YIELD CURVE FORECASTING WITH A LARGE MACROECONOMIC DATA SET USING TWO-STEP FACTORIZATION

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การพยากรณ์เส้นอัตราผลตอบแทนที่มีชุดข้อมูลเชิงเศรษฐศาสตร์มหภาคขนาดใหญ่โดยใช้การแยกตัวประกอบสองขั้นตอน

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การศึกษานี้เน้นความสำคัญของการปรับปรุงความสามารถในการพยากรณ์โครงสร้างอัตราผลตอบแทนด้วยกระบวนการคัดแยกตัวประกอบร่วมจากอนุกรมเวลาทางเศรษฐศาสตร์มหภาคที่มีขนาดใหญ่ให้มีความเหมาะสมมากยิ่งขึ้น การศึกษานี้เป็นการศึกษาเชิงประจักษ์โดยอาศัยหลักฐานจากตลาดพันธบัตรในประเทศสหรัฐอเมริกาและเยอรมนี ผู้ศึกษาได้ทำการตรวจสอบความสามารถในการพยากรณ์เส้นอัตราผลตอบแทนภายใต้วิธีการคัดแยกข้อมูลแบบวิธีวิเคราะห์องค์ประกอบหลัก (Principal Component Analysis) และวิธีการแยกตัวประกอบสองขั้นตอน (Two-Step Factorization) ซึ่งทั้งสองวิธีนี้อยู่ภายใต้ข้อกำหนดที่ว่า การเคลื่อนไหวของอัตราดอกเบี้ยระยะสั้นถูกจำลองด้วย Factor-Augmented Vector Autoregression (FAVAR) และโครงสร้างอัตราผลตอบแทนตามระยะเวลาไถ่ถอนหลักทรัพย์เกิดจากการใช้ตัวแปรที่ถูกกำหนดให้ไม่มีการค้ากำไรเกิดขึ้น(No-Arbitrage) จากนั้นผู้ศึกษาได้ใช้วิธีการวิเคราะห์องค์ประกอบหลักและการแยกตัวประกอบสองขั้นตอนคัดแยกตัวประกอบร่วม จากชุดข้อมูลทางเศรษฐศาสตร์ชุดเดียวกัน ความสามารถในการพยากรณ์โครงสร้างอัตราผลตอบแทนภายใต้วิธีการคัดแยกที่แตกต่างกันนี้จะถูกนำมาเปรียบเทียบกัน ผลการศึกษาพบว่า ตัวประกอบร่วม ที่ถูกคัดแยกโดยวิธีการแยกตัวประกอบสองขั้นตอนให้ผลการพยากรณ์ผลตอบแทนระยะกลางได้ดีกว่าวิธีการวิเคราะห์องค์ประกอบหลักและแบบจำลองสุ่มโดยเฉพาะเมื่อพยากรณ์ผลที่อยู่ห่างออกไปปานกลางและห่างออกไปมาก เมื่อนำเฉพาะกลุ่มตัวแปรเศรษฐศาสตร์ที่มีนัยสำคัญกับอัตราผลตอบแทนระยะสั้นมาใช้ในการแยกตัวประกอบสองขั้นตอน พบว่าผลการพยากรณ์อัตราผลตอบแทนดีขึ้นในการพยากรณ์ผลที่อยู่ห่างออกไปมาก แต่ความสามารถในการพยากรณ์ผลที่อยู่ห่างออกไปปานกลางลดลง แต่เมื่อนำเฉพาะกลุ่มตัวแปรเศรษฐศาสตร์ที่มีนัยสำคัญกับอัตราผลตอบแทนระยะยาวมาใช้พบว่าผลการพยากรณ์อัตราผลตอบแทนระยะยาวดีขึ้นแต่ผลการพยากรณ์ระยะสั้นลดลงในการพยากรณ์ผลที่อยู่ห่างออกไปในทุกระยะ เราสามารถอนุมานได้ว่าการกำหนดข้อจำกัดในวิธีการคัดแยกเพื่อใช้เลือกตัวประกอบอาจจะลดผลกระทบจากปัจจัยที่ไม่เกี่ยวข้อง แต่ในขณะเดียวกันก็อาจจะทำลายข้อมูลบางส่วนทำให้ความสามารถในการพยากรณ์ลดลงได้

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This study emphasizes on improving the performance of term structure forecasting with an appropriate methodology of extracting common factors from large macroeconomic time series. This empirical study has been conducted for the US and German bond markets. We investigate the yield curve forecast performance under the Principal Component Analysis (PCA) and the Two-Step Factorization techniques for extracting information. We assume that the dynamics of the short rate follow a Factor-Augmented Vector Autoregression (FAVAR) model and that the term structure implies no-arbitrage condition. The PCA technique and the Two-Step Factorization will be used to extract common factors from the same macroeconomic data set. Then, the yield curve forecast performance under the different approaches will be compared. The finding shows that the common factors extracted from the Two-Step Factorization outperform those extracted from the PCA technique and a random walk model in forecasting the intermediate yields in particular at the intermediate and long forecast horizons. By extracting factors only from macro variables that can explain short rates, the Two-Step Factorization method leads to better forecast performance for long forecast horizons. However, the performance for intermediate forecast horizons becomes worse. On the other hand, using factors extracted from macro variables that can explain a long term rate instead of the short rate leads to better forecast performance for the long term yields but it lowers the performance for the short term yields for all forecast horizons. This implies that the constraints put to the extracting method for selecting factors may help eliminate some noises, but at the same time they may also eliminate some of the information and hence lower the forecasting performance.

Department: Banking and Finance Student's Signature................................

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CHAPTER I
INTRODUCTION

As the central bank uses a large set of conditioning information when setting the short term interest rate, a broad range of conditioning information will become a key contribution to the term structure. According to this argument, there are several papers take a step toward bridging the joint dynamics of macroeconomic variables and bond prices in a factor model of the term structure. For example, Bernanke et al. (2005) examined the advantage of factor modeling based on estimation of the principal components and structural VAR analysis by estimating a joint vector-autoregression of the short-term interest rate and factors extracted from a large macroeconomic time series. This model is known as the Factor-Augmented Vector Autoregression (FAVAR).

To estimate the components from a large set of macroeconomic time series, the Principal Component Analysis (PCA) has been used by many researchers. As the FAVAR model directly extracts the components from a large set of macroeconomic information, some question may arise as many countries have different principals and policies used to calculate their macroeconomics variables. So, there are different numbers of macroeconomic time series for each macroeconomic variable. For example, there are many categories of time series that measure the quantity of GDP such as time series of durable goods, time series of non-durable goods, time series of industrial products, etc. Moreover, we found that the common factors directly extracted by the PCA technique from a large macroeconomic dataset typically have high correlation with group of macroeconomic variables that share the same character (highly correlated). On the other hands, the group of macro variables that do not share character (less correlated) with others is typically ignored even though they are considered as important variables.

Examining this question, we try to equalize the weights given to each of the macroeconomic categories to make sure that the important factors are not left out from our model due to a small number of time series included (small weighting). We have already known that a macroeconomic category can be measured by many time series. Accordingly, the average correlation between pairs of time series would be very high as they measure the same category. So, finding a factor that can capture the largest share of variation of the macroeconomic category will considerably be good proxy. We will apply this idea to equalize the weight given to each of the macroeconomic category by choosing only one factor from each group. This is considered as the main research question of this study which we expect that extracting the common factors from an equalized group data will improve the forecast performance of the yield curve. Moreover, we also develop our main research question further by imposing constraints to those equalized groups. Therefore, we will implement this idea in a term structure forecasting framework where the technique of principal component analysis is used to extract a set of common factors that will be included in a forecasting model.

To compare the forecast performance of each model, we set up three different approaches to extract the common factors from a large macroeconomic dataset. The first approach is FAVAR model in which a set of common factors will be extracted directly from a large macroeconomic time series variables. This approach is the typical PCA approach adopted by Bernanke at el. (2005). The second approach is Group Factor-Augmented Vector Autoregression (GFAVAR) in which one common factor is extracted from each group of macroeconomic variables. We call these common factors "first-layer" factors or group factor representatives. This approach is expected to perform well in the situation where there is high correlation between the time series in the groups' variables. According to this expectation, the first common factor extracted from a group in which all time series highly correlate with each other is considered as a good proxy because only one common factor that captures the largest variation of the group's variance sufficiently represents the total macroeconomic time series contained in the group. Therefore, we will extract one common factor from each group of macroeconomic variables. Then, to reduce the number of factors, another set of "second-layer" common factors are extracted from these "first-layer" common factors. This approach gives an equal weight to each group of macroeconomic variable when we form the final set of common factors. The performance of the GFAVAR model will answer the main research question of this study. Moreover, to further develop the model of GFAVAR, we propose a Significant Group Factor-Augmented Vector Autoregression (SGFAVAR) that is derived from the GFAVAR model where one common factor is extracted from each group of macroeconomic variables as in the GFAVAR approach but the "second-layer" common factors are extracted from those "first-layer" common factors that have a significant explanatory power to the short rate. This model focuses more on the factors that best explain the rate as we realize that the short rate is the main tool use by the monetarists to manipulate the economy. Moreover, the short rate is an important factor that drives the dynamic of the yields curves. So we decide to use the short rate as the criteria in extracting the common factors following the SGFAVAR model. This approach is expected to improve the forecast performance over the GFAVAR model because it contains only factors that significantly explain the short rate. In addition, we propose the LSGFAVAR model which is relatively similar to the SGFAVAR model except that the "second-layer" common factors are extracted from the "first-layer" common factors that significantly explain the long term yield instead of the short term. We propose this model because we expect that extracting the common factors from the groups that best explain the long term yields will improve the forecast performance of the long term yields. These SGFAVAR and LSGFAVAR models are expected to provide flexibility for researchers in term of a factors selection criterion to the term structure model.

In this study we use both the US and German zero-coupon yields as my test data in order to compare the consistency of the models. This study differs from many other works in that most normally study more on the development of the term structure. Therefore, our study, to the best of our knowledge, is the first study that applies the two-step factorization in the term structure modeling. This allows us to improve the performance of the yield curve forecasting through the method of extracting the common factors.

CHAPTER II

LITERATURE REVIEW

The term structures of interest rates have been used in finance and macroeconomics for different reasons. For monetary economists, they focus on understanding the relationship between interest rates, monetary policies and macroeconomic variables. Since short term interest rates represent a tool of monetary policy where the changes in the short-term policy rate will affect long-term yields. Therefore, it will affect the spending, saving and investment behavior of individuals and firms in the economy. On the other hand, the financial economists mainly focus on forecasting and pricing interest rate-related securities.

Nowadays, the development of the term structure modeling has occurred every year. As technology has advanced, the tools used to develop the term structure have been widely available and easy to use. The term structures of interest rates have been developed starting with the simplest version which is one-factor models where short-term interest rates are a single factor that drives the movements of the term structures. Examples of one-factor models are the models of Vasicek (1977) and Cox, Ingersoll and Ross (1985) which are also the pioneers of the affine term structure models. These models applied only short-term interest rates as the key factor to their term structures models. However, the one-factor models have been developed further as there are some arguments about the unrealistic properties of the models. Firstly, they are not able to generate all the shapes of the yield curves that are observed in practice. Secondly, the one-factor models do not allow for the twist of the yield curve such as the case when short-maturity yields move in the opposite direction of long-maturity yields. This drawback occurs because all yields are driven by a single factor, meaning that they have to be highly correlated. Therefore, this problem can be avoided by including more factors to the term structures.

With the objective to improve and correct the possible problems of single factor models, there are several papers that try to add different factors to one-factor models. Those factors are, for example, the volatility of the short rate, the macroeconomic factors and also many possible factors that are expected to capture the variation of the rates. In this study, we focus more on the effect of macroeconomic factors to the term structure model as many economists typically think that the economy was affected by monetary policy through the short term interest rate. In addition the central bank usually sets short term rates to stimulate the economy. Therefore the short rate is part of the economy that always reflects the changes in macroeconomic variables. Consequently, macroeconomic variables are considered as the possible factors used in the term structure modeling.

Ang and Piazzesi (2003) is an example of papers that applied the macroeconomic variables to the term structure. Moreover, their paper also used a joint dynamic of bond yields and macroeconomic variables in a vector autoregression. To estimate the term structures, they sort the macroeconomic variables into two groups. The first group is the variables related to various inflation measures. The second group is the time series related to variables that capture real activity. To reduce the dimensionality, they used the principal component analysis to extract the first common factor from each group. Each common factor represented the group of inflation and output respectively. Then, they used these two common factors and the short rate in the VAR model to estimate the term structures. So, their model is considered as a standard three-factor affine term structure that adds two macroeconomic factors. The results showed that models with macroeconomic factors forecasted with more accuracy than models with only unobservable factors. According to this reason, they implied that macroeconomic factors could capture a large share of the variation in interest rates and also improved the yield curves prediction. However, there are also some papers that criticized the use of output and inflation to the term structure.

As a selection of output and inflation might not be perfect factors that explained the most variation of the economy, Bernanke and Boivin (2003) argued that the central bank commonly worked with a data-rich environment. To test this argument, they extracted the factors from large data sets and used them to estimate the term structures. They also followed the early work of Stock and Watson (2002a, b) to reduce the dimension of the macroeconomic variables. The traditional principal component analysis (PCA) was applied to his work in order to extract the key forecasting information from large data sets. The results showed that their method offered potentially large improvements in the forecasting of important macroeconomic time series. Therefore, this was the evidence that supported the argument that the monetary policy authority based its decisions on a broad set of conditioning information rather than a few key macroeconomic aggregates.

Two years later, Bernanke et al. (2005) combined the advantages of factor modeling and structural VAR analysis by estimating a joint vector-autoregression of short-term interest rates and factors extracted from a large cross-section of macroeconomic time series. In this paper, they examined the performance of the two-step estimation approaches which were based on principal components extracted from large macroeconomic variables and the one-step approach which used Bayesian likelihood methods and Gibbs sampling. The results showed that using a few common factors extracted from a large number of macroeconomic time series variables and the interest rate (the two-step estimation approach) to estimate the parameter governing the dynamic of the state equation in a VAR model tend to produce more plausible results than the Bayesian method based on Gibbs (one-step approach).

Following the contributions acquired from previous papers, there are many works that applied the two-step estimation method to estimate the factors in the term structure modeling. For example, Moench (2008) forecasted the yield curve with a broad macroeconomic information set. This model used the short rate and common components extracted from a large number of macroeconomic variables as factors. This paper also took a step further from Bernanke et al (2005) which applied the two-step estimation approach to estimate the parameters in a VAR analysis. Moreover, the parameters of the model were restricted by no-arbitrage strategies which also used them to estimate the term structure. Therefore, the dynamics of the short rate were modeled by the No-Arbitrage Factor Augmented Vector Autoregression. The results showed that the No-Arbitrage FAVAR model based on macroeconomic factors and the short rate fitted the US yield curve well in-sample. More importantly, the model also showed a good ability to predict yields out-of-sample which provided superior forecasts to a number of benchmark models. Moreover, the No-Arbitrage FAVAR model significantly outperformed the random walk and a standard three factor affine model adopted by Bernanke et al. (2004).

As we have seen already, the development of the term structures of interest rates starts from the simplest models (one-factor model) that have only the short rate to the complicated models (multifactor models) that applied all knowledge, related to the improvement of the term structure. This study will also develop further in different directions by focusing more on the extracting method. As there are differences in principals and policies used by monetarists to calculate the macroeconomic variables, the numbers of macroeconomic variables in each country are not the same as the others. Therefore, applying the principal component analysis to directly extract the common factors from a large number of macroeconomic variables would be questionable.

As there is little evidence on the drawbacks of the principal component analysis related to a number of macroeconomic data, we will propose the new approach to improve the extracting method by giving equal weighting to each macroeconomic variable. This approach was created according to the observation that the common factors extracted by the PCA technique from large macroeconomic data sets typically have high correlation with the group of macroeconomic variables that share the same character (highest weighting). On the other hand, the group of macroeconomic variables that do not share character (small weighting) with others is typically ignored even though they are considered as important variables. Therefore, in this study we try to equalize the weights given to each of the macroeconomic categories in order to make sure that the important factors are not left out from our model due to a small number of time series included (small weighting). Moreover, we expected that our approaches will perform well in the situation where there is high correlation between the time series in the group of macroeconomic variables.

In brief, our paper tries to improve the classical extracting approach to extract the common factors by giving an equal weight to each macroeconomic variable. To extract the common factors following the assumption above, we propose the two-step factorization where the first layer factors are considered as the group factors representative and the second layer factors are also considered as the common factors that capture the total variation of the group representatives. To have common factors, we apply principal component analysis to extract the first common factor from each group of macroeconomic variables as we expect that the first factors extracted from a group that all time series highly correlate with each other is considered as a good proxy for group’s variable. This method is similar to Ang and Piazzesi (2003) who extracted the common factors from groups of output and inflation. From now we already have the first layer factors extracted from many groups of macroeconomic variables, not only the groups of output and inflation. To reduce the dimensionality, we apply principal component analysis again to extract the second layer factors that capture the total variation of the macroeconomic categories. According to the two-step factorization, we will obtain the common factors that equalized the weight to each category.

To examine the performance of the two-step factorization, we follow the method used by of Moench (2008) as the framework to forecast the yield curves. The yield curve forecast performance under the two-step factorization and the previous method, FAVAR model, which is typically the PCA method, are compared in terms of the root mean squared errors (RMSE). The interest rate term structure used in the Moench (2008) is an affine no-arbitrage term structure model using zero-coupon bond yields. This model started from the assumption of no-arbitrage. Moreover, they also had an explicit economic content that puts restrictions on the cross-section and time series behavior of bond prices and interest rates. To forecast the yield curves following this model, we first directly extract the common factors from large macroeconomic time series variables. Then we use these common factors and the short rate as the state variables in a VAR analysis to estimate the parameters. These parameters are then used to forecast the yield curves further. This procedure is typically the Factor Augmented Vector Autoregression (FAVAR). On the other hand, to forecast the yield curve following the proposed method, the two-step factorization has been used instead of the PCA technique to extract the common factors. Moreover, we also use the random walk model as a benchmark to compete with the model used in Moench (2008) and the two alternative models. The root mean square error of each model will be compared. To the best of our knowledge, there is no literature that studies the method of two-step factorization which gives an equal weight to each macroeconomic variable before.

CHAPTER III
DATA AND METHODOLOGY

A. Data

The main purpose in this study is to improve the performance of term structures forecasting with an appropriate methodology of extracting common factors from a large macroeconomic time series. For this study purpose, we collect various macroeconomics variables and the yields of the United States of America and Germany. Most German macroeconomic variables are downloaded from a database of Deutsche Bundesbank's website which contains a time series of various economic categories for Germany, from January 1991 to December 2008, with a 359 monthly time series. Moreover, the US's macroeconomic variables are collected from the DataStream and the Federal Department which contain a 341 monthly time series from January 1990 to December 2008. The economics variables include a large number of time series related to industrial production, employment-related variables, price indices, stock indices, exchange rates and various monetary aggregates. **Table 19-20**, in Appendix A, list the group of macroeconomic time series which are used for extracting the common factors. The details of macroeconomic time series in each group are provided in Appendix I.

Moreover, we follow the same criteria used in Moench (2008) to exclude all interest rate related series from the dataset. The reason is that if the factors of the no-arbitrage model were extracted from a dataset containing yields, the restrictions would have to be imposed on the factor loading parameters too. Therefore, to make it consistent with the assumption of no-arbitrage, we would exclude the interest rate related series.

As we employ the principal component analysis which requires stationarity of macroeconomic time series before extracting the common factors, we would apply the unit root test as a pre-adjustment to the time series in the dataset. Finally, we standardize all time series to have mean equal to zero and variance equal to one before we extract the common factors.

The dataset also contains zero - coupon yields that have maturities of 1, 3, 6, 9, 12, 24, 36, 48, 60, 84, 120 months, covering the short term, medium term and long term bonds.

For Germany, the German bond market is the largest market for publicly issued bonds in Europe, and the data set which is mainly used in the thesis i.e. macroeconomic variables, exchange rates, and production are easily accessible via the Deutsche Bundesbank website. Moreover, the yields on German government bonds are viewed as benchmark interest rates in Europe. Due to the important role of the German bond market, nominal spreads are typically computed relative to German government bonds (German bunds).

At the same time, the US's economy is considered as the world's largest economy and the US's GDP is almost a quarter of the total world GDP. They are also the world leading importer and their currency, the US dollar, is widely used around the world. So, the world's economy will dependent on the US's economy. Moreover, their treasury securities are kept in the form of international reserve funds for most countries around the world. Furthermore, their stock market, the New York Stock Exchange, is also the largest stock exchange in the world which provides a mean for investors around the world to buy or sell securities.

According to the above reasons, the German and US markets are chosen in this study to make a comparison which will be expected to see a big picture of the forecast results in the US and Euro zone countries.

B. Methodology

As this study focuses on the factors extracting methods in a large macroeconomic dataset whose results are expected to provide the appropriate factors for the yield curve forecasting, we will firstly examine the method used by Bernanke et al. (2005) which is a Factor Augmented Vector Autoregressive (FAVAR) whose factors are directly extracted from a large number of macroeconomic time series variables. There are two alternative methods that try to add on constraints to the PCA technique in order to recover any doubts of this method. The first alternative method is a Group Factor Augmented Vector Autoregressive (GFAVAR) whose factors are extracted from a group of macroeconomic variables. The second alternative method is a Significant Group Factor Augmented Vector Autoregressive (SGFAVAR). This method puts more constraints to the first alternative method that the common factors are only extracted from groups of macroeconomic variables that significantly explain the short rate. These two models are expected to perform well in the situation where there is high correlation between the time series in the group of macroeconomic variables as we realize that the time series containing in the group commonly measure the same variable

1) Factor Augmented Vector Autoregressive (FAVAR)

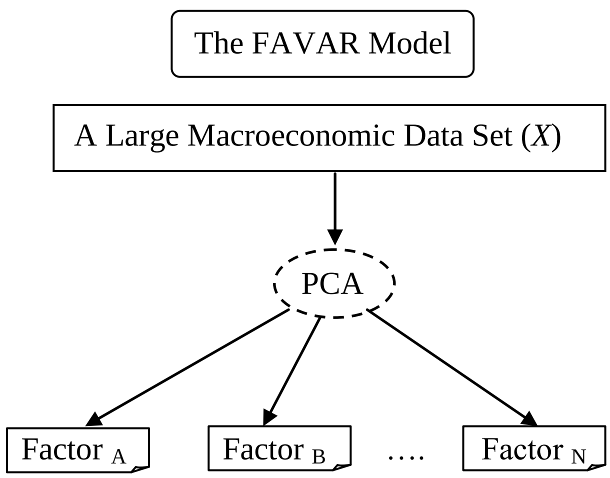
According to the important of the macroeconomic factors, Bernanke et al. (2005) extracted a few common factors from a large number of macroeconomic time series variables in order to estimate the term structure. Moreover, he also studied the mutual dynamics of monetary policy and the key economic factors by estimating a joint Vector Autoregressive (VAR) of the factors and the short rate. Their study analyzed the dynamics of the short rate on a board set of macroeconomic variables. Furthermore, Moench (2008) also studied an affine term structure model that starts from the assumption of the no-arbitrage and also having an explicit economic content that puts restrictions on the cross-section and time series behavior of bond prices and interest rates. This affine term structure has a Factor-Augmented Vector Autoregressive (FAVAR) as the state equation. Moreover, this model has the short rate and the common components of a large number of macro time series representing the factors that drive the variation of yields.

To estimate the Factor-Augmented VAR model, Moench (2008) examined the method use to estimate the parameters which is Kalman filter and maximum likelihood methods. However, if there are a large number of macroeconomic variables, the computation of these methods is infeasible. To combat this, Bernanke et al. (2005) studied two alternative estimation methods which are a single-step approach using Markov Chain Monte Carlo (MCMC) methods and a two-step approach in which first principal components techniques are used to estimate the common factors and then the parameters governing the dynamics of the state equation are obtained from standard classical methods for VAR. Comparing both methods in the context of an analysis of the effects of monetary policy shocks, Bernanke et al. (2005) found that the two-step approach yields more usable results meaning that this approach it easier to use on a computer.

In this study we will follow the method adopted by Bernanke (2005) as the main model to extract the common factors from a large number of macroeconomic time series variables. Moreover, we will also follow the term structure forecasting framework used in Moench (2008) to construct and forecast the yield curves. As you will see in the Table 15-16, in Appendix A, we collect various groups of macroeconomic time series variables from the creditable websites for research. These groups of variables are categorized by the central bank of each country which can be assumed that these variables may not contain any selection bias involving the number of macroeconomic time series contained in each category. So we decide to select the group of macroeconomic variables following the websites. The groups of German macroeconomic variable are already defined in the Deutsche Bundesbank's website which there is 16 groups of macroeconomic variables. For the groups of the US macroeconomic variables, they are already defined by the Federal Department which there is 14 groups of US macroeconomic variables.

Before estimating the common factors following the method adopted by Bernanke (2005), we need to combine macroeconomic time series from each group. Now, we extract the common factors from the macroeconomic data set. First we need to apply the unit root test to verify that each macroeconomic time series variable is stationary. Then we standardize all the time series variables to have mean equal to zero and standard deviation equal one.

Figure 1: The procedure to extract FAVAR's common factors from a large panel of macroeconomic time series



Once we have all stationary time series variables (*X*), we can now follow the procedure as shown in **Figure 1** which we will apply principal component analysis (PCA) to extract the common factors (*F*) that drive the dynamics of the term structure of interest rates following the equation (1).

$$X = VD^{\frac{1}{2}}F \tag{1}$$

where X denotes the T×N matrix of observed data each row of which corresponds to a time period and each column corresponds to a macroeconomic time series variable.

V denotes the N×N matrix whose columns are eigenvectors of the variance–covariance matrix of the data $X'X$.

D denotes the diagonal matrix of eigenvalues.

F denotes the T×N matrix of principal components

The detail explanation of the principal component analysis is presented in Appendix G. Moreover, the MATLAB's codes used to calculate these estimated factors are also shown in Appendix H.

After we have the factors extracted from a large number of macroeconomic time series variables (Factor $_A$ Factor $_B$ … Factor $_N$), we will choose only the first k^{th} optimal factors that can explain at least half of the total variation of macroeconomic time series. After that we will use these estimated factors and the short term interest rate as the state variables in a Vector Autoregression model (VAR model) to estimate the factor loading following equations (2).

$$\begin{pmatrix} F_t \\ r_t \end{pmatrix} = \mu + \phi_1 \begin{pmatrix} F_{t-1} \\ r_{t-1} \end{pmatrix} + \phi_2 \begin{pmatrix} F_{t-2} \\ r_{t-2} \end{pmatrix} + \cdots + \phi_p \begin{pmatrix} F_{t-p} \\ r_{t-p} \end{pmatrix} + \omega_t \tag{2}$$

where r_t denotes the short-term interest rate at time *t*,

F_t is the $k \times 1$ vector of period *t* observations of the common factors,

$\mu = (\mu_f', \mu_r)'$ is a $(k+1) \times 1$ vector of constants,

ϕ_j is a $(k + 1) \times (k + 1)$ matrix of factor loading for every j = 1,…, *p*,

ω_t is a $(k + 1) \times 1$ vector of error term with assuming that the error term are $IID \sim N(0, I)$ across time,

Ω denotes the $(k + 1)\times(k + 1)$ variance covariance matrix of an error term ω_t

To have all the parameters (μ, ϕ_j, ω, and Ω), we need to run the VAR model following the equation (2). Moreover, we apply the Bayesian Information Criteria

(BIC) with a maximum lag of 12 to indicate an optimal number of lags (p) for the VAR model. The MATLAB's codes for estimating a VAR model are shown in Appendix H. After we obtain the VAR model parameters, we will have all the parameters (μ, ϕ_j, ω, and Ω).

To estimate the yields curves following the no-arbitrage FAVAR model, we transform the VAR(p) above to VAR(1) of a new variable Z_t defined below. As a result, we can rewrite the VAR in equation (2) in companion form of VAR (1) as

$$Z_t \quad = \quad \boldsymbol{\mu} + \boldsymbol{\phi}\, Z_{t-1} + \boldsymbol{\omega_t}\ , \tag{3}$$

where $\quad Z_t = (F_t', r_t, F_{t-1}', r_{t-1}, \ldots, F_{t-p+1}', r_{t-p+1})'$,

$\boldsymbol{\mu}$, $\boldsymbol{\phi}$, $\boldsymbol{\omega}$, and $\boldsymbol{\Omega}$ denote the companion form equivalents of μ, ϕ_j , ω, and Ω respectively.

Moreover, the short rate r_t can also be written in term of Z_t as $r_t = \delta' Z_t$ where $\delta' = \left(0_{1\times k}, 1, 0_{1\times(k+1)(p-1)}\right)$. The MATLAB's codes for the transformation of these parameters are shown in appendix H. Once we have all the factors loadings ($\boldsymbol{\mu}$, $\boldsymbol{\phi}$, $\boldsymbol{\omega}$, and $\boldsymbol{\Omega}$) following the VAR(1) process, we can use these factors loadings to estimate the yield curves following equation (6).

As the method of No-Arbitrage FAVAR model imposed restriction on the parameters to control the impact of the state variables on the yields, the market price of risk, λ_t, was imposed into the bond pricing model in order to make sure that the model is consistent with the assumption of no-arbitrage which can be expressed as

$$\lambda_t = \lambda_0 + \lambda_1 Z_t$$

where $\quad \lambda_t$ is the market prices of risk, and

$$\lambda_0 = \left(\tilde{\lambda}_0', 0_{1\times(k+1)(p-1)}\right)'$$

$$\lambda_1 = \begin{pmatrix} \tilde{\lambda}_1 & 0_{(k+1)\times(k+1)(p-1)} \\ 0_{(k+1)(p-1)\times(k+1)} & 0_{(k+1)(p-1)\times(k+1)(p-1)} \end{pmatrix}.$$

Following this equation, only the $\tilde{\lambda}_0$ and $\tilde{\lambda}_1$ need to be estimated where $\tilde{\lambda}_0$ is of dimension $(k+1)$ and $\tilde{\lambda}_1$ is a $(k+1) \times (k+1)$ matrix. Following these approaches, the No-Arbitrage FAVAR model is guaranteed by computing A_n and B_n as the following equations. For further detail explanation of derivation of the bond pricing parameters, you can study in the Appendix F.

$$A_n = A_{n-1} + B'_{n-1}(\boldsymbol{\mu} - \boldsymbol{\Omega}\lambda_0) + \frac{1}{2}B'_{n-1}\boldsymbol{\Omega}B_{n-1} \quad (4)$$

$$B'_n = B'_{n-1}(\boldsymbol{\phi} - \boldsymbol{\Omega}\lambda_1) - \delta' \quad (5)$$

$$y_t^{(n)} = a_n + b'_n Z_t \quad (6)$$

where $\mathrm{a}_n = -\mathrm{A_n}/n$

$\mathrm{b}'_n = -\mathrm{B'_n}/n$

$y_t^{(n)}$ denotes the yield of an *n*-months to maturity zero-coupon bond at time *t*,

A_n and B_n denote the scalar and the coefficient vector which depend on the time to maturity *n* respectively,

If we replace A_n and B_n from equation (4) and (5) into equation (6), we found that

$$y_t^{(n)} = \frac{-(A_{n-1}+B'_{n-1}(\boldsymbol{\mu}-\boldsymbol{\Omega}\lambda_0)+\frac{1}{2}B'_{n-1}\boldsymbol{\Omega}B_{n-1})}{n} + \frac{-(B'_{n-1}(\boldsymbol{\phi}-\boldsymbol{\Omega}\lambda_1)-\delta')}{n} * Z_t \quad (7)$$

You will see that there are two unknown factors in the equation (7) which are λ_0 and λ_1. To estimate these two factors, minimizing the sum of squared fitting errors of the equation (7) where the sum of squared fitting errors is minimized with respect to λ_0 and λ_1 provided the possible results. That is, we minimize

$$S = \sum_{t=1}^{T}\sum_{n=1}^{N}(\hat{y}_t^{(n)} - y_t^{(n)})^2$$

where S denotes the sum of squared fitting errors

$y_t^{(n)}$denotes the yield of an *n*-months to maturity zero-coupon bond at time *t*,

$\hat{y}_t^{(n)}$denotes the estimated yield of an n-month to maturity zero-coupon bond at time *t*

To find good starting values and fast convergence, firstly we allow λ_0 to vary. Then set all elements of the matrix λ_1 to zero. Now we have only one unknown factor, λ_0, which can be calculated by minimizing the sum of squared fitting errors of the equation (7). After having the value of λ_0, we take these estimates of λ_0 as starting values for the second round. At this time we let all elements of λ_0 and λ_1 be estimated freely. As we have all the parameters used in the equation (7), now we can forecast the yield curves following the No-Arbitrage Factor Augmented VAR model. The method used to forecast the yield curves will be shown in the out of sample forecast results section.

You can see that this method is a two-step estimation approach in which the common factors and the short rate are used as the state variables in the VAR model to estimate the parameters (***μ, ϕ, ω,*** and **Ω**). Then the parameters from VAR are used to estimate the market price of risk (λ_0 and λ_1). As we have already mentioned that there is another method used to estimate the term structure, it is the maximum likelihood method which is a one-step approach. In this approach, the VAR's parameters (***μ, ϕ, ω,*** and **Ω**) and the market price of risk (λ_0 and λ_1) are estimated simultaneously. The maximum likelihood estimation differs from minimizing sum square error used in this study. Firstly, the log likelihood function has a weighted sum squared fitted error where the weigh for error of maturity is the reciprocal of the variance of yield error, while the two-step approach assumes an equally weighted in the sum square error. Secondly, the log likelihood function also has an extra term which is a non linear term in market price of risk, λ_t. Lastly, the VAR parameters (***μ, ϕ*** and **Ω**) are also included in the log likelihood function. As a result, the maximum likelihood estimation and the two-step approaches may yield different results. For further explanation of the maximum likelihood methods, please see the Appendix B of Ang and Piazzesi (2003).

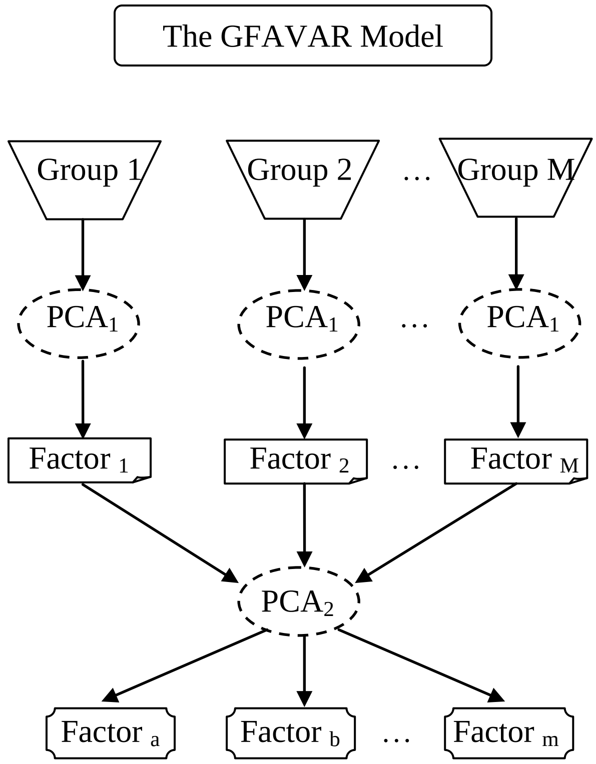
2) Group Factor Augmented Vector Autoregressive (GFAVAR)

As the macroeconomic time series variables in each country are calculated by using different principles and policies, applying the extracting method adopted by Bernanke et al. (2005) would be questionable. More precisely, the factors extracted from a large macroeconomic datasets typically have high correlation with variables in a group containing many variables that share the same character (highest weighting). The variables that do not share their characters with others are typically ignored (small weighting) even though they may be considered as important variables.

To reduce the effect of weighting of macroeconomic time series variables, we will follow the procedure to extract the common factors as shown in **Figure 2**. In this method, we do not combine all the groups of macroeconomic variables into one large group like the first method, FAVAR model, because we would like to equalize the weight given to each of the macroeconomic categories to make sure that the important factors are not left out from our model due to a small number of time series included. Moreover we realize that a macroeconomic category can be measured by many time series. The correlation between these time series would be very high as they measure the same category. So, finding a factor that can capture the largest share of variation of the macroeconomic category will considerably be good proxy for this category.

Let M denote the number of categories of the macroeconomic variables. The M categories or M groups are denoted by (Group 1, Group2… Group M). Like the FAVAR model, before we extract the common factors following the GFAVAR model, we need to apply the unit root test to each variable to verify that each macroeconomic time series is stationary. Then we standardize all the time series in each group to have mean equal to zero and standard deviation equal one.

