ซึ่งขอบเขตล่างของลากรานจ์นั้นจะมีค่าใกล้คำตอบมากกว่าคำตอบจาก Column Generation ในขั้นนั้น ส่งผลให้เวลาที่ใช้ในการหาคำตอบลดลงจากการนำมาใช้เป็นเงื่อนไขในการหยุด Branch-and-Price ช่วยให้ไม่สำรวจ Node ที่ไม่จำเป็น ในการทดลองนั้นจะดำเนินการโดยใช้ตัวอย่างที่สร้างขึ้นมาแบบสุ่มและตัวอย่างจากแผนที่จริง นอกจากนี้ได้เปรียบเทียบต้นทุนของการใช้เพียงรถบรรทุกและการใช้รถบรรทุกร่วมกับโดรนเพื่อให้เห็นถึงประโยชน์ของการใช้โดรนได้ชัดเจนยิ่งขึ้น

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Perawit Charoenwut : Lagrangian Relaxation Method for Integrated Vehicles and Drones Routing Model. Advisor: Assoc. Prof. MANOJ LOHATEPANONT, Sc.D. Co-advisor: Prof. Shin-ei TAKANO

Nowadays, e-commerce increases parcel transportation demand year by year. Many companies are seeking the way to improve performance of delivery service and cost-efficiency. One among many solutions is using a drone attached with a truck for the last mile delivery. A drone can be launched from either depot or truck and returned to the depot or wait for the truck to take them back from the docking station around the delivery area. The vehicle routing problem with drones considers truck- and drone-route simultaneously. It is an enormous problem that cannot be solved by a normal linear programming method. This research proposes the Lagrangian relaxation technique, in conjunction with the Branch-and-Price technique, to estimate the lower bound of the solution in each iteration of the Branch-and-Price. The lower bound from the Lagrangian relaxation is tighter than the solution from the Column generation which can improve the total solution time. The bound use in Branch-and-Price as a fathom condition, these help the search tree not to explore unnecessary nodes. The computational experiment includes randomly generated instances and real-world instances. Moreover, the operation cost of truck-based and truck-drone-based are compared to point out the benefit of using drones.

Field of Study:	Civil Engineering	Student's Signature ....
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Perawit Charoenwut

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Introduction

1.1 Background

The development of the internet and mobile applications make an easy access to stores around the world. The percentage of change in the e-commerce market is 28.0% and 22.9% in 2017 and 2018 respectively (Lipsman, 2019). In China, since 2014, the gross merchandise value of Black Friday, Cyber Monday, and T-Mall Single's Day were increased at least 30% year by year (Lai & Lui, 2019). In 2019, Alibaba Single's Day event made a total of $38.3 billion which is the world's largest shopping event (Yang, 2019). In the US, Amazon made $232.9 billion from net sales (Amazon, 2019b). In Amazon's Prime Day 2019, Amazon members around the world purchase more than 175 million items in this event and more than half of them were shipped on the same day or faster. From an operator perspective, this is a big challenge to handle this number of deliveries (Amazon, 2019a).

According to "The Last-Mile Delivery Challenge" (K. Jacobs et al., 2019), customers are giving precedence to delivery speed because their orders are usually fresh food or things from a crowded retail store that they don't want to waste their time with. Customers will change to more cost-efficient services such as same-day delivery. This is such a big challenge for the delivery company to maintain their quality of service (e.g., customer waiting time) that directly affects customer's loyalty while controlling their cost. An analysis showed that increasing delivery will decrease to a total profit of 26% in 3 years. Because the last-mile delivery can be cost up to 50% of the total delivery cost (Michał et al., 2019).

“Globally, more people live in urban areas than in rural areas, with 55 percent of the world’s population residing in urban areas in 2018. In 1950, 30 percent of the world’s population was urban, and by 2050, 68 percent of the world’s population is projected to be urban.” (United Nations Publications, 2019) Urbanization is always followed by massive road traffic. For example, In Japan, Japanese people waste their time more than 8 billion hours per year (Kono & Joshi, 2019). This undeniable problem directly affects the quality and cost of last-mile delivery. There were many ideas proposed, for example, Drop-shipping, Drop-off lockers, Autonomous vehicles, and Drones (Michał et al., 2019).

In 2013, the announcement of Jeff Bezos that Amazon is developing drones for small goods deliveries as a program called “Amazon Prime Air”. They told that at least 86% of the weight of their shipped package is below 5 pounds which is the weight carried limit of the drone, then the last-mile delivery process would take only 30 minutes (Popper, 2015). In 2016, They have succeeded in their first delivery flight in the suburbs of Cambridge, UK. After the announcement, many retails and logistics companies have turned their heads to this technology. Even non-logistics-based company, Alphabet Inc., the parent company of Google, developed their drone’s delivery project called Wing. In April 2019, the US Federal Aviation Administration (FAA) approved its operation. Then, the wing’s drone makes an operation in Virginia, US. In September the same year, they became a business partner with FedEx to operate delivery in Australia and the US. Wing said, “it would like to offer deliveries across the US in the future.” (Murphy, 2019),(Elias, 2019). This trend is not in the only western region. Recently, Rakuten and JD.com Japanese and Chinese retail companies have collaborated. JD.com started their drone program in 2015 and launched commercial flights in 2016 in Jiangsu, Shaanxi, and other provinces in China. While, Rakuten launch “Sora Raku” which is a golf club delivery service by drone (Rakuten, 2016), (Rakuten, 2019).

Figure 1 Amazon Prime Air (ONISHI, 2019)

In rural areas, the challenge is healthcare delivery. The use of drones is for delivering aid packages and medicines to the site. As well as delivering blood and specimens for a rural hospital to the laboratory which is usually located in the city center. The study of Médecins Sans Frontières (2014) concluded that drones could use delivery time only 25% of the time it took by land transportation. In Papua New Guinea, Tuberculosis is a serious problem. Especially in the Gulf Province which has hard to access health services. Médecins Sans Frontières and US company implement drone service to transport patient’s samples to the hospital and send back treatments (Atkinson & Mabey, 2019).

In the agriculture section, drones are set to fly over the field and capture images with the coordinator. It can be mapped in 2D. The map can be used for field monitoring and analysis. Moreover, drones can carry water and fertilizer and spray it into crops. This is more cost-efficient and easy to operate (Shakhatreh et al., 2019).

The advantage of using a drone is the ability to go anywhere without the concern of road congestion. A straightway of the drone's path could save a lot of time compared to travel along the congested road. However, there are also limitations. First, flight endurance and flight speed are constrained by battery technology. Second, the limitation of carried weight and size. Lastly, weather conditions can make drones fail in operation.

In the operation research area, there are a lot of mathematical models and methods to solve the truck-drone routing problem which will be reviewed in chapter 2. The proposed linear models can be categorized into 2 categories. The arc-based model which is the (Mixed) integer model contains a set of arcs from node to node as variables. The path-based model is the reformulation of the arc-based model. From the original arc-based variables turn to variables that contain a combination of arcs. The result is the high complexity constraints from the arc-based model are put into the step of generating variables. The simplex method is able to solve this problem but still consume a lot of computing resource due to a large number of variables. Many Authors use Column generation to solve this kind of problem (see Baldacci, Toth, and Vigo (2007), Feillet (2010)). The column generation is known as the successful iterative method to deal with a lot of variables. However, to obtain an integer solution, we need Branch-and-Price (Barnhart, Johnson, Nemhauser, Savelsbergh, & Vance, 1998). It is a combination between column generation and branch-and-bound. The branching rules are proposed by Ryan and Foster (1981).

The major disadvantage is slow convergence when the solution reaches the optimum point so-called tailing-off effect. The Lagrangian relaxation can provide the tight lower bound of the current node. In every explored node in the Branch-and-Bound tree, if the lower bound is greater than or equal to the current integer solution, that node is fathomed (Lübbecke & Desrosiers, 2005).

In this thesis, we use the Lagrangian relaxation as one of the fathom conditions in Branch-and-Price to solve the integrated vehicles and drones routing model. The expectation is the lower total computation time compared with the column generation without Lagrangian relaxation.

This thesis consists of the following chapter. In Chapter 2, the travelling salesman problem, vehicle routing problem, column generation, and Lagrangian relaxation works of literature are reviewed. In Chapter 3, the sub-network-based model is explained and the Lagrangian relaxation method is proposed. Chapter 4 is the experiment and result. The last chapter, chapter 5 is the conclusion and suggestions for future research.

1.2 Objectives of the study

- To develop a Lagrangian relaxation method for improving the speed of the Branch-and-Price framework for solving the integrated vehicles and drones routing model.
- To conduct a computational experiment to compare between the solutions which have only trucks in tours and the solutions which combined both types of vehicles.

1.3 Expected Benefit of Study

- The Branch-and-Price framework with the Lagrangian relaxation is faster than without the Lagrangian relaxation.
- In the same calculation time, Branch-and-Price framework with Lagrangian relaxation is able to handle a bigger problem size compared to without Lagrangian relaxation.
- The solution which combined both types of vehicles is better than using only trucks.

Chapter 2
Literature Review

2.1 Lagrangian Relaxation

Lagrangian relaxation is a relaxation method for estimates a solution of difficult Linear programming (LP) by removing some constraints which make the problem hard. Then, put into the objective function with a new multiplier vector μ. Each μ penalizes the solution if it does not satisfy the constraints.

Lagrangian relaxation is used for generating a lower bound (or upper bound in case of maximization problem). We recall the following theorem from Ahuja, Magnanti, and Orlin (1993).

Consider the following linear model:

$$z^* = min\, cx \qquad (1)$$

$$s.t. \quad Ax = b \qquad (2)$$

$$x \in X \qquad (3)$$

"*Lemma 1.1* (Lagrangian Bounding Principle). For any vector μ of the Lagrangian multipliers, the value L(μ) of the Lagrangian function is a lower bound on the optimal objective function value z* of the original optimization problem (P)."

"*Proof.* Since Ax = b for every feasible solution to (P), for any vector μ of Lagrangian multipliers, $z^* = \min\{cx: Ax = b, x \in X\}$ = $\min\{cx + \mu(Ax - b): Ax = b, x \in X\}$. Since removing the constraints Ax = b from the second formulation cannot lead to an increase in the value of the objective function (the value might decrease $z^* \geq \min\{cx + \mu(Ax - b): x \in X\} = L(\mu)$."

We can change the mentioned model to the Lagrangian relaxation by relaxing (2) as follow.

$$L(\mu) = min(cx + \mu(Ax - b) \quad (4)$$

$$s.t. \quad (2) \; and \; (3)$$

To get as close as possible to z^*, solving the Lagrangian multiplier is needed.

$$L^* = max_{\mu} L(\mu) \quad (5)$$

To find μ that makes L^* largest as possible, the most popular way is the sub-gradient method. First, initiate the Lagrangian multiplier. Second, improve it by using (6). If the relaxed constraint is greater than or equal to, then $\mu \geq 0$. On the other hand, If the relaxed constraint is greater than or equal to, then μ can be any number.

$$\mu^{k+1} = \mu^k + \theta^k(Ax - b) \quad (6)$$

Introducing step size θ^k by k is the number of iterations. There are two methods to calculate step size. First, the original one which $\theta^k = 1/k$. The second is from Held and Karp (1970) which $\theta^k = \frac{UB - L(\mu^k)}{\sum_1^n (Ax^k - b)^2}$ and UB is upper bound of the solution.

Lagrangian relaxation provides a tight lower bound of the solution of the minimization. The average gap of Lagrangian relaxation bound is from 0% to 10%. While, the LP relaxation gap is from 6% to 22% (Kwon, Kang, Lee, & Park, 1999). Moreover, it provides an upper bound or a good feasible solution (Fisher, 1985). The Branch-and-Bound algorithm mostly applies Lagrangian relaxation to get rid of waste calculation. (Tanaka & Araki, 2008)

Lagrangian relaxation can also mitigate the degeneracy of the Column generation or can be used to generate new columns (Huisman, Jans, Peeters, & Wagelmans, 1970). Many authors used this method in the routing problem (e.g. Kohl and Madsen (1997), Dell'Amico, Righini, and Salani (2006)). In the integrated vehicle and drone model, Z. Wang and Sheu (2019) use this method and weighted Dantzig-Wolfe decomposition for speed-up and stabilization.

2.2 Column Generation

Column generation was introduced by Dantzig and Wolfe (1960) and Gilmore and Gomory (1961). At that time, this method was proposed to solve the linear relaxation of the cutting stock problem which is the problem of cutting a piece of material into needed lengths. For example, they want to cut 13-metres woods into 3-meters, 5-meters, and 7-meters for a certain amount. The different lengths of wood can be made from the 13-meters. The variables are the pattern of cutting (e.g., 4 pieces of 3-meters, 2 pieces of 5-meters, a piece of 5 and 7-meters).

The number of combinations is a very large and inefficient way to solve by enumerating all possible combinations. Most of the variables will be zero (non-basic) in the optimal solution. Only a few variables are needed. They decided to break the master problem (MP) to the restricted master problem (RMP) which has the original constraints but consists of the subset of variables. Then, they introduced the sub-problem. The sub-problem is the problem for generating the new variable that has the potential to be a solution that has a negative reduced cost (in the minimization problem). The constraints of the sub-problem are the natural characteristic of the master problem. The objective function is the reduced cost of the new variables. On the other way, the column can be generated without sub-problem by price-out the reduced cost of every variable and add the column with negative reduced cost.

