CHAPTER 7

Discussions

In the present research a systematic technique for calculating the local and non-local harmonic-oscillator path integral has been developed. The main purpose of this technique is to obtain the prefactor and the classical action of the propagator simultaneously. In order to do this we have put the boundary points into the path integral and represented the paths as a cosine series. We have transformed the path integral to be the multiple integrals of the coefficients of the series. We restricted ourselves to the discrete-time assumption so that the evaluation of the transformed action can be performed by a summation instead of an integration as was done by Feynman and Devies. After performing some integrations we obtain a product series and the sum of the exponents. By taking care of all factors and taking the limit of N approaching infinity as the final step, we find that the product series converges to be the prefactor and the sums of exponent converge to be the classical action correctly. This demonotrates that the prefactor and the classical axtion can be obtained simultaniously.

Our method differs from that of Feynman He calculated the harmonic-oscillator prefactor by representing the paths as a Fourier sine series and by transforming the path integral to be the mulitple integrals of the Fourier coefficients. He performed the integrations but did not take proper care of all factors resulting the integrations. He combined all factors into a constant and evaluated the constant in the free-particle limit. He left the detailed calculation as an exercise (problem 3-13) in his book (10). Unfortunately, the problem given in the book is incorrect as can be seen from the expression $\mathcal{J}\sqrt{N}(T/\pi)^N \prod_{n=1}^{N}(1/n)$ which does not converge as $N \rightarrow \infty$. In Appendix E we show that by applying our techniques to this problem, the correct expression should be $\mathcal{J}\sqrt{N}\prod_{n=1}^{N-1}\left[\epsilon/4\sin^2\frac{n\pi}{2N}\right]^{1/2}$. The key point which lead Feynman getting the wrong expression is that he evaluated the transformed action by integration. Mathematically speaking, this meant that the limit $N \rightarrow \infty$ was taken at the point of integration. Some factors were, therefore, cancelled out before they should have been in the final step. In our techniques we took the limit at the final step and could obtain, therefore, the correct result.

In his technique, Davies obtained the exponent of the classical action but did not caculate the prefactor. Since he evaluated the transformed action function by using the integration he would not have been obtained the correct prefactor. This can be seen from the discussions in the previous paragraph.

There are also the other methods of calculating the harmonic-oscillator prefactor such as by using the Van Vleck-Pauli formula(13), (14)

$$F(T) = \left[\frac{1}{2\pi i\hbar}\left|\frac{\partial^2 S_{cl}}{\partial x_a \partial x_b}\right|\right]^{1/2} \qquad (7.1)$$

by using the method of integration in functional spaces develped by Gel'Fand and Yaglom (15), Papadopoulos (16).

Recently, Royer (17) had discussed the Fourier series representations of the path integrals, In his analysis, he tried to apply direct integrations to Feynman's method. However, he had to add some extra mathematical restriction, in order to get the correct result. He introduced the functional jacobian, in the calculation.

For applying our techniques to calculate the non-local harmonic oscillator propagator we have to linearize the memory terms in the action function. In order to do this we use an idea introduced by Stratonovich (12). This idea was that the effect of the interacting particles in a many particles system can be reduced to the investigation of non-interacting particles in a fluctuating external field. Since this problem is one body problem with memory effect this idea can be applied by increasing the dimension of the integration. After performing some integrations, we took the limit N approaches infinity , the non-local harmonic socillator propagator was then obtained.