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Appendix A

Transforming to Cosine Representation of the Kinetic-energy and Potential-energy terms

In representing the paths as the cosine series, the linear transformation equation is given by

$$x_j = \frac{1}{\sqrt{N\epsilon}} a_0 + \sqrt{\frac{2}{N\epsilon}} \sum_{n=1}^{N-1} a_n \cos \frac{n\pi}{N} j + \frac{(-1)^j}{\sqrt{N\epsilon}} a_N \qquad \text{(A.1)}$$

The kinetic-enery terms can be written as

$$\sum_{j=1}^{N} (x_j - x_{j-1})^2 = \frac{2}{N\epsilon} \sum_{n=1}^{N-1} \sum_{l=1}^{N-1} 4 a_n a_l \left[\sum_{j=1}^{N} \sin(2j-1)\frac{n\pi}{2N} \sin(2j-1)\frac{l\pi}{2N} \right] \sin\frac{n\pi}{2N} \sin\frac{l\pi}{2N}$$

$$+ \frac{8\sqrt{2}}{N\epsilon} \sum_{n=1}^{N-1} a_N a_n \left[\sum_{j=1}^{N} (-1)^{j+1} \sin(2j-1)\frac{n\pi}{2N} \right] \sin\frac{n\pi}{N} + \frac{4}{\epsilon} a_N^2 \qquad \text{(A.2)}$$

Using the identity

$$\sum_{j=1}^{N} (-1)^{j+1} \sin(2j-1)x = (-1)^{N+1} \frac{\sin 2Nx}{2\cos x} \qquad \text{(A.3)}$$

we have

$$\sum_{j=1}^{N} (-1)^{j+1} \sin(2j-1)\frac{n\pi}{2N} = 0 \qquad \forall n \neq N \qquad \text{(A.4)}$$

Then eq. (A.2) can be rewritten as

$$\sum_{j=1}^{N} (x_j - x_{j-1})^2 = \frac{2}{N\epsilon} \sum_{n=1}^{N-1} \sum_{l=1}^{N-1} 4 a_n a_l \left[\frac{1}{2} \sum_{j=1}^{N} \left(\cos(n-l)(2j-1)\frac{\pi}{2N} - \cos(n+l)(2j-1)\frac{\pi}{2N} \right) \right]$$

$$\times \sin\frac{n\pi}{2N} \sin\frac{l\pi}{2N} + \frac{4}{\epsilon} a_N^2 \qquad \text{(A.5)}$$

Using the identity

$$\sum_{j=1}^{N} \cos(2j-1)x = \frac{1}{2}\sin 2Nx \csc x \tag{A.6}$$

we have

$$\sum_{j=1}^{N}\left[\cos(m-l)(2j-1)\frac{\pi}{2N} - \cos(m+l)(2j-1)\frac{\pi}{2N}\right] = \begin{cases} 0 & \forall l \neq m \\ N & \forall l = m \end{cases} \tag{A.7}$$

Then eq. (A.5) becomes

$$\sum_{j=1}^{N}(x_j - x_{j-1})^2 = \sum_{m=1}^{N}\left[\frac{4}{\epsilon}\sin^2\frac{m\pi}{2N}\right] a_m^2 \tag{A.8}$$

For potential-energy terms we have

$$\begin{aligned} \sum_{j=0}^{N} x_j^2 = {} & \sum_{j=0}^{N}\frac{1}{N\epsilon}a_0^2 + \frac{2\sqrt{2}}{N\epsilon}a_0\sum_{m=1}^{N-1}a_m\left[\sum_{j=0}^{N}\cos\frac{m\pi}{N}j\right] \\ & + \sum_{j=0}^{N}\frac{1}{N\epsilon}a_N^2 + \frac{2\sqrt{2}}{N\epsilon}a_N\sum_{m=1}^{N-1}a_m\left[\sum_{j=0}^{N}\cos\frac{m\pi}{N}j\right] \\ & + \frac{2}{N\epsilon}\sum_{m=1}^{N-1}\sum_{l=1}^{N-1}a_m a_l\left[\frac{1}{2}\sum_{j=0}^{N}\left(\cos(l+m)\frac{\pi}{N}j + \cos(l-m)\frac{\pi}{N}j\right)\right] \end{aligned} \tag{A.9}$$