The procedure of the Column generation is described as following, 1) initiate the RMP with a subset of feasible variables (usually from the heuristic method). 2) Solve the RMP and obtain the value of the dual and put it into the sub-problem. 3) Solve sub-problem. 4) If the column with negative reduced cost is found, add the column into RMP and repeat steps 2 and 3 until no negative reduced cost column can be found. Then, the optimal solution is the solution of the last RMP.

2.3 Travelling Salesman Problem with Drones and Vehicle Routing Problem with Drones

The traveling salesman problem (TSP), the definition of itself is a problem of a salesman traveling to a given set of customers in the shortest distance, while the vehicle routing problem (VRP) is a generalization of TSP (Dantzig & Ramser, 1959). Given a number of customers, find the set of vehicle routes. Customers are served by each route depends on the total demand must not exceed the vehicle capacity limit. The objective is to minimize the total cost of delivery.

There are more than thousands of papers proposed variants of this problem such as the capacitated VRP, VRP with a time window, etc. and researchers always give either exact or heuristic algorithm to solve their problem (Golden, Raghavan, & Wasil, 2008).

The vehicle routing problem with drone (VRPD) is one of VRP's extensions which has more complexity from a large number of feasible combinations compared to the traditional one. The first papers of integrated vehicles and drones routing problem, "the flying sidekick traveling salesman problems" (FSTSP) by Murray and Chu (2015).

This is an extension of TSP. They focus only on the operation of a single truck attached to a single drone. The mixed-integer linear programming was formulated. Their objective is to minimize the time of the last vehicle (either a truck or a drone) back to the depot. Subject to covering constraint, sub tour elimination constraint, only once visited customer constraint, drone's launching/retrieving constraint, and drone's flight endurance constraint. Apparently, the TSP (also VRP) is the NP-hard problem (Kumar & Panneerselvam, 2012). Hence, it could be concluded that FSTSP is also an NP-hard problem. Therefore, A heuristic algorithm was proposed to solve a large-scale problem. They compared their solution with IP solution from MIP solver with 30 minutes time-limited, Savings algorithm, Nearest neighbor algorithm, and Sweep algorithm. The computational result, FSTSP's solution quality is better than the three algorithms mentioned and limited-time MIP solver's solution on average. In 2016, an extension of FSTSP, an alternative of the heuristic method, and modification of some constraints in FSTSP were proposed by Ponza (2016). Simulated Annealing (SA), a method for approximating optimum solution based on the Monte Carlo algorithm. This method gives a reasonable computing time and a good answer quality. In the same year, Agatz, Bouman, and Schmidt (2016) proposed an IP model called Traveling salesman problem with drone (TSP-D) and a route first–cluster second based on local search and dynamic programming. They work on finding the minimum time for a tour. This work is different from FSTSP. Due to the privacy regulation, they use the same set of routes for trucks and drone which is road network. A truck can be back to visited node for take drone to depot or launch it again if there are unserved customer nodes left. Furthermore, there is another extension of FSTSP proposed by Ha, Deville, Pham, and Ha (2018). All the works mentioned above were aimed to get the minimize of time, but in this work, they consider in the operational cost called min-cost TSP-D. Following by TSP-LS and GRASP which is heuristic approach.

X. Wang, Poikonen, and Golden (2016) and (Poikonen, Wang, & Golden, 2017) generalize the problem. Instead of a single truck and a single drone in an operation, they described VRPD and theoretically analysis on several worst-case scenarios and present the bound in case of using trucks and drones rather than trucks.

In 2019, Kitjacharoenchai, Ventresca, et al. (2019), The multiple traveling salesman problem with drone (mTSPD) were proposed. mTSPD was modifying and adding some constraints to FSTSP to perform multiple truck delivery and a drone can interact with more than one truck in the minimum time of the delivery process. Genetic algorithm, Combined K-means/nearest neighbor, and Random cluster/tour were used to initiate an mTSP tour and they performed an adaptive insertion heuristic to find the solution of mTSPD. The computing results showed that the best solution quality in limited time and lowest standard deviation are from a genetic algorithm. They also adjust the number of trucks per delivery which results in the more truck used the less time spent on the delivery process, but it has to trade-off with more computing time. After mTSPD, The researcher has proposed an improved version of mTSPD called Two Echelon Vehicle Routing Problem with Drone (Kitjacharoenchai, Min, & Lee, 2019). This paper extends the FSTSP as well as mTSPD but consider the capacity of vehicles and drone can carry and serve more than one customer. To solve the large size instance, Drone Truck Route Construction (DTRC) and Large Neighborhood Search (LNS) was proposed along with sensitivity analysis were conducted.

The assumption of mTSPD has disregarded the capacity of a truck that does not reflect the real-world scenario. Z. Wang and Sheu (2019) study the vehicle routing problem (VRPD). They take care of the capacity of trucks and customer demand. To be more realistic, a drone cannot return to the truck via a customer node but a docking node or a depot. They presented a typically arc-based model to explain the hardness of the problem itself. Then, they proposed a path-based model. The major difference from the arc-based model is 1) variable of the path-based model is a separately feasible complete path of each vehicle, while arc-based model's variable is an arc from one node to others. 2) Many constraints were used to satisfy in generating variables. Thus, there are only four instead of twenty-one constraints left in the path-based model. Besides, they developed the column generation and branch-and-price framework to solve it. Within 4 and a half hours they can obtain the exact solution for 15 nodes instance. The delivery cost can be saved up to 20% on average compared to the same set of customers but using no drones. They also conduct sensitivity analysis of drone flight endurance. The more max flight duration, the more customers that drone can serve. By double the flying duration, The total cost reduced almost 10%.

Chapter 3
Methodology

3.1 Problem Definition

3.1.1 Vehicle Routing Problem with Drone (Arc-Based and Path-Based)

The vehicle routing problem is a vehicle flow network problem defined as a graph $G = (N, A)$. A network contains two elements; a set of the depot and customer nodes $N = \{n_1,\ n_2, \dots, n_n\}$ and a set of arcs $A = \{(i, j)|i, j \in N, i \neq j\}$ which is a route of the vehicle from node to node. Each arc has a travel cost which can be distance or travel time or travel cost. Those are represented by a symmetrical matrix $C = (c_{ij})$. There are several major constraints of this model: 1) each customer node must be visited only once and must satisfy the customer demand. 2) All vehicle routes start and end at the depot. 3) Each vehicle has a capacity limit.

According to Z. Wang and Sheu (2019) work, the VRPD can be defined as a graph G as the same way of VRP. A set of docking hub node $O = \{o_1,\ o_2, \dots, o_m\}$ were introduced and be included in set N. The docking hub nodes are for the landing of a drone and a truck must collect the drone back to depot or re-launch. Both of vehicle type has a capacity limit. The drone has a maximum flying duration or distance due to the limited capacity of the battery.

In the sequence of the operation, all vehicles must start and back to the depot. a drone can serve in-range customers around the depot. For others, a truck will out from the depot and serve by itself or launch the drone and serve other customers instead. When the drone finished the operation, a drone will land at the docking hub and be collected by a truck. However, a truck can launch a spare drone if the drone they launch before is landing after it arrives at the docking hub. The time of battery swapping and parcel loading to the drone is neglect. The arc-based is the following model.

$$min\ F^T\left(\sum_{(i,j)\in A:i=o^s} \sum_{k\in K} x_{ijk} + \sum_{(i,j)\in A:i=o^s} \sum_{k\in K} u_{ijk}\right) + C^T \sum_{(i,j)\in A} \sum_{k\in K} t_{ij}^T(x_{ijk} + u_{ijk}) + C^D \sum_{(i,j)\in A} \sum_{d\in D} t_{ij}^D y_{ijd} \quad (7)$$

s.t.

$$\sum_{(i,j)\in A:i=o^s} \sum_{k\in K} x_{ijk} + \sum_{(i,j)\in A:i=o^s} \sum_{k\in K} u_{ijk} = \sum_{(i,j)\in A:j=o^t} \sum_{k\in K} x_{ijk} + \sum_{(i,j)\in A:j=o^t} \sum_{k\in K} u_{ijk} \quad (8)$$

$$\sum_{(i,j)\in A:i=o^s} \sum_{d\in D} y_{ijd} + \sum_{(i,j)\in A:i=\sigma^s} \sum_{k\in K} \sum_{d\in D} z_{ijkd} = \sum_{(i,j)\in A:j=o^t} \sum_{d\in D} y_{ijd} + \sum_{(i,j)\in A:j=o^t} \sum_{k\in K} \sum_{d\in D} z_{ijkd} \quad (9)$$

$$\sum_{(i,j)\in A} \sum_{k\in K} x_{ijk} + \sum_{(i,j)\in A} \sum_{d\in D} y_{ijd} + \sum_{(i,j)\in A} \sum_{k\in K} u_{ijk} = 1\ \forall j \in C \quad (10)$$

$$\sum_{(i,j)\in A} \sum_{k\in K} x_{ijk} + \sum_{(i,j)\in A} \sum_{d\in D} y_{ijd} + \sum_{(i,j)\in A} \sum_{k\in K} u_{ijk} = 1\ \forall i \in C \quad (11)$$

$$\sum_{j\in N} x_{ijk} + \sum_{j\in N} u_{ijk} = \sum_{j\in N} x_{jik} + \sum_{j\in N} u_{jik} \forall k \in K, i \in O \cup C \quad (12)$$

$$\sum_{j\in N} y_{ijd} + \sum_{k\in K} \sum_{j\in N} z_{ijkd} = \sum_{j\in N} y_{jid} + \sum_{k\in K} \sum_{j\in N} z_{jikd} \forall d \in D, i \in O \cup C \quad (13)$$

$$\sum_{d\in D} z_{ijkd} \le L^R \forall k \in K, (i,j) \in A \quad (14)$$

$$\sum_{(j,i)\in A} \sum_{d\in D} y_{jid} + \sum_{(j,i)\in A} \sum_{k\in K} \sum_{d\in D} z_{jikd} \le \sum_{(i,j)\in A} \sum_{k\in K} L^S(x_{ijk} + u_{ijk}) \forall j \in O \cup C \quad (15)$$

$$v_{id} \le T^D \forall d \in D, i \in N \quad (16)$$

$$v_{jd} \ge v_{id} + t_{ij}^D + (y_{ijd} - 1)M \forall d \in D, (i,j) \in A, i \in C \quad (17)$$

$$v_{jd} \ge t_{ij}^D + (y_{ijd} - 1)M \forall d \in D, (i,j) \in A, i \in O \cup \{0^s\} \quad (18)$$

$$w_{id}^D \le L^D \forall d \in D, i \in C \quad (19)$$

$$w_{jd}^D \ge w_{id}^D + g_j + (y_{ijd} - 1)M \forall d \in D, (i,j) \in A, j \in C \quad (20)$$

$$w_{ik}^T \le L^T \forall k \in K, i \in C \quad (21)$$

$$w_{jk}^T \geq w_{ik}^T + g_j + (x_{ijk} + u_{ijk} - 1)M \forall k \in K, (i,j) \in A, j \in C \quad (22)$$

$$w_{jk}^T \geq w_{ik}^T + (x_{ijk} + u_{ijk} - 1)M \forall k \in K, (i,j) \in A, j \in O \quad (23)$$

$$1 + (u_{ijk} - 1)M \leq \sum_{d \in D} z_{ijkd} \leq u_{ijk} M \forall k \in K, (i,j) \in A \quad (24)$$

$$x_{ijk} + u_{ijk} \leq 1 \forall k \in K, (i,j) \in A \quad (25)$$

$$y_{ijd} + \sum_{k \in K} z_{ijkd} \leq 1 \forall d \in D, (i,j) \in A \quad (26)$$

$$\sum_{(n,i) \in A} y_{nid} \leq (1 - z_{ijkd})M \forall (i,j) \in A, i \in C, k \in K, d \in D \quad (27)$$

$$x_{ijk}, y_{ijd}, z_{ijkd}, u_{ijk} \in 0,1 \quad (28)$$

$$v_{id}, w_{id}^D, w_{ik}^T \geq 0 \quad (29)$$

The variables are defined as follows.

- x_{ijk}: Equals 1 if truck k travels arc (i, j) ∈ A independently, and 0 otherwise.
- y_{ijd}: Equals 1 if the d[th] drone travels arc (i, j) ∈ A independently, and 0 otherwise.
- z_{ijkd}: Equals 1 if the truck k carries the drone d through an arc (i, j) ∈ A, and 0 otherwise.
- u_{ijk}: Equals 1 if the kth truck carries one or more drones through an arc (i, j) ∈ A, and 0 otherwise.
- v_{id}: Be the cumulative flying time at node i for the drone d after its last leave from the depot or a docking node.
- w_{id}^D: Be the cumulative weight units of customer parcels at node i that the drone d has dropped after its last leave from the depot or a docking node.
- w_{ik}^T: Be the cumulative weight units of customer parcels at node i that the kth truck has delivered

The objective function is (7) to minimize the transportation cost. Constraint (8) and (9) make the trucks and drones that leave the depot must return to the depot, respectively. Constraints (10) and (11) make in and outflow of a customer node are 1. Constraints (12) and (13) make each docking hub has equal drones in and out. Constraints (14) and (15) restrict loaded drone capacity on a truck. Constraints (16)-(18) are drone flying duration constraints. Constraints (19) and (20) are the drone capacity constraints. Constraints (21)-(23) are the truck capacity constraints. Constraints (24)-(26) define the relationships of binary variables. Constraint (27), landing on the customer node is disallowed.