Figure 2: The procedure to extract GFAVAR's common factor from group of macroeconomic variables



After we have all stationary time series in each group, we will apply equation (1), called the first-layer principal component analysis (PCA_1), to extract only one common factor from each group. The factors extracted from the PCA_1 (Factor $_1$, Factor $_2$ ... Factor $_M$) are considered as a group's factor representative where each group has only one component that accounts for a large variation of the total groups variance. The reason that we choose only one factor is to treat each group equally or give an equal weight to each group variables. Moreover, we also expect that if the time series in each group highly correlate with each other, the first factor that captures the largest variation of group's variance will be considered as a good proxy. So, the GFAVAR model is expected to perform well in the situation where there is high average correlation of macroeconomic time series within each group.

As you will see, each factor resulted from PCA1 can capture only a variation of their group's variance. To find the factors that capture a large variation of total group's variance, we will reapply second-layer principal component analysis (PCA2) to reduce the number of group factors representative in order to have the artificial factors that perfectly explain the total variation of the group of macroeconomic variables. Therefore, the factors resulted from PCA2 (Factor a, Factor b… Factors m) represent the common factors capturing the total variation of the group of macroeconomic variables. Even though, the number of factors resulted from PCA2 is equal to the number of factors resulted from PCA1, m=M, but the factors resulted from PCA2 are expected to explain a large variation of the total macroeconomic time series. Now we already have m common factors that are expected to capture the total variation of the macroeconomic time series. To reasonably compare the forecasting results with the FAVAR model, we will restrict the number of factors of GFAVAR model to be equal to the number of factors used in the FAVAR model. Another possible way to specify the number of the factors for the GFAVAR model is to choose the number of factors that can at least explain half of the variation of the first layer factors. However, this is not equivalent to being able to explain at least half of the variations of all the time series of macroeconomic variables as in the FAVAR model. Therefore, we choose to make the number of factors the same for both models.

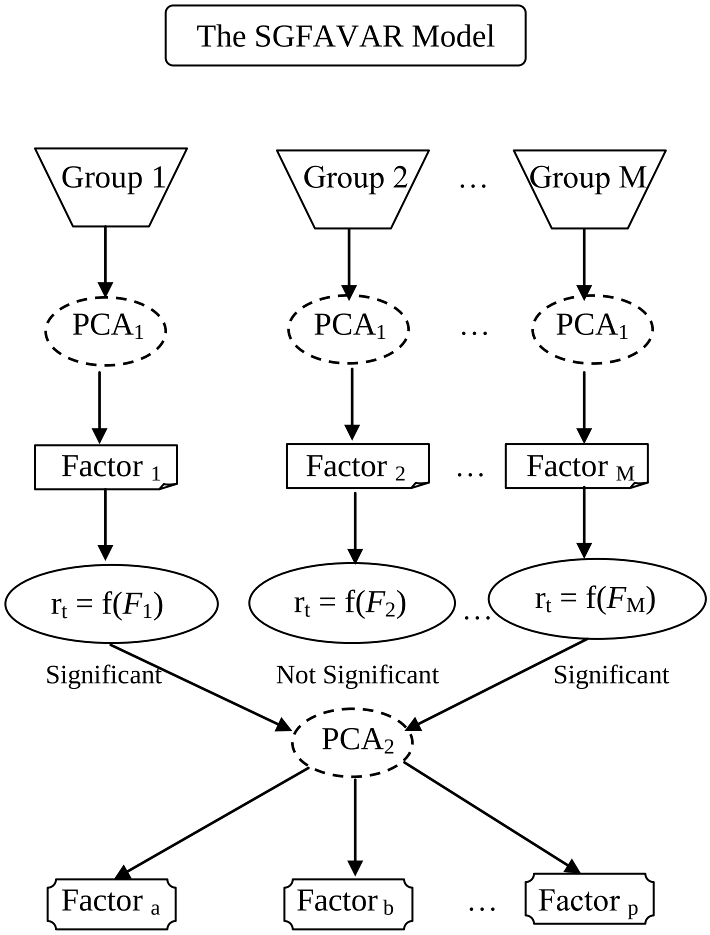
Next, we will follow the same method of estimating the factors loading from VAR model, equation (2) and (3), which is adopted by Bernanke (2005) and Moench (2008) that these estimated factors and the short term interest rate are used as the state variables in a VAR model. Moreover, we also apply the Bayesian Information Criteria (BIC) with a maximum lag of 12 to indicate an optimal number of lags (p) for the VAR model. Then, the VAR's parameters ($\boldsymbol{\mu}$, $\boldsymbol{\phi}$, $\boldsymbol{\omega}$, and $\boldsymbol{\Omega}$) would be used to find the optimal value of λ_0 and λ_1 by minimizing the sum of squared fitted errors of equation (7). Moreover, we also follow the same approach used by Moench (2008) to find the starting values for estimating the parameters λ_0 and λ_1. The optimal parameters resulting from minimizing the sum of squared fitted errors would then be used to estimate the yield curves in equation (7). The method used to forecast the yield curves will be explained in the out of sample forecast results section. Now we can forecast the yield curves following the GFAVAR model in which factors are extracted from groups of macroeconomic variables. This approach would be considered as the first alternative model.

3) Significant Group Factor Augmented Vector Autoregressive (SGFAVAR)

For the SGFAVAR model, the common factors are only extracted from groups of macroeconomic variables whose group representatives can well explain the short rate. This method was created because we would like to have the common factors that best explain the short rate as we realize that the short rate is the main tool use by the monetarists to manipulate the economy. Moreover, the short rate is an important factor that drives the dynamic of the yields curves. Therefore, we decide to use the short rate as the criteria in extracting the common factors following the SGFAVAR model. Logically, if we have the common factors that can well explain the short rate, it would be expected to well forecast the yield too.

This method, as summarized in Figure 3, is relatively similar to the GFAVAR model except that each group's factor representative (Factor $_1$, Factor $_2$…, Factor $_M$) needs to be tested further for explanatory power with a short rate in order to keep only the factors that can well explain the short rate. The reason for testing the explanatory power is that some macroeconomic variables cannot explain the short rate. Therefore, selecting only the significant groups of variables would be expected to have common factors that best explain the dynamic of the term structures. After we have the group's factor representatives that significantly explain the short rate, we will apply the second-layer principal component analysis (PCA_2) to reduce the number of these group's factors. The common factors resulted from the PCA_2 are considered as the artificial factors that capture the total variation of the first-layer factors that significantly explain the short rate (Factor $_a$, Factor $_b$, …, Factor $_p$). As you will see, the number of factors resulted from PCA_2 may not equal to the number of factors resulted from PCA_1 ($p \leq M$) because some group's factor representatives may not explain the short rate.

Figure 3: The procedure to extract SGFAVAR's common factor from a group of macroeconomic variables that can well explain the short rate.



From now, we have p common factors resulted from the PCA_2. So, we will follow the same approach of the GFAVAR model to forecast the yield curves in that we will restrict the number of factors to be the same as the number of factors used in the FAVAR model. Next, we use the common factors and the short rate to estimate the parameter in the VAR model following equation (2) and (3). Then, the VAR's results would be used to optimize in the equation (7) similar to the method of FAVAR and GFAVAR models. This approach would be considered as the second alternative model.

Obviously, the method of extracting the common factors from a data rich environment is the main difference between these three approaches. The FAVAR model directly extracts the common factors from a large number of macroeconomic time series. On the other hand, the GFAVAR model extracts the common factors from groups of macroeconomic variables which equalize the weight to each group. Lastly, the SGFAVAR model extracts the common factors only from groups of macroeconomic variables whose group representatives can well explain the short rate. Moreover, there is also a main similarity of these three models. The formulas used to forecast the yield curves starting from the VAR analysis (equation 2 and 3) to the equations of pricing bond parameters (equation 4-7) have been applied to all models in order to forecast the yield curves.

The competitor model:

4) Random Walk Model

As there are many paper pointed out that the random walk model could describe the movement of the interest rate, they apply the random walk model as the main competitor in order to evaluate the performance of their model. With a simple formula of the random walk model, it assumes that a rate in the future is represented as a today rate. In the paper of Moench 2008, they also applied the random walk model as the competitor model which is given by the following equation.

$$\hat{y}^{(n)}_{(t+h|t)} = y^{(n)}_t$$

where $\hat{y}^{(n)}_{(t+h|t)}$ denotes the *h* month ahead forecast of an *n*-maturity bond yield at time *t*,

$y^{(n)}_t$ denotes the *n*-maturity bond yield at time *t*,

To apply in this study, if today we would like to forecast the yield of a 3 year bond 1-month ahead, it would be directly represented as the today rate of a 3 year bond. According to the easily understandable formula and the acceptability by many other works, we will apply the random walk in this study to compete with our two alternative models as well.

CHAPTER IV
RESULTS

Empirical Results

A) The usefulness of the GFAVAR and SGFAVAR models

Before we estimate the term structure, we firstly examine the correlation between pairs of the macroeconomic time series contained in each group. The average correlation between pairs of the macroeconomic time series is used as the key indicator that measures the usefulness of the GFAVAR and SGFAVAR models whether they can capture a large share of variation from the group data.

Table 1: Summary statistics of the correlation between pairs of German macroeconomic time series in each group variable.

German's Macroeconomic Variables	Mean	Maximum	Minimum	No. of Pairs
Monetary Base	0.7483	0.9951	0.4622	3
Foreign Exchange Rate	0.7342	0.9785	0.4048	28
Stock Return Index	0.7127	0.9997	0.1000	78
Price Index	0.7904	0.9997	0.1327	28
Export – Import	0.6877	0.9979	0.1146	153
Employment	0.7499	0.9986	0.2857	10
Output	0.6930	0.9998	0.0939	120
Order Receive	0.6806	0.9984	0.1037	91
Pay Rate	0.8019	0.9997	0.5820	28
Retail Trade Turnover	0.7445	0.9978	0.2884	15
Factor Income & Services	0.7555	0.9997	0.1208	153
Household Sector	0.6907	0.9999	0.1103	1,035
General Government	0.6529	0.9998	0.0800	1,035
Monetary Financial Institution	0.5708	0.9999	0.0512	1,326
Non-Financial Corporation	0.5931	0.9998	0.0894	1,378
Other Financial Institution	0.7011	0.9997	0.1184	253

Table 2: Summary statistics of the correlation between pairs of the U.S. macroeconomic time series in each group variable.

US's Macroeconomic Variables	Mean	Maximum	Minimum	No. of Pairs
Reserve and Monetary Base	0.7276	0.9998	0.2355	28
Exchange Rate	0.6784	0.9717	0.1119	210
Price Index	0.8069	0.9999	0.3013	703
Stock Return Index	0.7553	0.9938	0.2003	55
Employment	0.6598	0.9987	0.1153	78
Industrial Production	0.6818	0.9996	0.1010	351
Capacity Utilization	0.6101	0.9939	0.0792	1,176
Pay Rate	0.7219	0.9979	0.3312	36
Export – Import	0.7076	0.9997	0.0953	666
Assets Liabilities Commercial Bank	0.8106	0.9999	0.2443	91
Consumer Credit	0.7577	0.9974	0.1100	91
Income payment and Receipts	0.7881	0.9999	0.2562	78
Monetary Aggregate	0.7082	0.9895	0.1509	36
Gross Domestic Product	0.8424	0.9998	0.4573	45

As you can see in **Table 1** and **Table 2**, the average correlations of each group of macroeconomic variable are all fairly high. Moreover, we found that the groups that contain a large number of pairs of macroeconomic variables show less average correlation than the groups that contain small number of pairs. So, we can imply from this finding that the large groups may contain the time series that represent significantly different pieces of information. Therefore, the correlation between these time series can be very low. For the groups of macroeconomic variables that have high average correlation between the pairs of macroeconomic time series, using only a few common factors extracted from each group of macroeconomic variables will be reasonably good proxy. According to these results, the usefulness of the proposed models, GFAVAR and SGFAVAR, would be possible.

Table 3: Percentage of variation explained by the first common factor extracted from each group of German macroeconomic Variables

Group of German Macroeconomic Variables	Percentage*	Group of German Macroeconomic Variables	Percentage*
Monetary Base	73.613	Pay Rate	68.557
Foreign Exchange Rate	81.538	Retail Trade Turnover	75.114
Stock Return Index	63.020	Factor Income & Services	52.440
Price Index	65.826	Household Sector	48.056
Export - Import	52.409	General Government	40.804
Employment	78.197	Monetary Financial Institution	45.866
Output	63.347	Non-Financial Corporation	47.948
Order Receive	60.819	Other Financial Institution	55.218

***The percentage is calculated from [100*diag(D)/sum(diag(D))] where D is the eigenvalue. The sample period is 1993:01 to 2008:12.**

Table 4: Percentage of variation explained by the first common factor extracted from each group of the U.S. macroeconomic Variables

Group of The U.S. Macroeconomic Variables	Percentage*	Group of The U.S. Macroeconomic Variables	Percentage*
Reserve and Monetary Base	52.656	Pay Rate	75.441
Exchange Rate	60.076	Export - Import	50.604
Price Index	62.933	Assets Liabilities Commercial Bank	55.125
Stock Return Index	70.682	Consumer Credit	54.503
Employment	61.056	Income payment and Receipts	50.782
Industrial Production	52.246	Monetary Aggregate	64.209
Capacity Utilization	47.242	Gross Domestic Product	78.380

***The percentage is calculated from [100*diag(D)/sum(diag(D))] where D is the eigenvalue. The sample period is 1993:01 to 2008:12.**

In **Table 3** and **Table 4**, we calculate the percentage of the variation of each group explained by its first common factor. For the group of German's macroeconomic variables, we found that more than 40 percent of the group's variation can be explained by their first common factor. Moreover, for the U.S. macroeconomic variables, we found that more than 47 percent of the group's variation can be explained by their first common factor as well. According to these results, we can imply that the first common factors that we extract from each group of the macroeconomic variables will capture a fairly large share of variation of the group's variables. However, a significant proportion of variations is not explained by its first factor. Therefore, the GFAVAR and SGFAVAR model may or may not be useful for the term structure forecasting.

B) Factor Estimate

In the first step of the estimation procedure, we extract the common factors from different approaches which firstly extract from a large panel of macroeconomic time series, secondly extract from groups of macroeconomic variables, and lastly extract from significant groups of macroeconomic variables. As the first four optimal factors from the FAVAR model can explain half of the total macroeconomic variation, applying four factors to all models would be reasonable to compare the results. Therefore, we restrict the number of factors to the first four principal components to all three models (FAVAR, GFAVAR and SGFAVAR).

Moreover**,** we apply the Bayesian Information Criterion (BIC) with a maximum lag of 12 to indicate an optimal number of lags for the VAR of factors and the short rate. This method will be applied to all three models for both in-sample fit and out-of-sample forecast the yield curves. Furthermore, for the out-of-sample forecast, the lag length of the model is re-estimated each time a forecast is made. The reason that we re-estimate the lag length every forecast period is that we would like to have the lag length that is more suitable for a period we make a forecast. **Tables 5-10** list the shares of variance explained by the first four factors from each model as well as the macroeconomic time series in the panel that strongly correlated with each factor. However, the factors estimated by principal components do not have a structural economic interpretation because they are just artificial factors that are linearly transformed in order to reduce the number of factors.

Tables 5-7 show the correlation of German's factors extracted from different methods and the associated time series of macroeconomic variables that are most correlated with the factors. **Table 5** shows the correlation of factors extracted from a large macroeconomic time series (FAVAR model) and the associated macroeconomic time series variable. The results show that the first factor highly correlates with a group of households which contain 46 macroeconomic time series. The group of households is considered as the third largest group of German macroeconomic time series dataset. Moreover, **Table 6** shows the correlation of German factors extracted from groups of macroeconomic variables (GFAVAR model) and the associated macroeconomic time series. We found that the first factor highly correlates with a group of import-export. This group contains only 18 macroeconomic time series. **Table 7** shows the correlation of the German factors extracted from significant groups of macroeconomic variables (SGFAVAR model) and the associated macroeconomic time series. We found that the first factor highly correlates with a group of output which is considered as the sixth largest group of macro time series dataset. This group contains 16 macroeconomic time series.

Tables 8-10 also show the correlation of the US's factors extracted from different methods and the associated time series of macroeconomic variables that are most correlated with the factors. **Table 8** shows the correlation of the first four factors extracted from a large panel of macroeconomic time series (FAVAR model) and the associated macroeconomic time series. We found that the first factor highly correlates with a group of capacity utilization which contains the largest macro time series for the US's dataset, 49 time series. On the other hand, **Table 9** shows the correlation of the first four factors extracted from groups of macroeconomic variables (GFAVAR model) and the associated macroeconomic time series variables. We found that the first factor highly correlates with a group of income payments which contains only 13 macroeconomic time series. Obviously, the time series that correlates with the first factor of the GFAVAR model are similar to those series that correlate with the first factor of the FAVAR model but are ranked in different order. For Germany, the time series that correlates with the first factor of FAVAR are not the same as the time series that correlates with the first factors of GFAVAR model like the results of US. Moreover, **Table 10** shows the correlation of the first four factors extracted from significant groups of macroeconomic variables (SGFAVAR model) and the macroeconomic time series. The first factor highly correlates with a group of import-export. We found that the macroeconomic time series containing in the first factor of SGFAVAR are totally different from the previous methods (FAVAR and GFAVAR).

Table 5: Correlations of FAVAR's factors on all individual German's macroeconomic time series

The first four FAVAR's factors sorted by their eigenvalue	correlation
Factor 1 (18.2680*% of Total Variance)	
Household: Total Loan (Private Household Stock Liability)	0.6602
Household: Total Long Term loan (Private Household Stock Liability)	0.6548
Household: Total Liability (Private Household Stock Liability)	0.6522
Monetary Financial Institute: Stock Financial Assets (Currency Gold & Special Draw)	0.5829
Monetary Financial Institute: Stock Financial Assets (Other Equity)	0.5414
Factor 2 (14.6069*% of Total Variance)	
Output: Production include Construction	0.8378
Output: Industry Production	0.8293
Output: Mining and Manufacturing	0.8286
Factor Income and Service: National (Gross Fixed Capital Formation)	0.8265
Output: Production exclude Construction	0.8245
Factor 3 (12.0547*% of Total Variance)	
Monetary Financial Institute: Stock Financial Assets (Mutual Share)	0.7737
Other Financial Intermediary: Stock Financial Assets (Share)	0.7028
Monetary Financial Institute: Stock Liability (Share)	0.6697
Household: Share (Household Stock Financial Assets)	0.6580
Other Financial Intermediary: Stock Liability (Mutual Share)	0.6550
Factor 4 (8.5048*% of Total Variance)	
Household: Total Claim Pension Commitment (Household Stock Financial Assets)	0.6908
Non Financial Corporation: Transaction External (Loan)	0.6718
Non Financial Corporation: Stock Liability (Claim Company Pension Commitment)	0.6443
Non Financial Corporation: Stock Financial Assets (Bond)	0.6011
Household: National (Private Consumption)	0.5853

* This eigenvalue represented the variance captured from 336 German's macroeconomic time series after transformation. Together, the first four factors explain about 53.4344% of the total variance of all variables in the dataset.

Table 6: Correlations of GFAVAR's factors on all individual German's macroeconomic time series

The first four GFAVAR's factors sorted by their eigenvalue	correlation
Factor 1 (25.5480% of Total Variance)**	
Import: Total	0.9531
Export: Total	0.9519
External Trade in Good Import	0.9443
External Trade in Good Export	0.9377
Foreign Exchange: NOK (Norway)	0.9067
Factor 2 (19.2805% of Total Variance)**	
Employment: Participant	0.5982
Employment: Short Time Worker	0.5982
Pay Rate: Pay All exclude Ancillary Benefit (hr)	0.5653
Household: Saving Deposit (Private Household Transaction Acquisition)	0.5301
Monetary Financial Institute Transaction External (Saving Deposit)	0.5276
Factor 3 (10.5250% of Total Variance)**	
Other Financial Intermediary: Transaction Acquisition (Currency & Deposit)	0.5677
Pay Rate: Pay Production exclude One-Off Payment (month)	0.5157
Pay Rate: Pay Production (month)	0.5092
Pay Rate: Pay Production exclude One-Off Payment (hr)	0.5087
Pay Rate: Pay Production (hr)	0.5062
Factor 4 (9.2795% of Total Variance)**	
Price Index: Other PI (Producer Price Industrial)	0.4848
Price Index: PPI	0.4815
Price Index: Other PI (Export Price)	0.4781
Household: National (Private Consumption)	0.4310
Employment: National (Labor Cast per Employee)	0.4291

** This eigenvalue represented the variance captured from 16 groups' factor representative of the German's macroeconomic variables. Together, the first four factors explain about 64.633% of the total variance of 16 groups of macroeconomic variables.

Table 7: Correlations of SGFAVAR's factors on all individual German's macroeconomics time series

The first four SGFAVAR's factors sorted by their eigenvalue	**correlation**
Factor 1 (31.4532*% of Total Variance)**	
Output: Mining and Manufacturing	0.8276
Output: Industry	0.8269
Output: Production exclude Construction	0.8256
Output: Capital Goods	0.8160
Output: Production include Construction	0.8086
Factor 2 (23.0463*% of Total Variance)**	
Monetary Aggregate: M2	0.8539
Monetary Aggregate: M3	0.6840
Household: Currency & Deposit (Household Stock Financial Assets)	0.6416
Monetary Aggregate: M1	0.6325
Household: Currency & Transaction Deposit (Household Stock Financial Assets)	0.5074
Factor 3 (18.5828*% of Total Variance)**	
Pay Rate: Pay Production exclude One-Off Payment (hr)	0.8733
Pay Rate: Pay Production exclude One-Off Payment (month)	0.8721
Pay Rate: Pay Production (hr)	0.8350
Pay Rate: Pay Production (month)	0.8313
Pay Rate: Pay All exclude Ancillary Benefit (month)	0.6158
Factor 4 (12.8646*% of Total Variance)**	
Household: Saving Certificate (Private Household Transaction Acquisition)	0.5202
Monetary Financial Institute: Transaction External (Saving Certificate)	0.5106
Monetary Financial institute Transaction External (Claim Company Pension Commitment)	0.4197
Household: Saving certificate (Household Stock Financial Assets)	0.3734
Monetary Financial Institute Stock Liability (Saving Certificate)	0.3697

*** This eigenvalue represented the variance captured from 6 significant groups (Group 1, 6, 7, 9, 10 and 13) whose factors have explanatory power to the short rate of the German macroeconomic time series. Together, the first four factors explain about 85.9469% of the total variance of 6 the groups of macroeconomic variables.

Table 8: Correlations of FAVAR's factors on all individual US's macroeconomic time series

The first four FAVAR's factors sorted by their eigenvalue	correlation
Factor 1 (21.9537*% of Total Variance)	
Capacity Utilization: Total ex. computers, communications eq.	0.9160
Capacity Utilization: Manufacturing ex. computers, communications	0.9017
Income Payment: Income Receipts	0.8627
Income Receipts on U.S. Assets Abroad	0.8625
Average Hourly Earnings: Total Private Industries	0.8485
Factor 2 (17.5620*% of Total Variance)	
All Employees: Nondurable Goods Manufacturing	0.7297
Industrial Production: Apparel and leather goods	0.7116
Employees on Nonfarm Payrolls: Manufacturing	0.6776
Imports of Goods and Services	0.6677
Exchange Rate: Nominal Broad Dollar Index	0.6509
Factor3 (6.1704*% of Total Variance)	
Currency Component of M1 Plus Demand Deposits	0.5322
Exchange Rate: DENMARK – KRONER	0.5173
Monetary Aggregate: M1 Money Stock	0.5157
Assets& Liability of Commercial Bank: Treasury and agency securities	0.5143
Monetary Aggregate: Total Checkable Deposits	0.5043
Factor4 (5.9333*% of Total Variance)	
U.S. Government Income Receipts on Assets Abroad	0.6664
Exchange Rate: SOUTH AFRICA - RAND	0.6208
Exchange Rate: INDIA – RUPEES	0.6099
Exchange Rate: SRI LANKA –RUPEES	0.6025
Imports of Services: Direct Defense Expenditures	0.5394

* This eigenvalue represented the variance captured from 273 US's macroeconomic time series after transformation. Together, the first four factors explain about 51.6194% of the total variance of all variables in the dataset.

Table 9: Correlations of GFAVAR's factors on all individual US's macroeconomic time series

The first four GFAVAR's factors sorted by their eigenvalue	correlation
Factor 1 (40.0533% of Total Variance)**	
Income Payment: Income Receipts	0.8975
Income Receipts on U.S. Assets Abroad	0.8974
Capacity Utilization: Manufacturing ex. computers, communications	0.8941
Capacity Utilization: Total ex. computers, communications eq.	0.8858
Average Hourly Earnings: Total Private Industries	0.8549
Factor 2 (19.8113% of Total Variance)**	
Imports of Goods and Services	0.7656
Exchange Rate: Nominal Broad Dollar Index	0.7491
Imports of Merchandise: Excluding Military	0.7412
All Employees: Nondurable Goods Manufacturing	0.7013
Imports of Goods, Services, and Income	0.6919
Factor3 (10.4182% of Total Variance)**	
Consumer Price Index for All Urban Consumers: Apparel	0.5573
Monetary Aggregate: Total Checkable Deposits	0.5352
Reserves of Depository Institutions, Required	0.5327
Monetary Aggregate: M1 Money Stock	0.4937
Monetary Aggregate: Other Checkable Deposits at Thrift Institutions	0.4189
Factor4 (6.8555% of Total Variance)**	
Capacity Utilization: Fabricated metal product	0.2978
Capacity Utilization: Durable manufacturing	0.2915
Capacity Utilization: Manufacturing	0.2888
Industrial Production: Machinery	0.2834
Exchange Rate: SWITZERLAND - FRANCS	0.2813

** This eigenvalue represented the variance captured from 14 groups' factor representative of the US macroeconomic variables. Together, the first four factors explain about 77.1383% of the total variance of 14 groups of macroeconomic variables.

Table 10: Correlations of SGFAVAR's factors on all individual US's macroeconomic time series

The first four SGFAVAR's factors sorted by their eigenvalue	correlation
Factor 1 (39.8633*% of Total Variance)**	
Imports of Goods, Services, and Income	0.7980
Monetary Aggregate: M1 Money Stock	0.7545
Imports of Merchandise: Excluding Military	0.7326
Monetary Aggregate: Total Checkable Deposits	0.7249
Imports of Goods and Services	0.7189
Factor 2 (22.3397*% of Total Variance)**	
Exchange Rate: NORWAY –KRONER	0.7106
Consumer Credit: Non-revolving Consumer Loans owned by Commercial Banks	0.6903
Exports of Services: Royalties and Licensing Fees	0.6841
Exchange Rate: DENMARK – KRONER	0.6564
Assets& Liability of Commercial Bank: Deposits, all commercial banks	0.6434
Factor 3 (13.9957*% of Total Variance)**	
Stock Return Index: CDAX Performance	0.6390
Stock Return Index: CDAX Price	0.6345
Stock Return Index: DAX Performance	0.6227
Stock Return Index: DAX Price	0.6176
Stock Return Index: CHINA Price	0.4444
Factor 4 (10.5272*% of Total Variance)**	
Assets& Liability of Commercial Bank: Loans and leases in bank credit, all commercial banks	0.3670
Capacity Utilization: Petroleum and coal products	0.3276
Producer Price Index: Crude Materials for Further Processing	0.3224
Assets& Liability of Commercial Bank: Other loans and leases, all commercial banks	0.3152
Producer Price Index: Crude Energy Materials	0.2939

*** This eigenvalue represented the variance captured from 6 significant groups (Group 1, 2, 4, 5, 9 and 13) whose factors have explanatory power to the short rate of the US. Together, the first four factors explain about 86.7259% of the total variance of 6 the groups of macroeconomic variables.

C) Preliminary Tests on the Role of Factors

Before estimating the term structure model, we run preliminary regressions in order to check whether the extracted macroeconomic factors are useful in a term structure model. Firstly, we apply a simple test to assess whether the factors extracted from each model provides a better fit than the output and inflation. Then we perform unrestricted regressions of yields on the model factors in order to explore the explanatory power for yields.

1) Do factors explain the short rate better than output and inflation?

There is an argument that the central banks normally base their monetary policy decisions on large macroeconomic information rather than using the output and inflation. Whether this argument holds true empirically, we will examine by comparing the fit of a policy rule based on output and inflation with a policy rule based on macroeconomic factors. The following equation is a policy rule based on the output and inflation.

$$r_t = c + \gamma_{CPI} CPI_t + \gamma_{GDP} GDP_t$$

where r_t denotes the short-term interest rate at time *t*,

c denotes the constant term,

γ_{CPI} and γ_{GDP} denote the coefficients of the CPI_t and GDP_t respectively,

CPI_t denotes the consumer price index at time *t*,

GDP_t denotes the gross domestic product at time *t*,

For the competitor model, we apply the a policy rule based on the four factors extracted from three different models which represent state variables in the No-Arbitrage FAVAR model, the GFAVAR model and the SGFAVAR model respectively.

$$r_t = c + \Psi'_F F_t$$

where r_t denotes the short-term interest rate at time *t*,

c denotes the constant term,

Ψ'_F denotes the coefficient of the common factors extracted from three different models,

F_t denotes the four macro factors extracted from different approaches (FAVAR, GFAVAR and SGFAVAR) at time *t*.

Table 11: Variation explained by factors and individual variables

Policy rule based	Germany	The USA
on the four factors extracted from FAVAR Model	49.808	67.158
on the four factors extracted from GFAVAR Model	47.322	65.086
on the four factors extracted from SGFAVAR Model	42.889	63.178
on output and inflation	42.412	61.408

This table reports the adjusted-R^2 of the estimation for policy rule based on the four factors extracted from different methods and the estimation for a policy rule based on output and inflation. The sample period for Germany is 1993:01 to 2008:12. For the US, the sample period is 1992:01 to 2008:12.

As indicated by the adjusted-R^2, **Table 11** shows that all factor-based policy rules fit the data slightly better than a standard Taylor-ruled based on output and inflation. Considering each factor-based equation of Germany, a policy rule based on the four factors of the FAVAR model fits the data slightly better than a policy rule based on the four factors of the GFAVAR model. Moreover, this result is also true for the US. Furthermore, a policy rule based on the SGFAVAR model also fits the data well but slightly poorer than the other two factor-based equations (FAVAR and GFAVAR) for both samples of US and Germany. This finding can be interpreted as evidence supporting the Fed that they commonly base their decision on a broad macroeconomic information rather than using output and inflation alone.

2) Unrestricted estimate of the term structure model

To further explore the question whether the factors from these three models have explanatory power for yields, we will apply a simple linear regression to test this question. The following **Table 12** provides estimates of an unrestricted regression of yields of different maturities onto a constant and the four macroeconomic factors from three different models.

$$Y_t = A + BF_t$$

where Y_t denotes the yields of different maturities at time *t*,

A denotes the constant term,

B and Π denote the coefficients of the common factors and the 1-month interest rate respectively,

F_t denotes the four common factors at time t extracted from different methods (FAVAR, GFAVAR and SGFAVAR) respectively.

Table 12: Variation of yields explained by factors extracted from different methods

	$\boldsymbol{y}^{(6)}$	$\boldsymbol{y}^{(12)}$	$\boldsymbol{y}^{(36)}$	$\boldsymbol{y}^{(60)}$	$\boldsymbol{y}^{(120)}$
Germany					
FAVAR Model	0.6807	0.6495	0.5618	0.5208	0.4716
GFAVAR Model	0.6405	0.6133	0.5189	0.4667	0.4140
SGFAVAR Model	0.6503	0.6214	0.4522	0.3951	0.3517
The USA					
FAVAR Model	0.7089	0.6910	0.6612	0.6493	0.6514
GFAVAR Model	0.6683	0.6473	0.6195	0.5696	0.5257
SGFAVAR Model	0.6976	0.6631	0.6430	0.5529	0.4779

This table summarizes the R^2 of an unrestricted regression of difference maturities yields on the four macro factors extracted from different methods.

Following **Table 12**, we found that the four factors extracted from the FAVAR model explain the variation of yields for all selected maturities better than the factors extracted from the other two alternative models for both samples of German and the U.S. Considering the two alternative models, we found that the four factors extracted from the SGFAVAR model explain the variation in the short yields of German and the U.S. ($\boldsymbol{y}^{(6)}$ and $\boldsymbol{y}^{(12)}$) better than the GFAVAR model.

For the variation in longer yields of German ($\boldsymbol{y}^{(36)}$, $\boldsymbol{y}^{(60)}$ and $\boldsymbol{y}^{(120)}$), the four factors from the FAVAR model explain on average 51.81% of the variation in the longer yields which is higher than the four factors from the GFAVAR model and the SGFAVAR model which can explain on average respectively, 46.65% and 39.96% of the variation. Moreover, the variation in longer yields of the U.S. ($\boldsymbol{y}^{(60)}$ and $\boldsymbol{y}^{(120)}$) also show the same pattern as the Germany's results that the four factors from the FAVAR model explain on average 65.03% of the variation in the longer yields which is also higher than the other two models whose four factors can explain on average only 54.76% and 51.40% for the GFAVAR and SGFAVAR models respectively.

Estimating the Term Structure Model

D) In-Sample fit

In this section, we report the in-sample fit of the term structure obtained from three different methods (FAVAR, GFAVAR and SGFAVAR) whose factors were extracted from a large macroeconomic data set. The factors in each model are used as the state variables for estimating the yield curves. The Germany's in-sample-fit's results are estimated from a period of January 1993 to December 2008. On the other hand, the in-sample-fit's period for the US started from January 1992 to December 2008. **Tables 13-16** report the mean and standard deviation of the five selected observed and model-implied yields of Germany and the USA.

Table 13: Mean of Germany's observed and model-implied yield for five selected interest rates following three different extracting models

		$y^{(1)}$	$y^{(6)}$	$y^{(12)}$	$y^{(36)}$	$y^{(120)}$
	Mean					
FAVAR Model	y_t	3.678	3.752	3.828	4.053	5.011
	$\hat{y}_t$	3.678	3.772	3.784	4.060	5.008
	$\lvert y_t - \hat{y}_t\rvert$	0.000	0.137	0.226	0.372	0.521
GFAVAR Model	y_t	3.678	3.752	3.828	4.053	5.011
	$\hat{y}_t$	3.678	3.770	3.803	4.032	5.002
	$\lvert y_t - \hat{y}_t\rvert$	0.000	0.169	0.249	0.469	0.694
SGFAVAR Model	y_t	3.678	3.752	3.828	4.053	5.011
	$\hat{y}_t$	3.678	3.687	3.785	4.059	4.996
	$\lvert y_t - \hat{y}_t\rvert$	0.000	0.170	0.288	0.490	0.704

This table summarizes means of Germany's observed and fitted yields. Yields are reported in percentage terms. The first and second row in each panel report the mean of observed yield and fitted values under different models while the third row shows the mean of absolute fitting errors.

For the in-sample-fit's results of German, **Table 13** presents that the FAVAR model whose factors directly extracted from a large panel of macroeconomic time series fits the data better than the other two models whose factors extracted from a group of macroeconomic variables (GFAVAR) and a significant group of macro variables (SGFAVAR) for all selected maturities ($y^{(1)}$, $y^{(6)}$, $y^{(12)}$, $y^{(36)}$ and $y^{(120)}$). For the results of the USA, **Table 14** shows that the GFAVAR model fits the data well in the short and medium of the curves ($y^{(1)}$, $y^{(6)}$, $y^{(12)}$ and $y^{(36)}$) whereas the long end of the curves is dominated by the FAVAR model.

Table 14: Mean of the US observed and model-implied yield for five selected interest rates following three different extracting models

		$y^{(1)}$	$y^{(6)}$	$y^{(12)}$	$y^{(36)}$	$y^{(120)}$
	Mean					
FAVAR Model	y_t	3.682	3.938	4.086	4.620	5.386
	$\hat{y}_t$	3.682	3.915	4.094	4.635	5.382
	$\|y_t - \hat{y}_t\|$	0.000	0.239	0.294	0.413	0.416
GFAVAR Model	y_t	3.682	3.938	4.086	4.620	5.386
	$\hat{y}_t$	3.682	3.928	4.082	4.631	5.392
	$\|y_t - \hat{y}_t\|$	0.000	0.218	0.275	0.402	0.544
SGFAVAR Model	y_t	3.682	3.938	4.086	4.620	5.386
	$\hat{y}_t$	3.682	3.893	4.095	4.651	5.378
	$\|y_t - \hat{y}_t\|$	0.000	0.253	0.330	0.524	0.661

This table summarizes means of the US observed and fitted yields. Yields are reported in percentage terms. The first and second rows in each panel report the mean of observed yield and fitted values under different models while the third row shows the mean of absolute fitting errors.

Moreover, consider the performance of the two alternative approaches, we found that the GFAVAR model whose factors extracted from groups of macroeconomic variables fits slightly better than the model whose factors extracted from groups of significant variables (the SGFAVAR model). Obviously, the No-Arbitrage FAVAR model whose factors directly extracted from a large panel of macroeconomic time series provides a good fit to the long end of the yield curves for both the US and German yields.

Table 15 and Table 16 also show that these three models cannot capture some of the variation in longer maturities as we found that the standard deviations of fitted interest rates are lower than the standard deviations of the observed yields at the long end of the curve for both samples. This can be seen in **Figure 4 and Figure 5** which plot the time series for a selection of observed and model-implied yields of both Germany and the United State of America.