Using the identity

$$\sum_{j=0}^{N}\cos jx = \cos\frac{Nx}{2}\sin(N+1)\frac{x}{2}\csc\frac{x}{2} \tag{A.10}$$

we have

$$\sum_{j=0}^{N}\cos\frac{m\pi}{N}j = \cos^2\frac{m\pi}{2} = \begin{cases} 1 & \text{for } m = \text{even} \\ 0 & \text{for } m = \text{odd} \end{cases} \tag{A.11}$$

Using the identity

$$\sum_{j=0}^{N}(-1)^{j}\cos jx = \frac{1}{2} + \frac{(-1)^{N}\cos\left[(2N+1)x/2\right]}{2\cos(x/2)} \tag{A.12}$$

we have

$$\sum_{j=0}^{N}(-1)^{j}\cos j\frac{\pi m}{N} = \frac{1}{2} + \frac{(-1)^{N}\cos m\pi}{2} \tag{A.13}$$

Then eq. (A.9) can be rewritten as

$$\begin{aligned}\sum_{j=0}^{N} x_j^2 = {} & \frac{1}{N\epsilon}(N+1)a_0^2 + \frac{1}{N\epsilon}(N+1)a_N^2 + \frac{2\sqrt{2}}{N\epsilon}\sum_{m=1}^{N-1} a_0 a_m \cos^2 m\frac{\pi}{2} \\ & + \frac{2\sqrt{2}}{N\epsilon}\sum_{m=1}^{N-1} a_m a_N \frac{1}{2}\left[1 + (-1)^N \cos m\pi\right] + \frac{1}{N\epsilon}(N+1)\sum_{m=1}^{N-1} a_m^2 \\ & + \frac{2}{N\epsilon}\sum_{m\neq}^{N-1}\sum_{l=1}^{N-1} a_m a_l \frac{1}{2}\left[\cos^2 (m+l)\frac{\pi}{2} + \cos^2 (m-l)\frac{\pi}{2}\right]\end{aligned} \tag{A.14}$$

In eq. (A.14) we compare the coefficients of the off-diagonal terms with the coefficients of the diagonal terms ; it is obvious that they are of the order $2/N$ and $1/\epsilon$ respectively. Since we are dealing with the case of large N as N approaches to infinity we can neglect all the off-diagonal terms. Hence eq. (A.14) can be rewritten as

$$\sum_{j=0}^{\prime N} x_j^2 \approx \sum_{m=0}^{N} \frac{1}{\epsilon} a_m^2 \tag{A.15}$$

Appendix B

The Jacobian of Transformation (Cosine Representation)

From the linear transformation equation

$$x_j = \frac{1}{\sqrt{N\epsilon}} a_0 + \sqrt{\frac{2}{N\epsilon}} \sum_{m=1}^{N-1} a_m \cos m\frac{\pi}{N} j + \frac{(-1)^j}{\sqrt{N\epsilon}} a_N \tag{B.1}$$

the jacobian matrix $M = \left[\frac{\partial x_j}{\partial a_n} \right]$ can be written as

$$M = \frac{1}{\sqrt{N\epsilon}} \begin{bmatrix} 1 & \sqrt{2} & \sqrt{2} & - & - & - \\ 1 & \sqrt{2}\cos\frac{\pi}{N} & \sqrt{2}\cos 2\frac{\pi}{N} & - & - & - \\ 1 & \sqrt{2}\cos 2\frac{\pi}{N} & \sqrt{2}\cos 4\frac{\pi}{N} & - & - & - \\ \vdots & \vdots & \vdots & - & - & - \\ 1 & -\sqrt{2} & \sqrt{2} & - & - & - \end{bmatrix} \tag{B.2}$$

For the sake of simplicity we let N = odd integer and M^T be the transpose of the jacobian matrix. Then we have

$$M^T M = \frac{1}{N\epsilon} \begin{bmatrix} \sum_{j=1}^{N} 1 & \cdots & \sqrt{2}\sum_{j=0}^{N} \cos m\frac{\pi}{N} j & \cdots \\ \vdots & & \vdots & \\ \sqrt{2}\cos mj\frac{\pi}{N} & \cdots & 2\sum_{j=0}^{N} \cos lj\frac{\pi}{N} \cos mj\frac{\pi}{N} & \cdots \\ \vdots & & \vdots & \\ \sum_{j=1}^{N} (-1)^j & \cdots & \sqrt{2}\sum_{j=0}^{N} (-1)^j \cos mj\frac{\pi}{N} & \cdots \end{bmatrix} \tag{B.3}$$

where n = 1, 2,..., N-1 and ℓ = 1, 2,..., N-1.