As mentioned in chapter 2, This model is NP-Hard. To solve this with a big number of customers, a huge computing resource is required. Because of many constraints have weak linear relaxation. To get better performance, they reformulate into the Path-Based model. There are two types of variables: 1) Truck path, the complete tour from depot to customers and/or to docking hub, then, back to the depot. 2) Drone path, the complete tour from the depot to the customer(s) and back to the depot or the complete tour from the depot to the truck's served customer(s), launch by truck and back to the depot by itself or landed to docking hub and back to the depot together with a truck. The optimal solution is the combination of truck path and drone path with the lowest total cost. The path-based is the following.

$$min \sum_{r \in R^T} (c_r + F^T) x_r + \sum_{r \in R^D} c_r \cdot y_r \quad (30)$$

s.t.

$$\sum_{r \in R^T} \delta_i^r x_r + \sum_{r \in R^D} \delta_i^r y_r \geq 1 \forall i \in C \quad (31)$$

$$\sum_{r \in R^T} L^S \theta_i^r x_r \geq \sum_{r \in R^D} \theta_i^r y_r \forall i \in O \quad (32)$$

$$\sum_{r \in R^T} L^R \vartheta_a^r x_r \geq \sum_{r \in R^D} \varphi_a^r y_r \forall a \in A^0 \quad (33)$$

$\sum_{r \in R^T} x_r \leq |K|$ *(34)*

$x_r, y_r \in \{0,1\}\ \forall r \in R^T, r' \in R^D$ *(35)*

Let R^T and R^D be the set of all truck and drone feasible paths start from o^s to o^t , respectively. x_r and y_r are decision variables.

The objective function is (30) to minimize the total transportation cost. Constraint (31) make each customer is served only once. δ_i^r equals 1 if r travels to node i either truck or drone node, and 0 otherwise. Constraints (32) and (33) make a truck cannot collaborate with more drone(s) than the restricted one. θ_i^r equals 1 if r travels to docking or truck node, and 0 otherwise. Constraint (34) is the maximum truck that can be used. ϑ_a^r and φ_a^r equals 1 if r travels to truck and drone arc an in A^0 which is truck arc without drone node, and 0 otherwise.

The characteristic of the Path-Based model is a huge number of variables (Columns). The researchers developed a branch-and-price algorithm (See Barnhart et al. (1998)). First, they generated a first restricted master problem by select some columns using the solution of the Saving heuristic then solve. Secondly, solve a pricing sub-problem to generate the negative reduced cost columns then put the column into the restricted master problem then solve. If the solution is an integer solution, return it; otherwise, use the branch-and-bound framework to obtain the integer solution.

3.1.2 Vehicle Routing Problem with Drone (Sub-network Based)

According to a working paper of Pichayavet, Charoenwut, and Lohatepanont (2019), The sub-network-based model was proposed which is heavily inspired by the Path-Based model of Z. Wang and Sheu (2019). Each variable represents the feasible tour of trucks and drones start from the depot to all customers and back to the depot again. Following this methodology, the VRPD is reformed to set covering formulation (e.g., see Winston and Goldberg (2004)). The number of variables of this model is more than the path-based model and extremely higher than the arc-based model, but the set covering has tight linear relaxation which benefits the procedure of searching for an integer solution in a branch and bound tree.

The sub-network-based assumptions are on the following.

- Any drone can perform only one cycle of delivery per tour.
- The maximum number of trucks per tour is 2.
- A drone can land only on a docking hub or a depot.
- The charging and swapping the battery time are neglected. All drones are homogenous and work properly while delivering a parcel.

A feasible tour is defined as follows.

- A tour can have a maximum number of trucks up to 2 trucks.
- As illustrated in Fig. 1, a drone can depart from either the depot node or from a truck in any customer node. After the delivery process, a drone can either fly back to the depot or land on the docking hub.
- A tour must satisfy the capacity of both vehicles and the maximum distance limit of a drone.

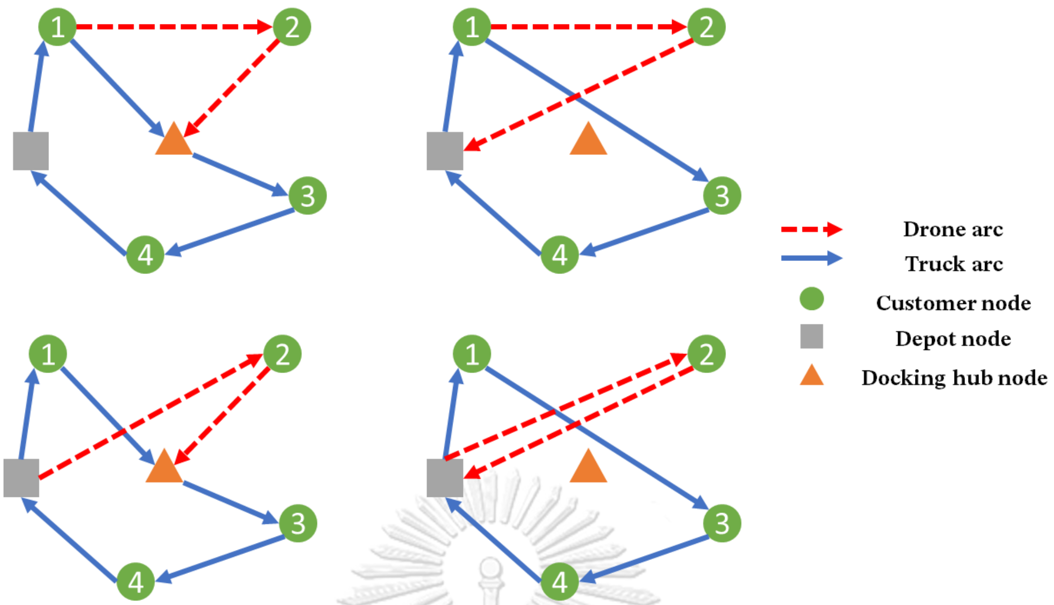

Figure 2 Illustration of feasible tours given the node 2 is served by a drone.

3.2 Mathematical Model

Given a graph $G = (N, A)$. Let N be a set of nodes, containing the depot node which are o_s and o_t for the origin and the destination of a tour respectively, docking hub node $O = \{o_1, o_2 \dots, o_m\}$ and customer nodes $C = \{c_1, c_2, \dots, c_n\}$. Let $A = \{(i, j)|i, j \in N, i \neq j\}$ be a set of arcs (i, j) represent a route of a vehicle from node i to node j. The mathematical model can be formulated as follow.

$$min \ \sum_{r \in R_c} (c_r + F^T) x_r \tag{36}$$

$$s.t. \ \sum_{r \in R_c} \delta_i^r x_r = 1; \forall i \in C \tag{37}$$

$$\sum_{r \in R_c} k_r^T x_r \leq |K| \tag{38}$$

$$x_r \in \{0,1\}, \forall r \in R_c \tag{39}$$

The decision variables x_r is defined to represent which tour $r \in R_c$ is chosen as a solution. The objective function (36) is to minimize the total cost: c_r transportation cost of tour r and F^T fixed cost of trucks. Constraint (37) is the set coverage constraint that makes every customer be served once. When R_c is the set of all feasible tours. δ_i^r be 1 if $r \in R_c$ has any vehicle visit node i, and be 0 otherwise. Constraint (38) is the number of trucks used. Lastly, the number of trucks employed for a tour k_r^T must not exceed the maximum number of trucks K.

3.3 Column Generation and Branch-and-Price Framework

In column generation, we need the sub-problem to add a new variable in each iteration. Referring to the working paper of (Pichayavet et al., 2019), Let graph $G = (N, A)$. Let N be a set of the depot node, docking hub node O and customer nodes C. The customer nodes C has three layers which is a customer visited by any of the two trucks and any drone. $T^1 = \{c_1, c_2, \ldots, c_n\}$ and $T^2 = \{c_1, c_2, \ldots, c_n\}$ are defined as a set of customer nodes served by the first and second truck and $D = \{c_1, c_2, \ldots, c_n\}$ defined as a set of customer nodes served by a drone. The docking hub nodes and depot remain the same as the master problem. The arcs $A_{\{(i,j)\}}$ are also replicated into three layers: a truck arc $a_{(i,j)}^{T_n}$ where $n \in \{1,2\}$ and a drone arc $a_{(i,j)}^{D}$. A truck arc of each layer represents an arc that traveled by each truck. All possible $a_{(i,j)}^{T_n}$ is a combination of two nodes from $\{o_s, o_t\} \cup O \cup T^n$ without connecting between layers. A drone arc represents an arc that traveled by a drone itself. According to the feasible tour definitions, a drone can visit three types of nodes: departing node, visiting, and landing node. As any drone can either departing from a truck or a depot by itself, the start node i of a drone arc can be any type of node in $\bigcup_{n=1}^{2} T^n \cup D \cup O \cup \{o_s\}$. For visiting nodes, they can be a drone's customer node (D), a docking hub (O), or a depot. As a drone is not allowed to land on any customer node, the destination of a drone j can be $(D \cup O \cup \{o_t\})$ but not T^n. Therefore, the sets of all possible truck arcs and drone arcs can be defined as: A^{T_n} is a set of all possible arcs for n^{th} truck. A^T is a set of all possible arcs for every truck. A^D is a set of all possible drone arcs.

$$A^{T_n} = \{(i,j)|i \neq j, i \in T^n \cup O \cup \{o_s\}, j \in T^n \cup O \cup \{o_t\}\} \ ;n \in \{1,2\} \qquad (40)$$

$$A^T = \bigcup_{n=1}^{2} A^{T_n} \qquad (41)$$

$$A^D = \left\{(i,j)|i \neq j, i \in \ \bigcup_{n=1}^{2} T^n \cup O \cup \{o_s\}, j \in D \cup O \cup \{o_t\}\right\} \qquad (42)$$

The variables are defined as follows.

- $a^{Tn}_{(i,j)}$: equals 1 if a truck passed arc $(i,j) \in A^{T_n}$, and 0 otherwise.
- $a^{D}_{(i,j)}$: equals 1 if a drone passed arc $(i,j) \in A^D$, and 0 otherwise.
- θ_i: be the number of arcs from node $i \in N$.
- β_i: be the number of arcs to node $i \in N$.
- y_i^n: be the cumulative weight unit of the n^{th} truck at node $i \in \{o_s, o_t\} \cup O \cup T^n$.
- g_i: be the cumulative distance traveled by truck at node $i \in \{o_s, o_t\} \cup O \cup T^n$.
- z_i: be the cumulative weight unit of a drone at node $i \in N$.
- υ_i: be the drone cumulative flying distance at node $i \in N$.
- α_i^n: equals 1 if the n^{th} truck passed node $i \in \{o_s, o_t\} \cup O \cup T^n$, and equals 0 otherwise.
- γ_i^n: be a binary variable used in if-then constraint for $i \in O \cup T^n$.

The parameters are defined as follows.

- q_i: be a customer demand of each node $i \in N$ (depot and docking hub are set to be 0).
- $d^T_{(i,j)}$: be the distance of a truck arc $(i, j) \in A^{T_n}$.
- $d^D_{(i,j)}$: be the distance of a drone arc $(i, j) \in A^D$.
- $c^T_{(i,j)}$: be a travel cost of a truck arc $(i, j) \in A^{T_n}$.
- $c^D_{(i,j)}$: be a travel cost of a drone arc $(i, j) \in A^D$.
- L_t: be the capacity for a truck.
- L_d: be the capacity for a drone.
- L_r: be the maximum number of drones that a truck can carry in one route.
- D_d: be the flying distance limit of a drone.