Table 15: Standard Deviation of Germany's observed and model-implied yield for five selected interest rates following three different extracting models

		$y^{(1)}$	$y^{(6)}$	$y^{(12)}$	$y^{(36)}$	$y^{(120)}$
	Standard Deviation					
FAVAR Model	y_t	1.312	1.226	1.155	1.061	1.126
	$\hat{y}_t$	1.312	1.193	1.094	0.962	0.865
	$\lvert y_t - \hat{y}_t\rvert$	0.000	0.109	0.176	0.282	0.369
GFAVAR Model	y_t	1.312	1.226	1.155	1.061	1.126
	$\hat{y}_t$	1.312	1.184	1.079	0.929	0.640
	$\lvert y_t - \hat{y}_t\rvert$	0.000	0.131	0.186	0.356	0.481
SGFAVAR Model	y_t	1.312	1.226	1.155	1.061	1.126
	$\hat{y}_t$	1.312	1.181	1.058	0.896	0.585
	$\lvert y_t - \hat{y}_t\rvert$	0.000	0.142	0.188	0.378	0.511

This table summarizes standard deviations of Germany's observed and fitted yields. Yields are also reported in percentage terms. The first and second row in each panel report the standard deviation of observed yield and fitted values under different models while the third row shows the standard deviation of absolute fitting errors.

Table 16: Standard Deviation of the US observed and model-implied yield for five selected interest rates following three different extracting models

		$y^{(1)}$	$y^{(6)}$	$y^{(12)}$	$y^{(36)}$	$y^{(120)}$
	Standard Deviation					
FAVAR Model	y_t	1.556	1.645	1.610	1.468	1.123
	$\hat{y}_t$	1.556	1.554	1.525	1.369	0.895
	$\lvert y_t - \hat{y}_t\rvert$	0.000	0.220	0.249	0.349	0.385
GFAVAR Model	y_t	1.556	1.645	1.610	1.468	1.123
	$\hat{y}_t$	1.556	1.647	1.541	1.376	0.834
	$\lvert y_t - \hat{y}_t\rvert$	0.000	0.187	0.246	0.333	0.440
SGFAVAR Model	y_t	1.556	1.645	1.610	1.468	1.123
	$\hat{y}_t$	1.556	1.662	1.486	1.317	0.618
	$\lvert y_t - \hat{y}_t\rvert$	0.000	0.283	0.320	0.416	0.485

This table summarizes standard deviations of the US observed and fitted yields. Yields are also reported in percentage terms. The first and second row in each panel report the standard deviation of observed yield and fitted values under different models while the third row shows the standard deviation of absolute fitting errors.

Figure 4: Observed and Model-implied yield of Germany. This figure provides plots of observed and model-implied time series for four selected interest rates, the 6-month yield, the 12-month yield and the 3-and 10-year yields.

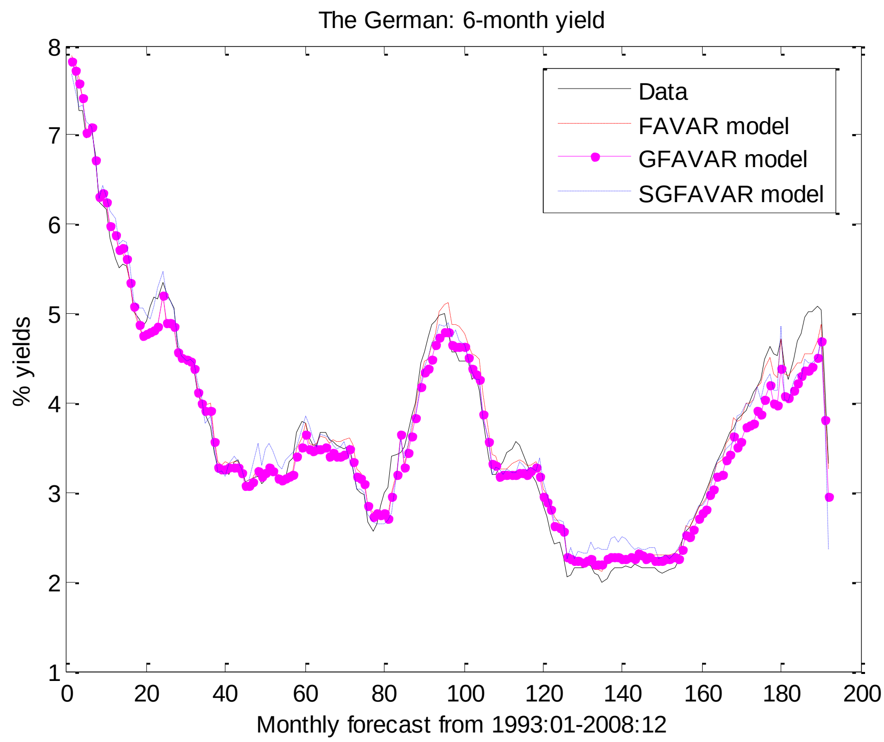


Figure 4a

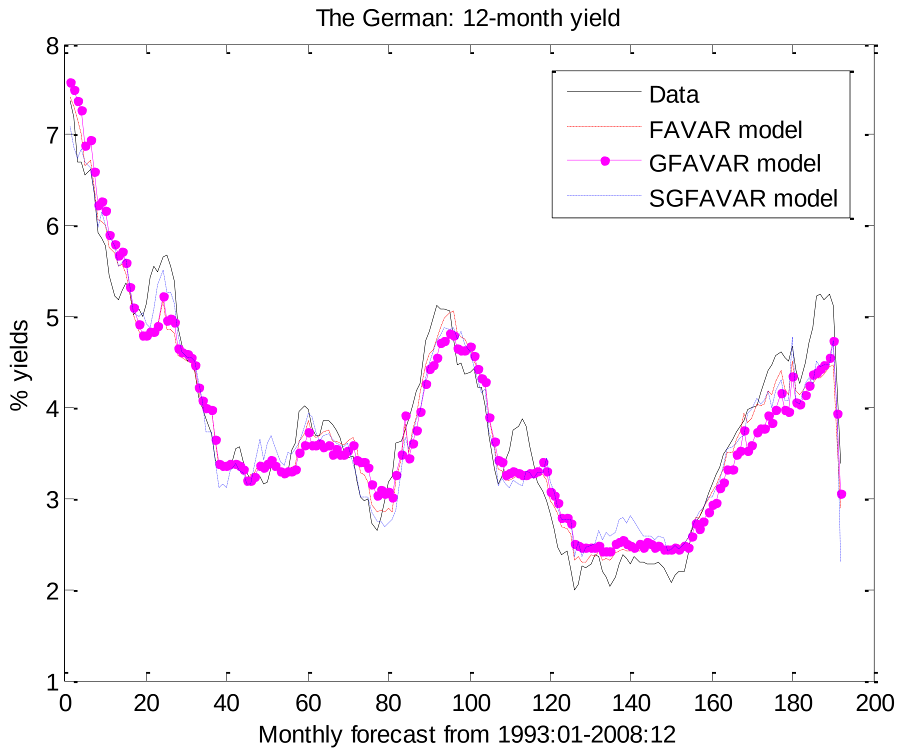


Figure 4b

Figure 4: Observed and Model-implied yield of Germany. This figure provides plots of observed and model-implied time series for four selected interest rates, the 6-month yield, the 12-month yield and the 3-and 10-year yields.

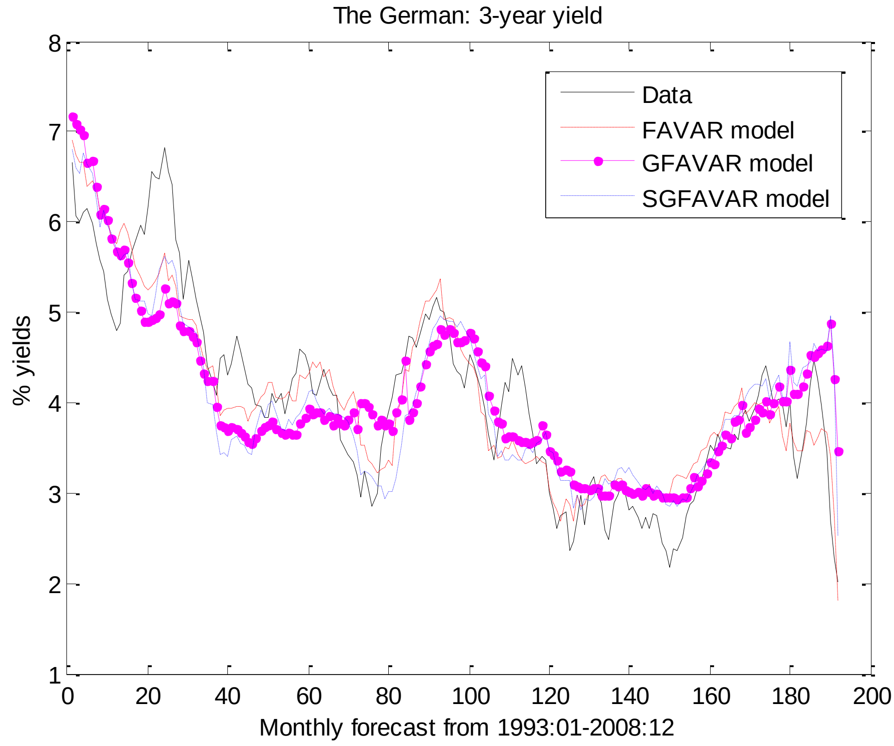


Figure 4c

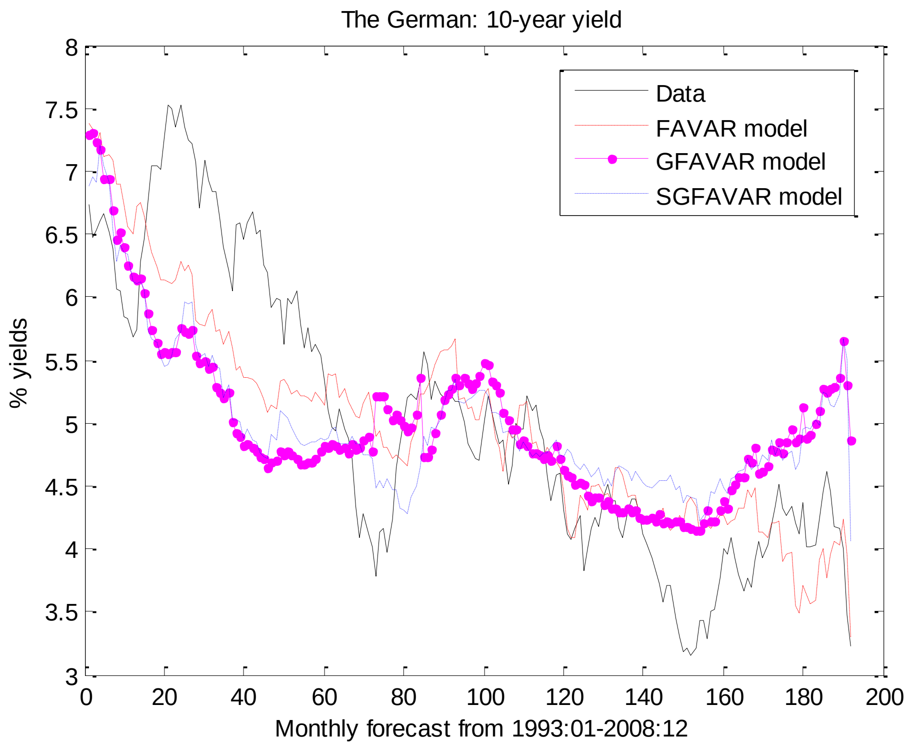


Figure 4d

Figure 5: Observed and Model-implied yields of The United States of America. This figure provides plots of observed and model-implied time series for four selected interest rates, the 6-month yield, the 12-month yield and 3 and 10 year yields.

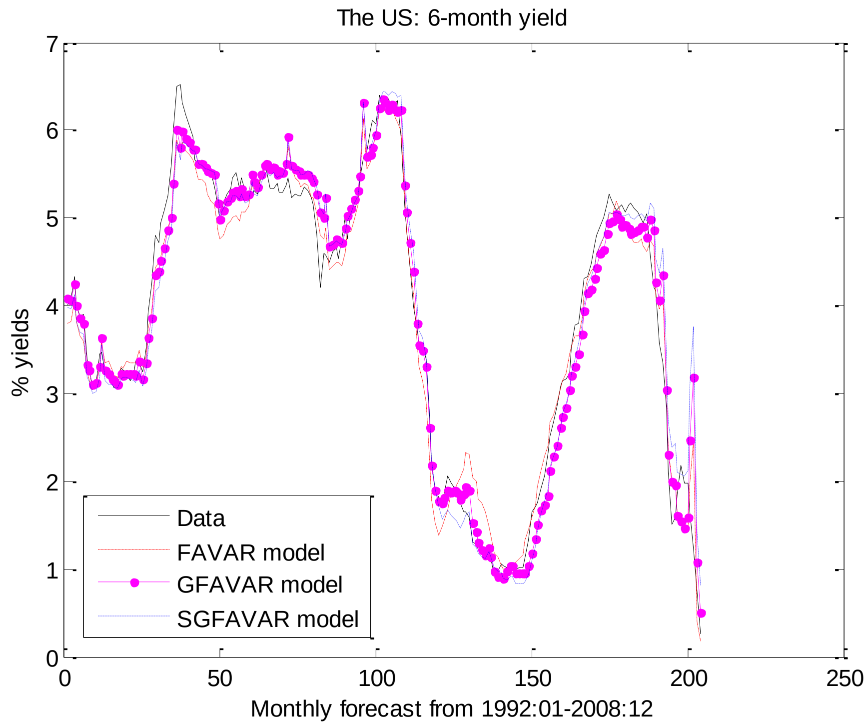


Figure 5a

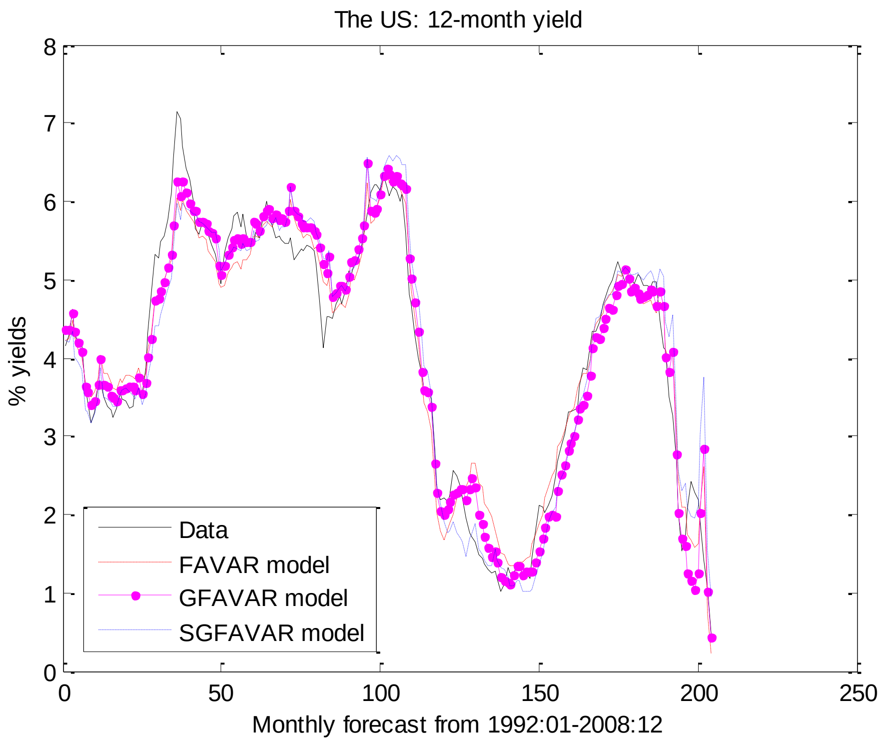


Figure 5b

Figure 5: Observed and Model-implied yields of The United States of America. This figure provides plots of observed and model-implied time series for four selected interest rates, the 6-month yield, the 12-month yield and 3 and 10 year yields.

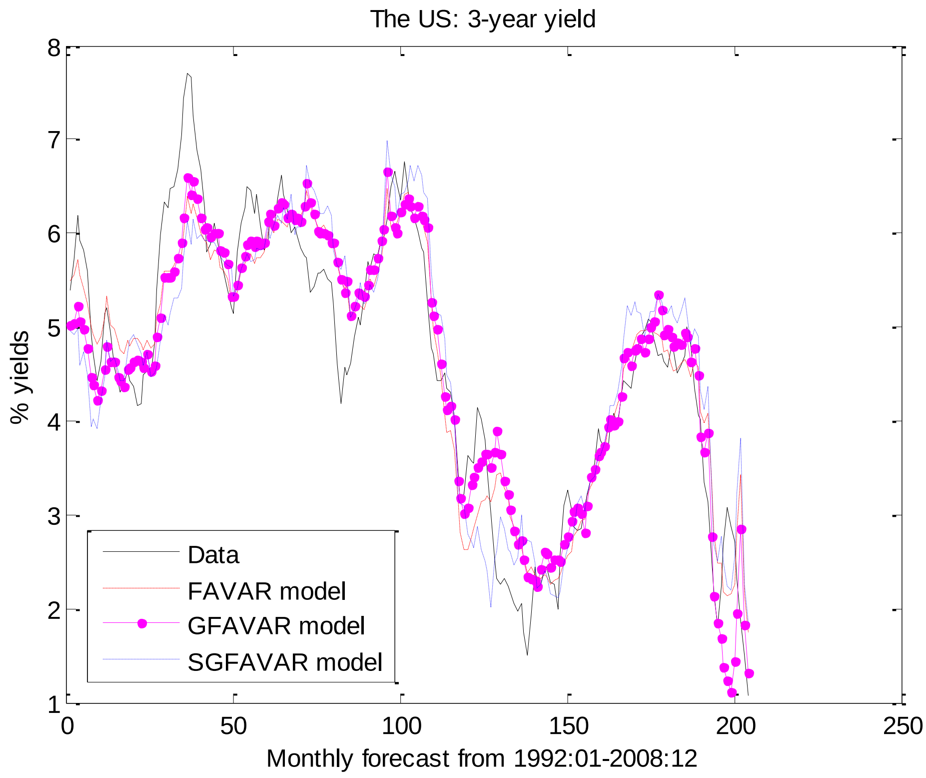


Figure 5c

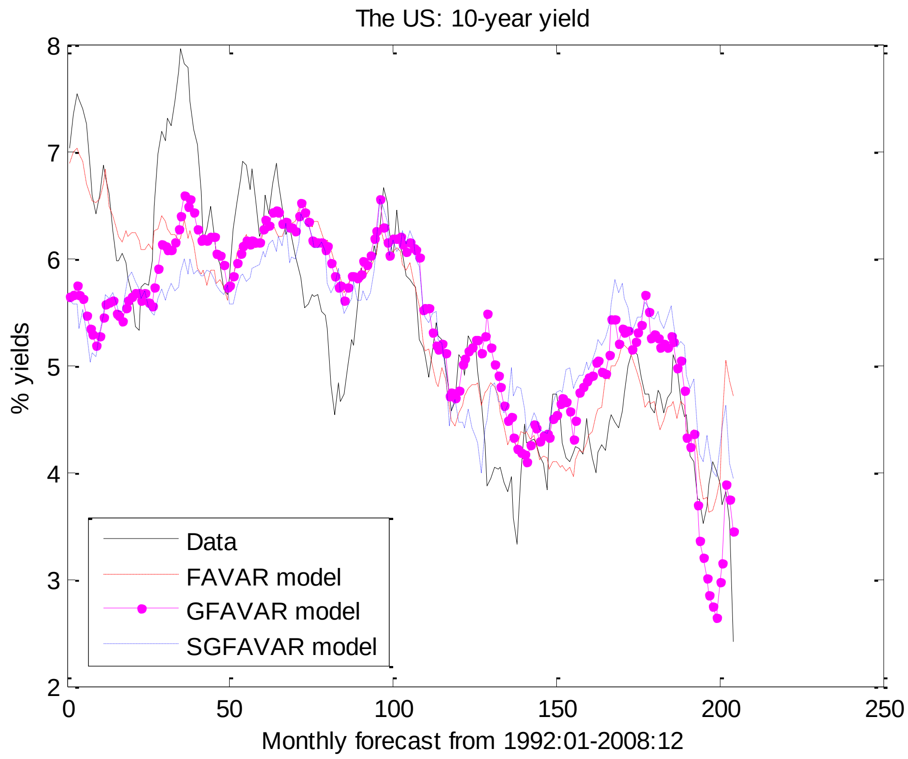


Figure 5d

Out of Sample Forecast

1) The Behavior of Factors

To study the behavior of factors extracted from the three different models, we calculate the correlation between the forecasted factors and the group factor representatives. These values will tell us about the group factors representatives that they are mostly correlated with. **Figure 6** show the correlation of the forecasted factors and the group factor representative that they are mostly correlated with during the forecast period of 2003:01-2008:12.

For Germany's sample, these figures show that the first factor of the FAVAR model, whose factors are directly extracted from a large panel of German macroeconomics time series, highly correlate with a group of "factor income & service" for the first fifty months of the forecasting period, 2003:01-2007:02. Moreover, for the period of 2007:03-2008:12, the first factor of the FAVAR model changes to correlate with a group of "monetary base" the most. Therefore, the first factors of the FAVAR model changes only once along the forecast period of 72 months. On the other hand, the first factor of the GFAVAR model whose factors are extracted from groups of German macroeconomic variables and the first factor of the SGFAVAR model whose factors are extracted from significant groups of German macroeconomic variables correlate most with several groups in the dataset during the forecast period. Therefore, we can conclude that the factors of the FAVAR model which are directly extracted from a large German macroeconomic data set consistently rely on a particular group than the other two alternative models whose factors extracted from a group of German macroeconomic variables, GFAVAR, and a significant group of German macro variables, SGFAVAR,.

Similarly, the first factor of the FAVAR model whose factors are directly extracted from a large number of the US macroeconomic time series highly correlates with a group of "capacity utilization" for the first thirty months, 2003:01-2005:06, and changes to correlate with a group of "income payment and receipts" for the next twenty five months, 2005:07-2007:07. Then it turns back to correlate with the group of "capacity utilization" again until the last month of the forecast period (2007:08-2008:12). Therefore, the first factor of the FAVAR model changes two times. On the other hand, for the first factors of the GFAVAR and SGFAVAR model, they correlate with a number of groups of the US macroeconomic variables along the forecast period, 2003:01-2008:12. The first factor of each alternative model (GFAVAR and SGFAVAR model) correlates with at least three different groups during the total forecasted periods, 72 months.

Figure 6: The variation of the forecasted factors on groups of macroeconomic variables for the forecast period 2003:01-2008:12 which is measured by the maximum correlation of model factors on the groups of macroeconomic variables in each country**.**

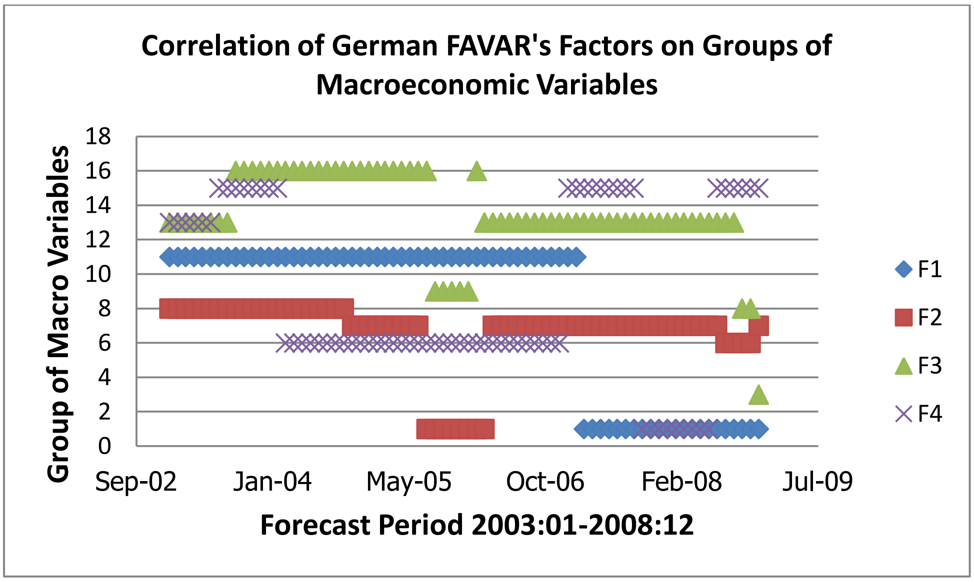


Figure 6a: *Plot of the correlation of forecasted FAVAR's factors on groups of German macroeconomic variables during the forecast period*

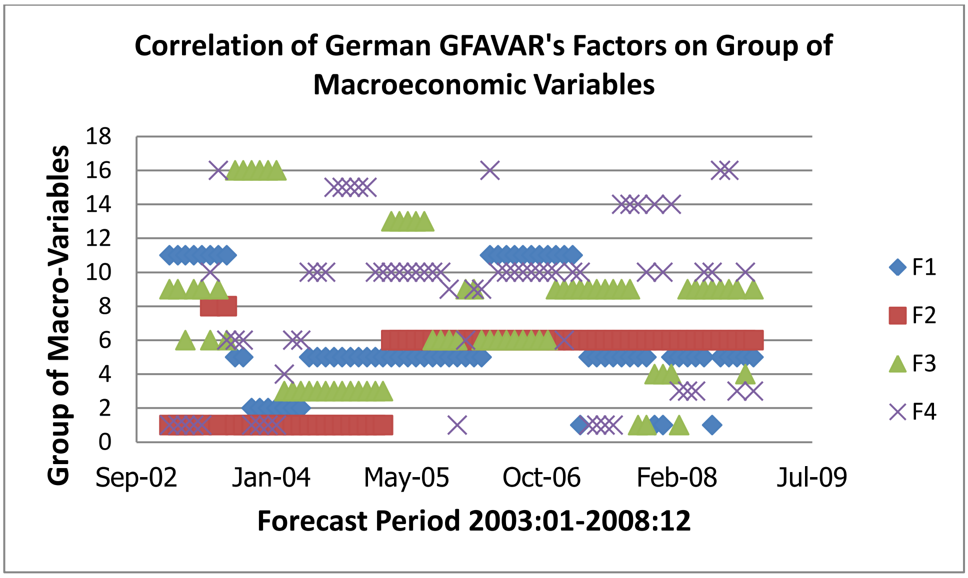


Figure 6b: *Plot of the correlation of forecasted GFAVAR's factors on group of German macroeconomic variables during the forecast period*

Figure 6: The variation of the forecasted factors on groups of macroeconomic variables for the forecast period 2003:01-2008:12 which is measured by the maximum correlation of model factors on the groups of macroeconomic variables in each country**.**

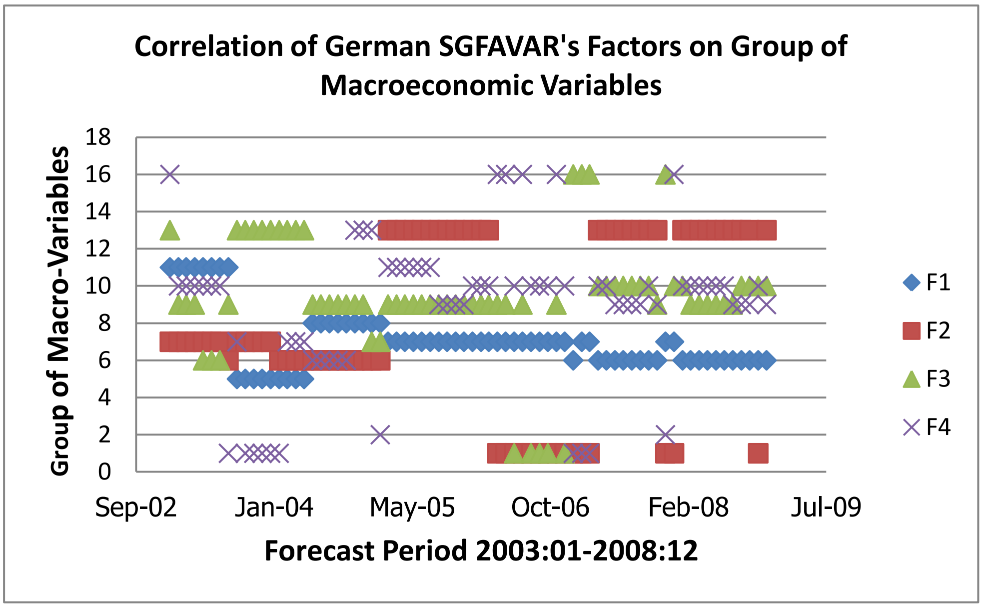


Figure 6c: *Plot of the Correlation of forecasted SGFAVAR's factors on groups of German macroeconomic variables during the forecast period*

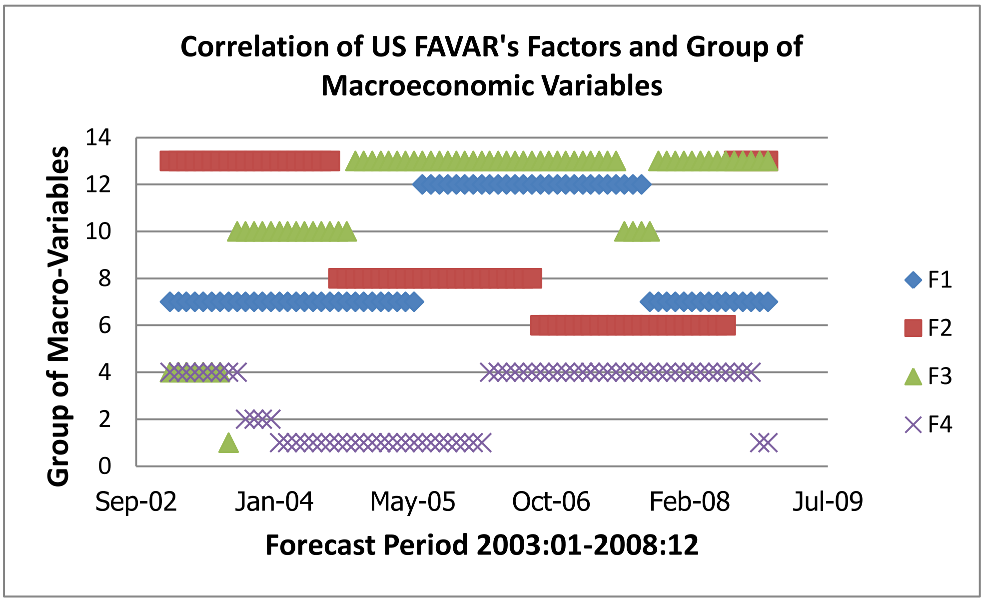


Figure 6d: *Plot of the correlation of forecasted FAVAR's factors on groups of US macroeconomic variables during the forecast period*

Figure 6: The variation of the forecasted factors on groups of macroeconomic variables for the forecast period 2003:01-2008:12 which is measured by the maximum correlation of model factors on the groups of macroeconomic variables in each country.

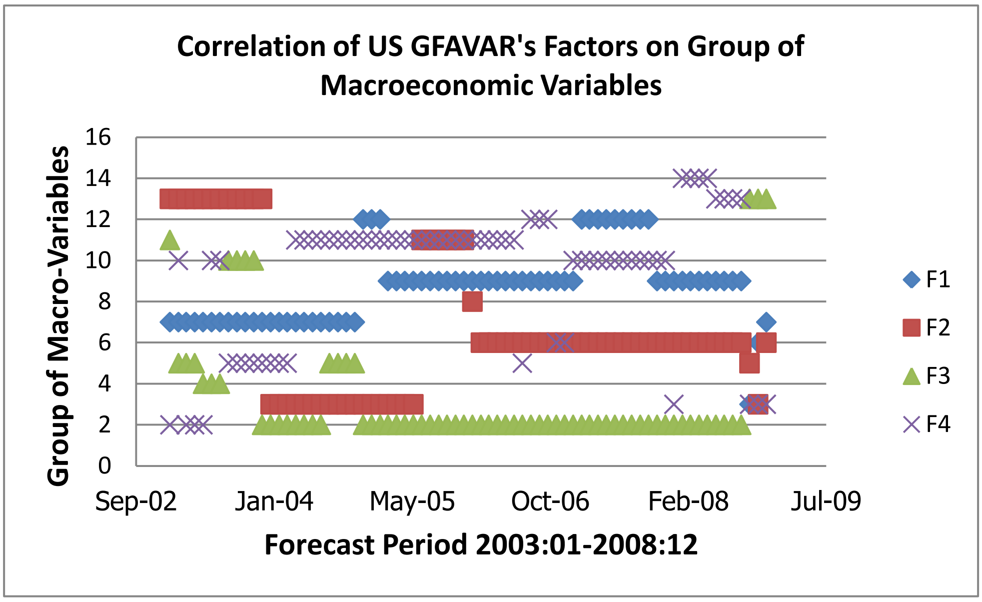


Figure 6d: *Plot of the Correlation of forecasted GFAVAR's factors on groups of US macroeconomic variables during the forecast period*

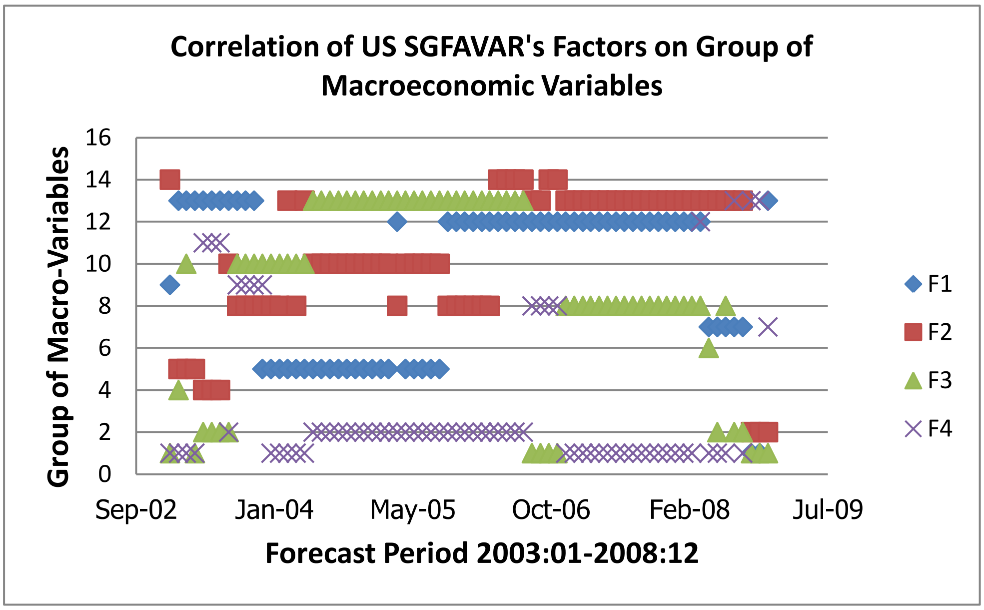


Figure 6f: *Plot of the Correlation of forecasted SGFAVAR's factors on groups of US macroeconomic variables during the forecast period*

We can imply that the factors of the FAVAR model have a small chance of changing the group during the forecast periods. As the FAVAR's factors commonly capture the variation of total macroeconomic time series, a small change in time series data may not affect the power of factors in order to correlate with the other group factor representatives. So, the effect of changing a time series is not captured by the FAVAR's factors. On the other hand, GFAVAR's factors and SGFAVAR's factors whose factors extracted from group factor representatives and significant group factor representatives will have a large chance to correlate with several groups because the effect of changing a time series is sensitively captured by the factors. Therefore, the factors with extracting constraints do not rely on a particular group of macroeconomic variables. They can correlate with several groups of macroeconomic variables. According to this result, we can conclude that the methods used to extract the common factors from a group of macroeconomic variables provide an equal weight to each variable and can be equally selected as the common factors in the term structure.

These findings can be summarized that the factors of the FAVAR model basically capture the variation of a large number of time series, 341 time series for the US and 359 time series for Germany, so that they relatively have a small chance to change to the other groups of variables compared to the two alternative models, GFAVAR and SGFAVAR, when the new data came. In contrast, the factors of the GFAVAR and factors of the SGFAVAR capture the variation of a small number of time series, 14 groups for the US and 16 groups for Germany, compared to the factors of the FAVAR. Therefore, their factors can easily change to correlate with the other groups once the new data comes. Moreover, the GFAVAR's factors and the SGFAVAR's factors do not rely much on a particular group of variables. The groups that have a small number of macroeconomic time series (small weighting) will have an equal chance to be selected as the state variables used to forecast the yield curve.