Using the identities

$$\sum_{j=0}^{N} \cos jx = \cos N\frac{x}{2} \sin(N+1)\frac{x}{2} \csc\frac{x}{2} \tag{B.4}$$

and

$$\sum_{j=0}^{N} (-1)^j \cos jx = \frac{1}{2} + \frac{(-1)^N}{2}\cos(2N+1)\frac{x}{2} \sec\frac{x}{2} \tag{B.5}$$

we have

$$\sum_{j=0}^{N} \cos j\frac{n\pi}{N} \cos j\frac{\ell\pi}{N} = \frac{1}{2}\left[\cos^2(\ell+n)\frac{\pi}{2} + \cos^2(\ell-n)\frac{\pi}{2}\right]$$

$$= \begin{cases} 0 & \forall\ \ell \neq n,\ (\ell \pm n) = \text{odd} \\ 1 & \forall\ \ell \neq n,\ (\ell \pm n) = \text{even} \end{cases} \tag{B.6}$$

$$= \frac{1}{2}(N+2) \qquad \forall\ \ell = n \tag{B.7}$$

and

$$\sum_{j=0}^{N} (-1)^j \cos j\frac{n\pi}{N} = \begin{cases} 0 & \forall\ n = \text{even} \\ 1 & \forall\ n = \text{odd.} \end{cases}$$

Thus eq. (B.3) can be rewritten as

$$M^TM = \frac{1}{N\epsilon}\begin{bmatrix} (N+1) & 0 & 2 & 0 & \cdots \\ 0 & (N+2) & 0 & 2 & \cdots \\ 2 & 0 & (N+2) & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \\ 2 & 0 & 2 & 0 & \cdots \\ 0 & 2 & 0 & 2 & \cdots \end{bmatrix} \tag{B.8}$$

The matrix in eq. (B.9) can be reduced to be

$$M^TM = \frac{1}{N\epsilon}\begin{bmatrix} N & 0 & 0 & \cdots & -\frac{N}{2} & 0 \\ 0 & N & 0 & \cdots & 0 & -2N \\ 0 & 0 & N & \cdots & -N & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 2N & 0 \\ 0 & 0 & 0 & \cdots & 0 & 2N \end{bmatrix} \tag{B.9}$$

which is a triangular matrix. Then the jacobian of transformation can be obtained directly ; $J = 2/(\epsilon)^{\frac{N+1}{2}}$.

Appendix C

Transforming to Sine Representation of the Kinetic-energy and Potential-energy terms

By using the linear transformation equation

$$y_j = \sqrt{\frac{2}{N\varepsilon}} \sum_{m=1}^{N-1} a_m \sin \frac{m\pi}{N} j \qquad \text{(C.1)}$$

the kinetic-energy terms can be written as

$$y_j - y_{j-1} = \sqrt{\frac{2}{N\varepsilon}} \sum_{m=1}^{N-1} a_m \left[2\cos(2j-1)\frac{m\pi}{2N} \sin\frac{m\pi}{2N} \right] \qquad \text{(C.2)}$$

Thus

$$\sum_{j=1}^{N} (y_j - y_{j-1})^2 = \frac{2}{N\varepsilon} \sum_{m=1}^{N-1} \sum_{l=1}^{N-1} a_m a_l \left(4\sin\frac{l\pi}{N} \sin\frac{m\pi}{N}\right)\left[\frac{1}{2}\sum_{j=1}^{N} \cos(l+m)(2j-1)\frac{\pi}{2N} + \frac{1}{2}\sum_{j=1}^{N} \cos(l-m)(2j-1)\frac{\pi}{2N} \right] \qquad \text{(C.3)}$$

Using the identity

$$\sum_{k=1}^{N} \cos(2k-1)x = \frac{1}{2}\sin 2Nx \csc x \qquad \text{(C.4)}$$

we get

$$\sum_{j=1}^{N} \cos(l \pm m)(2j-1)\frac{\pi}{2N} = 0 \qquad \forall\, l \neq m \qquad \text{(C.5)}$$