The objective function of the sub-problem is minimizing the reduced cost of the tour. Let π and σ be the dual variables of constraints (37) and (38) respectively. i_T and i_D represent the index of a customer node visited by the n^{th} truck and a drone, respectively. Let $\widetilde{c_r}$ be the reduced cost of a combined path.

$$\widetilde{c_r} = c_r - \sum_{i \in C} \pi_i \left(\theta_{i_D} + \sum_{n=1}^{2} \alpha^n_{i_T}\right) - \sigma k^T_r \ ; r \in R_c \tag{43}$$

$$min\, \widetilde{c_r} \tag{44}$$

$$s.t. \sum_{(i,j) \in A^{T_n}} a^T_{(i,j)} - \sum_{(j,k) \in A^{T_n}} a^T_{(j,k)} = 0; \forall j \in T^n \cup O, \forall n \in \{1,2\} \tag{45}$$

$$y^n_j \geq y^n_i + q_j a^T_{(i,j)} - \boldsymbol{M}\left(1 - a^T_{(i,j)}\right); \forall (i,j) \in A^{T_n}, \forall n \in \{1,2\} \tag{46}$$

$$g^n_j \geq g^n_i + d^T_{(i,j)} a^T_{(i,j)} - \boldsymbol{M}\left(1 - a^T_{(i,j)}\right); \forall (i,j) \in A^{T_n}, \forall n \in \{1,2\} \tag{47}$$

$$y^n_j \leq L_t; \forall i \in \{o_t\} \cup T^n \cup O, \forall n \in \{1,2\} \tag{48}$$

$$\theta_i = \sum_{(i,j) \in \boldsymbol{A^T}} a^T_{(i,j)} + \sum_{(i,j) \in A^D} a^D_{(i,j)}; \forall i \in \{o_s\} \cup \bigcup_{n=1}^{2} T^n \cup O \cup D \tag{49}$$

$$\beta_j = \sum_{(i,j)\in \boldsymbol{A^T}} a^T_{(i,j)} + \sum_{(i,j)\in A^D} a^D_{(i,j)} \,;\forall j \in \{o_t\} \cup \bigcup_{n=1}^{2} T^n \cup O \cup D \qquad (50)$$

$$\alpha_i^n = \sum_{(i,j)\in A^{T^n}} a^T_{(i,j)} \,;\forall i \in \{o_s\} \cup T^n \cup O, \forall n \in \{1,2\} \qquad (51)$$

$$\sum_{(i,j)\in \boldsymbol{A^T}} a^T_{(i,j)} \ge 1; \forall i \in \{o_s\} \qquad (52)$$

$$\sum_{(i,j)\in A^D} a^D_{(i,j)} - \sum_{(j,k)\in A^D} a^D_{(j,k)} = 0; \forall j \in D \qquad (53)$$

$$z_j \ge z_i + q_j a^D_{(i,j)} - \boldsymbol{M}\left(1 - a^D_{(i,j)}\right) ;\forall (i,j) \in A^D \qquad (54)$$

$$\upsilon_j \ge \upsilon_i + d^D_{(i,j)} a^D_{(i,j)} - \boldsymbol{M}\left(1 - a^D_{(i,j)}\right) ;\forall (i,j) \in A^D \qquad (55)$$

$$z_i \le L_D \,;\forall i \in D \qquad (56)$$

$$\upsilon_i \le D_D \,;\forall i \in D \cup O \cup \{o_t\} \qquad (57)$$

$$\theta_i \le 1 \,;\forall i \in D \qquad (58)$$

$$\sum_{i\in T^n} (\theta_i - \alpha_i^n) \le L_R \,;\forall n \in \{1,2\} \qquad (59)$$

$$\sum_{i\in O} \left(\beta_i - \sum_{n=1}^{2} \alpha_i^n\right) \le L_R \qquad (60)$$

$$\sum_{(i,j)\in A^D} a^D_{(i,j)} \le \boldsymbol{M}\alpha_i^n \,;\forall i \in T^n, n \in \{1,2\} \qquad (61)$$

$$\sum_{(i,j)\in A^D} a^D_{(i,j)} \le \boldsymbol{M}\left(\sum_{n=1}^{2} \alpha_i^n\right) ;\forall i \in O \qquad (62)$$

$$\sum_{(i,j)\in A^D} a^D_{(i,j)} \le \boldsymbol{M}\left(1 - \sum_{n=1}^{2} \alpha_k^n\right) ;\forall (j,k) \in \{(j,k) \mid j = k\} \qquad (63)$$

$$\sum_{(i,j)\in A^D} a^D_{(i,j)} \le \boldsymbol{M}\left(\sum_{n=1}^{2} \alpha_i^n\right) ;\forall j \in O \qquad (64)$$

$$\sum_{(i,j)\in A^D} a^D_{(i,j)} \le \boldsymbol{M}(1 - \gamma_i^n) \,;\forall i \in T^n \cup O, \forall n \in \{1,2\} \qquad (65)$$

$$g_j^n - g_i^n \le \boldsymbol{M}\gamma_i^n \,;\forall (i,j) \in \{(i,j) \mid i \ne j, i \in T^n \cup O, j \in O\}, \forall n \in \{1,2\} \qquad (66)$$

Constraint (45) is the conservation of flow constraints for ensuring the number of trucks in and out of the node is equal. Constraints (46) and (47) satisfy the truck capacity feasibility and the cumulative distance in which a cumulative weight variable y_j and a cumulative distance variable g_j must be higher or equal to those of the previous node i. Constraint (48) ensures the capacity limit of each truck. Constraints (49)-(51)

define the variables θ, β, and α. Constraint (52) ensures that each path must release at least one truck. Constraint (53), the conservation of flow, ensuring the number of drones in and out of the node is equal. Constraints (54) and (55) satisfy the drone capacity feasibility and the flying distance in which a cumulative weight variable z_j and a cumulative flying distance variable υ_j must be higher or equal to those of the previous node i. Constraint (56) ensures the capacity limit of each drone. Constraint (57) ensures the flying distance limit of each drone. Constraint (58) ensures that each customer can be served by drone only once and preventing the multiple visited drone's nodes. Constraint (59), (60) ensure that a truck cannot carry drones more than restricted for any arc exists.

Constraints (61)-(62) guarantee the feasibility of departing and landing a drone in a tour. If the drone departs from the truck node or the docking hub node then, the truck must be visiting that node too. Constraint (63) prevents a double visit of a drone and a truck. Constraint (64) ensures that a drone on a docking hub needs a truck to visit. Constraints (65), (66) prevent the infeasible tour in which a drone lands on a docking hub where a truck already passed. The node that the drone departing from must has a cumulative distance of a truck less than or equal to the cumulative distance of the truck in the docking hub node.

Since we cannot guarantee that the solution from column generation is an integer, The branch-and-price framework is used. The algorithm has shown in Figure 3.

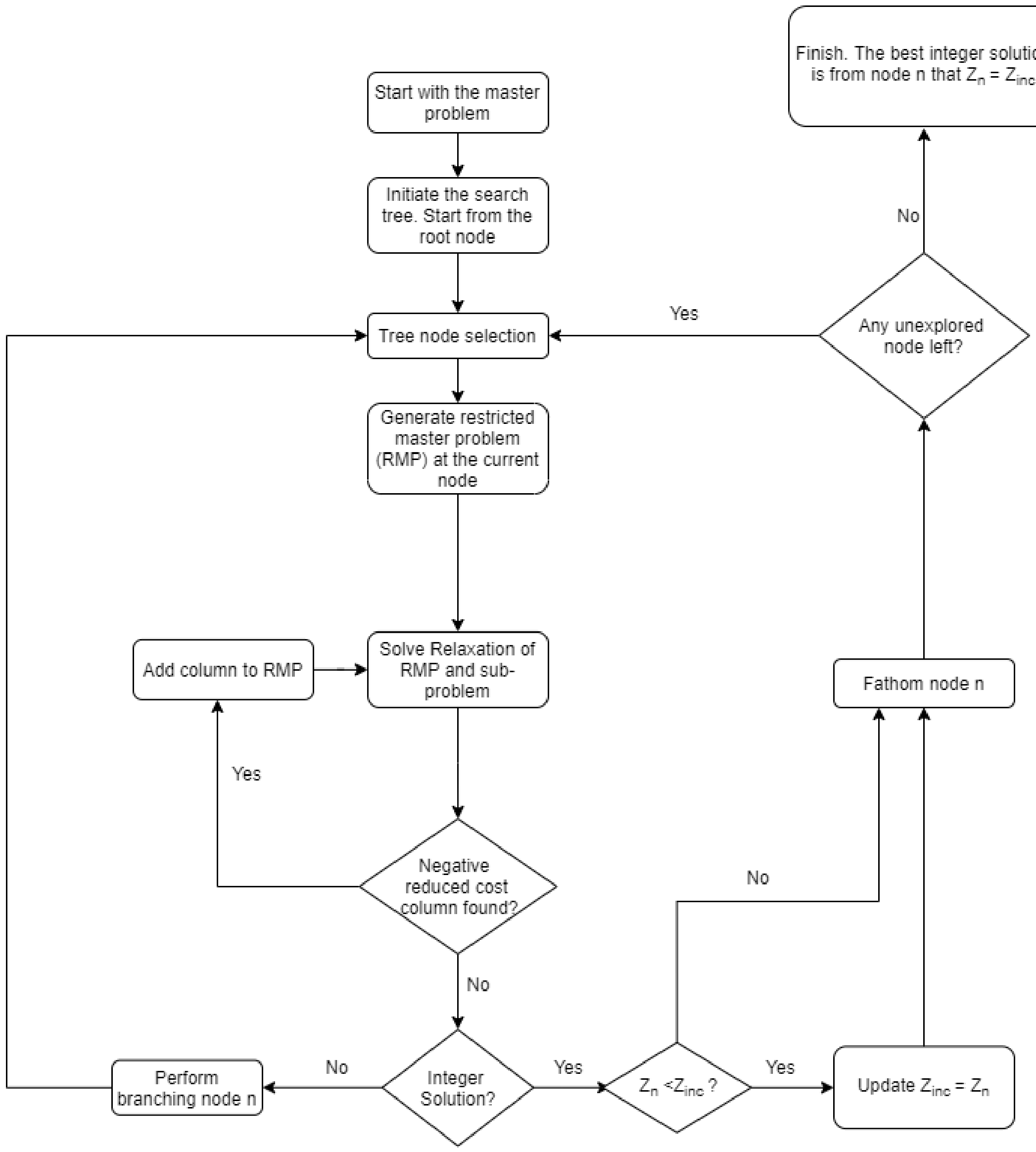

Figure 3 The flow diagram of the Branch-and-Price Algorithm (Pichayavet et al., 2019).

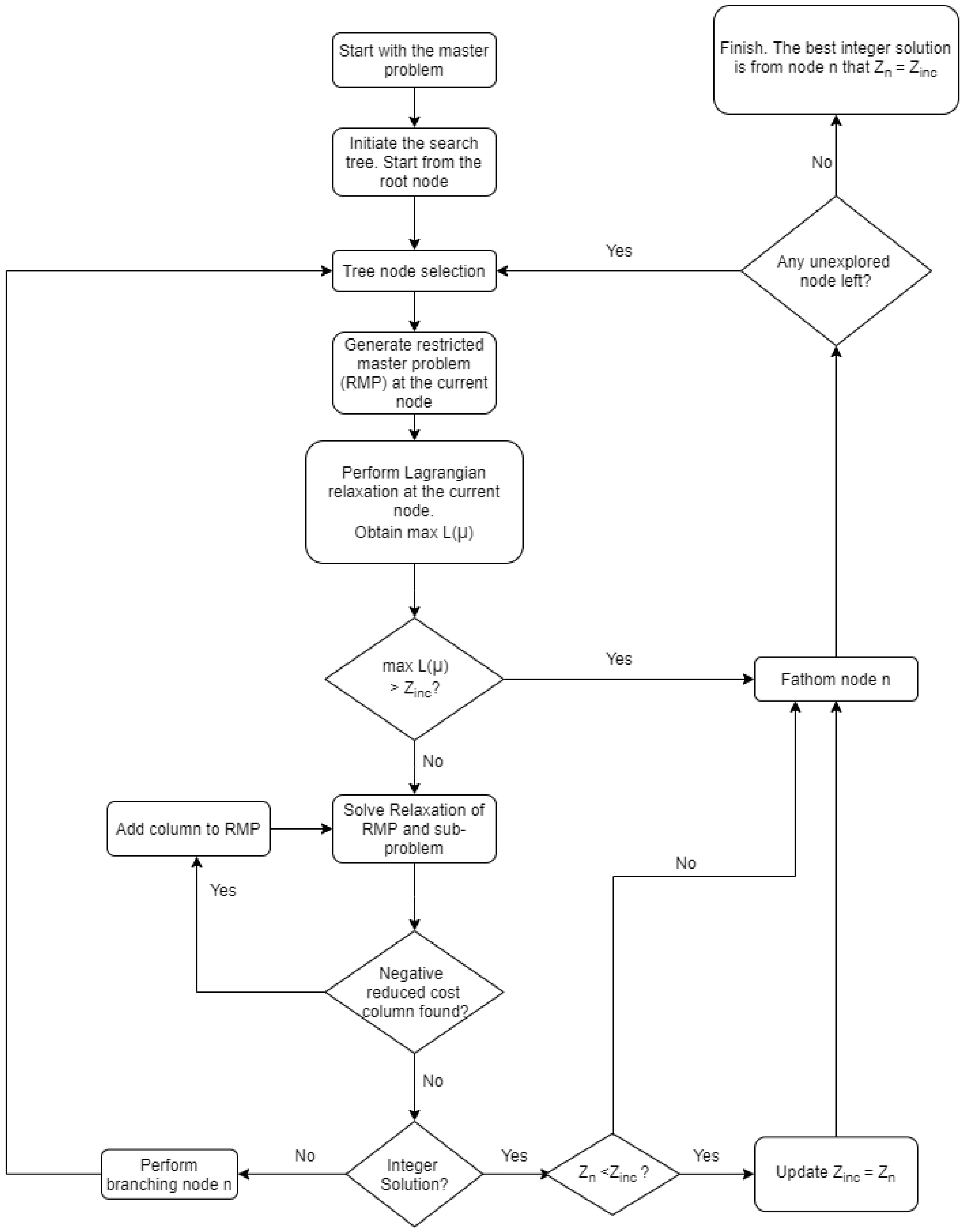

Figure 4The flow diagram of the Branch-and-Price Algorithm with Lagrangian relaxation. (Pichayavet et al., 2019)

In Figure 4, we execute Lagrangian relaxation after branching a new node. Z_n is the solution of node n. Z_{inc} is the incumbent solution. L(μ) is the objective value of Lagrangian relaxation. This algorithm will stop if explored node reaches 2000 nodes or the calculation time is more than 6 hours.