2) Out of Sample Forecast Results

In the previous section, it has been shown that each model provides a good in-sample fit to both German and the U.S. yields data. In this section, we will study the forecast performance of the No-Arbitrage FAVAR model and the other two alternative models which are the GFAVAR model and the SGFAVAR model. To forecast the yields curves, we follow the model-implied forecasts of the No-Arbitrage FAVAR model which is obtained as follows.

$$\hat{Z}_{(t+h|t)} = \widehat{\boldsymbol{\phi}}^{\boldsymbol{h}} Z_{\boldsymbol{t}} + \sum_{i=0}^{h-1} \widehat{\boldsymbol{\phi}}^{i}\, \widehat{\boldsymbol{\mu}} \tag{8}$$

$$\hat{y}^{(n)}_{(t+h|t)} = \hat{a}_n + \hat{b}'_n \hat{Z}_{(t+h|t)} \tag{9}$$

where $\hat{y}^{(n)}_{(t+h|t)}$ denotes the h month ahead forecast of an n-maturity bond yield at time t,

$\widehat{Z}_{(t+h|t)}$ denotes the h month ahead forecast of the state variables at time t,

$Z_t = (F'_t, r_t, F'_{t-1}, r_{t-1}, \ldots, F'_{t-p+1}, r_{t-p+1})'$

$\hat{a}_n$ and $\hat{b}'_n$ denote the estimated constant term and the coefficient of the h-month ahead forecast of the state variables at time t

$\widehat{\boldsymbol{\phi}}$ and $\widehat{\boldsymbol{\mu}}$ are the parameters estimated from the VAR model on the states equation.

To forecast the yields curves in this study, we start from estimating the h month ahead of the state variables ($\widehat{Z}_{(t+h|t=2002:12)}$) at time t = 2002:12. To estimate $\widehat{Z}_{(t+h|t=2002:12)}$, we need to estimate three input parameters which are the factors F_t and the other two parameters $\widehat{\boldsymbol{\phi}}$ and $\widehat{\boldsymbol{\mu}}$. These variables are estimated from a period of 1993:01-2002:12 for German's sample and from a period of 1992:01-2002:12 for the US's sample. As we have already demonstrated in the methodology section, the factors F_t are the macro factors extracted from different approaches (FAVAR, GFAVAR and SGFAVAR). Once we have the estimated factors F_t, we can use them and the short rate as the state variable in a VAR model in order to estimate the VAR's parameters $\widehat{\boldsymbol{\phi}}$ and $\widehat{\boldsymbol{\mu}}$. Now we can estimate $\widehat{Z}_{(t+h|t=2002:12)}$ based on equation (8). Once we have the estimated $\widehat{Z}_{(t+h|t=2002:12)}$, we can use it in equation (9) to forecast the n-maturity bond yield in the next h month from t = 2002:12.

For the next period of the forecast, $t+1$= 2003:01, we will re-estimate the variable ($\widehat{Z}_{(t+h|t=2003:01)}$) in order to have the appropriate factors for the forecasting period. Now we already have the variable $\hat{Z}_{(t+h|t=2003:01)}$ to be used in the equation (9) to forecast the n-maturity bond yield in the next h month from t = 2003:01. We will apply this procedure to all three models (FAVAR, GFAVAR and SGFAVAR model) respectively.

For Germany, the out-of-sample forecasts are carried out over the time interval 2003:01–2008:12 so the starting values for the parameters are estimated from the period of 1993:01–2002:12. Therefore, the forecast sample covers a period of six years. We first estimate the model which factors are directly extracted from a large panel of German macroeconomics time series, the FAVAR model. Then, we estimate the model which factors are extracted from groups of macroeconomic variables, the GFAVAR model, and lastly the model which factors are extracted from the groups of macroeconomic variables significantly explain the short rate, the SGFAVAR model, respectively.

Similarly, the US's out-of-sample forecast results are then estimated following the same sequence as German. The US's out-of-sample forecasts are carried out over the time interval 2003:01–2008:12 which cover a period of six years. The period of 1992:01–2002:12 are used to estimate the starting values for the parameters.

Table 17 and Table 18 summarize the root mean squared errors obtained from these forecasts. According to these tables, we obviously see that the FAVAR, GFAVAR and SGFAVAR model outperform the random walk model for most maturities in forecasting 6-month and 12-month ahead of the forecast. This can be implied that the use of macroeconomic information when forecasting the yield curves will improve the forecast performance in an intermediate and long forecast horizon.

Three main observations can be made. Firstly, at 1 month ahead of the forecast horizon, the random walk model outperforms the three macroeconomic-based VAR models for yield of all maturities. **Figure 7-12** provide the plot of the forecasted yields from different models including the random walk. From these figures, we can obviously see that when the yield curves have a small change in yields, the yield curves predicted from the random walk model is close to the true value. As the no-change forecast of the individual yields is the main assumption of the random walk, the period where yield curves have a small change is outperformed by the random walk model.

In the absence of the random walk model, we found that the FAVAR model whose factors are directly extracted from a large panel of macroeconomic time series outperforms the others for all yields forecasted. Moreover, the GFAVAR model whose factors are extracted from groups of macroeconomic variables performs better than the SGFAVAR model whose factors are extracted from groups of macroeconomic variables that significantly explain the short rate. These results are also consistent to both sample of US and Germany.

Table 17: German's Out-of-sample RMSE – Forecast Period 2003:01-2008-12

$y^{(n)}$	FAVAR	GFAVAR	SGFAVAR	Random Walk
1 month ahead forecast				
1	0.2530	0.2821	0.3026	**0.1925**
6	0.2769	0.3007	0.3229	**0.1780**
12	0.3072	0.3292	0.3539	**0.1883**
36	0.3466	0.3824	0.4014	**0.2423**
60	0.3964	0.4101	0.4314	**0.2162**
120	0.4101	0.4457	0.4618	**0.1691**
6 month ahead forecast				
1	0.3935	**0.3699**	0.4104	0.4246
6	**0.4156**	0.4273	0.4501	0.4643
12	**0.4340**	0.4523	0.4726	0.4975
36	0.5083	**0.4878**	0.5227	0.6037
60	0.5342	0.5360	0.5504	**0.5271**
120	0.5780	0.5593	0.5949	**0.4105**
12 month ahead forecast				
1	**0.5410**	0.5818	0.5602	0.7092
6	0.6135	0.6327	**0.5998**	0.7448
12	0.6754	0.6878	**0.6490**	0.7391
36	0.8310	0.8559	0.8090	**0.6836**
60	0.9033	0.9196	0.8793	**0.6069**
120	1.0171	1.2754	0.9821	**0.5277**

This table summarizes the German's root mean squared errors obtained from out-of-sample yield forecasts. The models were estimated using data from 1993:01 until the period when the forecast is made. The forecasting period is 2003:01-2008:12.

Secondly, at 6-months ahead of forecast, **Table 17** shows that the GFAVAR model dominates the FAVAR model for forecasting the 1-month and 3-year yields. However, the FAVAR model outperforms the two alternative models and the random walk model in forecasting the 6-month and 12-month yields. For the long term yields 60-month and 120-month yields, the random walk model dominates all the macro-based FAVAR models. This result is also consistent with **Table 18** that shows the RMSE's results of the US's yields forecasted from different models. The FAVAR model outperforms in forecasting the 6-month yield. Moreover, the GFAVAR model performs better than the others in forecasting the 1-month, 12-month and 3-year yields. For the longer yields forecast, 5-year and 10-year yields, the random walk model still outperforms all the macro-based FAVAR models. Without the random walk, the GFAVAR model performs the best in forecasting the longer term yields.

We can imply that the FAVAR model which factors are directly extracted from a large panel of macroeconomic time series performs better than the others in forecasting the short term yields. On the other hand, the GFAVAR model which factors are extracted from groups of macroeconomic variables performs better than others in forecasting the intermediate yields.

Table 18: The U.S.'s Out-of-sample RMSE – Forecast Period 2003:01-2008-12

$y^{(n)}$	FAVAR	GFAVAR	SGFAVAR	Random Walk
1 month ahead forecast				
1	0.3914	0.4176	0.4307	**0.3893**
6	0.4386	0.4599	0.4705	**0.2317**
12	0.4858	0.4999	0.5217	**0.2355**
36	0.5057	0.5285	0.5402	**0.2689**
60	0.5304	0.5499	0.5686	**0.2617**
120	0.5699	0.5520	0.5801	**0.2441**
6 month ahead forecast				
1	0.6786	**0.6573**	0.7098	0.9676
6	**0.6987**	0.7226	0.7529	0.9760
12	0.7644	**0.7426**	0.7937	0.9224
36	0.8013	**0.7887**	0.8299	0.8181
60	0.8389	0.8097	0.8502	**0.6765**
120	0.8959	0.8300	0.8699	**0.5017**
12 month ahead forecast				
1	**0.9899**	1.1124	1.0649	1.6685
6	**1.0719**	1.1877	1.1256	1.7811
12	1.1911	1.2372	**1.1502**	1.6384
36	1.2282	1.2799	**1.1920**	1.2520
60	1.2675	1.3184	1.2317	**0.9501**
120	1.3169	1.3597	1.2715	**0.6124**

This table summarizes the U.S.'s root mean squared errors obtained from out-of-sample yield forecasts. The models were estimated using data from 1992:01 until the period when the forecast is made. The forecasting period is 2003:01-2008:12.

Thirdly, at 12-months ahead of forecast, **Table 17** shows that the FAVAR model performs the best in forecasting the 1-month yield. On the other hand, the

SGFAVAR model dominates the FAVAR model and the GFAVAR model in forecasting the 6-month and 12-month yields. Moreover, the SGFAVAR model is also better than the random walk model for the 6-month and 12-month yields forecast. Therefore, we can conclude that the factors extracted following the SGFAVAR model outperform in forecasting the intermediate yields. **Table 18** also shows that the US's factors from FAVAR model dominate the others in forecasting the 1-month and 6-month yields whereas the US's factors from SGFAVAR model outperform in forecasting the 1-year and 3-year yields. This result is also consistent with Germany that is an evidence support the result of the SGFAVAR in forecasting the intermediate yields. Moreover, the random walk model still outperforms in forecasting the long term yields for both countries. Without the random walk, the SGFAVAR model whose factors extracted from a significant group of macroeconomic variables provides the best forecast performance of the long term yields (5-year and 10-year yields).

According to **Table 17** and **Table 18**, the results are rather consistent to both the US and Germany that the FAVAR model provides a better forecast performance in the short term yields for both 6-month and 12-month forecast horizons. While the GFAVAR model provides a better forecast performance in the intermediate yields for the 6-month forecast horizon. Moreover, the SGFAVAR model provides a better forecast performance in the intermediate yields for the 12-month forecast horizon. Therefore, we can imply that, for the short and intermediate yields, the three macro-based FAVAR models have smaller out of sample root mean square forecast error than a benchmark, random walk model. For the longer term yields 60-month and 120-month yields, the random walk model always outperforms the three macro-based FAVAR models. Moreover, at 1 month forecast horizon, the random walk model always outperforms the three macro-based FAVAR models for both samples of the US and Germany.

Even though the forecast results of this study cannot tell exactly whether which models perform the best as their root mean square forecast error are relatively close to each other. However, we can directly imply that the constraints imposed to the extracting method for selecting the common factors may eliminate some of the information that best describe the short term yield but they well describe the intermediate term yields instead.

Figure 7: **Observed and Predicted yields 1 month ahead of Germany. This figure provides plots of observed and 1-month ahead prediction time series of the 1-month, 12 month, and the 3- and 10-year maturities.**

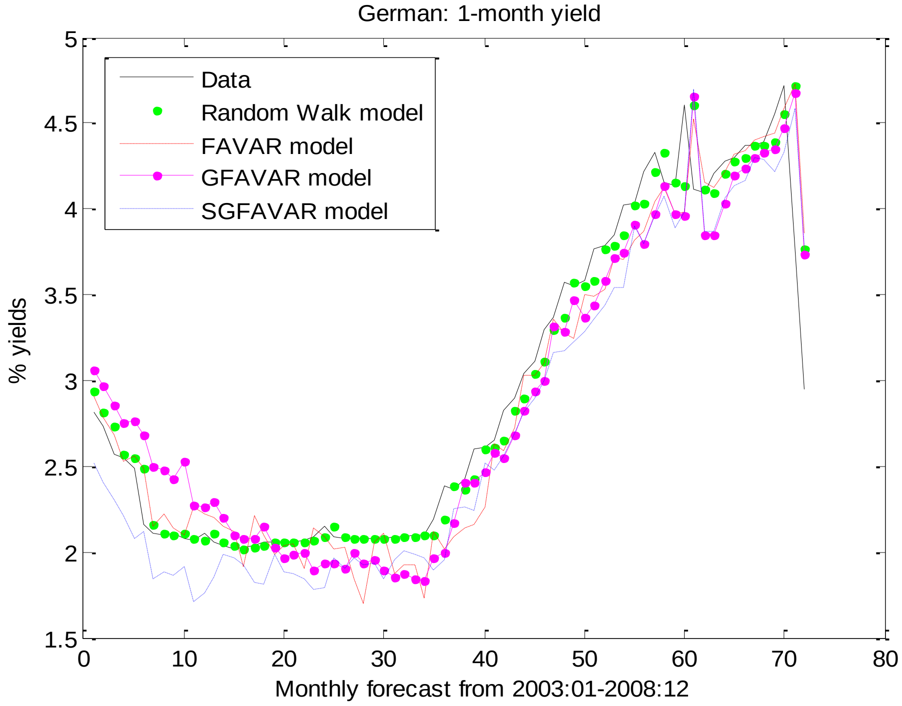


Figure 7a

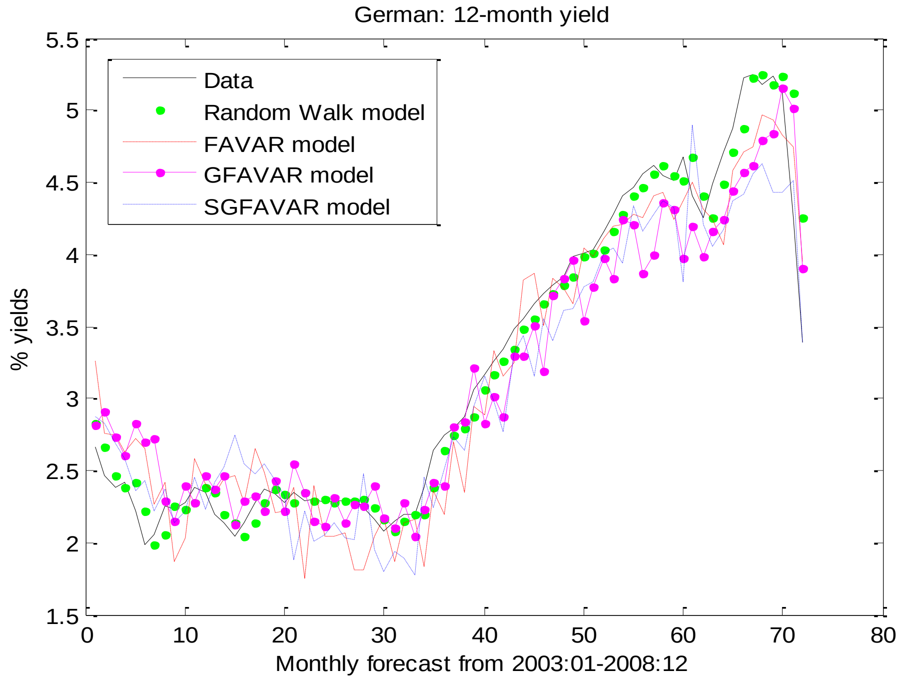


Figure 7b

Figure 7: Observed and Predicted yields 1 month ahead of Germany. This figure provides plots of observed and 1-month ahead prediction time series of the 1-month, 12 month, and the 3- and 10-year maturities.

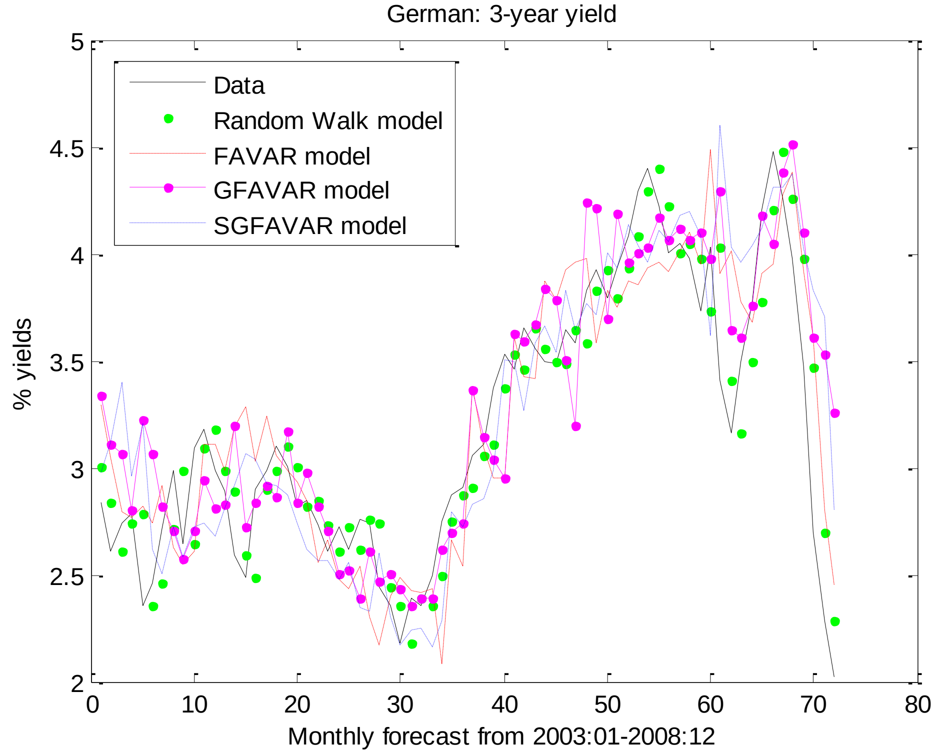


Figure 7c

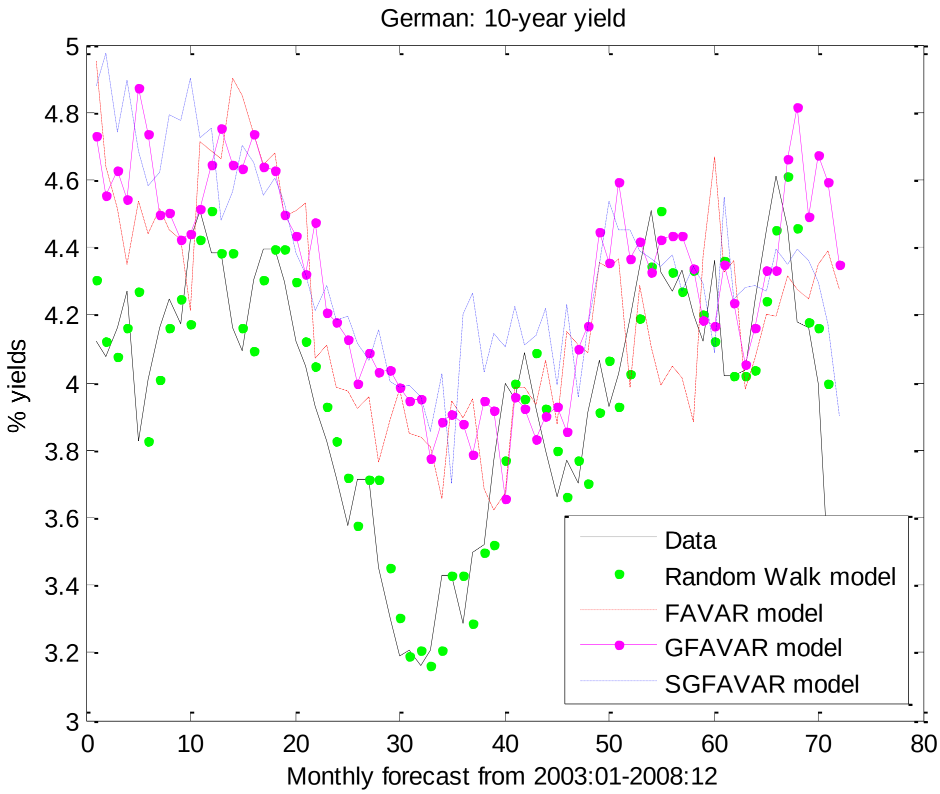


Figure 7d

Figure 8: **Observed and Predicted yields 6 month ahead of Germany. This figure provides plots of observed and 6-month ahead prediction time series of the 1-month, 12 month, and the 3- and 10-year maturities.**

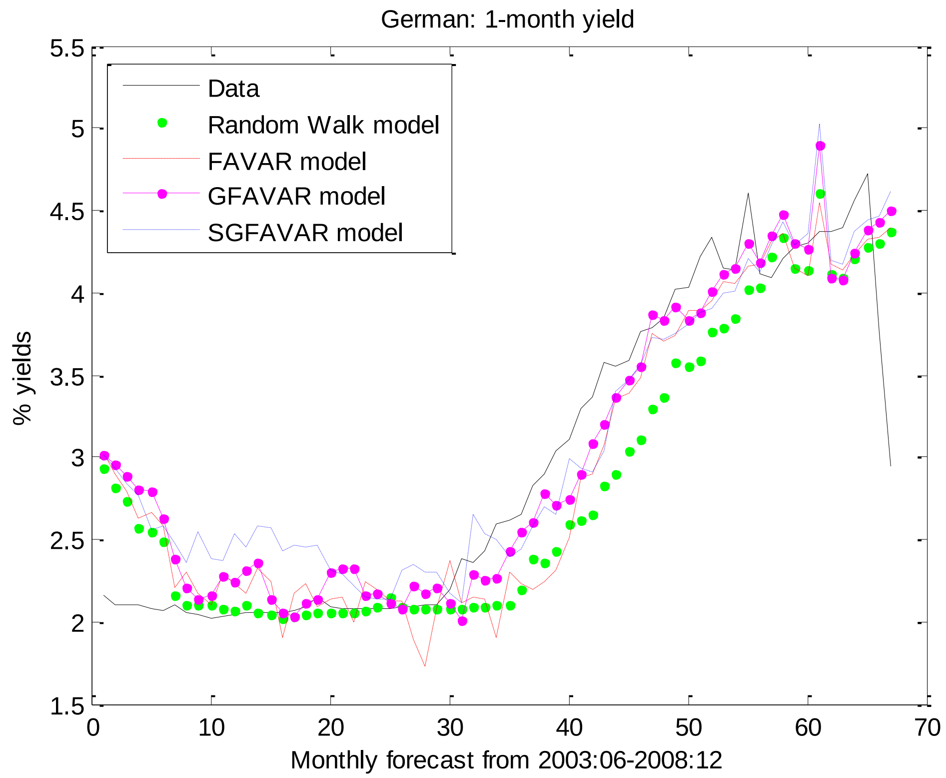


Figure 8a

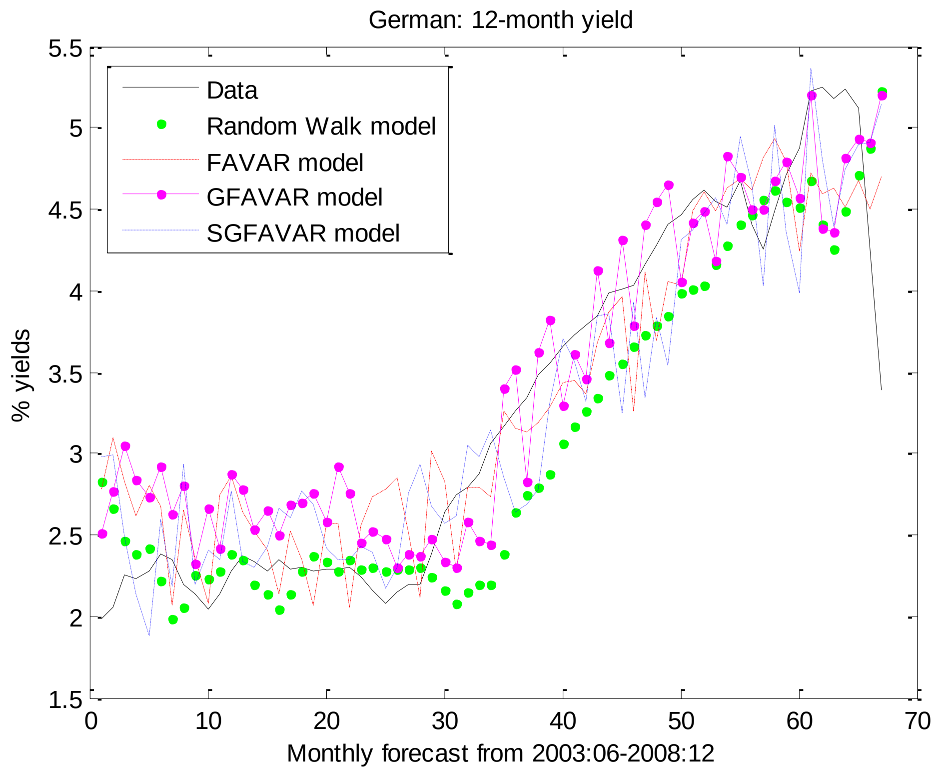


Figure 8b

Figure 8: **Observed and Predicted yields 6 month ahead of Germany. This figure provides plots of observed and 6-month ahead prediction time series of the 1-month, 12 month, and the 3- and 10-year maturities.**

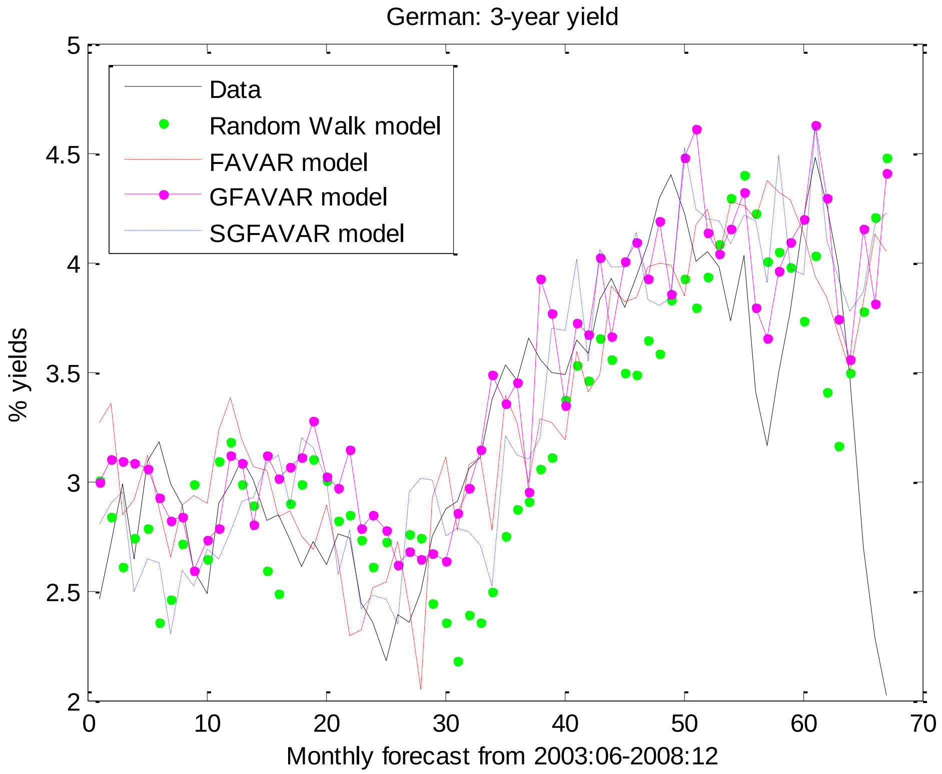


Figure 8c

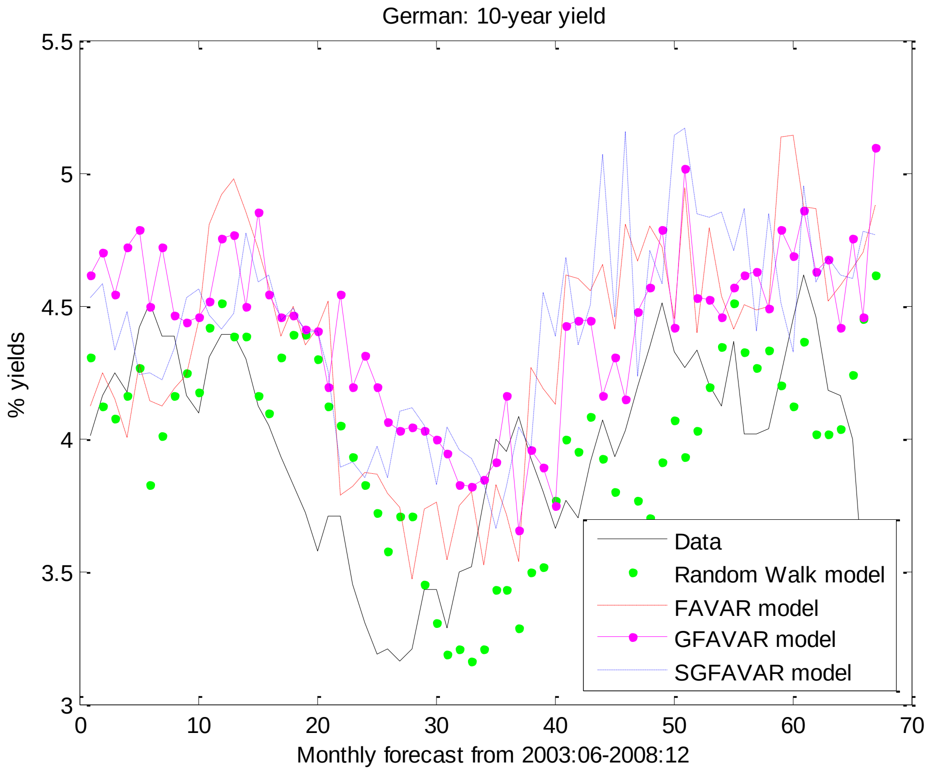


Figure 8d

Figure 9: Observed and Predicted yields 12 month ahead of Germany. This figure provides plots of observed and 12-month ahead prediction time series of the 1-month, 12 month, and the 3- and 10-year maturities.

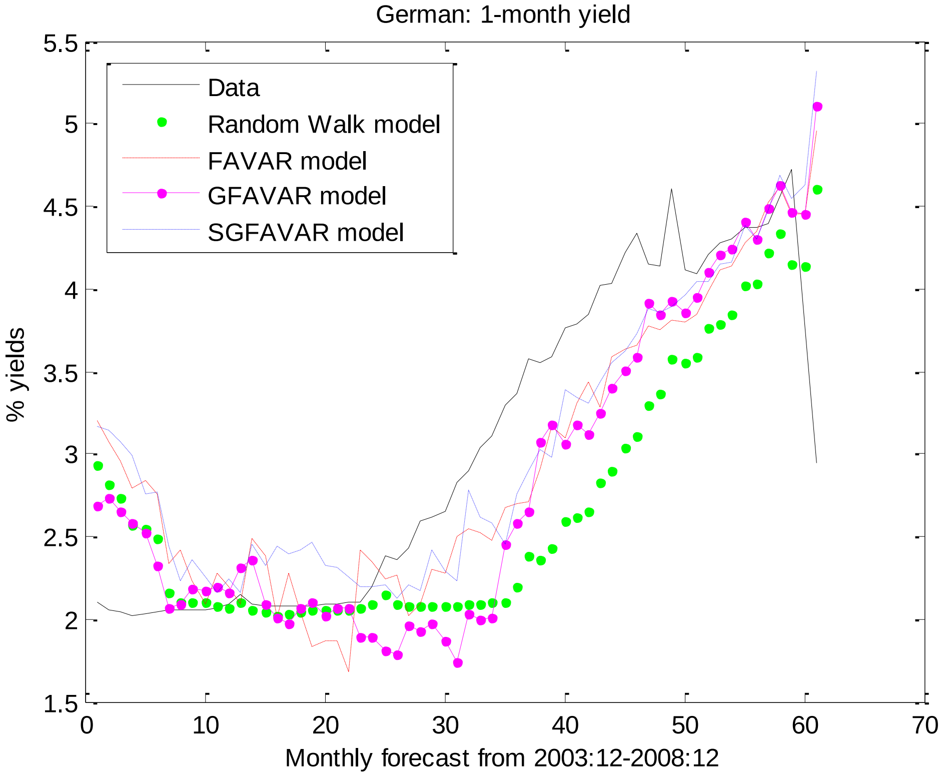


Figure 9a

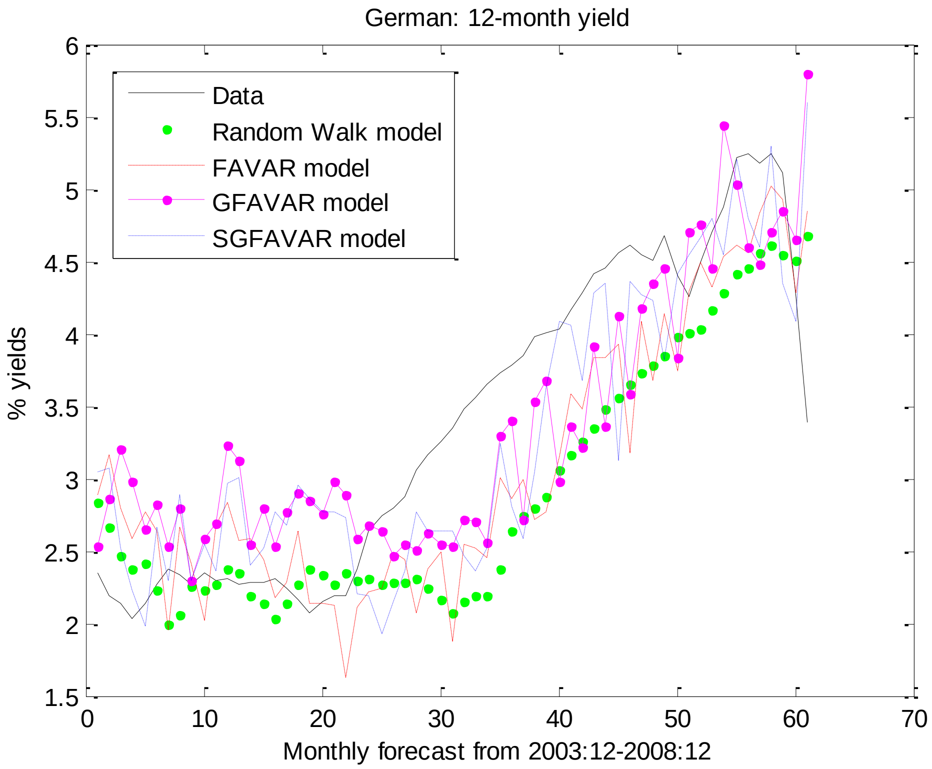


Figure 9b

Figure 9: Observed and Predicted yields 12 month ahead of Germany. This figure provides plots of observed and 12-month ahead prediction time series of the 1-month, 12 month, and the 3- and 10-year maturities.