Then eq. (C.3) becomes

$$\sum_{j=1}^{N} (y_j - y_{j-1})^2 = \sum_{m=1}^{N-1} \left(\frac{4}{\epsilon} \sin^2 \frac{m\pi}{2N} \right) a_m^2 \qquad \text{(C.6)}$$

For the potential-energy terms we have

$$y_j^2 = \frac{2}{N\epsilon} \sum_{m=1}^{N-1} \sum_{l=1}^{N-1} a_m a_l \sin \frac{m\pi}{N} j \sin \frac{l\pi}{N} j \qquad \text{(C.7)}$$

$$\sum_{j=0}^{N} y_j^2 = \frac{2}{N\epsilon} \sum_{m=1}^{N-1} \sum_{l=1}^{N-1} a_m a_l \left[\sum_{j=0}^{N} \frac{1}{2} \left(\cos(l-m) \frac{\pi}{N} j - \cos(l+m) \frac{\pi}{N} j \right) \right] \qquad \text{(C.8)}$$

Using the identity

$$\sum_{j=0}^{N} \cos jx = \cos \frac{Nx}{2} \sin (N+1) \frac{x}{2} \csc \frac{x}{2} \qquad \text{(C.9)}$$

we get

$$\sum_{j=0}^{N} \cos \frac{m\pi}{N} j = \begin{cases} 1 & \text{for } m = \text{even} \\ 0 & \text{for } m = \text{odd} \end{cases} \qquad \text{(C.10)}$$

Then eq. (C.8) can be written as

$$\sum_{j=0}^{N} y_j^2 = \sum_{m=1}^{N-1} \frac{1}{\epsilon} a_m^2 \qquad \text{(C.11)}$$

Appendix D

The Jacobian of Transformation (Sine Representation)

From the linear transformation equation

$$y_j = \sqrt{\frac{2}{N_E}} \sum_{m=1}^{N-1} a_m \sin \frac{m\pi j}{N} \qquad \text{(D.1)}$$

the jacobian matrix $M = \left[\frac{\partial y_j}{\partial a_m} \right]$ can be written as

$$M = \sqrt{\frac{2}{N_E}} \begin{bmatrix} \sin\frac{\pi}{N} & \sin\frac{2\pi}{N} & \sin\frac{3\pi}{N} & \cdots \\ \sin\frac{2\pi}{N} & \sin\frac{4\pi}{N} & \sin\frac{6\pi}{N} & \cdots \\ \sin\frac{3\pi}{N} & \sin\frac{6\pi}{N} & \sin\frac{9\pi}{N} & \cdots \\ \vdots & \vdots & \vdots & \cdots \\ \vdots & \vdots & \vdots & \cdots \end{bmatrix} \qquad \text{(D.2)}$$

Let M^T be the transpose of the jacobian matrix, then we have

$$M^T M = \frac{2}{N_E} \begin{bmatrix} \sum_{j=1}^{N-1} \sin^2 j\frac{\pi}{N} & \sum_{j=1}^{N-1} \sin j\frac{\pi}{N} \sin 2j\frac{\pi}{N} & \cdots \\ \sum_{j=1}^{N-1} \sin j\frac{\pi}{N} \sin 2j\frac{\pi}{N} & \sum_{j=1}^{N-1} \sin^2 2j\frac{\pi}{N} & \cdots \\ \vdots & \vdots & \cdots \\ \vdots & \vdots & \cdots \\ \vdots & \vdots & \cdots \\ \sum_{j=1}^{N-1} \sin j\frac{\pi}{N} \sin (N-1)j\frac{\pi}{N} & \sum_{j=1}^{N-1} \sin 2j\frac{\pi}{N} \sin (N-1)j\frac{\pi}{N} & \cdots \end{bmatrix} \qquad \text{(D.3)}$$

Using eq. (C.10) with the fact that

$$\sin \frac{l\pi j}{N} \sin \frac{m\pi j}{N} = \frac{1}{2}\left[\cos(l-m)\frac{\pi j}{N} - \cos(l+m)\frac{\pi j}{N}\right] \tag{D.4}$$

We can rewrite eq. (D.3) as

$$M^T M = \frac{1}{N\epsilon}\begin{bmatrix} N & & & & 0 \\ & N & & & \\ & & N & & \\ & & & \ddots & \\ 0 & & & & N \end{bmatrix} \tag{D.5}$$

Then the jacobian of transformation can be obtained by

$$J = \left(\det[M^T M]\right)^{1/2} = \left(\frac{1}{\epsilon}\right)^{\frac{N-1}{2}} \tag{D.6}$$