The branching rule was proposed by Ryan and Foster (1981) then Barnhart et al. (1998) proposed in the Branch-and-Price framework. In every fractional solution, there must have at least a pair of constraints which

$$0 < \sum_{k:y_{rk}=1,y_{sk}=1} x_k < 1 \qquad (67)$$

Therefore, branching constraints of the zero-branch and one-branch can be defined as,

Zero-branch: $$\sum_{k:y_{rk}=1,y_{sk}=1} x_k = 0 \qquad (68)$$

One-branch: $$\sum_{k:y_{rk}=1,y_{sk}=1} x_k = 1 \qquad (69)$$

This branching rule implies that the integer solution must solve from the RMP that cannot identify any branching pair. Then, each branching iteration eliminates a large number of variables from consideration.

3.4 Lagrangian Relaxation

The sub-network-based model, the Lagrangian objective function can be formulated as follows.

We relax (37), put it into the objective function (36), and introduce μ_i as Lagrangian multipliers.

$$L(\mu) = min \sum_{r\in R_c}(c_r + F^T)\, x_r - \sum_{i\epsilon C} \mu_i \sum_{r\in R_c}(\delta_i^r x_r - 1) \qquad (70)$$

$$s.t. \quad (2)\ and\ (3)$$

Then (70) derived to

$$L(\mu) = \min \sum_{i\in C} \sum_{r\in R_c} (c_r + F^T - \delta_i^r \mu_i)\, x_r + \sum_{i\epsilon C} \mu_i \quad (71)$$

$s.t.$ (38) and (39)

Solving the Lagrangian multiplier problem using sub-gradient optimization is the following algorithm.

1. Set the initial value of the vector μ (in this thesis is 1) and solve the Lagrangian multiplier problem.
2. In each iteration, update μ at each iteration k with the step size parameter $\theta^k = 1/k$ by using the following equation. $\mu^{k+1} = \mu^k + \theta^k\left(\sum_{r\epsilon R_c} \delta_i^r x_r - 1\right)$
3. After the 100^{th} iteration kept the value of maxL(μ) as the lower bound of the current node in the Branch-and-Price tree.

Chapter 4
Experiment Result

4.1 Computational Result in Random instances

The experiment conducts with random data to measure the performance of the method. The instance generation rules are following the working paper from Pichayavet et al. (2019). The instances have 3 types: the type 1 instance has every node uniformly distributed; the type 2 instance is the same as type 1, but the depot node is located at the centroid of the other nodes cluster; The type 3 instance, the location of customer node and docking hub are generated in coordinates that the radius and the angle are random from 0 to 10 and 0 to 2π respectively. The depot is fixed at the origin. We generate four different sizes of instances including 8 and 10 customer nodes with 1 and 2 dock nodes. Every instance has 1 depot node. We generate 3 instances for each type and size.

For all random instances, a truck route and a drone route are computed using the Manhattan distance and the Euclidean distance between two nodes, respectively. The customer's demands are randomly selected from 10, 20, 30, 40, and 50.

We generate real-world data to see the behavior of the truck-drone solutions and calculate cost savings. The coordinates and truck distances are obtained from OpenStreetMap. The depot node is the main post office of those areas. The customer nodes are randomly picked within the residential zones of those areas. A drone distances use Euclidian distance, the same way as random instances. We choose 5 difference places, 1) Sapansung, Bangkok, 2) Nongchang, Uthaitani, 3) Sapporo, Hokkaido, 4) Patumwan, Bangkok, 5) Wattana, Bangkok. Each instance has 10 customer nodes, 1 docking-hub node, and 1 depot.

A truck and drone speed are the same at 40 km/hr. An operation cost is set to 20 USD plus 0.0083 USD per minute for a truck and 0.0021 USD per minute for a drone. A truck can carry up to 100 kg of parcels and 5 drones with unlimited travel distance while a drone can carry up to 20 kg and be able to fly up to 20 km. (Z. Wang & Sheu, 2019)

We compare the proposed algorithm with and without Lagrangian relaxation to see the improvement in the calculation time measure from the model initiation to the end of the Branch-and-Price. Furthermore, we compare the solution which combined both types of vehicles if it is better than using only trucks in random instances and the real-world instances.

The implementation uses Python 3.8.8 on Ryzen 7 4800H with 16 GB of ram. Gurobi 9.1 is the MIP solver. The results of the experiment are in the following table.

Table 1Result of The Experiment

No.	Type	Customer Node, Dock Node	Node Explored (Without LG, With LG)	Time (sec)		% Time Saving
				Without LG	With LG	
1	1	8,1	5, 5	19.96	19.68	1.40
2	1	8,1	11, 7	38.31	25.62	33.12
3	1	8,1	57, 53	126.68	99.43	21.51
4	2	8,1	37, 37	88.00	84.8	3.64
5	2	8,1	15, 15	49.18	45.22	8.05
6	2	8,1	19, 19	167.30	128.04	23.47
7	3	8,1	3, 3	25.10	19.63	21.79
8	3	8,1	15, 13	30.35	27.50	9.39
9	3	8,1	5, 5	26.12	23.70	9.26
10	1	10,1	23, 21	368.59	221.74	39.84

11	1	10,1	61, 39	294.58	198.27	32.69
12	1	10,1	7, 7	345.35	322.63	6.58
13	2	10,1	3, 3	239.98	163.28	31.96
14	2	10,1	7, 7	53.30	43.20	18.95
15	2	10,1	11,11	351.93	188.92	46.32
16	3	10,1	2000, 79	604.96	173.73	71.28
17	3	10,1	41, 35	288.03	198.20	31.19
18	3	10,1	3, 3	44.39	37.87	14.69
19	1	8, 2	9, 9	72.10	71.61	0.68
20	1	8, 2	7, 5	42.92	25.74	40.03
21	1	8, 2	21,21	710.08	596.40	16.01
22	2	8, 2	9, 7	1225.66	683.92	44.20
23	2	8, 2	55, 61	124.12	96.00	22.66
24	2	8, 2	15, 15	238.84	192.98	19.20
25	3	8, 2	9, 9	30.88	24.99	19.07
26	3	8, 2	5, 5	21.71	22.04	-1.52
27	3	8, 2	7, 7	54.67	38.86	28.92
28	1	10, 2	19, 13	1490.64	795.24	46.65
29	1	10, 2	13, 19	5979.45	5755.86	3.74
30	1	10, 2	21, 21	3293.98	3025.56	8.15
31	2	10, 2	15, 7	4960.64	2810.51	43.34
32	2	10, 2	719, 71	21600.00	9743.27	54.89
33	2	10, 2	15, 15	3825.63	2785.65	27.18
34	3	10, 2	9, 9	203.10	184.62	9.10
35	3	10, 2	19, 17	679.70	561.24	17.43
36	3	10, 2	1, 1	87.78	86.31	1.67
R1		10, 1	9, 9	279.9	236.78	15.41
R2		10, 1	3, 3	58.09	44.81	22.86
R3		10, 1	1, 1	174.08	176.59	-1.44
R4		10, 1	35, 11	273.09	183.24	32.90
R5		10, 1	3, 3	296.63	303.67	-2.37

It can be observed from Table 1 that there are results that explored nodes between with and without Lagrangian relaxation are equal (Figure 7). The Lagrangian relaxation provides a tighter bound in some nodes then the slower column generation is not used. Furthermore, some Branch-and-Price nodes which must explore in the normal method must not explore if the node above it was fathomed by Lagrangian relaxation (Figure 9). Figure 8 and Figure 10 show the improvement in L(μ) by sub-gradient search. Only 100 iterations with the original step size are good enough to provide a bound for this framework.

The Lagrangian relaxation improves the calculation time by 21.80 % on average. Instance no. 16 has the most improvement up to 71.28 %. The Lagrangian relaxation method stops explore Branch-and-Price at 79 nodes while the normal approach uses more than 2000 nodes to confirm optimality.

Instance no. 26 shown that even it has some nodes pruned by the Lagrangian relaxation, but the additional Lagrangian relaxation calculation time is more than the normal method. The calculation time of instance no.32 without Lagrangian relaxation exceeds the 6 hours limit. The time-saving percentage should be more than 54.89 %. Instance no. 36 is solved in 1 Branch-and-Price node. Therefore, Lagrangian relaxation cannot improve its performance.

In between types of instances, they have no significant difference in improvement from Lagrangian relaxation (Figure 5) but instance type 3 has a lower range of results than type 1 and 2. The average calculation time of types 1, 2, and 3 are 1065.22, 2143.71(without no.32, the average will be 1029.5), and 174.73. It is obvious that instance type 3 is easier to solve than type 1 or 2 then. It has not so much room for improvement.

An observation from Figure 6, there is a trend that the mean percentage of improvement increases from 8,1 to 10,1 and decrease at 10,2. These show the larger of instance is, the more branch of Branch-and-Price which can be more improve by Lagrangian relaxation.