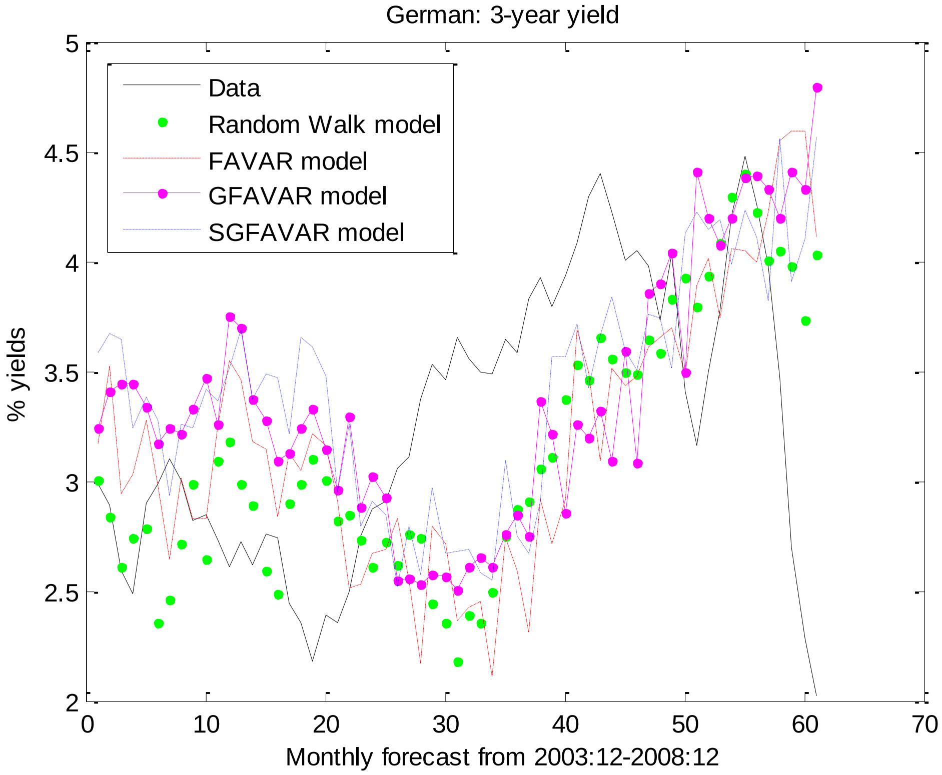


Figure 9c

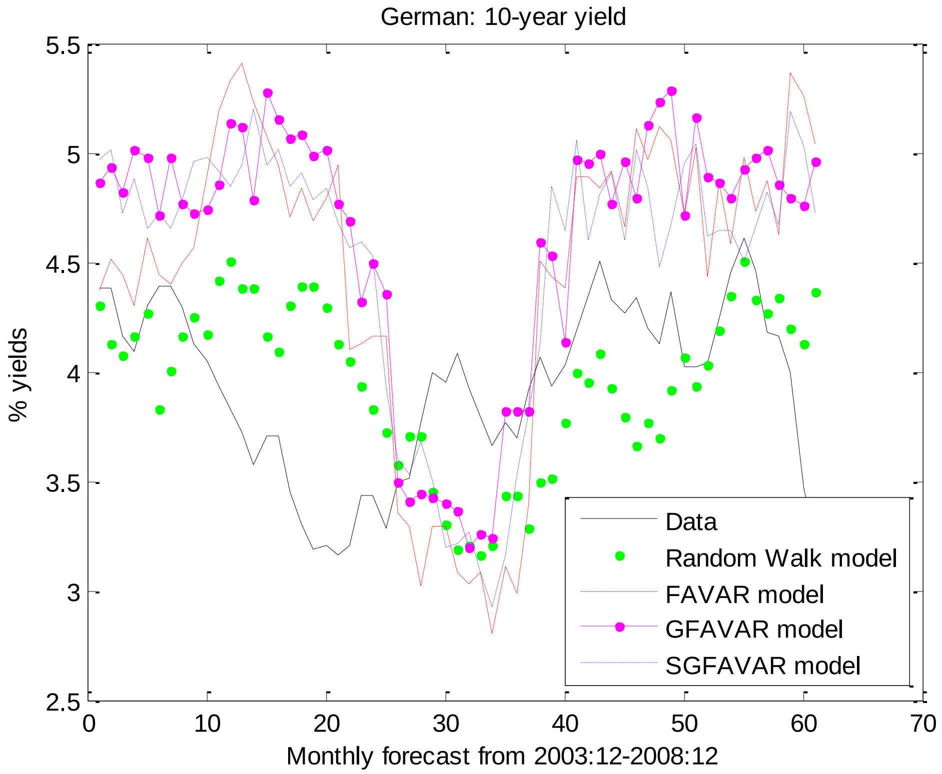


Figure 9d

Figure 10: Observed and Predicted yields 1 month ahead of the United States of America. This figure provides plots of observed and 1-month ahead prediction time series of the 1-month, 12 month, and the 3- and 10-year maturities.

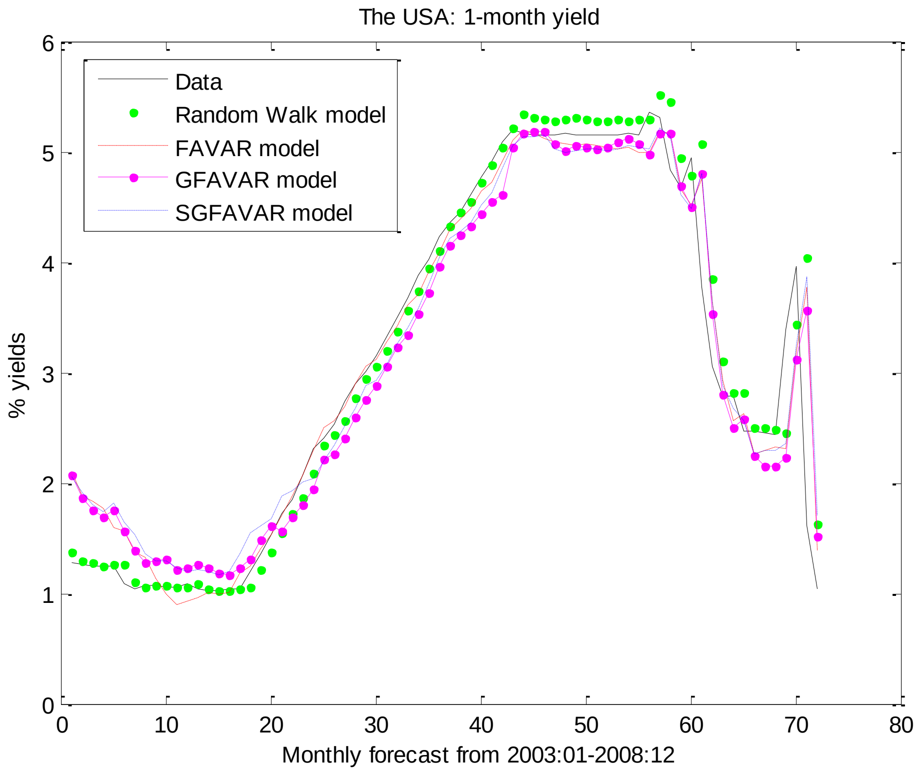


Figure 10a

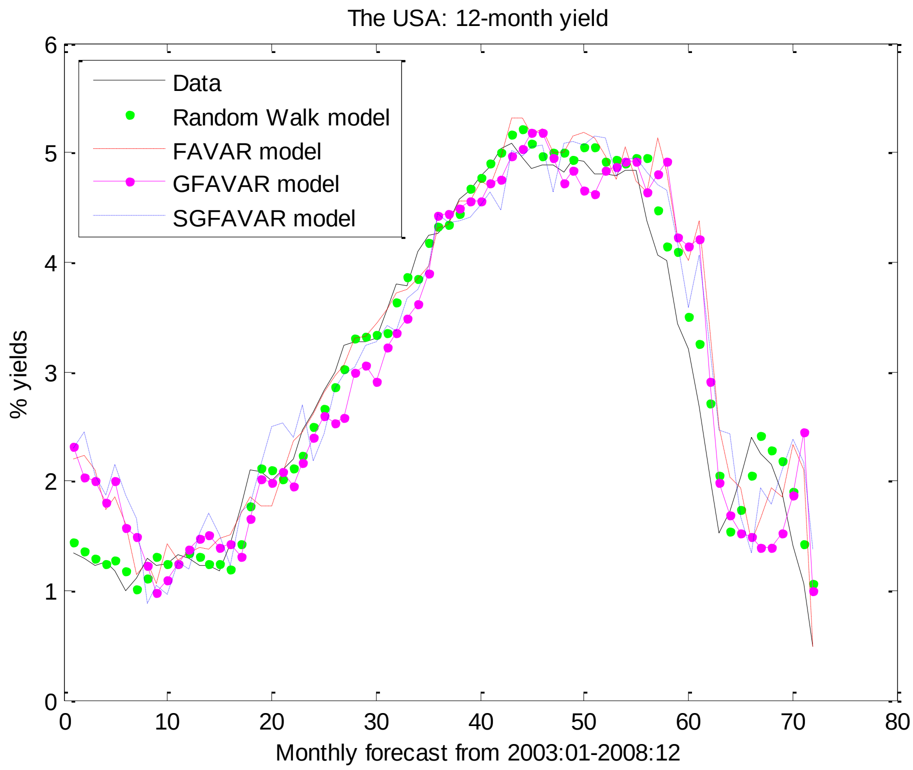


Figure 10b

Figure 10: Observed and Predicted yields 1 month ahead of the United States of America. This figure provides plots of observed and 1-month ahead prediction time series of the 1-month, 12 month, and the 3- and 10-year maturities.

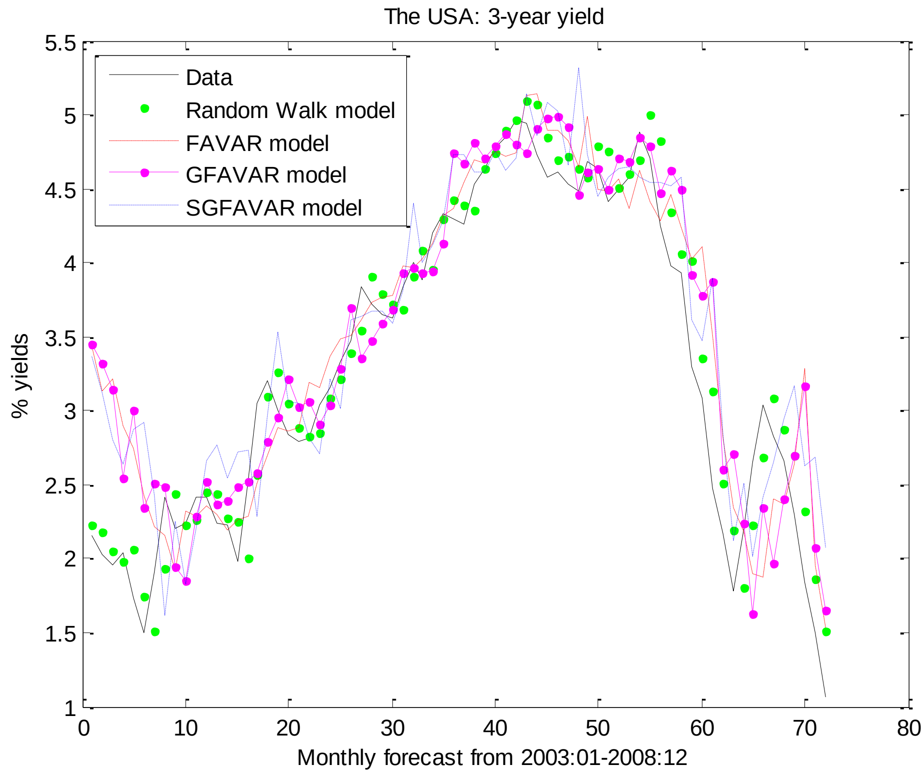


Figure 10c

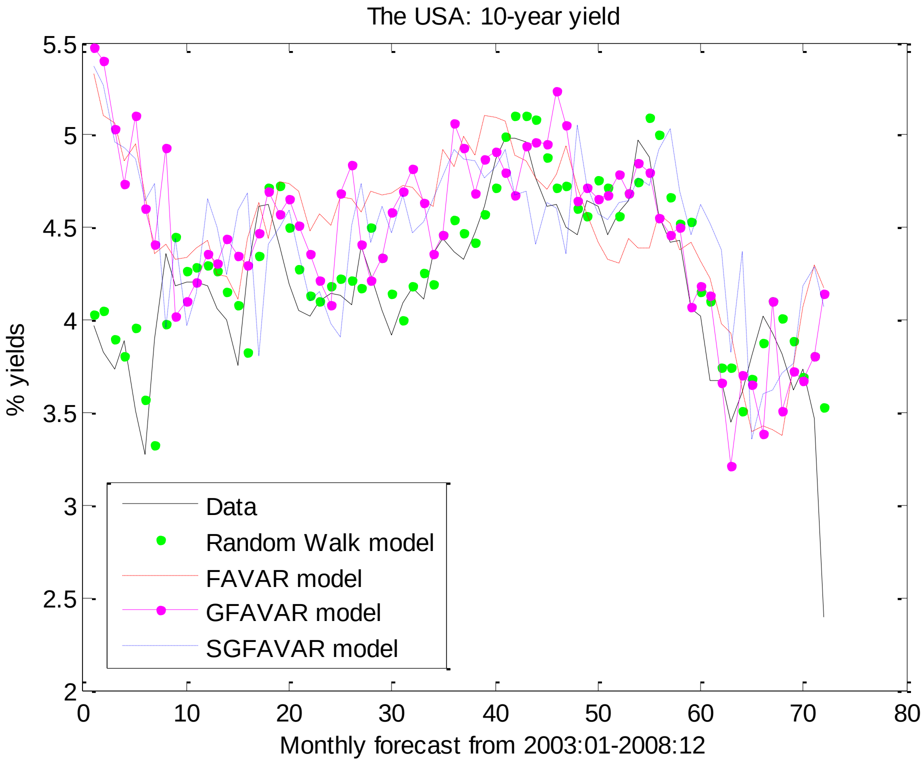


Figure 10d

Figure 11: Observed and Predicted yields 6 month ahead of the United States of America. This figure provides plots of observed and 6-month ahead prediction time series of the 1-month, 12 month, and the 3- and 10-year maturities.

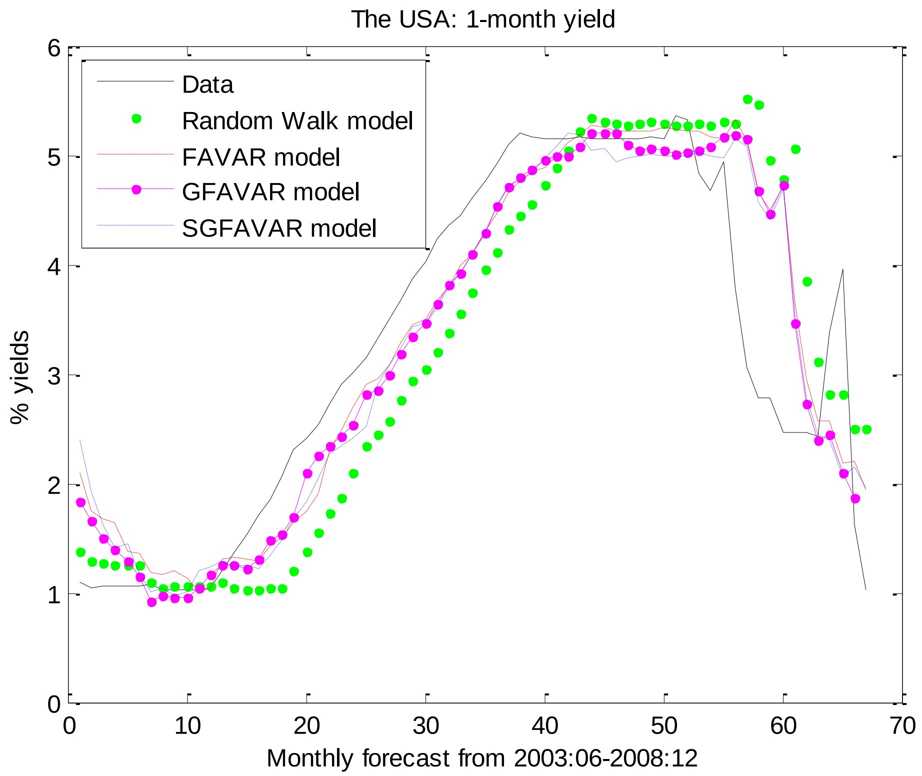


Figure 11a

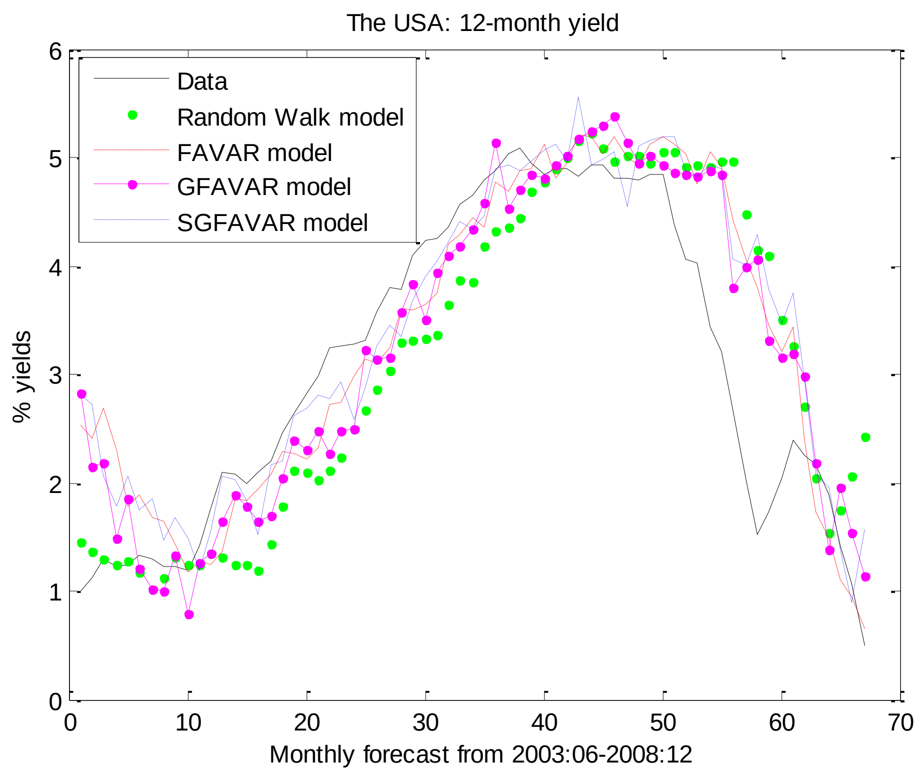


Figure 11b

Figure 11: Observed and Predicted yields 6 month ahead of the United States of America. This figure provides plots of observed and 6-month ahead prediction time series of the 1-month, 12 month, and the 3- and 10-year maturities.

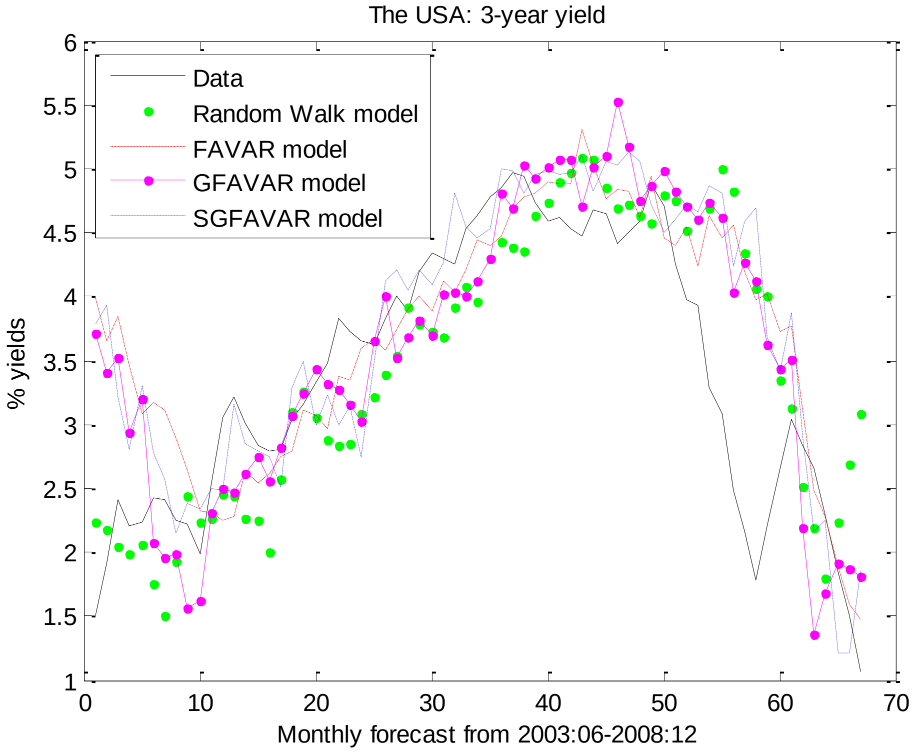


Figure 11c

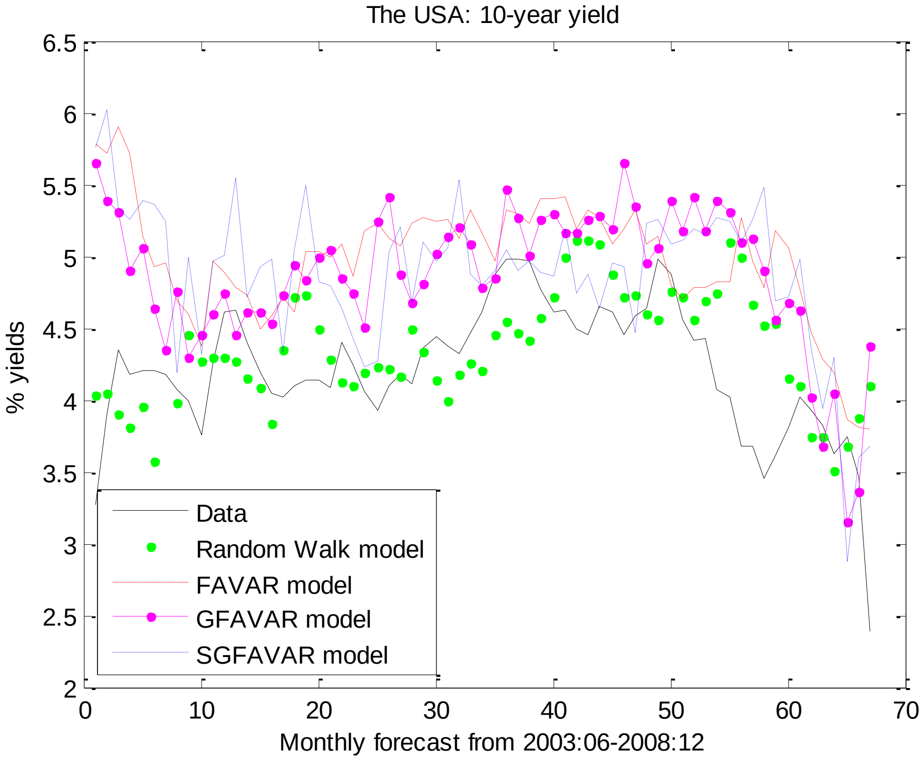


Figure 11d

Figure 12: Observed and Predicted yields 12 month ahead of the United States of America. This figure provides plots of observed and 12-month ahead prediction time series of the 1-month, 12 month, and the 3- and 10-year maturities.

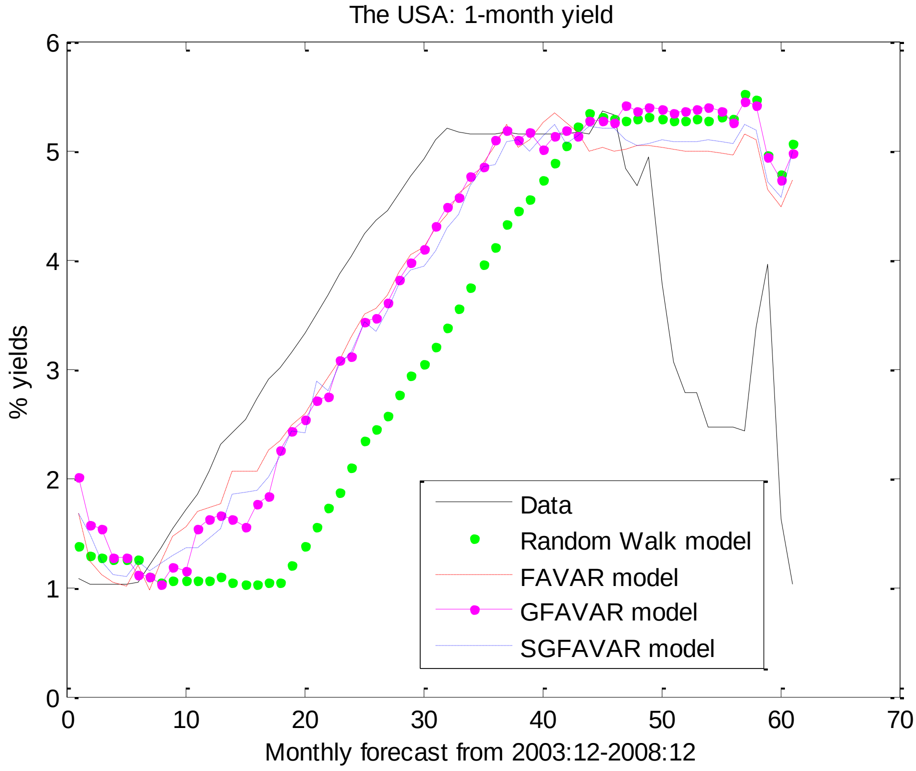


Figure 12a

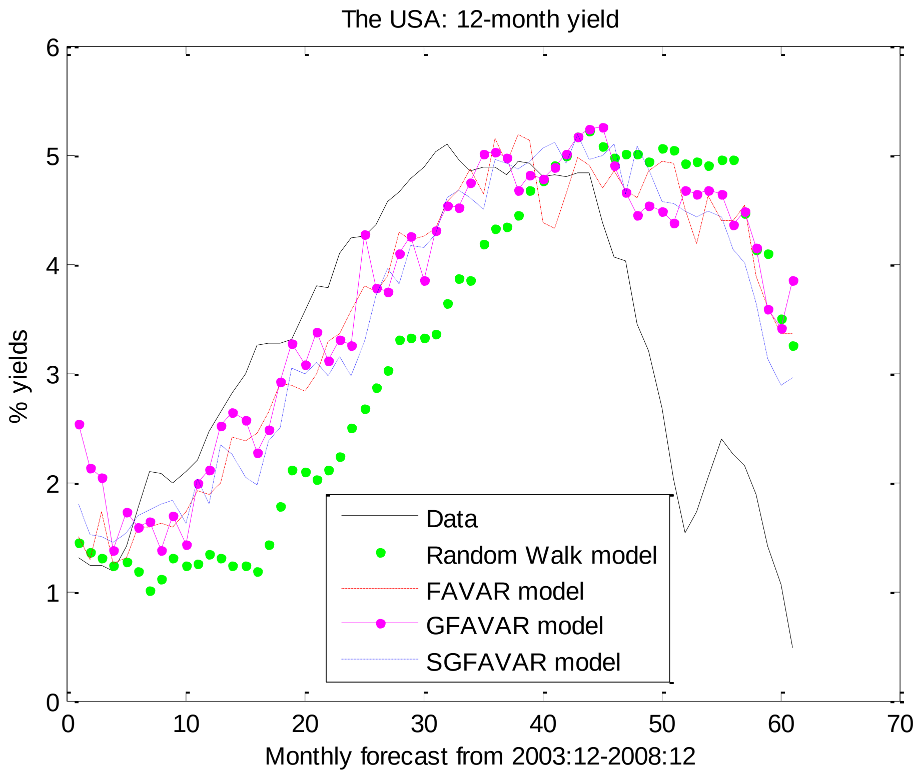


Figure 12b

Figure 12: Observed and Predicted yields 12 month ahead of the United States of America. This figure provides plots of observed and 12-month ahead prediction time series of the 1-month, 12 month, and the 3- and 10-year maturities.

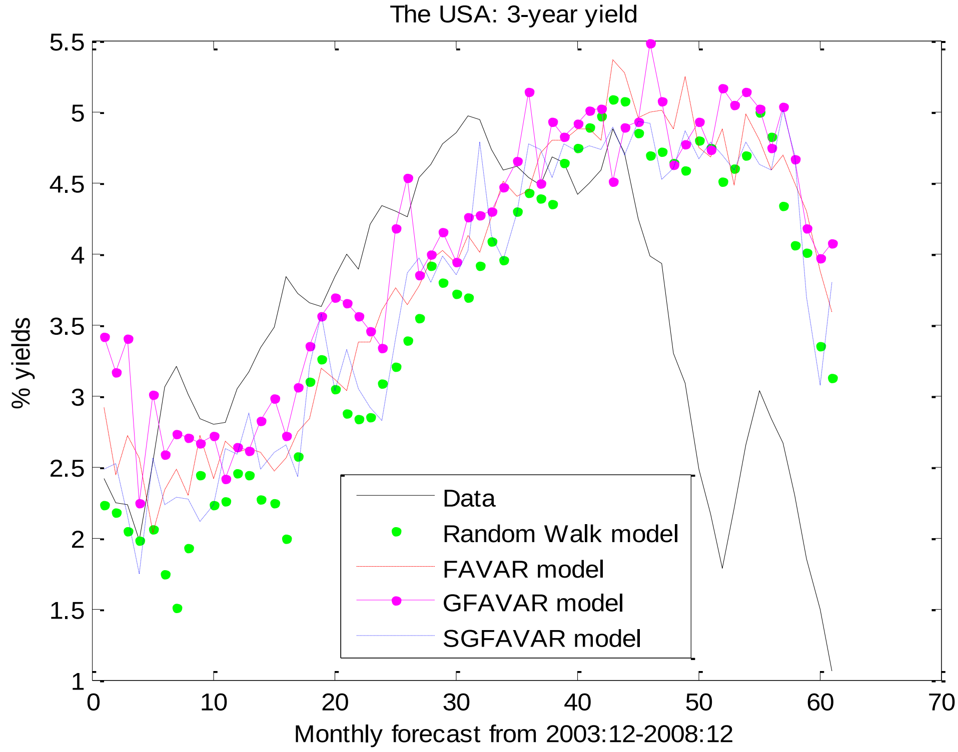


Figure 12c

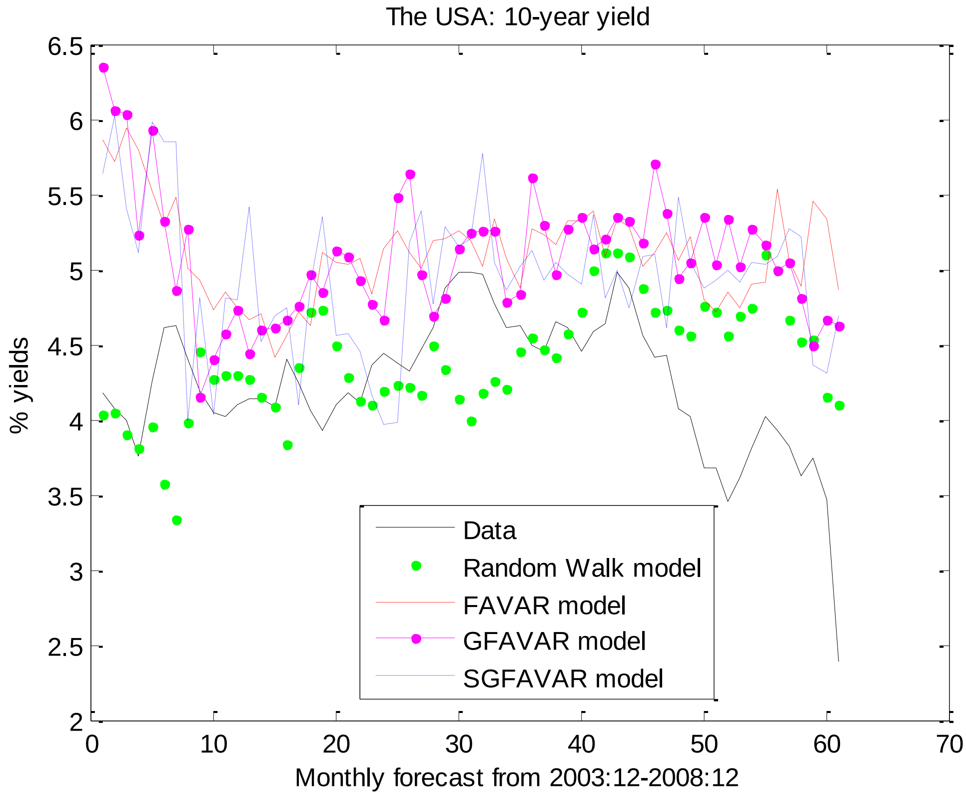


Figure 12d

3) A Variant of SGFAVAR Model

As no model can beat the random walk for the forecast of the long term yields, we will extend the model of SGFAVAR in this section to examine whether extracting the common factors from the groups of macroeconomic variables that well explain the long term rate (10 year yield) instead of the short term interest rate will improve the forecasting results of the long term yields. Therefore, we create the model named "Long Term Significant Group Factor Augmented VAR" or "LSGFAVAR". This model relatively similar to the SGFAVAR except that the factors of the LSGFAVAR model are extracted from the groups of macroeconomic variables whose group representatives can well explain the long term yield (10 year yield) instead of the short rate. As we would like to provide flexibility for researchers in term of a factors selection criterion to the term structure model, we hope that the LSGFAVAR model would provide a better result in forecasting the long term yields.

Before we examine the forecast results of the LSGFAVAR, we also perform a preliminary regressions the same as the other models in previous sections to test whether the factors acquired from the LSGFAVAR model are useful for the term structure model. Firstly we calculate the correlation of German and the U.S.'s factors extracted from the groups of macroeconomic variables that significantly explain the 10 year yield (LSGFAVAR model) and the associated time series of macroeconomic variables that are most correlated with the factors. These results are shown in the following **Table 19** and **Table 20.** We found that the first factor of German highly correlates with a group of "output" which is considered as the sixth largest group of macroeconomic time series dataset. This group contains only 16 macroeconomic time series. Moreover, the first factor of the U.S. highly correlates with a group of "gross domestic product". This group contains only 10 macroeconomic time series. This result is consistent with the GFAVAR and SGFAVAR models in that each group of macroeconomic variables has an equal chance to be selected as the common factors.

Table 19: Correlations of LSGFAVAR's factors on all individual German's macroeconomic time series

The first four LSGFAVAR's factors sorted by their eigenvalue	Correlation
Factor 1 (29.9432**% of Total Variance)**	
Monetary Base: M1	0.8145
Price Index: Producer price index industry	0.7831
Output: Consumer Goods	0.7655
Output: Construction	0.7416
Retail Trade Turnover: Total Value	0.7362
Factor 2 (20.7762**% of Total Variance)**	
Order Receive: Construction	0.7436
Monetary Base: M1	0.7109
Factor Income & Services: Service travel receive	0.6915
Order Receive: Housing construction	0.6825
Other Financial Institution: Time deposit	0.6588
Factor 3 (17.9420**% of Total Variance)**	
Monetary Base: M2	0.6429
Factor Income & Services: Change reserve assets bundes	0.6107
General Government: External financing (money market paper)	0.5969
Output: Construction	0.5710
Other Financial Institution: Current & transaction deposit	0.5633
Factor 4 (13.2372**% of Total Variance)**	
Factor Income & Services: Balance of unclassifiable transaction	0.5610
Order Receive: Total Domestic	0.5213
Other Financial Institution: Money Market papers	0.5184
Output: Production include Construction	0.5069
General Government: External financing (Long-term loan)	0.4988

**** This eigenvalue represented the variance captured from 8 significant groups (Group 1, 6, 7, 9, 10, 13, 14 and 16) whose factors have explanatory power to the 10 year rate of the German. Together, the first four factors explain about 81.8986% of the total variance of 8 the groups of macroeconomic variables.

Table 20: Correlations of LSGFAVAR's factors on all individual US's macroeconomic time series

The first four LSGFAVAR's factors sorted by their eigenvalue	Correlation
Factor 1 (42.9556**% of Total Variance)**	
Gross Domestic Product: Final Sales to Domestic Purchasers	0.7629
Monetary Aggregate: Currency Component of M1	0.7418
Consumer Credit: Total consumer loans owned by commercial banks	0.7211
Monetary Aggregate: Currency Component of M1 Plus Demand Deposits	0.7182
Exchange Rate: Nominal Broad Dollar Index	0.6944
Factor 2 (21.2094**% of Total Variance)**	
Income Payment and Receipts: Compensation of Employees	0.6852
Export-Import: Exports of Goods and Services	0.6604
Export-Import: Exports of Merchandise: excluding Military	0.6388
Income Payment and Receipts: U.S. government pensions and others transfers	0.6061
Gross Domestic Product: Gross National Product	0.5864
Factor 3 (7.4442**% of Total Variance)**	
Assets Liabilities Commercial Bank: Loans and leases in bank credit	0.5725
Gross Domestic Product: Real Potential Gross Domestic Product	0.5609
Income Payment and Receipts: Income receipts on U.S. assets abroad	0.5322
Export-Import: Exports of Services	0.5101
Pay Rate: Average Hourly Earnings: Construction	0.5063
Factor 4 (6.6228**% of Total Variance)**	
Capital Utilization: Manufacturing	0.4883
Export-Import: Exports of Goods, Services and Income	0.4541
Gross Domestic Product: Real Change in Private Inventories	0.4672
Pay Rate: Average Hourly Earnings: Manufacturing	0.4267
Capital Utilization: Finished processing	0.4009

**** This eigenvalue represented the variance captured from 13 significant groups (Group 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 13 and 14) whose factors have explanatory power to the 10 year rate of the US. Together, the first four factors explain about 78.2320% of the total variance of 13 the groups of macroeconomic variables.

In this section, we examine the explanatory power of the LSGFAVAR's factors whether they are useful for the term structure model. Firstly, we regress the LSGFAVAR factors to estimate the short rate, and then the variances explained by the LSGFAVAR are compared with the previous models. Moreover we also regress the factors of LSGFAVAR to estimate the yields for different maturities. These results will tell us about the usefulness of the LSGFAVAR factors to the term structure model.

Table 21: Variation explained by the factors of four macro-based FAVAR model and individual variables

Policy rule based	Germany	The USA
on the four factors extracted from FAVAR Model	49.808	67.158
on the four factors extracted from GFAVAR Model	47.322	65.086
on the four factors extracted from SGFAVAR Model	42.889	63.178
on the four factors extracted from LSGFAVAR Model	42.640	62.091
on output and inflation	42.412	61.408

This table reports the adjusted-R^2 of the estimation for policy rule based on the four factors extracted from different methods and the estimation for a policy rule based on output and inflation. The sample period for Germany is 1993:01 to 2008:12. For the US, the sample period is 1992:01 to 2008:12.