Appendix E

Harmonic-Oscillator Prefactor

The harmonic-oscillator prefactor can be written as

$$F(T) = \lim_{N\to\infty}\left(\frac{1}{A}\right)^{N}\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty} dy_1\cdots dy_{N-1}\exp\left\{-\frac{m}{2i\hbar\epsilon}\left[\sum_{j=1}^{N}(y_j-y_{j-1})^2-\omega^2\epsilon^2\sum_{j=0}^{N}y_j^2\right]\right\} \qquad (E.1)$$

where $\frac{1}{A} = \left(\frac{m}{2\pi i\hbar\epsilon}\right)^{1/2}$, $y_0 = y_N = 0.$

Since the boundary points of the path integral in eq. (E.1) vanish, we can represent the paths $y(t)$ as a sine series. We restrict ourselves to the discrete-time assumption so that we have

$$y_j = \sqrt{\frac{2}{N\epsilon}}\sum_{m=0}^{N} a_m \sin\frac{m\pi j}{N} \qquad (E.2)$$

By using the eq. (E.2) as the linear transformation equation, eq. (E.1) can be transformed to become

$$F(T) = \lim_{N\to\infty}\left(\frac{1}{A}\right)^{N} J\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty} da_1\cdots da_{N-1}\exp\left\{-\frac{m}{2i\hbar\epsilon^2}\sum_{n=1}^{N-1}\left(4\sin^2\frac{n\pi}{2N}-\omega^2\epsilon^2\right)a_n^2\right\} \qquad (E.3)$$

where J is the jacobian of transformation which equal to $\left[1/\epsilon\right]^{\frac{N-1}{2}}$.

Since the integration over each a_n can be performed separately, we have

$$\int_{-\infty}^{\infty} da_n \exp\left\{-\frac{m}{2i\hbar\epsilon^2}\left(4\sin^2\frac{n\pi}{2N}-\omega^2\epsilon^2\right)a_n^2\right\} = \left[\frac{2\pi i\hbar\epsilon^2}{m\left(4\sin^2\frac{n\pi}{2N}-\omega^2\epsilon^2\right)}\right]^{1/2} \qquad (E.4)$$

Thus

$$F(T) = \lim_{N\to\infty} \left(\frac{1}{A}\right)^N J \left[\frac{2\pi i\hbar\epsilon^2}{m}\right]^{\frac{N-1}{2}} \prod_{n=1}^{N-1} \left[4\sin^2\frac{n\pi}{2N} - \omega^2\epsilon^2\right]^{-1/2} \qquad (E.5)$$

On substituting the value of $1/A$ into eq. (E.5) we obtain

$$F(T) = \lim_{N\to\infty} \left[\frac{m}{2\pi i\hbar T}\right]^{1/2} \sqrt{N}\, J \prod_{n=1}^{N-1}\left[\frac{\epsilon}{4\sin^2\frac{n\pi}{2N}}\right]^{1/2} \prod_{n=1}^{N-1}\left[1 - \frac{\omega^2\epsilon^2}{4\sin^2\frac{n\pi}{2N}}\right]^{-1/2} \qquad (E.6)$$

Since we know that $J = \left(1/\epsilon\right)^{\frac{N-1}{2}}$ and $\prod_{n=1}^{N-1}\left[4\sin^2\frac{n\pi}{2N}\right]^{-1/2} = \frac{1}{\sqrt{N}}$

then we have

$$F(T) = \lim_{N\to\infty} \left[\frac{m}{2\pi i\hbar T}\right]^{1/2} \prod_{n=1}^{N-1}\left[1 - \frac{\omega^2\epsilon^2}{4\sin^2\frac{n\pi}{2N}}\right]^{-1/2} \qquad (E.7)$$

As N approaches infinity $\sin\frac{n\pi}{2N}$ approaches $\frac{n\pi}{2N}$, the harmonic-oscillator prefactor can be written as

$$F(T) = \left[\frac{m}{2\pi i\hbar T}\right]^{1/2} \prod_{n=1}^{\infty}\left[1 - \frac{\omega^2 T^2}{n^2\pi^2}\right]^{-1/2}$$

$$= \left[\frac{m\omega}{2\pi i\hbar \sin\omega T}\right]^{1/2} \qquad (E.8)$$

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