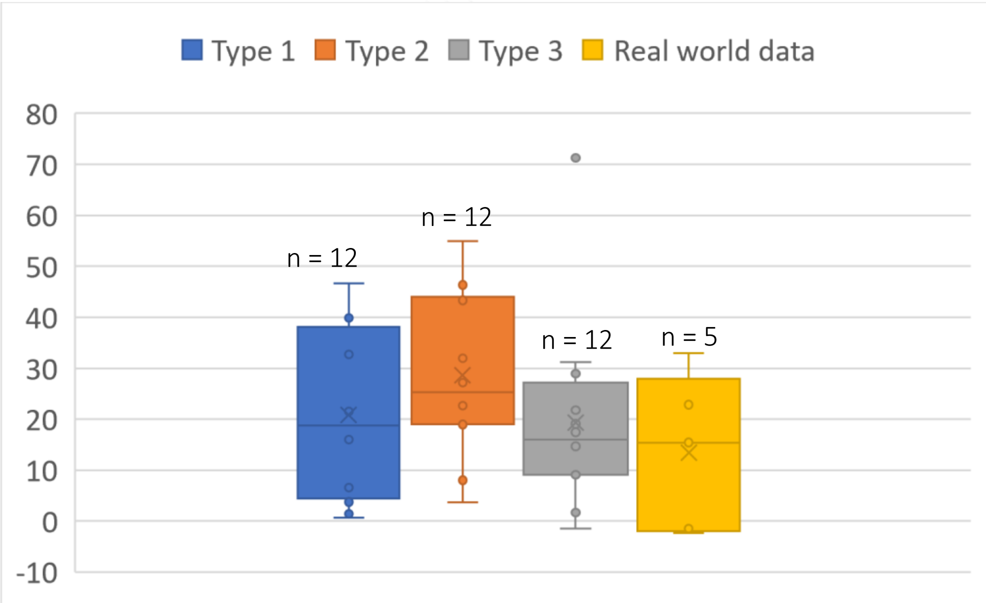

Figure 5 Percentage Improvement of Each Instance Size

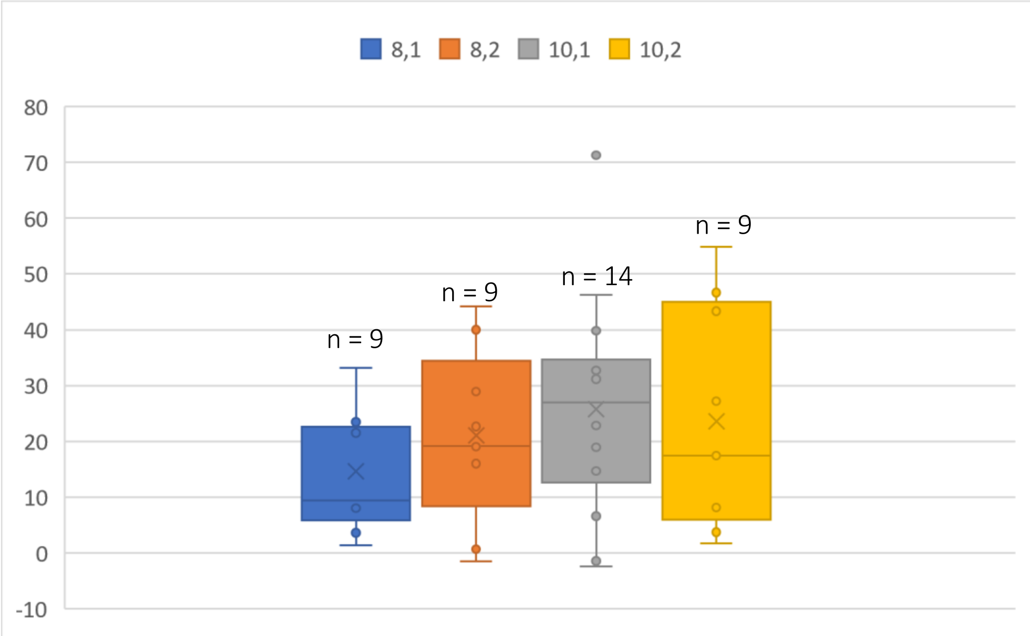

Figure 6 Percentage Improvement of Each Instance Type

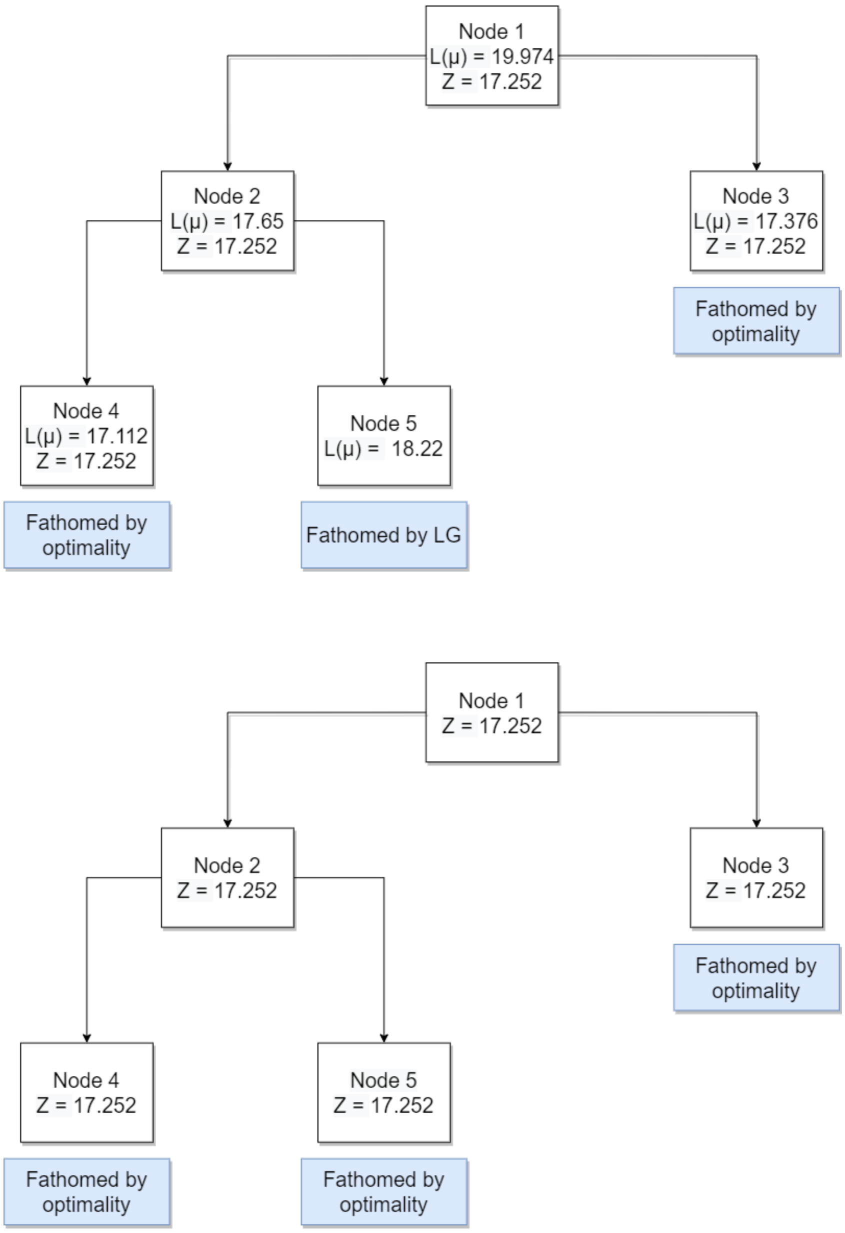

Figure 7 Branch-and-Price tree of instance 9 with and without LG

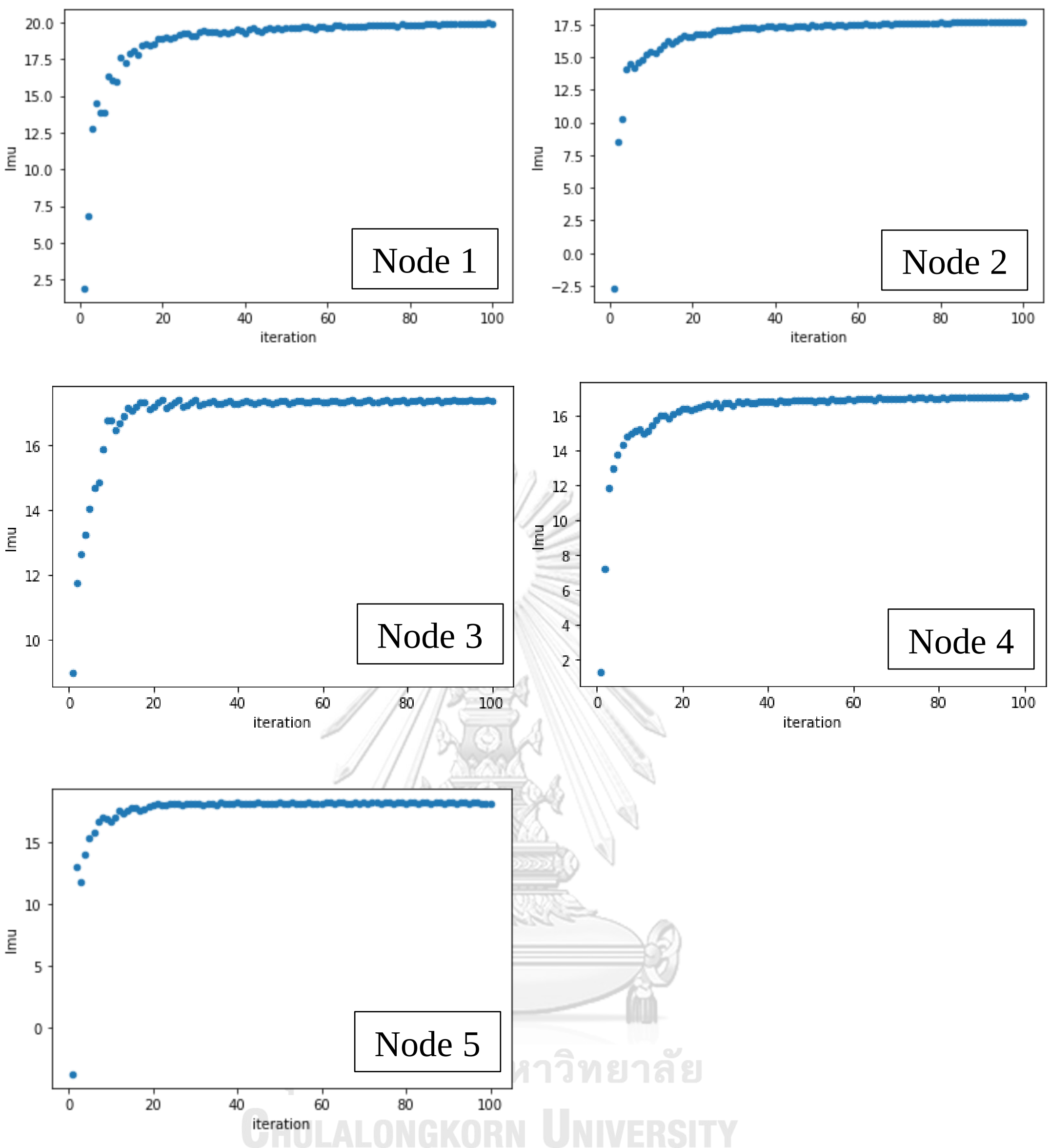

Figure 8 Relationship between $L(\mu)$ and iteration in every node of instance no. 9's solution.