As indicated by the adjusted-R^2, **Table 21** shows that the four factors extracted from the LSGFAVAR fit the data slightly better than a standard Taylor-ruled based on output and inflation. Comparing the LSGFAVAR with the other three factor-based equations of Germany, we found that a policy rule based on the four factors of the LSGFAVAR model cannot beat the others in explaining the variations of the short rate. These results can be implied that the LSGFAVAR's factors may contain small information about the short rate as they are extracted from only group of macroeconomic variables that well explain the long term rate. Moreover, these results are also true for the US's sample. However, the results of LSGFAVAR still support the Fed that they commonly base their decision on a large set of macroeconomic information rather than using output and inflation alone.

Table 22: Variation of yields explained by four factors extracted from four different methods

	$y^{(6)}$	$y^{(12)}$	$y^{(36)}$	$y^{(60)}$	$y^{(120)}$
Germany					
FAVAR Model	0.6807	0.6495	0.5618	0.5208	0.4716
GFAVAR Model	0.6405	0.6133	0.5189	0.4667	0.4140
SGFAVAR Model	0.6503	0.6214	0.4522	0.3951	0.3517
LSGFAVAR Model	0.6625	0.6319	0.5436	0.4778	0.4544
The USA					
FAVAR Model	0.7089	0.6910	0.6612	0.6493	0.6514
GFAVAR Model	0.6683	0.6473	0.6195	0.5696	0.5257
SGFAVAR Model	0.6976	0.6631	0.6430	0.5529	0.4779
LSGFAVAR Model	0.7015	0.6882	0.6600	0.6124	0.5834

This table summarizes the R^2 of an unrestricted regression of difference maturities yields on the four macro factors extracted from different methods.

According to **Table 22,** we found that the four factors extracted from the LSGFAVAR model explain the variation of yields for all selected maturities better than the factors extracted from the other two alternative models (GFAVAR and SGFAVAR models) for both samples of German and the U.S. However, the factors extracted from the LSGFAVAR still cannon beat the FAVAR model in explaining the variation of the yields for all selected maturities.

From now we already know that the LSGFAVAR factors are useful for the term structure model. Then we estimate the yields curves following the LSGFAVAR approach and compare the in-sample fit results with the other three macro-based FAVAR models. **Table 23** shows the in-sample fit results of the German yields. We found that the LSGFAVAR model fits the long end of the curve ($y^{(36)}$ and $y^{(120)}$) better than the GFAVAR and SGFAVAR models respectively. However, the LSGFAVAR model cannot beat the FAVAR model in fitting the long end of the curves. Moreover, for the short end of the curve ($y^{(6)}$ and $y^{(12)}$), the LSGFAVAR model cannot fit the data better than the other three macro-based FAVAR models. In addition, **Table 24** shows the U.S. in-sample fit results which we found that the LSGFAVAR model fits the 10-year yield ($y^{(120)}$) better than the GFAVAR and SGFAVAR models. However, the LSGFAVAR model still cannot beat the FAVAR model in fitting the long end of the curves. For fitting the 6-month and 3-year yields, the LSGFAVAR model performs better than the SGFAVAR but cannot beats the GFAVAR and FAVAR models. For the 1-year yield, the LSGFAVAR model cannot beat the other three macro-based FAVAR models.

Table 23: Mean of Germany's observed and model-implied yield for five selected interest rates following four different extracting models

		$y^{(1)}$	$y^{(6)}$	$y^{(12)}$	$y^{(36)}$	$y^{(120)}$
	Mean					
FAVAR Model	y_t	3.678	3.752	3.828	4.053	5.011
	$\hat{y}_t$	3.678	3.772	3.784	4.060	5.008
	$\|y_t - \hat{y}_t\|$	0.000	0.137	0.226	0.372	0.521
GFAVAR Model	y_t	3.678	3.752	3.828	4.053	5.011
	$\hat{y}_t$	3.678	3.770	3.803	4.032	5.002
	$\|y_t - \hat{y}_t\|$	0.000	0.169	0.249	0.469	0.694
SGFAVAR Model	y_t	3.678	3.752	3.828	4.053	5.011
	$\hat{y}_t$	3.678	3.687	3.785	4.059	4.996
	$\|y_t - \hat{y}_t\|$	0.000	0.170	0.288	0.490	0.704
LSGFAVAR Model	y_t	3.678	3.752	3.828	4.053	5.011
	$\hat{y}_t$	3.678	3.695	3.894	4.048	5.005
	$\|y_t - \hat{y}_t\|$	0.000	0.185	0.290	0.434	0.622

This table summarizes means of Germany's observed and fitted yields. Yields are reported in percentage terms. The first and second row in each panel report the mean of observed yield and fitted values under different models while the third row shows the mean of absolute fitting errors.

Table 24: Mean of the US observed and model-implied yield for five selected interest rates following four different extracting models

		$y^{(1)}$	$y^{(6)}$	$y^{(12)}$	$y^{(36)}$	$y^{(120)}$
	Mean					
FAVAR Model	y_t	3.682	3.938	4.086	4.620	5.386
	$\hat{y}_t$	3.682	3.915	4.094	4.635	5.382
	$\|y_t - \hat{y}_t\|$	0.000	0.239	0.294	0.413	0.416
GFAVAR Model	y_t	3.682	3.938	4.086	4.620	5.386
	$\hat{y}_t$	3.682	3.928	4.082	4.631	5.392
	$\|y_t - \hat{y}_t\|$	0.000	0.218	0.275	0.402	0.544
SGFAVAR Model	y_t	3.682	3.938	4.086	4.620	5.386
	$\hat{y}_t$	3.682	3.893	4.095	4.651	5.378
	$\|y_t - \hat{y}_t\|$	0.000	0.253	0.330	0.524	0.661
LSGFAVAR Model	y_t	3.682	3.938	4.086	4.620	5.386
	$\hat{y}_t$	3.682	3.862	4.097	4.644	5.380
	$\|y_t - \hat{y}_t\|$	0.000	0.248	0.339	0.418	0.429

This table summarizes means of the US observed and fitted yields. Yields are reported in percentage terms. The first and second rows in each panel report the mean of observed yield and fitted values under different models while the third row shows the mean of absolute fitting errors.

Table 25: Standard Deviation of Germany's observed and model-implied yield for five selected interest rates following four different extracting models

		$y^{(1)}$	$y^{(6)}$	$y^{(12)}$	$y^{(36)}$	$y^{(120)}$
	Standard Deviation					
FAVAR Model	y_t	1.312	1.226	1.155	1.061	1.126
	$\hat{y}_t$	1.312	1.193	1.094	0.962	0.865
	$\lvert y_t - \hat{y}_t \rvert$	0.000	0.109	0.176	0.282	0.369
GFAVAR Model	y_t	1.312	1.226	1.155	1.061	1.126
	$\hat{y}_t$	1.312	1.184	1.079	0.929	0.640
	$\lvert y_t - \hat{y}_t \rvert$	0.000	0.131	0.186	0.356	0.481
SGFAVAR Model	y_t	1.312	1.226	1.155	1.061	1.126
	$\hat{y}_t$	1.312	1.181	1.058	0.896	0.585
	$\lvert y_t - \hat{y}_t \rvert$	0.000	0.142	0.188	0.378	0.511
LSGFAVAR Model	y_t	1.312	1.226	1.155	1.061	1.126
	$\hat{y}_t$	1.312	1.197	1.070	0.959	0.709
	$\lvert y_t - \hat{y}_t \rvert$	0.000	0.149	0.181	0.350	0.421

This table summarizes standard deviations of Germany's observed and fitted yields. Yields are also reported in percentage terms. The first and second row in each panel report the standard deviation of observed yield and fitted values under different models while the third row shows the standard deviation of absolute fitting errors.

Table 26: Standard Deviation of the US observed and model-implied yield for five selected interest rates following four different extracting models

		$y^{(1)}$	$y^{(6)}$	$y^{(12)}$	$y^{(36)}$	$y^{(120)}$
	Standard Deviation					
FAVAR Model	y_t	1.556	1.645	1.610	1.468	1.123
	$\hat{y}_t$	1.556	1.554	1.525	1.369	0.895
	$\lvert y_t - \hat{y}_t \rvert$	0.000	0.220	0.249	0.349	0.385
GFAVAR Model	y_t	1.556	1.645	1.610	1.468	1.123
	$\hat{y}_t$	1.556	1.647	1.541	1.376	0.834
	$\lvert y_t - \hat{y}_t \rvert$	0.000	0.187	0.246	0.333	0.440
SGFAVAR Model	y_t	1.556	1.645	1.610	1.468	1.123
	$\hat{y}_t$	1.556	1.662	1.486	1.317	0.618
	$\lvert y_t - \hat{y}_t \rvert$	0.000	0.283	0.320	0.416	0.485
LSGFAVAR Model	y_t	1.556	1.645	1.610	1.468	1.123
	$\hat{y}_t$	1.556	1.651	1.502	1.342	0.861
	$\lvert y_t - \hat{y}_t \rvert$	0.000	0.293	0.348	0.374	0.410

This table summarizes standard deviations of the US observed and fitted yields. Yields are also reported in percentage terms. The first and second row in each panel report the standard deviation of observed yield and fitted values under different models while the third row shows the standard deviation of absolute fitting errors.

According to **Table 23** and **Table 24**, we can imply that the LSGFAVAR model improves the performance of the two alternative models in fitting the 10 year yield. Moreover, **Table 25** and **Table 26** show the standard deviation of the observed and model-implied yield for five selected interest rates following four different extracting models. We found that all models cannot capture some of the variation in the long end of the curves while the FAVAR model captures the variation of the long maturities yields better than the LSGFAVAR, GFAVAR and SGFAVAR.

In the in-sample fit section, even though the LSGFAVAR model cannot beat the FAVAR model, we expect that they will improve the performance of the out-of sample forecast. Following **Table 27** and **Table 28** show the root mean squared error of the four different extracting methods of German and the US respectively.

Table 27: German's Out-of-sample forecast of the four different extracting methods measured by RMSE - Forecast Period 2003:01-2008-12

$y^{(n)}$	FAVAR	GFAVAR	SGFAVAR	LSGFAVAR	Random Walk
1 month ahead forecast					
1	0.2530	0.2821	0.3026	0.3124	**0.1925**
6	0.2769	0.3007	0.3229	0.3558	**0.1780**
12	0.3072	0.3292	0.3539	0.3655	**0.1883**
36	0.3466	0.3824	0.4014	0.3997	**0.2423**
60	0.3964	0.4101	0.4314	0.4302	**0.2162**
120	0.4101	0.4457	0.4618	0.4692	**0.1691**
6 month ahead forecast					
1	0.3935	**0.3699**	0.4104	0.4181	0.4246
6	**0.4156**	0.4273	0.4501	0.4656	0.4643
12	**0.4340**	0.4523	0.4726	0.4798	0.4975
36	0.5083	**0.4878**	0.5227	0.5110	0.6037
60	0.5342	0.5360	0.5504	0.5409	**0.5271**
120	0.5780	0.5593	0.5949	0.5821	**0.4105**
12 month ahead forecast					
1	**0.5410**	0.5818	0.5602	0.5647	0.7092
6	0.6135	0.6327	**0.5998**	0.6012	0.7448
12	0.6754	0.6878	0.6490	**0.6394**	0.7391
36	0.8310	0.8559	0.8090	**0.6749**	0.6836
60	0.9033	0.9196	0.8793	0.8372	**0.6069**
120	1.0171	1.2754	0.9821	0.9127	**0.5277**

This table summarizes the German's root mean squared errors obtained from out-of-sample yield forecasts. The models were estimated using data from 1993:01 until the period when the forecast is made. The forecasting period is 2003:01-2008:12.

At 1-month ahead forecast, we found that the LSGFAVAR model cannot beat the other three macro-based FAVAR models (FAVAR, GFAVAR and SGFAVAR models) for most maturities. Moreover, the random walk model still outperforms the others in forecasting the yields of all maturities. These results are true for both German and the U.S samples.

Table 28: The U.S.'s Out-of-sample forecast of the four different extracting methods measured by RMSE - Forecast Period 2003:01-2008-12

$y^{(n)}$	FAVAR	GFAVAR	SGFAVAR	LSGFAVAR	Random Walk
1 month ahead forecast					
1	0.3914	0.4176	0.4307	0.4516	**0.3893**
6	0.4386	0.4599	0.4705	0.4856	**0.2317**
12	0.4858	0.4999	0.5217	0.5273	**0.2355**
36	0.5057	0.5285	0.5402	0.5388	**0.2689**
60	0.5304	0.5499	0.5686	0.5602	**0.2617**
120	0.5699	0.5520	0.5801	0.5816	**0.2441**
6 month ahead forecast					
1	0.6786	**0.6573**	0.7098	0.7184	0.9676
6	**0.6987**	0.7226	0.7529	0.7599	0.9760
12	0.7644	**0.7426**	0.7937	0.7772	0.9224
36	0.8013	**0.7887**	0.8299	0.8049	0.8181
60	0.8389	0.8097	0.8502	0.8430	**0.6765**
120	0.8959	0.8300	0.8699	0.8751	**0.5017**
12 month ahead forecast					
1	**0.9899**	1.1124	1.0649	1.1017	1.6685
6	**1.0719**	1.1877	1.1256	1.1485	1.7811
12	1.1911	1.2372	**1.1502**	1.1639	1.6384
36	1.2282	1.2799	1.1920	**1.1874**	1.2520
60	1.2675	1.3184	1.2317	1.2101	**0.9501**
120	1.3169	1.3597	1.2715	1.2686	**0.6124**

This table summarizes the U.S.'s root mean squared errors obtained from out-of-sample yield forecasts. The models were estimated using data from 1992:01 until the period when the forecast is made. The forecasting period is 2003:01-2008:12.

At 6-month ahead forecast, we found from the German out-of sample forecast results that the LSGFAVAR model outperforms the SGFAVAR in forecasting the intermediate and longer term yields ($y^{(36)}$, $y^{(60)}$ and $y^{(120)}$). However, the SGFAVAR model outperforms the LSGFAVAR in forecasting the short term yields instead ($y^{(1)}$, $y^{(6)}$ and $y^{(12)}$). Moreover, the U.S. results also show that the LSGFAVAR model outperforms the SGFAVAR in forecasting the 1-year, 3-year and 5-year yields. In contrast, the SGFAVAR model performs better than the LSGFAVAR in forecasting the 1-month, 6-month and 10-year yields. Even though the LSGFAVAR model can improve the performance of the SGFAVAR in forecasting the longer term yields for most maturities, they cannot perform better than the random walk model in forecasting the 5-year and 10-year yields for both samples.

At 12-month ahead forecast, the LSGFAVAR model still outperforms the SGFAVAR in forecasting the intermediate and long term yields ($y^{(36)}$, $y^{(60)}$ and $y^{(120)}$) for both samples. Considering all the models, we found that the LSGFAVAR model performs the best in forecasting the 1-year and the 3-year yields of German. These results are relatively the same as the U.S. in that the LSGFAVAR model performs the best in forecasting the 3-year yield. However, the long term yields are still dominated by the random walk model.

According to **Table 26** and **Table 27**, we can conclude that the LSGFAVAR model outperforms the SGFAVAR in forecasting the intermediate yields and the long term yields. On the other hand, the SGFAVAR model performs better than the LSGFAVAR in forecasting the short term yields instead. Even though we try to improve the performance of the SGFAVAR model in forecasting the longer term yield, but we still cannot beat the random walk model.

CHAPTER V
CONCLUSION

This study develops the extracting method used to model the term structures based on the idea that the central bank commonly uses a large set of conditioning information when setting the short term interest rate. With the question that many countries have different principals and policies used to calculate their macroeconomics variables, extracting common factors from a large macroeconomic data set following the FAVAR model may be questionable as the macroeconomic variables that have a large number of macroeconomic time series (highest weighting) relatively have more chance to be extracted as the common factors. To examine this question, we collect the macroeconomic time series based on their character which the group of macroeconomic variables are already defined by the central bank of sample countries. The macroeconomic time series that share the same character are grouped together. As we realize that one macroeconomic category can be measured by many macroeconomic time series, the correlation between these time series could be very high if they measure similar piece of information. So, finding a factor that can capture the largest share of variation of the macroeconomic category could be good proxy. Therefore, we then extract one common factor from each group to be a group factor representative. From now we already equate each group to have only one factor where they will have an equal chance to be extracted as the common factors. This method is named as the GFAVAR model. To further develop the previous model, we impose constraints to the group of macroeconomic variables so that only the groups that significantly explain the short rate will be selected to be extracted as the common factors. This method is name as the SGFAVAR model. As we have different extracting methods of the common factors, the yield curve forecast performance of each model has been examined to identify the best method to extract the common factors.

The usefulness of the extracted macroeconomic factors from the two alternative models to the term structures has been firstly studied. We found that each group of macroeconomic variables has fairly high average correlation between pairs of the macroeconomic time series. Moreover, the first common factors extracted from group of macroeconomic variables can explain a large share of their group's variance. According to these results, we can imply that the common factors extracted from GFAVAR and SGFAVAR model can capture significant proportion of information of the data set.

Moreover we also examine the explanatory power of the extracted macroeconomic factors over the output and inflation. We found that the models based on macroeconomic factors explain the short rate better than the model based on output and inflation. This findings support the Fed that they normally base their decision on a broad macroeconomic information rather than using output and inflation alone. According to these results, the macroeconomic factors are potentially useful as the state variables for a term structure model.

By using the common factors extracted from different models and the short rate as the state variables in the Factor-Augmented VAR approach with restrictions implied by no-arbitrage to model the dynamics of the short-term interest rate, we can construct the yield curves. The mean and standard deviation of absolute errors have been used to compare the in sample fit results for each model. We found that the GFAVAR model fits the US data well in the short and medium of the curve whereas the long end of the curve is dominated by the FAVAR model. On the other hand, the FAVAR model fits the German data better than the other two models for all selected maturities. Moreover, the FAVAR model provides a good fit to the long end of the yield curves for both US and German yields. This finding can be concluded that the entire macro-based FAVAR model fits the data well.

For the out-of-sample forecast, we applied the root mean square forecast errors (RMSEs) to compare the forecast performance of each model. We found that the FAVAR model exhibits a good ability to predict the yield curve out-of-sample especially the short term yields for both 6-month and 12-month forecast horizons while the GFAVAR model provides better forecast performance in the intermediate yields for the 6-month forecast horizon. Moreover, the SGFAVAR model also provides better forecast performance for the intermediate yield at the 12-month forecast horizon. For the long term yields, the random walk model outperforms the three macro-based FAVAR models. Moreover, at 1 month forecast horizon, the random walk model also outperforms the three macro-based FAVAR models for both the samples of the US and Germany.

As no models can beat the random walk in forecasting the long term yields, we create the LSGFAVAR model. This model is relatively similar to the SGFAVAR model except that the common factors are extracted from the group of macroeconomic variables that significantly explain the long term yield instead of the short term as we expect that extracting the common factors from the groups that best explain the long term yields will improve the forecast performance of the long term yield too. For the results of the LSGFAVAR, we found that the factors of LSGFAVAR are useful for the term structure model as they provide both a good in-sample fit and out-of sample forecast results. In the in-sample fit section, we found that the LSGFAVAR model fits the long end of curves better than the GFAVAR and SGFAVAR for both samples. However, they still cannot beat the FAVAR model. For the out of sample forecast results, the LSGFAVAR model performs the best in forecasting the intermediate yields at the 12-month ahead forecast. These results provide an improvement on the SGFAVAR model in that the factors extracted based on the LSGFAVAR model forecast the longer term yields better than the SGFAVAR.

As the no-change forecast of the individual yields is the main assumption of the random walk model, we can explain some of the implication of the random walk that forecasting the yields in a period where the yield curves have a small change in yields is outperformed by the random walk model.

As there is a small difference in the RMSEs of each model, we cannot clearly justify which model is the best model in term of forecasting the yield curves. However, we can summarize from this study that the constraints imposed to the extracting method may be the two-edged sword in that they help us to select the appropriate factors describing the intermediate term yields but they may eliminate some of the information that best describe the short term yields. Therefore, the appropriate model used to forecast the yield curve is subjected to the researcher's objective.

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APPENDICES

APPENDIX A

Table 29: Group of German's Macroeconomics Time Series

Group of German Macroeconomics' Variables					
		No. of Series			**No. of Series**
1	Monetary Base	3	9	Pay Rate	8
2	Foreign Exchange Rate	8	10	Retail Trade Turnover	6
3	Stock Return Index	13	11	Factor Income & Services	18
4	Price Index	8	12	Household Sector	46
5	Export - Import	18	13	General Government	46
6	Employment	5	14	Monetary Financial Institution	52
7	Output	16	15	Non-Financial Corporation	53
8	Order Receive	14	16	Other Financial Institution	23

This table summarizes the group of German macroeconomic time series and their number of time series containing in each group.

Table 30: Group of the US's Macroeconomic Time Series

Group of US's Macroeconomics variable					
		No. of series			**No. of Series**
1	Reserve and Monetary Base	8	**8**	Pay Rate	9
2	Exchange Rate	21	**9**	Export - Import	37
3	Price Index	38	**10**	Assets Liabilities Commercial Bank	14
4	Stock Return Index	11	**11**	Consumer Credit	14
5	Employment	13	**12**	Income payment and Receipts	13
6	Industrial Production	27	**13**	Monetary Aggregate	9
7	Capacity Utilization	49	**14**	Gross Domestic Product Component	10

This table summarizes the group of US macroeconomic time series and their number of time series containing in each group.

APPENDIX B

Table 31: Policy Rule Based on Individual Variables of Germany

cst	*GDP*	*INF*
-9.5840	0.2174	-0.0922
[-6.2583]	[10.6642]	[-7.3601]

This table reports the estimation of short rate base on the output and inflation, where r denotes the short rate, GDP the monthly rate of real GDP, and INF the monthly rate of consumer price index. The sample period is 1993:01 to 2008:12. Test statistics are in brackets. The R^2 Adjusted of this regression is 0.42412

Table 32: Policy Rule Based on Individual Variables of the US

cst	*GDP*	*INF*
-3.8914	0.4557	-0.067
[-4.1237]	[16.6419]	[-3.7651]

This table reports the estimation of short rate base on the output and inflation, where r denotes the federal fund rate, GDP the monthly rate of real GDP, and INF the monthly rate of consumer price index. The sample period is 1992:01 to 2008:12. Test statistics are in brackets. The R^2 Adjusted of this regression is 0.614083

Table 33: Policy Rule Base on FAVAR's Factors of German

cst	*UF1*	*UF2*	*UF3*	*UF4*
3.0846	-0.1766	0.1084	-0.3552	0.3969
[6.4864]	[-4.2202]	[2.2590]	[-7.4939]	[9.0141]

This table reports the estimation of short rate base on the four factors directly extracted from a large panel of macroeconomic time series, where r denotes the federal fund rate and UF1 to UF4 the four macro factors directly extracted from a panel of about 337 monthly time series for German. The sample period is 1993:01 to 2008:12. Test statistics are in parentheses. The R^2 Adjusted of this regression is 0.498082

Table 34: Policy Rule Base on FAVAR's Factors of the US

cst	*UF1*	*UF2*	*UF3*	*UF4*
3.9444	0.0803	0.1868	-1.3646	0.1771
[8.1967]	[1.1815]	[2.7495]	[-9.0839]	[2.6064]

This table reports the estimation of short rate base on the four factors directly extracted from a large panel of macroeconomic time series, where r denotes the federal fund rate and UF1 to UF4 the four macro factors directly extracted from a panel of about 341 monthly time series for the USA. The sample period is 1992:01 to 2008:12. Test statistics are in parentheses. The R^2 Adjusted of this regression is 0.671581

Table 35: Policy Rule Base on GFAVAR's Factors of German

cst	*GF1*	*GF2*	*GF3*	*GF4*
3.0598	0.1592	-0.1896	0.2586	-0.2187
[6.0056]	[2.9251]	[-2.7933]	[4.3549]	[-3.0299]

This table reports the estimation of short rate base on the four factors extracted from groups of macroeconomic variables, where r denotes the short rate and GF1 to GF4 the four macro factors extracted from groups of 16 macroeconomic variables which all contain 337 monthly time series for Germany. The sample period is 1993:01 to 2008:12. Test statistics are in brackets. The R^2 Adjusted of this regression is 0.473222

Table 36: Policy Rule Base on GFAVAR's Factors of the US

cst	*GF1*	*GF2*	*GF3*	*GF4*
3.9444	0.2611	0.0804	-0.4775	-0.3576
[5.8345]	[2.3660]	[0.7288]	[-4.3272]	[-3.2405]

This table reports the estimation of short rate base on the four factors extracted from groups of macroeconomic variables, where r denotes the federal fund rate and GF1 to GF4 the four macro factors extracted from groups of 14 macroeconomic variables which all contain 341 monthly time series for the USA. The sample period is 1992:01 to 2008:12. Test statistics are in brackets. The R^2 Adjusted of this regression is 0. 650863

Table 37: Policy Rule Base on SGFAVAR's Factors of German

cst	*SF1*	*SF2*	*SF3*	*SF4*
3.1594	0.1123	-0.0664	-0.0693	0.1639
[8.6465]	[0.9599]	[-0.8229]	[-0.9025]	[2.4874]

This table reports the estimation of short rate base on the four factors extracted from groups of German macroeconomic variables that significantly explain the short rate, where r denotes the short rate and SF1 to SF4 the four macro factors extracted from 6 groups of macroeconomic variables which significantly explain the short rate. The sample period is 1993:01 to 2008:12. Test statistics are in brackets. The R^2 Adjusted of this regression is 0.428898

Table 38: Policy Rule Base on SGFAVAR's Factors of the US

cst	*SF1*	*SF2*	*SF3*	*SF4*
3.9444	-0.4914	-0.5726	-0.3670	0.0236
[8.0514]	[-4.7287]	[-5.5101]	[-3.5319]	[0.2273]

This table reports the estimation of short rate base on the four factors extracted from groups of US macroeconomic variables that significantly explain the short rate, where r denotes the federal fund rate and SF1 to SF4 the four macro factors extracted from 6 groups of macroeconomic variables which significantly explain the short rate. The sample period is 1992:01 to 2008:12. Test statistics are in brackets. The R^2 Adjusted of this regression is 0. 631785

Table 39: Policy Rule Base on LSGFAVAR's Factors of German

cst	*LF1*	*LF2*	*LF3*	*LF4*
3.2029	-0.0637	-0.0416	-0.0735	0.2037
[4.9901]	[-0.6168]	[-0.5591]	[-1.1672]	[2.6444]

This table reports the estimation of short rate base on the four factors extracted from groups of German macroeconomic variables that significantly explain the 10-year rate, where r denotes the short rate and LF1 to LF4 the four macro factors extracted from 8 groups of macroeconomic variables which significantly explain the 10-year rate. The sample period is 1993:01 to 2008:12. Test statistics are in brackets. The R^2 Adjusted of this regression is 0.426404

Table 40: Policy Rule Base on LSGFAVAR's Factors of the US

cst	*LF1*	*LF2*	*LF3*	*LF4*
3.9444	0.2518	0.1037	-0.5325	0.7721
[4.4714]	[2.5777]	[1.0618]	[-5.4506]	[7.9024]

This table reports the estimation of short rate base on the four factors extracted from groups of US macroeconomic variables that significantly explain the 10-year rate, where r denotes the federal fund rate and LF1 to LF4 the four macro factors extracted from 13 groups of macroeconomic variables which significantly explain the 10-year rate. The sample period is 1992:01 to 2008:12. Test statistics are in brackets. The R^2 Adjusted of this regression is 0. 620916

APPENDIX C

Table 41: Unrestricted Regressions of German Yields on FAVAR's Factors

	$y^{(6)}$	$y^{(12)}$	$y^{(36)}$	$y^{(60)}$	$y^{(120)}$
cst	3.7523	3.8283	4.0526	4.4281	5.0109
	[4.2418]	[7.8938]	[9.7594]	[8.5902]	[8.6809]
UF1	-0.1139	-0.1366	-0.0374	0.0105	0.0775
	[-1.3399]	[-1.7049]	[-0.5505]	[0.1617]	[1.2599]
UF2	-0.2512	-0.1639	-0.1199	-0.2101	-0.3465
	[-2.9545]	[-2.0452]	[-1.7639]	[-3.2462]	[-5.6331]
UF3	-0.0468	-0.0266	0.4082	0.5207	0.6133
	[-0.5501]	[-0.3314]	[6.0034]	[8.0438]	[9.9712]
UF4	0.2707	0.2941	0.2801	0.2683	0.2396
	[3.1829]	[3.6695]	[4.1195]	[4.1456]	[3.8956]
R-square	**0.6807**	**0.6495**	**0.5618**	**0.5208**	**0.4716**

This table summarizes the results of an unrestricted prediction of yields of different maturities on the four macro factors directly extracted from a large panel of German macroeconomic time series. The estimate period is 1993:01 to 2008:12. Test statistics are in brackets.

Table 42: Unrestricted Regressions of the US Yields on FAVAR's Factors

	$y^{(6)}$	$y^{(12)}$	$y^{(36)}$	$y^{(60)}$	$y^{(120)}$
cst	3.9376	4.0865	4.6202	4.9665	5.3858
	[6.7269]	[6.5914]	[7.4515]	[9.8959]	[11.8424]
UF1	0.2196	0.2518	0.2991	0.2900	0.2696
	[3.4890]	[3.9708]	[4.9367]	[5.2949]	[5.7351]
UF2	0.2870	0.3594	0.5944	0.6547	0.6485
	[4.5602]	[5.6660]	[9.8114]	[11.9525]	[13.7942]
UF3	-1.3185	-1.2384	-0.9210	-0.6439	-0.3361
	[-13.9528]	[-12.5259]	[-10.2021]	[-11.7549]	[-7.1496]
UF4	0.2238	0.2529	0.3667	0.4221	0.4647
	[3.5560]	[3.9880]	[6.0525]	[7.7059]	[9.8851]
R-square	**0.7089**	**0.6910**	**0.6612**	**0.6493**	**0.6514**

This table summarizes the results of an unrestricted VAR of yields of different maturities on the four macro factors directly extracted from a large panel of US macroeconomic time series. The estimate period is 1992:01 to 2008:12. Test statistics are in brackets.

Table 43: Unrestricted Regressions of German Yields on GFAVAR's Factors

	$y^{(6)}$	$y^{(12)}$	$y^{(36)}$	$y^{(60)}$	$y^{(120)}$
cst	3.7523	3.8283	4.0526	4.4281	5.0109
	[6.6292]	[5.2935]	[6.1139]	[8.8944]	[7.7694]
GF1	0.1162	0.1501	0.0935	0.0507	-0.0170
	[1.4405]	[1.9661]	[1.4293]	[0.7862]	[-0.2626]
GF2	0.3055	0.2201	0.1446	0.2199	0.3340
	[3.7865]	[2.8837]	[2.2103]	[3.4126]	[5.1708]
GF3	0.0400	0.0581	-0.2030	-0.2358	-0.2511
	[0.4957]	[0.7607]	[-3.1031]	[-3.6594]	[-3.8867]
GF4	0.4219	0.4127	0.5043	0.5364	0.5583
	[5.2296]	[5.4070]	[7.7089]	[8.3236]	[8.6428]
R-square	**0.6405**	**0.6133**	**0.5189**	**0.4667**	**0.4140**

This table summarizes the results of an unrestricted prediction of yields of different maturities on the four macro factors (GF1, GF2, GF3 and GF4) extracted from the group of German macroeconomic variables. The estimate period is 1993:01 to 2008:12. Test statistics are in brackets.

Table 44: Unrestricted Regressions of the US Yields on GFAVAR's Factors

	$y^{(6)}$	$y^{(12)}$	$y^{(36)}$	$y^{(60)}$	$y^{(120)}$
cst	3.9376	4.0865	4.6202	4.9665	5.3858
	[7.9290]	[4.8480]	[5.8042]	[7.7987]	[9.4516]
GF1	0.4209	0.4639	0.5289	0.5009	0.4445
	[4.0445]	[4.6261]	[6.1441]	[6.8210]	[7.3636]
GF2	0.1530	0.2197	0.4656	0.5496	0.5727
	[1.4702]	[2.1906]	[5.4090]	[7.4837]	[9.4876]
GF3	-0.4937	-0.4807	-0.3611	-0.2291	-0.0845
	[-4.7437]	[-4.7929]	[-4.1948]	[-3.1195]	[-1.3996]
GF4	-0.3280	-0.3075	-0.2328	-0.1548	-0.0669
	[-3.1513]	[-3.0665]	[-2.7049]	[-2.1086]	[-1.1087]
R-square	**0.6683**	**0.6473**	**0.6195**	**0.5696**	**0.5257**

This table summarizes the results of an unrestricted prediction of yields of different maturities on the four macro factors (GF1, GF2, GF3 and GF4) extracted from the group of US macroeconomic variables. The estimate period is 1992:01 to 2008:12. Test statistics are in brackets.

Table 45: Unrestricted Regressions of German Yields on SGFAVAR's Factors

	$y^{(6)}$	$y^{(12)}$	$y^{(36)}$	$y^{(60)}$	$y^{(120)}$
cst	3.7523	3.8283	4.0526	4.4281	5.0109
	[5.1017]	[8.2620]	[8.9100]	[7.6532]	[8.6888]
SF1	-0.6557	-0.5450	-0.4452	-0.4981	-0.5800
	[-9.6044]	[-8.2721]	[-7.5498]	[-8.3760]	[-9.4303]
SF2	-0.2519	-0.2853	-0.2793	-0.2891	-0.2855
	[-3.6891]	[-4.3307]	[-4.7373]	[-4.8608]	[-4.6420]
SF3	-0.3551	-0.3432	-0.4362	-0.4177	-0.3766
	[-5.2009]	[-5.2098]	[-7.3985]	[-7.0231]	[-6.1230]
SF4	0.1117	0.1611	0.0933	0.0574	-0.0072
	[1.6357]	[2.4459]	[1.5820]	[0.9648]	[-0.1171]
R-square	**0.6503**	**0.6214**	**0.4522**	**0.3951**	**0.3517**

This table summarizes the results of an unrestricted prediction of yields of different maturities on the four macro factors (SF1, SF2, SF3 and SF4) extracted from the group of German macroeconomic variables that significantly explain the short rate. The estimate period is 1993:01 to 2008:12. Test statistics are in brackets.

Table 46: Unrestricted Regressions of the US Yields on SGFAVAR's Factors

	$y^{(6)}$	$y^{(12)}$	$y^{(36)}$	$y^{(60)}$	$y^{(120)}$
cst	3.9376	4.0865	4.6202	4.9665	5.3858
	[4.7711]	[5.3370]	[9.5250]	[7.6686]	[8.5477]
SF1	-0.5923	-0.5891	-0.4808	-0.3678	-0.2503
	[-6.2682]	[-6.5196]	[-6.1786]	[-5.3682]	[-4.1051]
SF2	-0.6447	-0.6698	-0.7533	-0.7146	-0.6140
	[-6.8222]	[-7.4124]	[-9.6817]	[-10.4305]	[-10.0703]
SF3	-0.3966	-0.3856	-0.3293	-0.2815	-0.2236
	[-4.1973]	[-4.2679]	[-4.2318]	[-4.1081]	[-3.6673]
SF4	-0.0794	-0.1467	-0.2096	-0.2078	-0.1804
	[-0.8397]	[-1.6230]	[-2.6936]	[-3.0337]	[-2.9579]
R-square	**0.6976**	**0.6631**	**0.6430**	**0.5529**	**0.4779**

This table summarizes the results of an unrestricted prediction of yields of different maturities on the four macro factors (SF1, SF2, SF3 and SF4) extracted from the group of US macroeconomic variables that significantly explain the short rate. The estimate period is 1992:01 to 2008:12. Test statistics are in brackets.

Table 47: Unrestricted Regressions of German Yields on LSGFAVAR's Factors

	$y^{(6)}$	$y^{(12)}$	$y^{(36)}$	$y^{(60)}$	$y^{(120)}$
cst	3.7523	3.8283	4.0526	4.4281	5.0109
	[7.2241]	[6.2971]	[9.5972]	[7.5009]	[8.6290]
LF1	0.5572	0.4641	0.4520	0.5228	0.6162
	[8.4749]	[7.2908]	[7.7429]	[8.8915]	[9.1350]
LF2	-0.0877	-0.1390	-0.1766	-0.1886	-0.1850
	[-1.3343]	[-2.1830]	[-3.0243]	[-3.2069]	[-3.0427]
LF3	-0.0935	-0.0662	0.1665	0.1825	0.1736
	[-1.4224]	[-1.0393]	[2.8519]	[3.1032]	[2.8549]
LF4	0.6071	0.5805	0.4741	0.4266	0.3628
	[9.2349]	[9.1187]	[8.1202]	[7.2548]	[5.9677]
R-square	**0.6625**	**0.6319**	**0.5436**	**0.4778**	**0.4544**

This table summarizes the results of an unrestricted prediction of yields of different maturities on the four macro factors (LF1, LF2, LF3 and LF4) extracted from the group of German macroeconomic variables that significantly explain the 10-year rate. The estimate period is 1993:01 to 2008:12. Test statistics are in brackets.