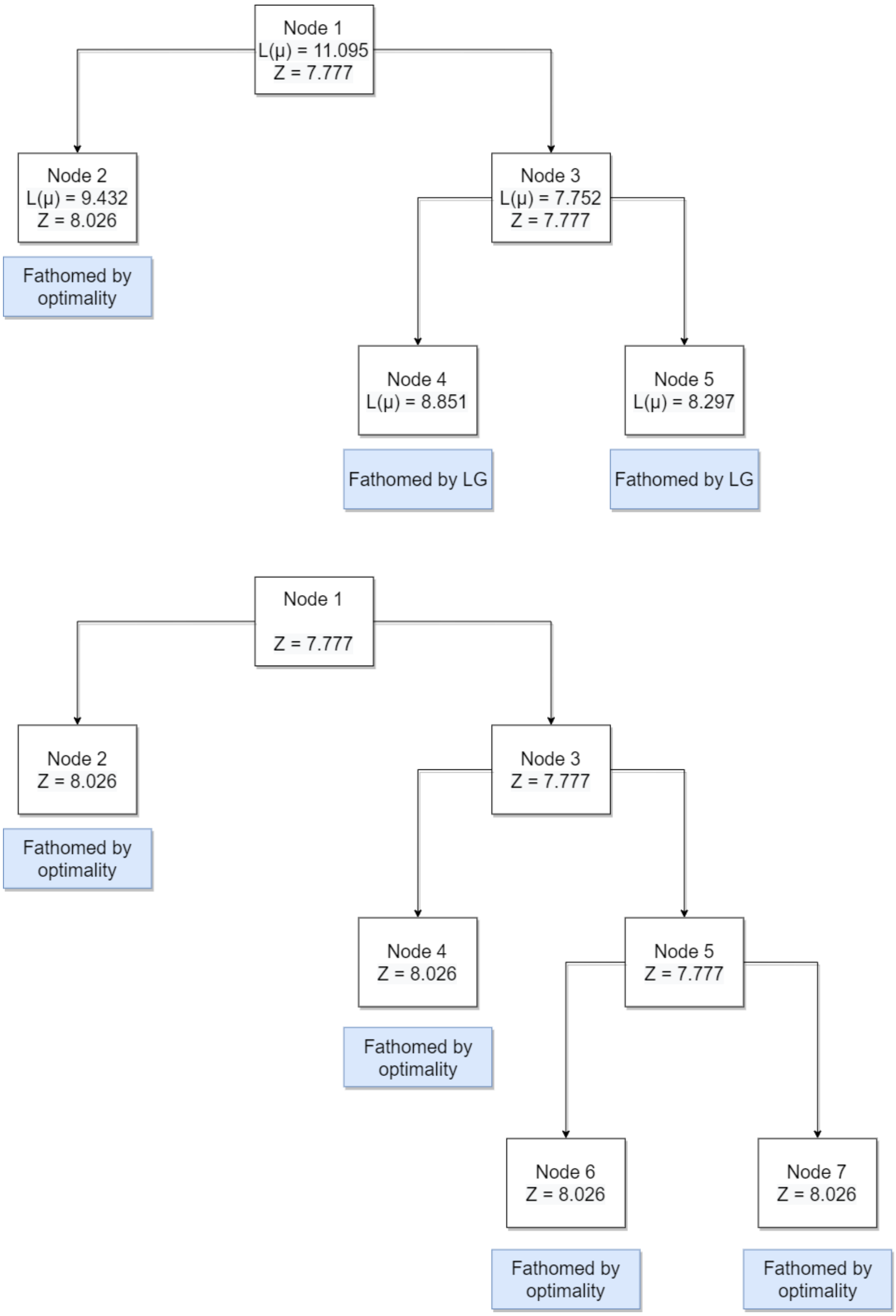

Figure 9 Branch-and-Price tree of instance 20 with and without LG

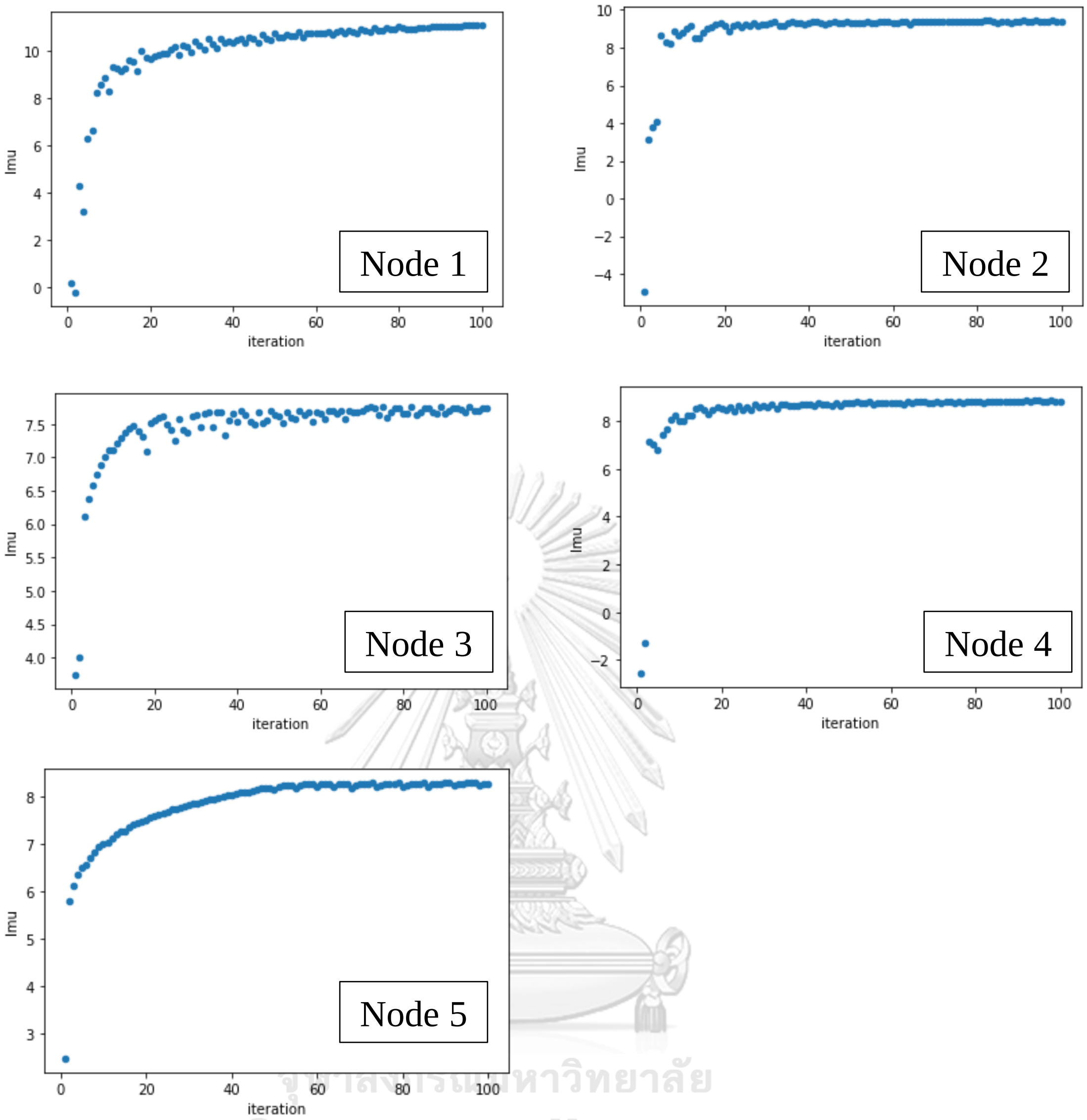

Figure 10 Relationship between $L(\mu)$ and iteration in every node of instance no. 20's solution.

4.2 Truck-Drone Solution and Cost Savings

We compare the fuel cost between truck-based and drone-based solutions. It varied from 0% to 33.33% and 10.35% on average. The percentage of cost-saving equals zero because no drone can satisfy any demand within their flight range.

Table 2 Result of Fuel Cost Saving

No.	Cost without Drone	Cost with Drone	% Cost Saving
1	7.47	4.98	33.33
2	9.46	7.83	17.23
3	9.84	9.43	4.17
4	10.33	9.37	9.29
5	10.33	9.5	8.03
6	13.07	12.43	4.90
7	14.19	13.26	6.55
8	19.89	19.06	4.17
9	17.38	17.25	0.75
10	12.32	10.82	12.18
11	11.2	9.49	15.27
12	12.69	10.48	17.42
13	9.83	8.34	15.16
14	10.83	7.67	29.18
15	9.46	8.61	8.99
16	18.99	17.04	10.27
17	22.2	22.11	0.41
18	12.32	9.33	24.27
19	11.57	9.9	14.43
20	9.46	8.03	15.22
21	13.07	13.07	0.00
22	15.90	15.28	3.90
23	8.96	8.5	5.13
24	11.33	10.96	3.27
25	13.62	12.85	5.65
26	13.98	11.23	19.67
27	12.2	11.39	6.64
28	13.94	12.7	8.90

29	13.57	12.96	4.50
30	11.08	10.12	8.66
31	12.2	10.09	17.30
32	18.55	18.3	1.35
33	12.94	10.86	16.07
34	16.03	12.75	20.46
35	16.01	16.01	0.00
36	6.9	6.9	0.00
R1	6.23	5.09	18.30
R2	3.56	2.743	22.95
R3	3.47	2.88	17.00
R4	3.57	3.3	7.56
R5	3.85	2.67	30.65

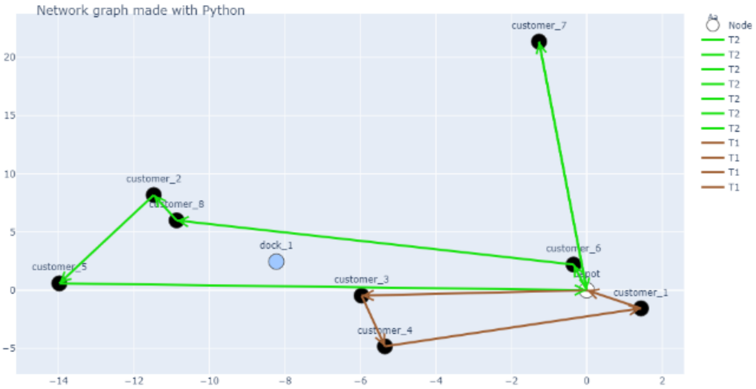

Figure 11 Solution of instance no. 7 without drone

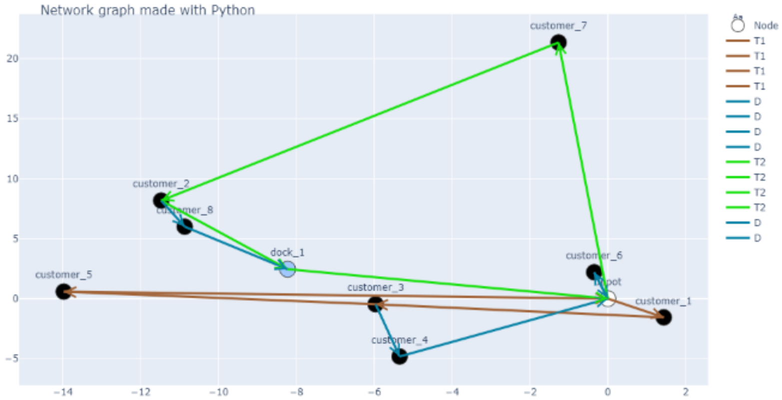

Figure 12 Solution of instance no.7 with drone

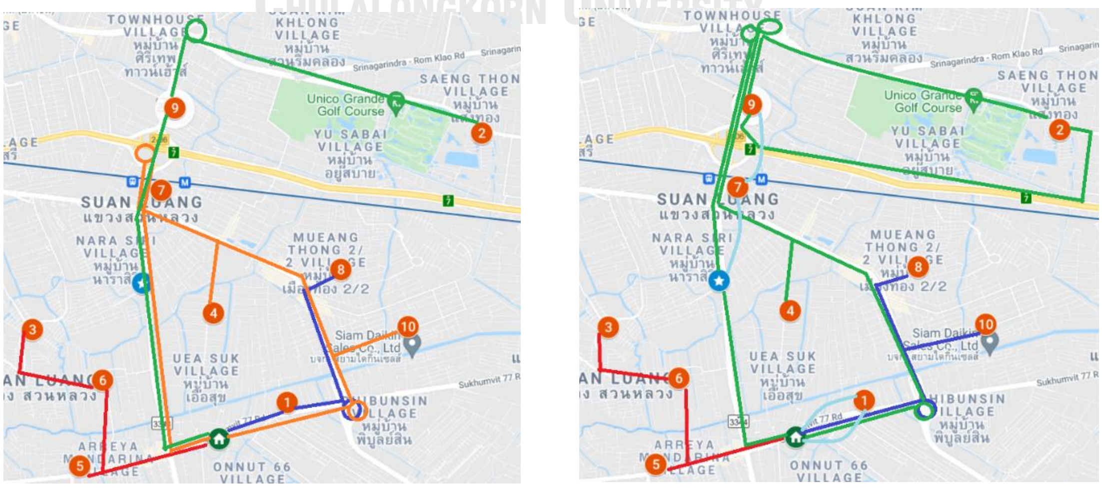

Figure 13 Illustration of instance R1's solutions (with and without drone)

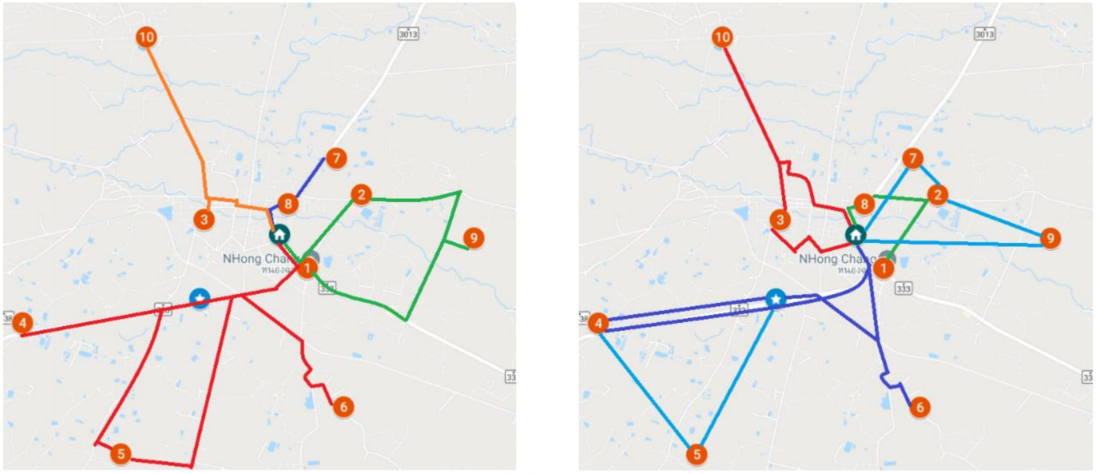

Figure 14 Illustration of instance R2's solutions (with and without drone)

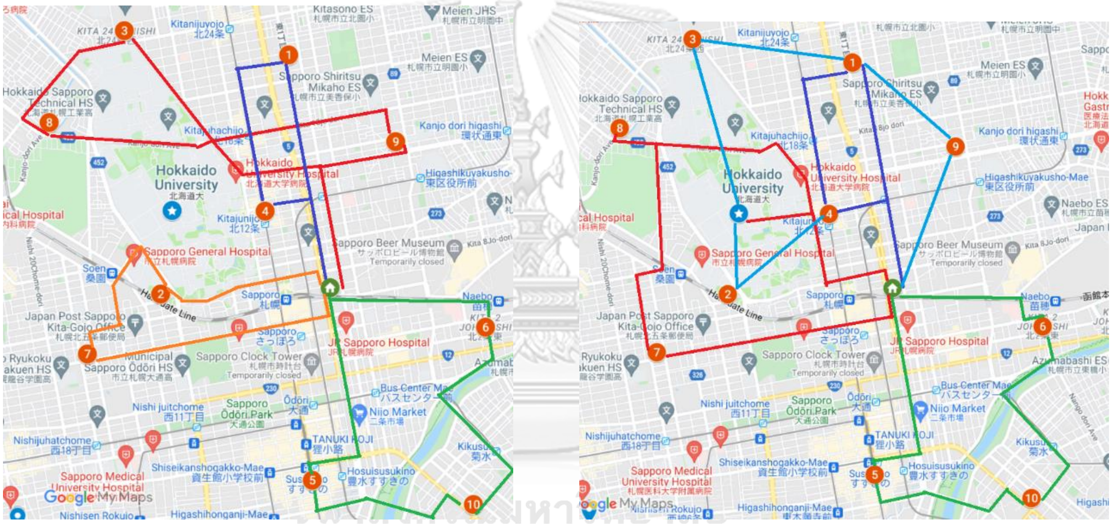

Figure 15 Illustration of instance R3's solutions (with and without drone)

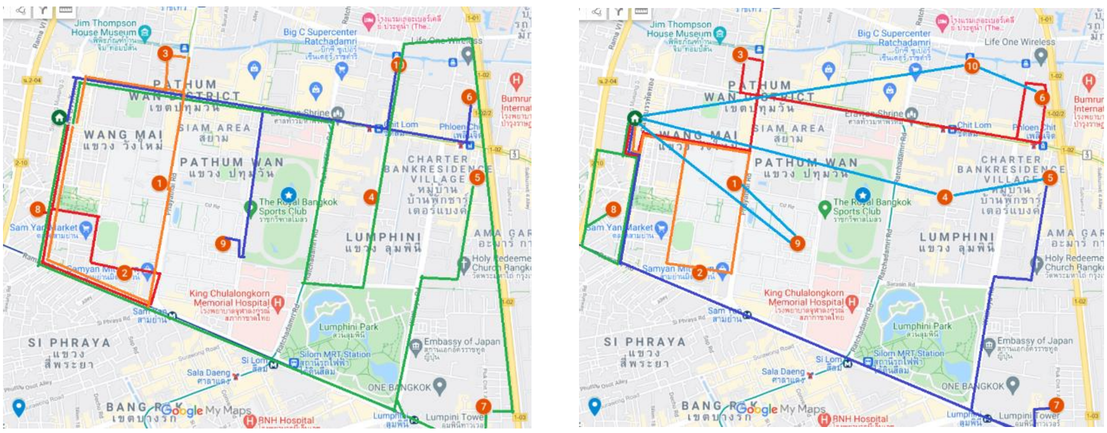

Figure 16 Illustration of instance R4's solutions (with and without drone)

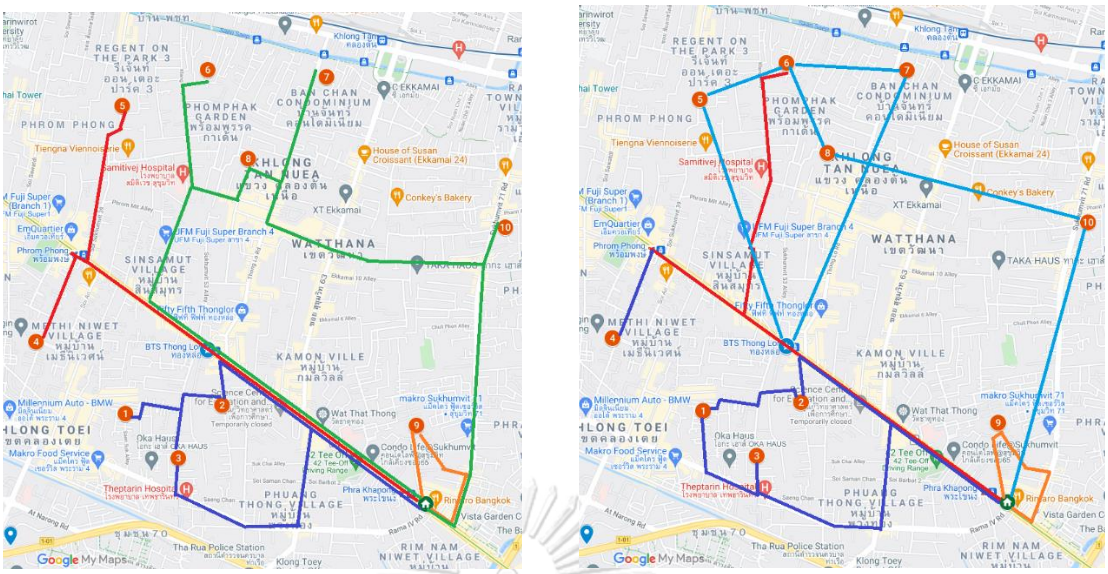

Figure 17 Illustration of instance R5's solutions (with and without drone)

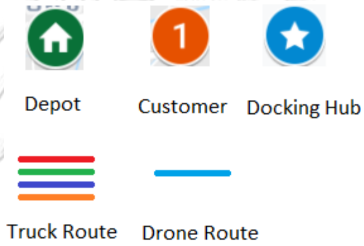

Figure 18 Symbol Explanation

We compare the transportation cost between truck-based and drone-based solutions. From Table 2, It varies from 0% to 33.33%. 11.44% on average. The percentage of cost-saving equals zero because a drone cannot satisfy any demand within its flight range. From the illustration of the real-world solutions, we can obtain some observations. First, the urban area like Bangkok (Figure 13, Figure 16, and Figure 17) has a lot of small roads connected to the main road and a lot of dead-end roads. A city plan like this made a problem for the truck to maintain the quality of delivery. Drones help the truck to stay mostly on the main road. Second, as the same as an urban area, the rural area (Figure 14) has a wide distribution of residential zone. Drones also serve customers who stay far away from the main road if demands are not exceeding the limit

Table 3 Result of Total Cost Saving

No.	No Drone Cost	With Drone Cost	%Saving	Number of Truck No Drone	Number of Truck with Drone	Truck No. Difference
1	67.47	64.98	3.7	3	3	0
2	69.46	67.83	2.3	3	3	0
3	89.84	69.43	22.7	4	3	1
4	70.33	49.37	29.8	3	2	1
5	70.33	69.5	1.2	3	3	0
6	73.07	72.43	0.9	3	3	0
7	74.19	53.26	28.2	3	2	1
8	79.89	79.06	1.0	3	3	0
9	77.38	77.25	0.2	3	3	0
10	72.32	70.82	2.1	3	3	0
11	91.2	69.49	23.8	4	3	1
12	92.69	90.48	2.4	4	4	0
13	69.83	68.34	2.1	3	3	0
14	90.83	67.67	25.5	4	3	1
15	69.46	68.61	1.2	3	3	0
16	78.99	77.04	2.5	3	3	0
17	102.2	102.11	0.1	4	4	0
18	72.32	69.33	4.1	3	3	0
19	91.57	69.9	23.7	4	3	1
20	69.46	48.03	30.9	3	2	1
21	73.07	73.07	0.0	3	3	0
22	75.9	75.28	0.8	3	3	0
23	68.96	68.49	0.7	3	3	0
24	91.33	90.96	0.4	4	4	0
25	73.62	53.85	26.9	3	2	1

26	73.98	71.23	3.7	3	3	0
27	72.2	51.39	28.8	3	2	1
28	93.94	92.7	1.3	4	4	0
29	93.57	92.96	0.7	4	4	0
30	71.08	70.12	1.4	3	3	0
31	112.2	50.09	55.4	5	2	3
32	118.55	93.3	21.3	5	4	1
33	93.07	50.86	45.4	4	2	2
34	76.03	52.75	30.6	3	2	1
35	116.1	116.1	0.0	5	5	0
36	46.9	46.9	0.0	2	2	0
R1	86.23	65.09	24.5	4	3	1
R2	83.56	62.743	24.9	4	3	1
R3	83.47	62.88	24.7	4	3	1
R4	83.57	83.3	0.3	4	4	0
R5	83.85	62.67	25.3	4	3	1

Table 3 shows the total cost of transportation including truck fix cost, truck variable cost, and drone variable cost. It can be noticed that drones can reduce the usage of trucks in some instances. In this experiment, the best we can do is use 2 trucks and 3 drones instead of 4 trucks. (Figure 19 and Figure 20) which the total cost saving is 45.4%. It depends on the customer demand and range, if drones can satisfy that then the truck usage can be 0.

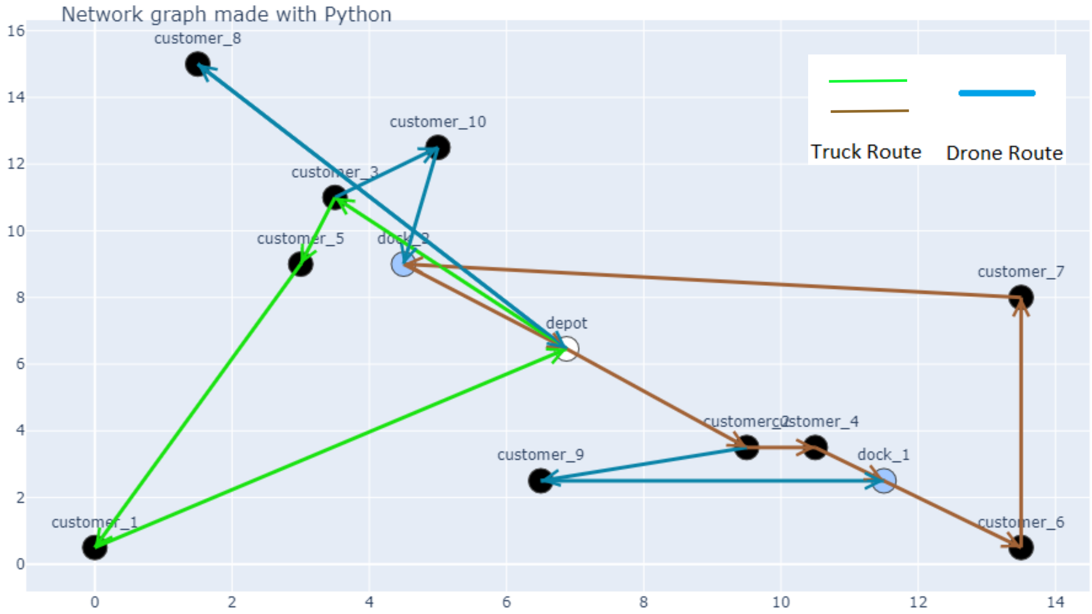

Figure 19 Solution of instance no. 33 without drone

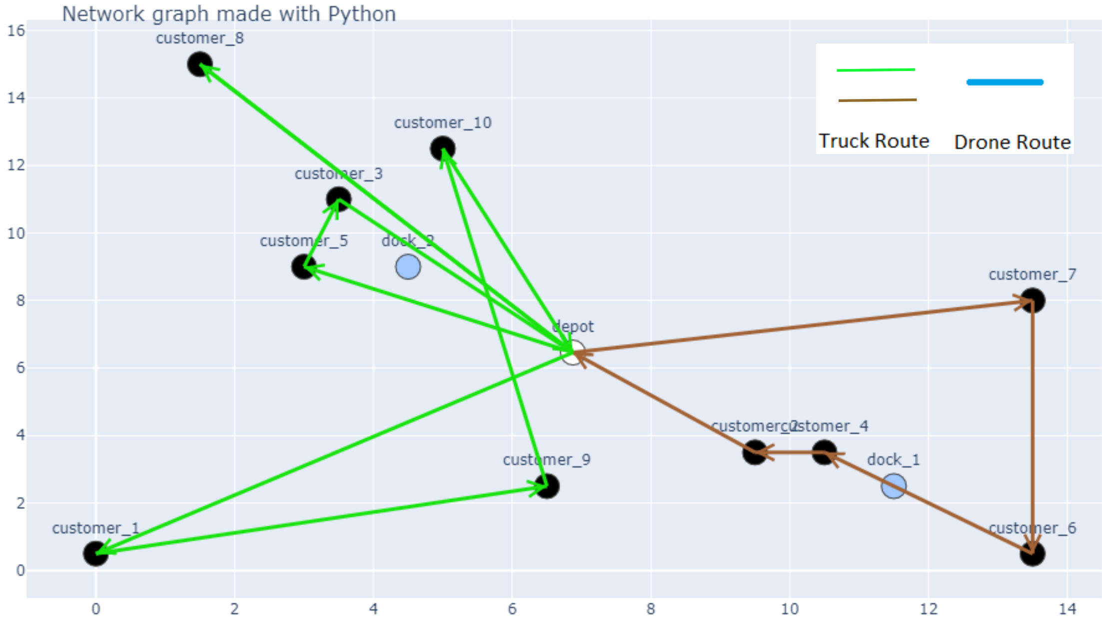

Figure 20 Solution of instance no. 33 with drones

Chapter 5
Conclusion

5.1 Summary and Discussion

We proposed the Lagrangian relaxation model and use the sub-gradient method to solve it. This Lagrangian relaxation model can be solved much faster than the MIP model, but it does not provide the exact solution. It provides the lower bound instead. If the Lagrangian lower bound is greater or equal to the incumbent solution, that node is fathomed. We have tested in 3 types and 4 different sizes of instance. The calculation time with Lagrangian relaxation is reduced up to 71.28% and 22.96% on average. The CPU time is in the range of 19.68 to 9743.27 seconds and 19.96 seconds to 6 hours, with and without Lagrangian relaxation for all instances.

We compare the solution of truck-drone to truck-only model. The fuel cost saving is 11.44% on average. The total cost saving is 12.81% on average. The collaboration of trucks and drones reduce truck route and truck distance which benefits in the total cost of operation in a real-world case. The drone advantage is helping a truck stays on the main road which easy to pass through than a small street in an urban area or a poor road in a rural area.

5.2 Suggestions for Future Research

In the experiment process, there are some points to be noticed. First, the problem which can be solved in one node of Branch-and-Price cannot speedup by this proposed method. This mostly happened with small size instances (less than 10 nodes). Second, there is one instance that Lagrangian relaxation cannot improve calculation time even there are some nodes that were fathomed by Lagrangian relaxation. It shows that in some rare cases LP relaxation is faster than Lagrangian relaxation.

This research did not consider all real-world conditions. According to Thailand drone regulation, “during flight, must not fly over cities, villages, communities or areas where people are gathered” which means drone cannot fly over most of the Bangkok area (The Civil Aviation Authority of Thailand, 2015). The assumption of truck capacity, a small truck can carry more than 1000 kilograms which is much higher than our assumption.

There is a huge room for improvement in this research. Lagrangian relaxation can be used to generate columns along with duals in column generation. Other stabilization methods can be used to speed up this model. Moreover, another cost of the last-mile delivery process is not included in this model. The depot and the docking hub node position, size and drone operation require effective management at minimum cost. The real-world data in this research is only a real map route but demand quantity and position do not reflex the real-world situation. If the model can solve faster, it can include real-time traffic conditions to provide the best route while in operation.

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