Table 48: Unrestricted Regressions of the US Yields on LSGFAVAR's Factors

	$y^{(6)}$	$y^{(12)}$	$y^{(36)}$	$y^{(60)}$	$y^{(120)}$
cst	3.8495	3.9932	4.5068	4.8394	5.2402
	[4.9247]	[8.5594]	[6.2248]	[7.4390]	[9.8030]
LF1	0.3978	0.4383	0.5024	0.4776	0.4253
	[4.6305]	[5.3164]	[6.9192]	[7.5258]	[7.8378]
LF2	0.1713	0.2341	0.4609	0.5309	0.5417
	[1.9940]	[2.8401]	[6.3475]	[8.3654]	[9.9833]
LF3	-0.4960	-0.4711	-0.3564	-0.2412	-0.1128
	[-5.7747]	[-5.7141]	[-4.9082]	[-3.7998]	[-2.0788]
LF4	0.7893	0.7656	0.5819	0.4231	0.2474
	[9.1888]	[9.2869]	[8.0138]	[6.6664]	[4.5585]
R-square	**0.7015**	**0.6882**	**0.6600**	**0.6124**	**0.5834**

This table summarizes the results of an unrestricted prediction of yields of different maturities on the four macro factors (LF1, LF2, LF3 and LF4) extracted from the group of US macroeconomic variables that significantly explain the 10-year rate. The estimate period is 1992:01 to 2008:12. Test statistics are in brackets.

APPENDIX D

Table 49: German's In Sample Fit of FAVAR Model: Observed and Model-Implied Yield for a Large Panel of German Time series

	$y^{(1)}$	$y^{(3)}$	$y^{(6)}$	$y^{(9)}$	$y^{(12)}$	$y^{(24)}$	$y^{(36)}$	$y^{(48)}$	$y^{(60)}$	$y^{(84)}$	$y^{(120)}$
Mean											
y_t	3.678	3.729	3.752	3.782	3.828	3.843	4.053	4.251	4.428	4.714	5.011
$\hat{y}_t$	3.678	3.728	3.772	3.795	3.784	3.860	4.060	4.246	4.411	4.730	5.008
$\|y_t - \hat{y}_t\|$	0.000	0.078	0.137	0.188	0.226	0.313	0.372	0.418	0.452	0.491	0.521
Standard Deviation											
y_t	1.312	1.283	1.226	1.179	1.155	1.071	1.061	1.069	1.082	1.107	1.126
$\hat{y}_t$	1.312	1.277	1.193	1.135	1.094	0.975	0.962	0.969	0.972	0.952	0.865
$\|y_t - \hat{y}_t\|$	0.000	0.080	0.109	0.138	0.176	0.258	0.282	0.304	0.317	0.341	0.369

This table summarizes means and standard deviations of German observed and fitted yields. Yields are reported in percentage terms. The first and second row in each panel report the perspective moment of observed yield and fitted values implied by the No-Arbitrage FAVAR model while the third row the mean and standard deviation of absolute fitting errors are reported, respectively.

Table 50: German's In Sample Fit of GFAVAR Model: Observed and Model-Implied Yield for Group of German Macroeconomic Data

	$y^{(1)}$	$y^{(3)}$	$y^{(6)}$	$y^{(9)}$	$y^{(12)}$	$y^{(24)}$	$y^{(36)}$	$y^{(48)}$	$y^{(60)}$	$y^{(84)}$	$y^{(120)}$
Mean											
y_t	3.678	3.729	3.752	3.782	3.828	3.843	4.053	4.251	4.428	4.714	5.011
$\hat{y}_t$	3.678	3.661	3.770	3.786	3.803	3.885	4.032	4.228	4.431	4.731	5.002
$\lvert y_t - \hat{y}_t \rvert$	0.000	0.097	0.169	0.210	0.249	0.386	0.469	0.536	0.590	0.655	0.694
Standard Deviation											
y_t	1.312	1.283	1.226	1.179	1.155	1.071	1.061	1.069	1.082	1.107	1.126
$\hat{y}_t$	1.312	1.262	1.184	1.130	1.079	0.973	0.929	0.891	0.850	0.753	0.640
$\lvert y_t - \hat{y}_t \rvert$	0.000	0.109	0.131	0.149	0.186	0.314	0.356	0.384	0.398	0.424	0.481

This table summarizes means and standard deviations of German observed and fitted yields. Yields are reported in percentage terms. The first and second row in each panel report the perspective moment of observed yield and fitted values implied by GFAVAR model while the third row the mean and standard deviation of absolute fitting errors are reported, respectively.

Table 51: German's In Sample Fit of SGFAVAR Model: Observed and Model-Implied Yield for Significant Group of German Macroeconomic Data

	$y^{(1)}$	$y^{(3)}$	$y^{(6)}$	$y^{(9)}$	$y^{(12)}$	$y^{(24)}$	$y^{(36)}$	$y^{(48)}$	$y^{(60)}$	$y^{(84)}$	$y^{(120)}$
Mean											
y_t	3.678	3.729	3.752	3.782	3.828	3.843	4.053	4.251	4.428	4.714	5.011
$\hat{y}_t$	3.678	3.745	3.687	3.736	3.785	3.938	4.059	4.205	4.385	4.767	4.996
$\lvert y_t - \hat{y}_t \rvert$	0.000	0.111	0.170	0.235	0.288	0.421	0.490	0.548	0.594	0.674	0.704
Standard Deviation											
y_t	1.312	1.283	1.226	1.179	1.155	1.071	1.061	1.069	1.082	1.107	1.126
$\hat{y}_t$	1.312	1.246	1.181	1.107	1.058	0.971	0.896	0.840	0.796	0.725	0.585
$\lvert y_t - \hat{y}_t \rvert$	0.000	0.111	0.142	0.167	0.188	0.344	0.378	0.414	0.441	0.456	0.511

This table summarizes means and standard deviations of German observed and fitted yields. Yields are reported in percentage terms. The first and second row in each panel report the perspective moment of observed yield and fitted values implied by SGFAVAR model while the third row the mean and standard deviation of absolute fitting errors are reported, respectively.

Table 52: German's In Sample Fit of LSGFAVAR Model: Observed and Model-Implied Yield for Long Term Significant Group of German Macroeconomic Data

	$y^{(1)}$	$y^{(3)}$	$y^{(6)}$	$y^{(9)}$	$y^{(12)}$	$y^{(24)}$	$y^{(36)}$	$y^{(48)}$	$y^{(60)}$	$y^{(84)}$	$y^{(120)}$
Mean											
y_t	3.678	3.729	3.752	3.782	3.828	3.843	4.053	4.251	4.428	4.714	5.011
$\hat{y}_t$	3.678	3.802	3.695	3.701	3.894	3.920	4.048	4.225	4.419	4.728	5.005
$\|y_t - \hat{y}_t\|$	0.000	0.126	0.185	0.267	0.290	0.411	0.434	0.538	0.545	0.619	0.622
Standard Deviation											
y_t	1.312	1.283	1.226	1.179	1.155	1.071	1.061	1.069	1.082	1.107	1.126
$\hat{y}_t$	1.312	1.269	1.197	1.166	1.070	0.973	0.959	0.897	0.814	0.792	0.709
$\|y_t - \hat{y}_t\|$	0.000	0.135	0.149	0.173	0.181	0.348	0.350	0.374	0.406	0.418	0.421

This table summarizes means and standard deviations of German observed and fitted yields. Yields are reported in percentage terms. The first and second row in each panel report the perspective moment of observed yield and fitted values implied by LSGFAVAR model while the third row the mean and standard deviation of absolute fitting errors are reported, respectively.

Table 53: US's In Sample Fit of FAVAR Model: Observed and Model-Implied Yield for a Large Panel of US Macroeconomic Time series

	$y^{(1)}$	$y^{(3)}$	$y^{(6)}$	$y^{(12)}$	$y^{(24)}$	$y^{(36)}$	$y^{(60)}$	$y^{(84)}$	$y^{(120)}$
Mean									
y_t	3.682	3.770	3.938	4.086	4.416	4.620	4.966	5.215	5.386
$\hat{y}_t$	3.682	3.845	3.915	4.094	4.395	4.635	4.977	5.222	5.382
$\lvert y_t - \hat{y}_t \rvert$	0.000	0.207	0.239	0.294	0.378	0.413	0.430	0.423	0.416
Standard Deviation									
y_t	1.556	1.633	1.645	1.610	1.558	1.468	1.305	1.221	1.123
$\hat{y}_t$	1.556	1.615	1.554	1.525	1.509	1.369	1.119	1.068	0.895
$\lvert y_t - \hat{y}_t \rvert$	0.000	0.205	0.220	0.249	0.326	0.349	0.356	0.372	0.385

This table summarizes means and standard deviations of the US observed and fitted yields. Yields are reported in percentage terms. The first and second row in each panel report the perspective moment of observed yield and fitted values implied by the No-Arbitrage FAVAR model while the third row the mean and standard deviation of absolute fitting errors are reported, respectively

Table 54: US's In Sample Fit of GFAVAR Model: Observed and Model-Implied Yield for Group of US Macroeconomic Data

	$y^{(1)}$	$y^{(3)}$	$y^{(6)}$	$y^{(12)}$	$y^{(24)}$	$y^{(36)}$	$y^{(60)}$	$y^{(84)}$	$y^{(120)}$
Mean									
y_t	3.682	3.770	3.938	4.086	4.416	4.620	4.966	5.215	5.386
$\hat{y}_t$	3.682	3.876	3.928	4.082	4.413	4.631	4.967	5.199	5.392
$\lvert y_t - \hat{y}_t \rvert$	0.000	0.195	0.218	0.275	0.356	0.402	0.476	0.510	0.544
Standard Deviation									
y_t	1.556	1.633	1.645	1.610	1.558	1.468	1.305	1.221	1.123
$\hat{y}_t$	1.556	1.645	1.647	1.541	1.529	1.376	1.187	0.939	0.834
$\lvert y_t - \hat{y}_t \rvert$	0.000	0.124	0.187	0.246	0.314	0.333	0.347	0.402	0.440

This table summarizes means and standard deviations of the US observed and fitted yields. Yields are reported in percentage terms. The first and second row in each panel report the perspective moment of observed yield and fitted values implied by the GFAVAR model while the third row the mean and standard deviation of absolute fitting errors are reported, respectively.

Table 55: US's In Sample Fit of SGFAVAR Model: Observed and Model-Implied Yield for Significant Group of US Macroeconomic Data

	$y^{(1)}$	$y^{(3)}$	$y^{(6)}$	$y^{(12)}$	$y^{(24)}$	$y^{(36)}$	$y^{(60)}$	$y^{(84)}$	$y^{(120)}$
Mean									
y_t	3.682	3.770	3.938	4.086	4.416	4.620	4.966	5.215	5.386
$\hat{y}_t$	3.682	3.812	3.893	4.095	4.381	4.651	4.951	5.232	5.378
$\lvert y_t - \hat{y}_t \rvert$	0.000	0.234	0.253	0.330	0.466	0.524	0.589	0.638	0.661
Standard Deviation									
y_t	1.556	1.633	1.645	1.610	1.558	1.468	1.305	1.221	1.123
$\hat{y}_t$	1.556	1.661	1.662	1.486	1.392	1.317	1.052	0.821	0.618
$\lvert y_t - \hat{y}_t \rvert$	0.000	0.224	0.283	0.320	0.384	0.416	0.457	0.465	0.485

This table summarizes means and standard deviations of the US observed and fitted yields. Yields are reported in percentage terms. The first and second row in each panel report the perspective moment of observed yield and fitted values implied by the SGFAVAR model while the third row the mean and standard deviation of absolute fitting errors are reported, respectively.

Table 56: US's In Sample Fit of LSGFAVAR Model: Observed and Model-Implied Yield for Long Term Significant Group of US Macroeconomic Data

	$y^{(1)}$	$y^{(3)}$	$y^{(6)}$	$y^{(12)}$	$y^{(24)}$	$y^{(36)}$	$y^{(60)}$	$y^{(84)}$	$y^{(120)}$
Mean									
y_t	3.682	3.770	3.938	4.086	4.416	4.620	4.966	5.215	5.386
$\hat{y}_t$	3.682	3.824	3.862	4.097	4.385	4.644	4.956	5.229	5.380
$\lvert y_t - \hat{y}_t \rvert$	0.000	0.211	0.248	0.339	0.397	0.418	0.420	0.426	0.429
Standard Deviation									
y_t	1.556	1.633	1.645	1.610	1.558	1.468	1.305	1.221	1.123
$\hat{y}_t$	1.556	1.639	1.651	1.502	1.486	1.342	1.142	0.945	0.861
$\lvert y_t - \hat{y}_t \rvert$	0.000	0.235	0.293	0.348	0.362	0.374	0.395	0.401	0.410

This table summarizes means and standard deviations of the US observed and fitted yields. Yields are reported in percentage terms. The first and second row in each panel report the perspective moment of observed yield and fitted values implied by the LSGFAVAR model while the third row the mean and standard deviation of absolute fitting errors are reported, respectively.

APPENDIX E

Figure 13: Observed and Model-implied yields of German. This figure provides plots of observed and model-implied time series for all interest rates data, the 3-, 6-, 9-month yield, the 12-month yield and 2, 3, 4, 5, 8 and 10 year yields.

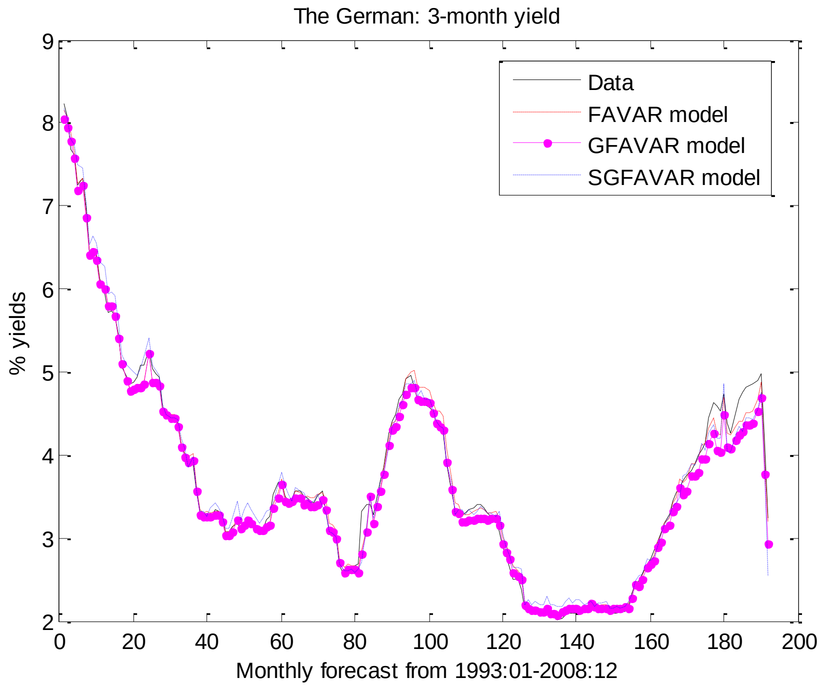

Figure 13a

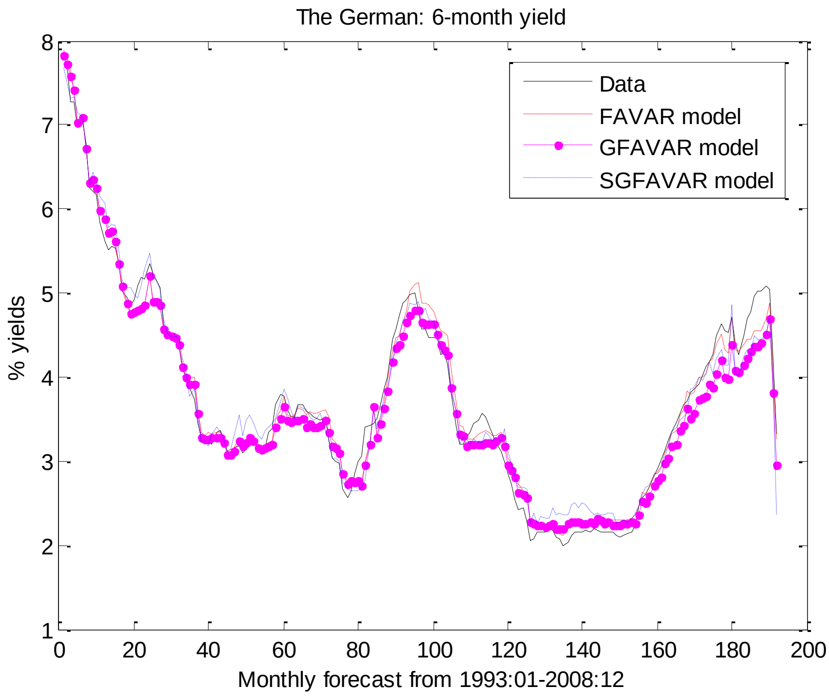

Figure 13b

Figure 13: Observed and Model-implied yields of German. This figure provides plots of observed and model-implied time series for all interest rates data, the 3-, 6-, 9-month yield, the 12-month yield and 2, 3, 4, 5, 8 and 10 year yields.

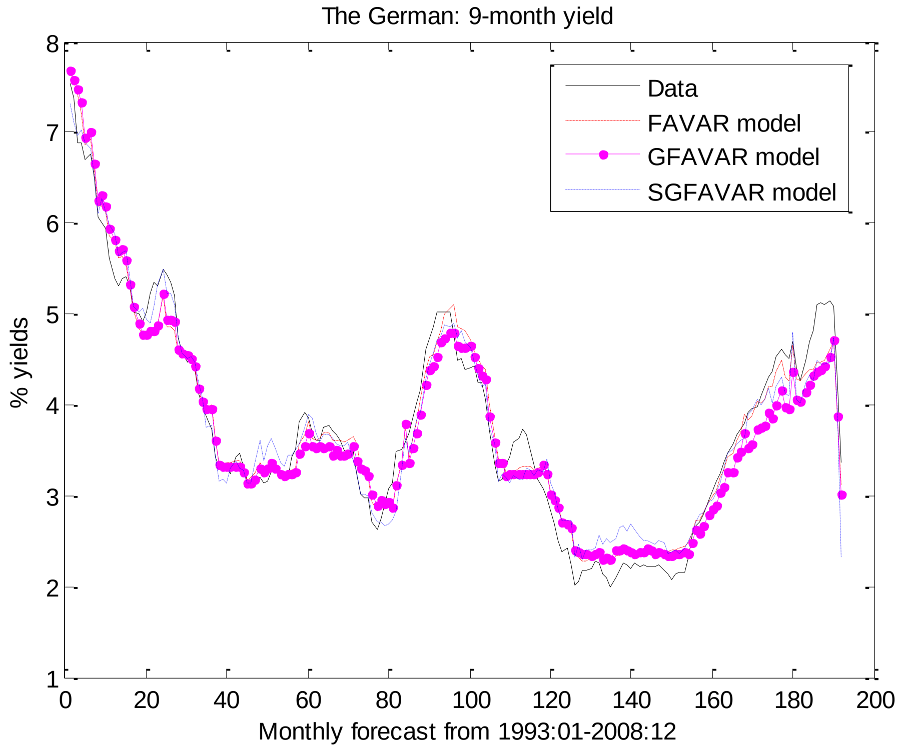

Figure 13c

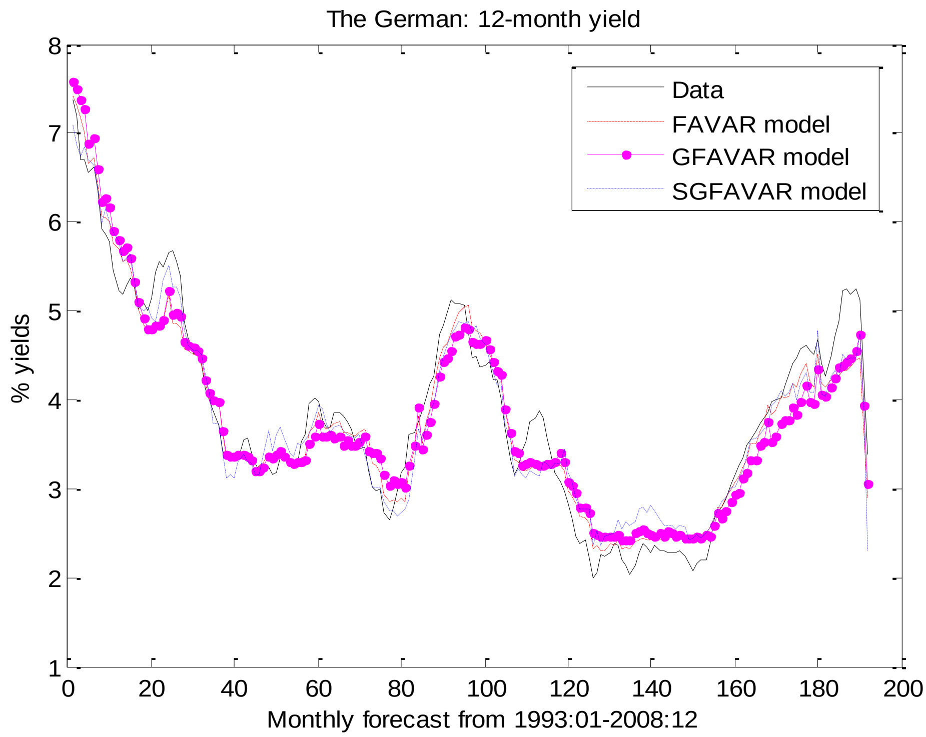

Figure 13d

Figure 13: Observed and Model-implied yields of German. This figure provides plots of observed and model-implied time series for all interest rates data, the 3-, 6-, 9-month yield, the 12-month yield and 2, 3, 4, 5, 8 and 10 year yields.

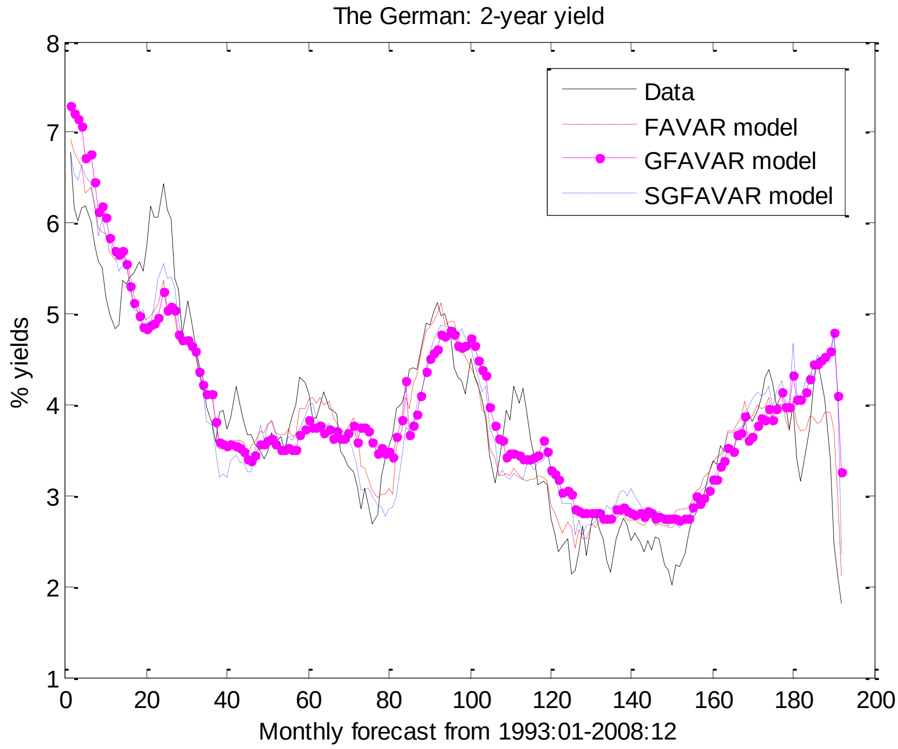

Figure 13e

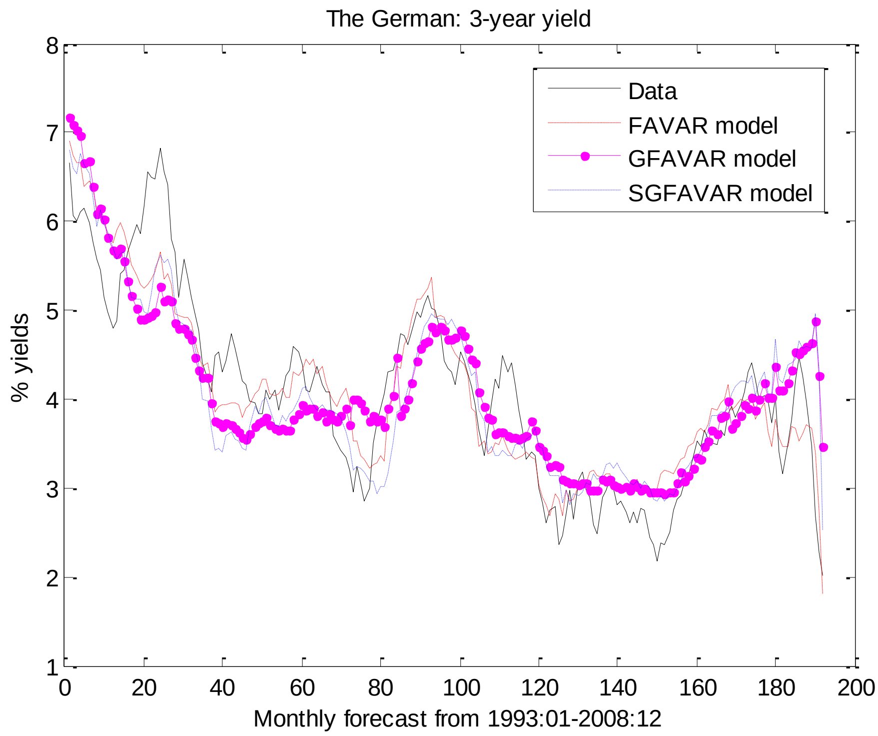

Figure 13f

Figure 13: Observed and Model-implied yields of German. This figure provides plots of observed and model-implied time series for all interest rates data, the 3-, 6-, 9-month yield, the 12-month yield and 2, 3, 4, 5, 8 and 10 year yields.

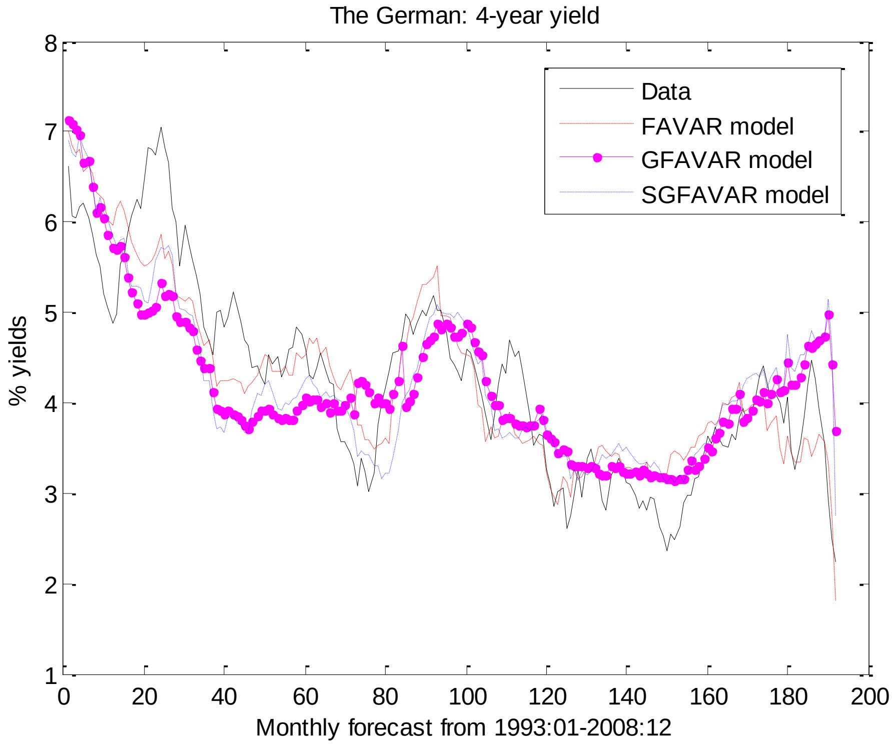

Figure 13g

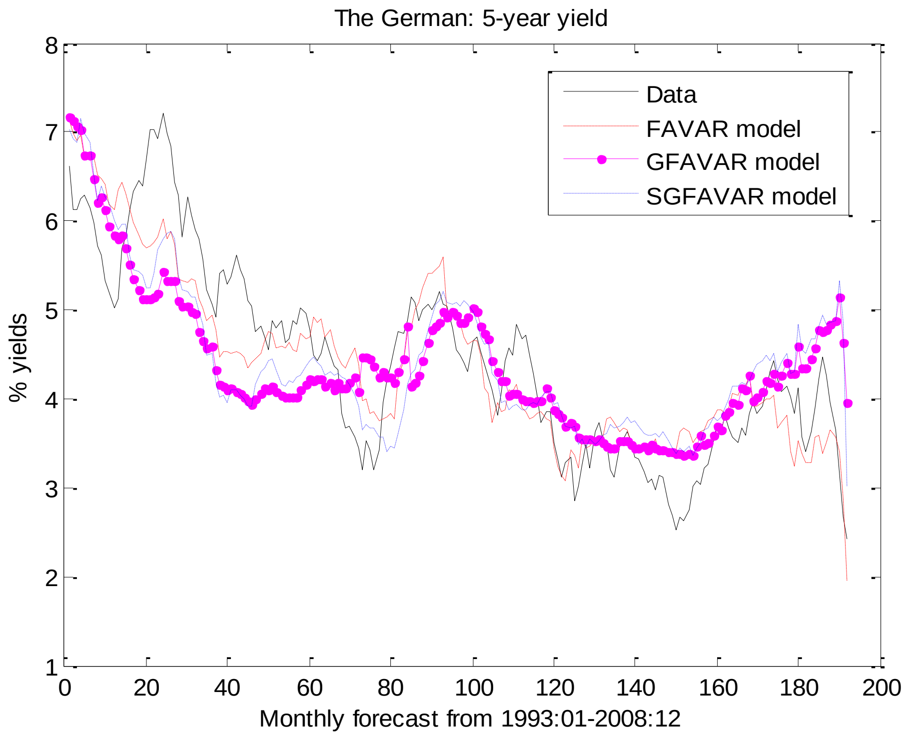

Figure 13h

Figure 13: Observed and Model-implied yields of German. This figure provides plots of observed and model-implied time series for all interest rates data, the 3-, 6-, 9-month yield, the 12-month yield and 2, 3, 4, 5, 8 and 10 year yields

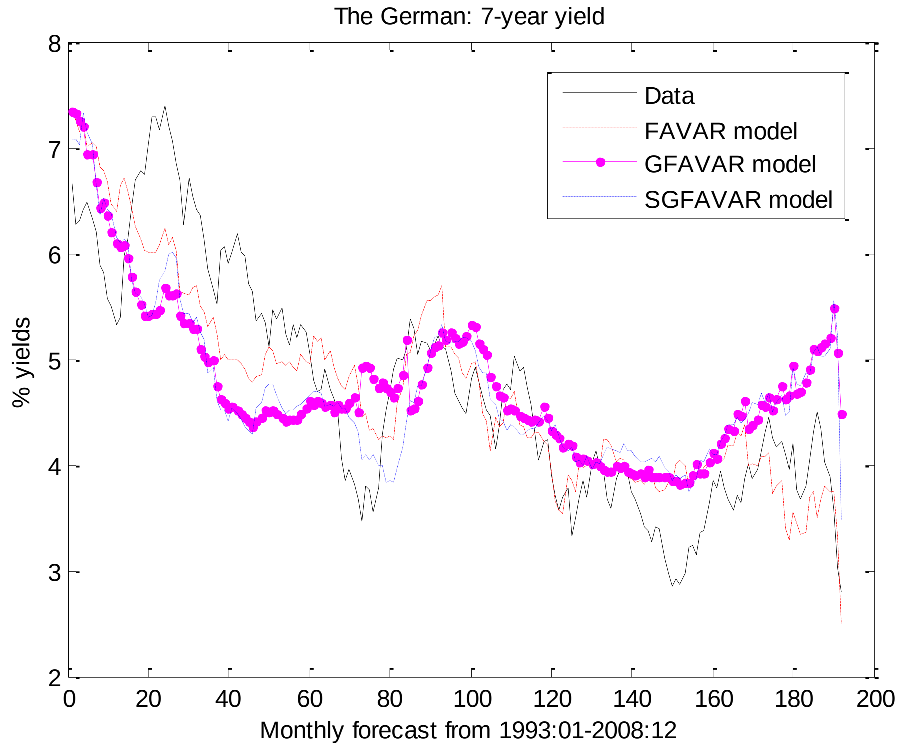

Figure 13i

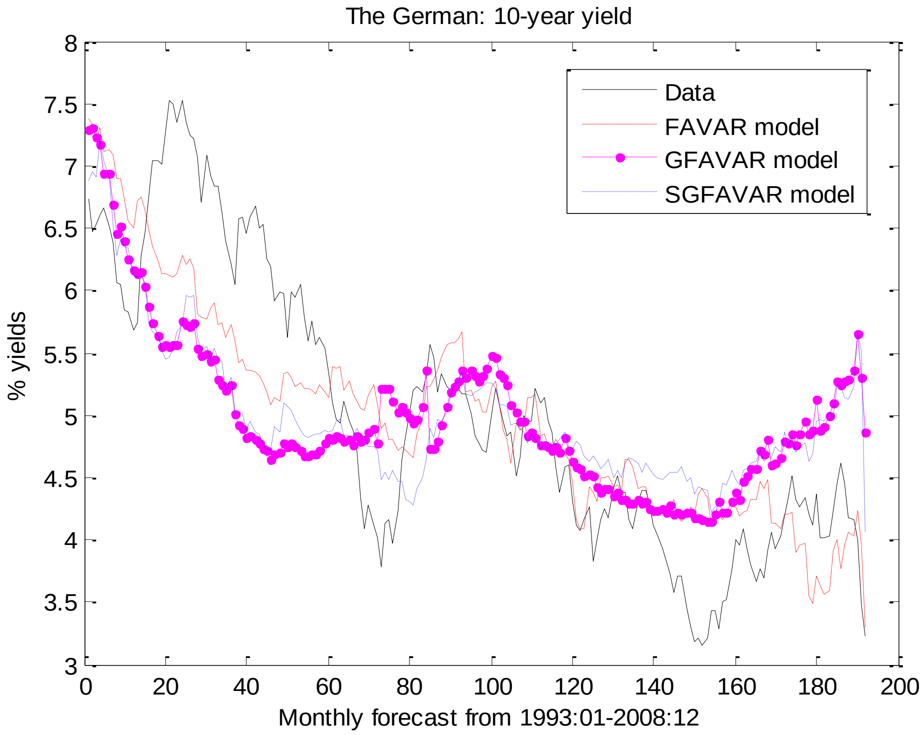

Figure 13j

Figure 14: Observed and Model-implied yields of the United State of America. This figure provides plots of observed and model-implied time series for all interest rates data, the 3-, 6-month yield, the 12-month yield and 2, 3, 5, 7 and 10 year yields.

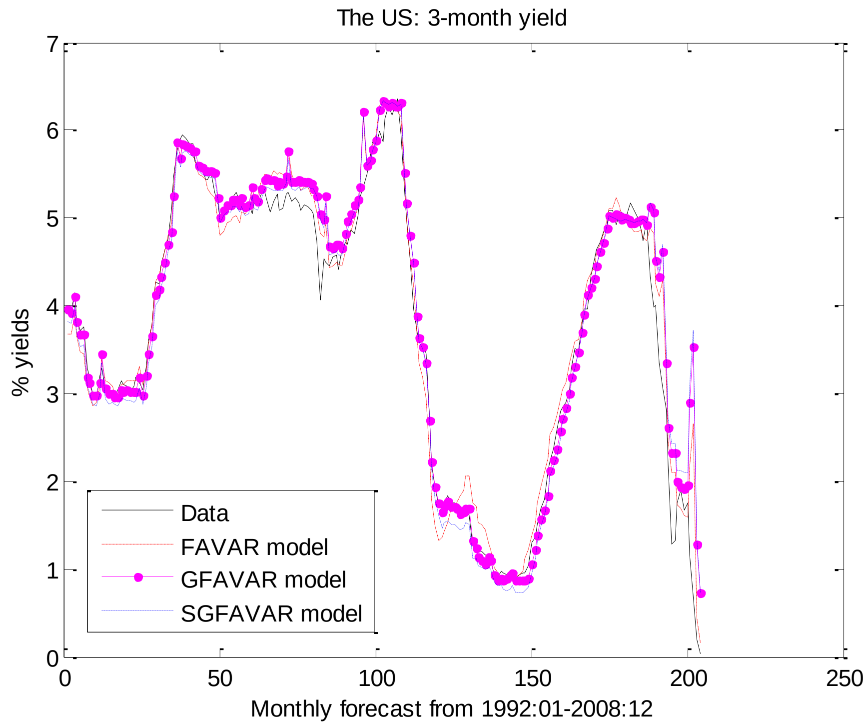

Figure 14a

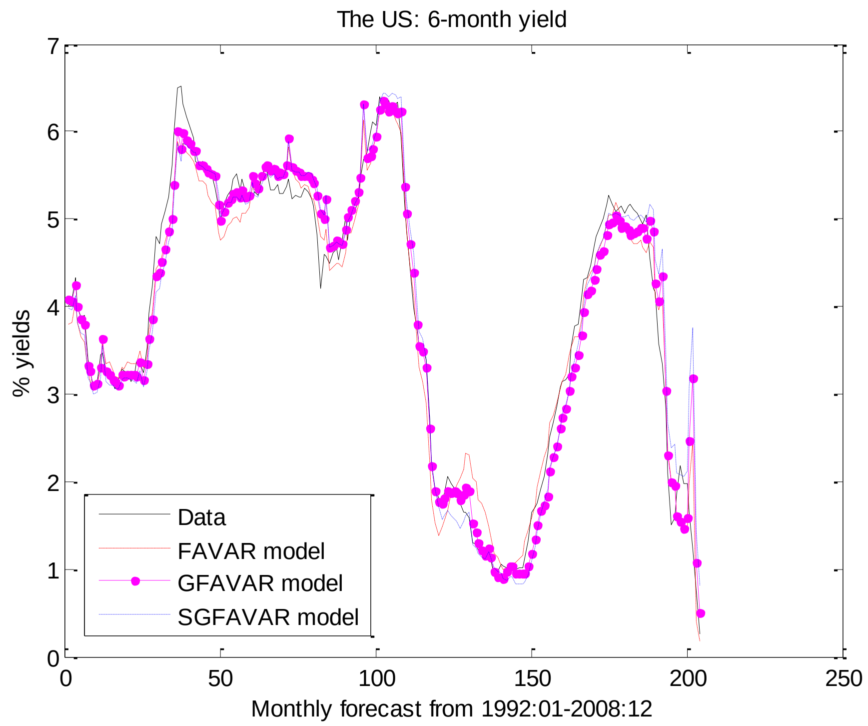

Figure 14b

Figure 14: Observed and Model-implied yields of the United State of America. This figure provides plots of observed and model-implied time series for all interest rates data, the 3-, 6-month yield, the 12-month yield and 2, 3, 5, 7 and 10 year yields.

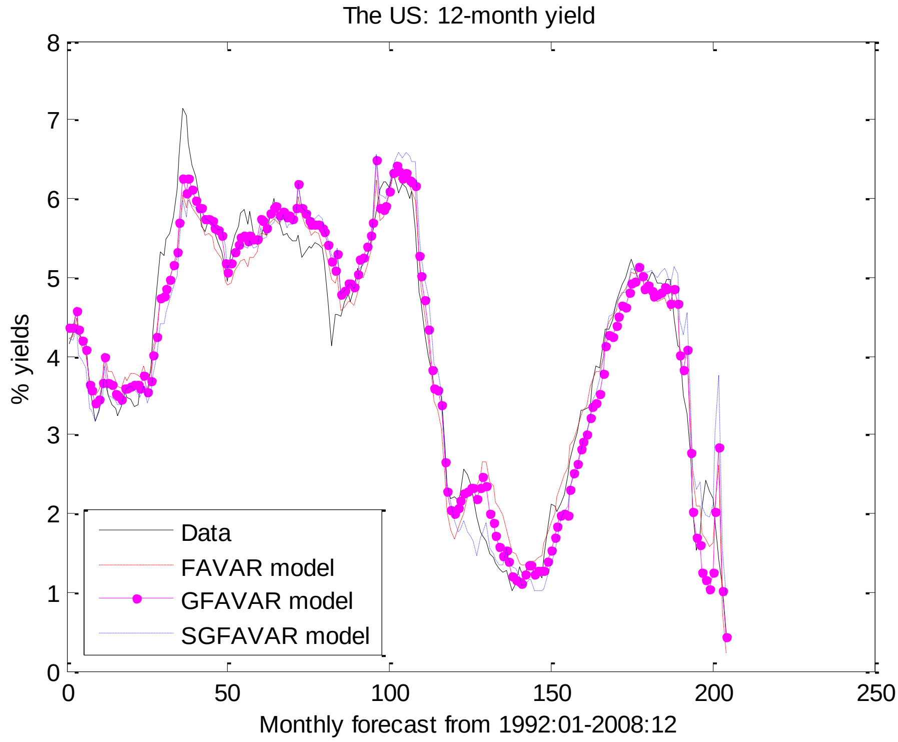

Figure 14c

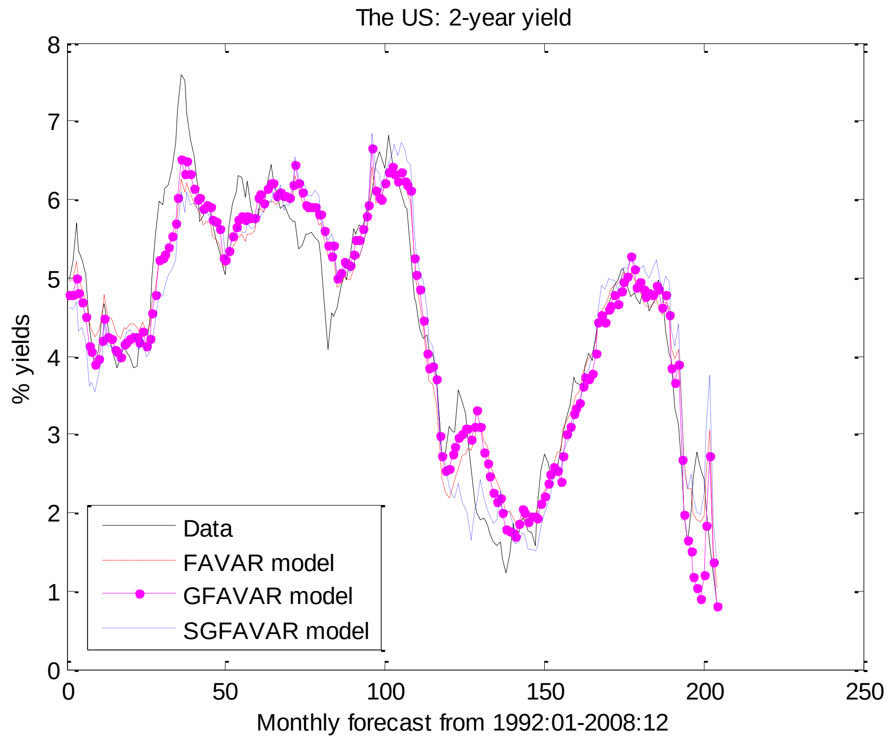

Figure 14d

Figure 14: Observed and Model-implied yields of the United State of America. This figure provides plots of observed and model-implied time series for all interest rates data, the 3-, 6-month yield, the 12-month yield and 2, 3, 5, 7 and 10 year yields.

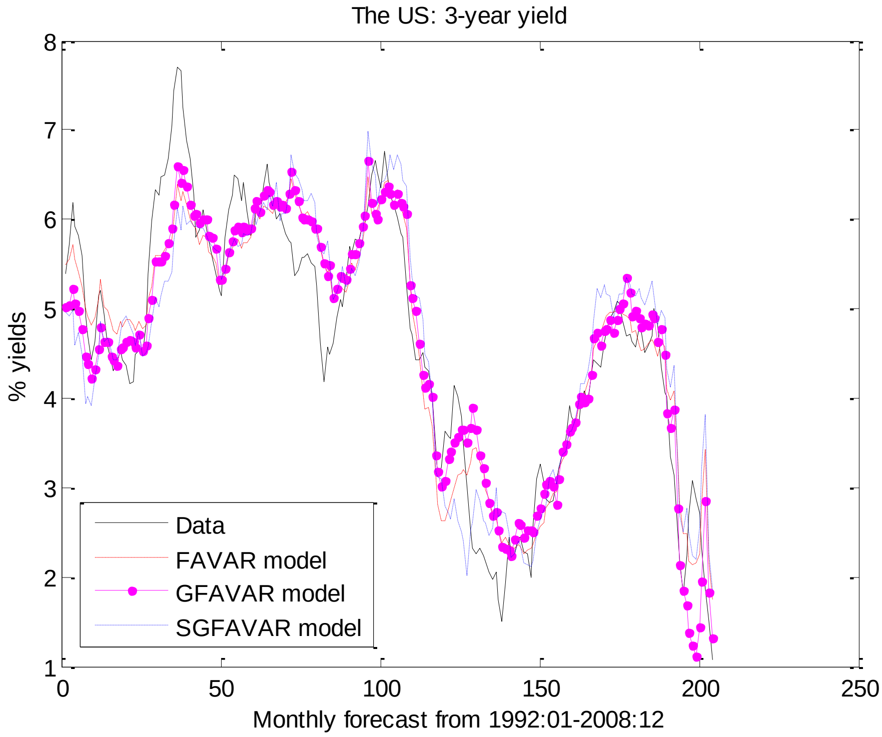

Figure 14e

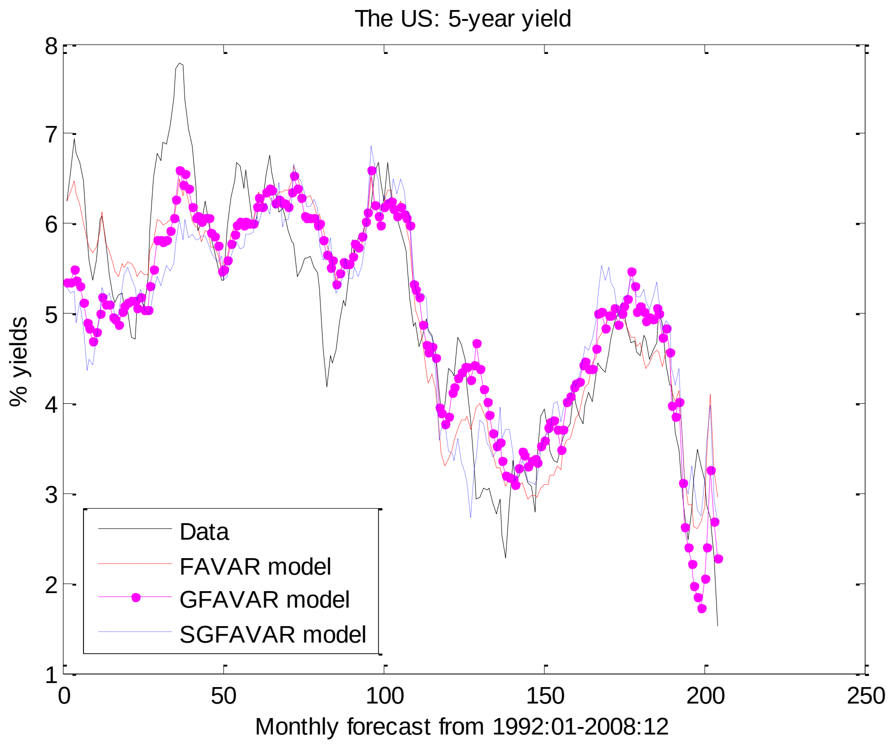

Figure 14f

Figure 14: Observed and Model-implied yields of the United State of America. This figure provides plots of observed and model-implied time series for all interest rates data, the 3-, 6-month yield, the 12-month yield and 2, 3, 5, 7 and 10 year yields.

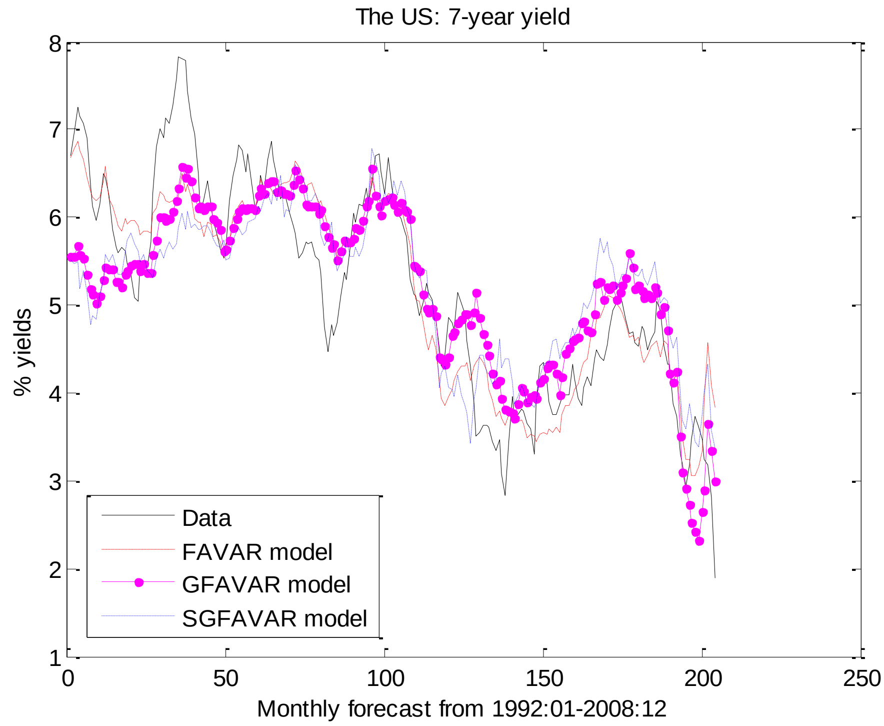

Figure 14g

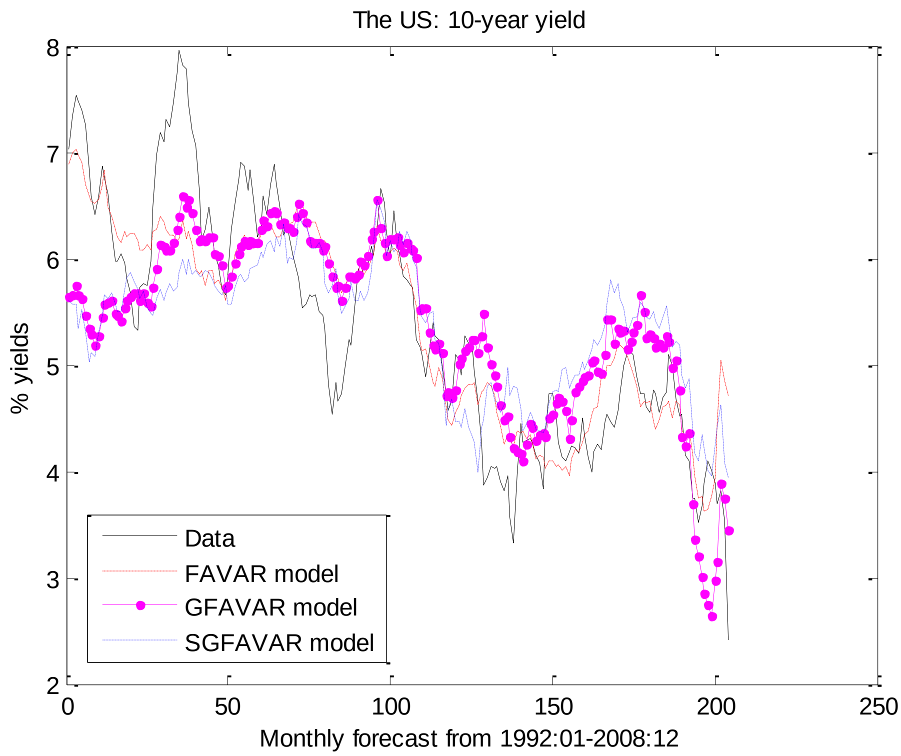

Figure 14h

APPENDIX F

Derivation of the bond pricing parameters

In this appendix we repeat the derivation of the bond pricing parameters as described in Appendix A in "Forecasting the yield curve in a data-rich environment: A no-arbitrage factor-augmented VAR approach" of Moench (2008). He showed that the no-arbitrage is guaranteed by computing following this procedure.

The no-arbitrage between bonds of different maturity implies the existence of the stochastic discount factor *M* such that

$$P_t^{(n)} = E_t[M_{t+1}P_{t-1}^{(n-1)}]$$

The price of an *n*-month to maturity bond in month *t* must equal the expected discounted price of an *(n−1)*-month to maturity bond in month *(t+1)*. Following Ang and Piazzesi (2003), the derivation of the recursive bond pricing parameters starts by assuming that the nominal pricing kernel M takes the form

$$M_{t+1} = \exp\left(-r_t - \frac{1}{2}\lambda_t'\mathbf{\Omega}\lambda_t - \lambda_t'\boldsymbol{\omega}_{t+1}\right)$$

and by guessing that bond prices P are exponentially affine in the state variables Z,

$$P_t^{(n)} = \exp\,(A_n + B_n'Z_t)$$

Substituting the above expressions for P and M into the first relation, one obtains

$$\begin{aligned}
P_t^{(n)} &= E_t[M_{t+1}P_{t+1}^{(n-1)}] \\
&= E_t[\exp\left(-r_t - \tfrac{1}{2}\lambda_t'\mathbf{\Omega}\lambda_t - \lambda_t'\boldsymbol{\omega}_{t+1}\right)\exp(A_{n-1} + B_{n-1}'Z_{t+1})] \\
&= \exp\,(-r_t - \tfrac{1}{2}\lambda_t'\mathbf{\Omega}\lambda_t + A_{n-1}) \times E_t[\exp\left(-\lambda_t'\boldsymbol{\omega}_{t+1} + B_{n-1}'(\boldsymbol{\mu} + \boldsymbol{\phi}Z_t + \boldsymbol{\omega}_{t+1})\right)] \\
&= \exp\,(-r_t - \tfrac{1}{2}\lambda_t'\mathbf{\Omega}\lambda_t + A_{n-1} + B_{n-1}'\boldsymbol{\mu} + B_{n-1}'\boldsymbol{\phi}Z_t) \\
&\quad \times E_t[\exp\left((-\lambda_t' + B_{n-1}')\boldsymbol{\omega}_{t+1}\right)]
\end{aligned}$$

Since the innovations $\boldsymbol{\omega}$ of the state variable process are assumed Gaussian with variance-covariance matrix $\boldsymbol{\Omega}$, it is obvious that

$$lnE_t\left[\exp\left((-\lambda_t' + B_{n-1}')\boldsymbol{\omega}_{t+1}\right)\right] = E_t[\ln\left(\exp\left((-\lambda_t' + B_{n-1}')\boldsymbol{\omega}_{t+1}\right)\right)]$$

$$+\frac{1}{2}Var_t(\ln(\exp((-\lambda_t' + B_{n-1}')\boldsymbol{\omega}_{T+1})))$$

$$=\frac{1}{2}[\lambda_t'\boldsymbol{\Omega}\lambda_t - 2B_{n-1}'\boldsymbol{\Omega}\lambda_t + B_{n-1}'\boldsymbol{\Omega}B_{n-1}]$$

$$=\frac{1}{2}\lambda_t'\boldsymbol{\Omega}\lambda_t - B_{n-1}'\boldsymbol{\Omega}\lambda_t + \frac{1}{2}B_{n-1}'\boldsymbol{\Omega}B_{n-1}$$

Hence, $E_t\left[\exp\left((-\lambda_t' + B_{n-1}')\boldsymbol{\omega}_{t+1}\right)\right] = \exp\,(\frac{1}{2}\lambda_t'\boldsymbol{\Omega}\lambda_t - B_{n-1}'\boldsymbol{\Omega}\lambda_t + \frac{1}{2}B_{n-1}'\boldsymbol{\Omega}B_{n-1})$

and thus

$$P_t^{(n)} = \exp\,(-r_t - \frac{1}{2}\lambda_t'\boldsymbol{\Omega}\lambda_t + A_{n-1} + B_{n-1}'\boldsymbol{\mu} + B_{n-1}'\boldsymbol{\phi}Z_t + \cdots + \frac{1}{2}\lambda_t'\boldsymbol{\Omega}\lambda_t - B_{n-1}'\boldsymbol{\Omega}\lambda_t + \frac{1}{2}B_{n-1}'\boldsymbol{\Omega}B_{n-1})$$

Using the relations $r_t = \delta' Z_t$ and $\lambda_t = \lambda_0 + \lambda_1 Z_t$, and matching coefficients finally yields

$$P_t^{(n)} = \exp(A_n + B_n' Z_t),$$

where

$$A_n = A_{n-1} + B_{n-1}'(\boldsymbol{\mu} - \boldsymbol{\Omega}\lambda_0) + \frac{1}{2}B_{n-1}'\boldsymbol{\Omega}B_{n-1}$$

$$B_n' = B_{n-1}'(\boldsymbol{\phi} - \boldsymbol{\Omega}\lambda_1) - \delta'$$

These are the recursive equations of the pricing parameters stated in (6) and (7).

APPENDIX G

Mathematics of Principal Component Analysis

In this appendix we repeat the mathematics of principal component analysis as described in Jolliffe, I.T. (2002), Principal Component Analysis, second edition, Springer series in statistic. Principal component analysis is a variable reduction procedure. It is useful when we have obtained data on a number of variables, possibly a large number of variables.

Given X denotes the vector of observed data. Each row of X corresponds to a set of measurements from one particular trial. Each columns of X corresponds to all measurement of a particular type. We can define the linear transformation from the vector *F* to X by

$$X = AF$$

where *F* is the vector of principal components, *A* is the matrix of principal component loadings. To estimate the common factors *F*, we start from calculating the covariance matrix of the observed data C_x

$$C_x = Var(X) = Var(AF)$$

$$X'X = A\,Var(F)A'$$

$$Var(F) = I$$

The diagonal terms of C_x are the variance of the measurement types. The off diagonal terms of C_x are the covariance between measurement types. The covariance matrix C_x can be decomposed as

$$C_x = VDV'$$

where V is the matrix whose columns are eigenvectors of $X'X$, D is the diagonal matrix of eigenvalues of $X'X$ whose i^{th} entry corresponds to the i^{th} colume of *V*. As the eigenvalue matrix is normalized so that the length of each eigenvector is one, $VV' = I$. Moreover, the diagonal entries of eigenvalue are ordered from the largest eigenvalue to the smallest. Furthermore, if the $Var(F) = I$ (i.e., F_j has standard deviation *1* for all *j* and F_j and F_k are uncorrelated for all $j \neq k$), the resulting covariance matrix of observed data will be consistent with the original covariance matrix C_x. Therefore,

$$X = VD^{\frac{1}{2}}D^{\frac{-1}{2}}V'X$$

where $A = VD^{\frac{1}{2}}$ and $F = D^{\frac{-1}{2}}V'X$. The $D^{1/2}$ is the diagonal matrix which each element is the square-roots of the eigenvalues. Moreover, the $D^{-1/2}$ is the diagonal matrix that each element is the reciprocals of the square-roots of the eigenvalues.

To prove the consistent of the covariance matrix of observed data $Var(F)$, it can be verified by

$$Var(F) = Var(D^{\frac{-1}{2}}V'X)$$

$$Var(F) = D^{\frac{-1}{2}}V'(VDV')VD^{\frac{-1}{2}} = I$$

APPENDIX H

The Transformation of the Macroeconomic Time Series (Unit Root Test)

```
function [B] = transform(A)

[row col] = size(A);
B = zeros(row,1);

for i = 1:col
    H(i) = dfARTest(A(:,i),0,0.05);
    temp = A(:,i);
    while H(i) == 0,
    [row1 col1] = size(temp);
    temp = temp(13:row1)-temp(1:row1-12);
    H(i) = dfARTest(temp,0,0.05);
    end

[row1 col1] = size(temp);
temp = [zeros(row-row1,1); temp];
B = [B temp];
end
end
```

Standardize Time Series Data

```
function [B] = standardize(A)

[row col] = size(A);
xbar = mean(A);
sd = std(A,0,1);
for i = 1:col
    B(:,i) = (A(:,i)-xbar(i))/sd(i);
end
end
```

Principal Component Analysis

```
function [F,Explained,A,V,D,CV] = pca(X)

X = X';
[M,N] = size(X);
meanx = mean(X,2);
x = X - repmat(meanx,1,N);
CV = (1 / (N-1)) * (x * x');
[V, D] = eig(CV);
[B, k] = sort(-1*diag(D));
D = D(k,k);
V = V(:,k);
A = V * sqrt(D);
Explained = 100*diag(D)/sum(diag(D));
F = A \ x;
F = F';
end
```

Best lag length and Lag length Criteria

```
function [bic_lag]=best_lag(Zall,MaxNLag)

 [row,col]=size(Zall);
  bic_lag=inf*ones(MaxNLag,1);

    for i=1:MaxNLag,

        Spec=vgxset('nAR',i,'n',col,'Constant',true);
        NumParams = col + i*col^2 + col*(col+1)/2;
        NumObs = row - i;

        [EstSpec,EstStdErrors,LLF] = vgxvarx(Spec,Zall);

        [AIC,BIC] = aicbic(LLF,NumParams,NumObs);

         bic_lag(i)=BIC;
     end
end
```

Vector Autoregressive

```
function [mu,phi,ohm,best_lag,EstStdErrors]=estvar(bic_lag,Zall)

[row,col]=size(Zall);
[min_bic,best_lag] = min(bic_lag);

phi=zeros(col*best_lag,col*best_lag);

Spec=vgxset('nAR',best_lag,'n',col,'Constant',true);
[EstSpec,EstStdErrors] = vgxvarx(Spec,Zall,[],[],'StdErrType','all');

mu = [EstSpec.a; zeros(col*(best_lag-1),1)];
     for j=1:best_lag
         phi(1:col,(j-1)*col+1:j*col)=EstSpec.AR{j,1};
     end
phi(col+1:end,1:col*(best_lag-1))=eye(col*(best_lag-1));
ohm=[EstSpec.Q zeros(col,col*(best_lag-1)); zeros(col*(best_lag-
1),col*best_lag)];
end
```

Testing the Significant Explanatory Variables

```
function [stats_ans]=linear_test(group,y)

 [row col]=size(group);
  for i=1:col
      x=[ones(row,1) group(:,i)];
      [b,bint,r,rint,stats] = regress(y,x);
      stats_ans(i)=stats(1,3);
  end
end
```

```
function [group_new]=setup_some(stats_ans,group)

[row col]=size(stats_ans);
k=1;
for i=1:col
    if stats_ans(i)<=0.05
        group_new(:,k)=group(:,i);
        k=k+1;
    end
end
```

Minimization of the fitted errors assuming risk premia are constant

```
function [w] = insamplefit_w(mu,phi,ohm,delta,Zall,ydata,best_lag)

options=optimset('MaxFunEvals',40000,'MaxIter',4000,'TolFun',1e-
12,'TolX',1e-12);

A=[];
B=[];
A(1) = 0;
B(:,1) = zeros(5*best_lag,1);

lamda0 = rand(5,1);
lam0 = [lamda0;zeros(5*(best_lag-1),1)];

lamda1 = zeros(5,5);
lam1 = [lamda1 zeros(5,5*(best_lag-1));zeros(5*(best_lag-
1),5*best_lag)];

w0 = lamda0;

TTM = [1,3,6,12,24,36,60,84,120];
selectTTM = [1,2,3,4,5,6,7,8,9];
selecttime = 1:1:132;
TTM=TTM(1,selectTTM);
ydata = ydata(selectTTM,selecttime);
Zall = Zall(:,selecttime);

w = fminunc(@MinFunction1,w0,options);

function f = MinFunction1(w)
    yhat = [];
     a = [];
     b = [];
    [row,colume] = size(Zall);
    lamda0 = w;

% Loop for all maturity of the interest rate
       for n = 1:max(TTM)

              A(n+1) = A(n) + (B(:,n)'*(mu - (ohm *
[lamda0;zeros(5*(best_lag-1),1)]))) + (B(:,n)' * ohm * B(:,n))/2;
```

```
            B(:,n+1) = ((B(:,n)'*(phi - (ohm * [lamda1
zeros(5,5*(best_lag-1));zeros(5*(best_lag-1),5*best_lag)]))) -
delta')';

                 a(n+1) = -A(n+1)/n;

                 b(:,n+1) = (-B(:,n+1)'/n)';
       end

       % Loop for all period of time
       for t = 1:colume-best_lag+1

                 Z = Zall(:,t:t+best_lag-1);
                 [k,m] = size(Z);
                 Z = reshape(Z,k*m,1);

           for n = TTM

                 yhat(n,t)= a(n+1) + (b(:,n+1)' * Z);

           end
       end

  Sn = zeros(length(TTM),colume-best_lag+1);

% Loop for the "n" bond that match with the available interest rate
data
  for i=1:length(TTM)

% Loop for the "t" maturity that match with the yhat
        for j=1:colume-best_lag+1

          Sn(i,j) = (yhat(TTM(1,i),j) - ydata(i,j)).^2;

        end
  end
         f = sqrt(mean(mean(Sn)));
end
end
```

<u>**Minimization of the fitted errors with let the risk premia be estimated freely**</u>

```
function [h,yhat] =
insamplefit_h(mu,phi,ohm,delta,Zall,ydata,w,best_lag)

options = optimset('MaxFunEvals',60000,'MaxIter',5000,'TolFun',1e-
30,'TolX',1e-30);

A=[];
B=[];
A(1) = 0;
B(:,1) = zeros(5*best_lag,1);

lamda0 = w;
lam0 = [lamda0;zeros(5*(best_lag-1),1)];
```

```
lamda1 = rand(5,5);
lam1 = [lamda1 zeros(5,5*(best_lag-1));zeros(5*(best_lag-
1),5*best_lag)];

h0 = [lamda0 lamda1];
TTM = [1,3,6,12,24,36,60,84,120];
selectTTM = [1,2,3,4,5,6,7,8,9];
selecttime = 1:1:132;
TTM = TTM(1,selectTTM);
ydata = ydata(selectTTM,selecttime);
Zall = Zall(:,selecttime);

h = fminunc(@MinFunction1,h0,options);

function f = MinFunction1(h)
    yhat = [];
     a = [];
     b = [];
    [row,colume] = size(Zall);
    lamda0 = h(:,1);
    lamda1 = h(:,2:end);

% Loop for all maturity of the interest rate included
       for n = 1:max(TTM)

              A(n+1) = A(n) + (B(:,n)'*(mu - (ohm *
[lamda0;zeros(5*(best_lag-1),1)]))) + (B(:,n)' * ohm * B(:,n))/2;

              B(:,n+1) = ((B(:,n)'*(phi - (ohm * [lamda1
zeros(5,5*(best_lag-1));zeros(5*(best_lag-1),5*best_lag)]))) -
delta')';

                 a(n+1) = -A(n+1)/n;

                 b(:,n+1) = (-B(:,n+1)'/n)';
       end

       % Loop for all period of time
       for t = 1:colume-best_lag+1

                 Z = Zall(:,t:t+best_lag-1);
                 [k,m] = size(Z);
                 Z = reshape(Z,k*m,1);

           for n = TTM

                 yhat(n,t)= a(n+1) + (b(:,n+1)' * Z);

           end
       end

  Sn = zeros(length(TTM),colume-best_lag+1);

% Loop for the "n" bond that match with the available interest rate
data
```

```
  for i=1:length(TTM)

% Loop for the "t" maturity that match with the yhat
        for j=1:colume-best_lag+1

          Sn(i,j) = (yhat(TTM(1,i),j) - ydata(i,j)).^2;

        end

  end
         f = sqrt(mean(mean(Sn)))

end
end
```

Forecasting the interest rate from optimal value of risk premia

```
function
[yhat1 yhat6 yhat12] =
forecast(mu,phi,ohm,delta,Zall,ydata,h,best_lag)

A=[];
B=[];
A(1) = 0;
B(:,1) = zeros(5*best_lag,1);

lamda0 = h(:,1);
lam0 = [lamda0;zeros(5*(best_lag-1),1)];

lamda1 = h(:,2:end);
lam1 = [lamda1 zeros(5,5*(best_lag-1));zeros(5*(best_lag-
1),5*best_lag)];

TTM = [1,3,6,12,24,36,60,84,120];
selectTTM = [1,2,3,4,5,6,7,8,9];

[fac obs]=size(Zall);
selecttime = 1:1:obs;

TTM=TTM(1,selectTTM);
ydata = ydata(selectTTM,selecttime);
Zall = Zall(:,selecttime);

yhat = [];
a = [];
b = [];
[row,colume] = size(Zall);

       for n = 1:max(TTM)

              A(n+1) = A(n) + B(:,n)'*(mu - ohm *
[lamda0;zeros(5*(best_lag-1),1)]) + (B(:,n)' * ohm * B(:,n))/2;

              B(:,n+1) = (B(:,n)'*(phi - ohm * [lamda1
zeros(5,5*(best_lag-1));zeros(5*(best_lag-1),5*best_lag)]) -
delta')';
```

```
                a(n+1) = -A(n+1)/n;

                b(:,n+1) = (-B(:,n+1)'/n)';
     end

                Z = Zall(:,best_lag);
                [k,m] = size(Z);
                Z = reshape(Z,k*m,1);

                Zhat1 = phi * Z + mu;

                Zhat6 = phi*Z + (mu + phi*mu + phi^2*mu + phi^3*mu +
phi^4*mu + phi^5*mu);

                Zhat12 = phi*Z + (mu + phi*mu + phi^2*mu + phi^3*mu
+ phi^4*mu + phi^5*mu + phi^6*mu + phi^7*mu + phi^8*mu + phi^9*mu +
phi^10*mu + phi^11*mu);




    for n = TTM

                yhat1(n)= a(n+1) + (b(:,n+1)' * Zhat1);

                yhat6(n)= a(n+1) + (b(:,n+1)' * Zhat6);

                yhat12(n)= a(n+1) + (b(:,n+1)' * Zhat12);
     end

end
```

APPENDIX I

Table 56: The United State of America's Macroeconomic Time Series Data

Reserve and Monetary Base	
1	Monetary base; seasonally adjusted, break adjusted
2	Reserves of depository institutions, total; seasonally adjusted, break adjusted
3	Vault cash, total; not seasonally adjusted
4	Reserves of depository institutions, non-borrowed; seasonally adjusted, break adjusted
5	Reserves of depository institutions, non-borrowed plus extended credit; seasonally adjusted, break adjusted
6	Reserves of depository institutions, required; seasonally adjusted, break adjusted
7	Vault cash, used to satisfy required reserves; not seasonally adjusted
8	Vault cash, surplus; not seasonally adjusted
9	Net carryover of reserve balances; not seasonally adjusted
10	Reserve balance with F.R. Banks; not seasonally adjusted
11	Total borrowings from the Federal Reserve; not seasonally adjusted
12	Other borrowing from the Federal Reserve, total; not seasonally adjusted
13	Other borrowing from the Federal Reserve, seasonal; not seasonally adjusted
14	St. Louis Adjusted Monetary Base
15	Board of Governors Monetary Base, Adjusted for Changes in Reserve Requirements
16	Reserve Adjustment Magnitude (RAM)
17	St. Louis Source Base
18	Nominal Broad Dollar Index
19	Nominal Major Currencies Dollar Index
20	Nominal Other Important Trading Partners Dollar Index
Exchange Rate	
21	Australia -- Spot Exchange Rate, US$/AUSTRALIAN $
22	New Zealand -- Spot Exchange Rate, US$/NZ$
23	South Africa -- Spot Exchange Rate, US$/RAND
24	United Kingdom -- Spot Exchange Rate, US$/POUND STERLING
25	Canada -- Spot Exchange Rate, CANADIAN $/US$
26	China -- Spot Exchange Rate, YUAN/US$
27	Denmark -- Spot Exchange Rate, KRONER/US$
28	HONG KONG -- Spot Exchange Rate, HK$/US$

Table 56: The United State of America's Macroeconomic Time Series Data (continue)

Exchange Rate (continue)	
29	India -- Spot Exchange Rate, RUPEES/US$
30	Japan -- Spot Exchange Rate, YEA/US$
31	Korea -- Spot Exchange Rate, WON/US$
32	Malaysia - Spot Exchange Rate, RINGGIT/US$
33	Norway -- Spot Exchange Rate, KRONER/US$
34	Sweden -- Spot Exchange Rate, KRONOR/US$
35	Singapore - Spot Exchange Rate, SINGAPORE $/US$
36	Sri Lanka -- Spot Exchange Rate, RUPEES/US$
37	Switzerland -- Spot Exchange Rate, FRANCS/US$
38	Taiwan -- Spot Exchange Rate, NT$/US$
39	Thailand -- Spot Exchange Rate -- THAILAND
Price Indices	
40	CPI all urban consumer
41	CPI all urban consumer old base
42	CPI urban wage earner
43	CPI all urban less food & energy
44	CPI urban wage less food & energy
45	CPI commodity finish good
46	PPI commodity finish less food & energy
47	PPI commodity finish energy
48	PPI commodity finish consumer
49	Import price index
50	Export price index
51	Consumer Price Index for All Urban Consumers: Apparel
52	Consumer Price Index for All Urban Consumers: All Items
53	Consumer Price Index for All Urban Consumers: Energy
54	Consumer Price Index for All Urban Consumers: Food and Beverages
55	Consumer Price Index for All Urban Consumers: Housing
56	Consumer Price Index for All Urban Consumers: All Items Less Energy
57	Consumer Price Index for All Urban Consumers: All Items Less Food & Energy
58	Consumer Price Index for All Urban Consumers: Medical Care
59	Consumer Price Index for All Urban Consumers: Other Goods and Services
60	Consumer Price Index for All Urban Consumers: Transportation
61	Consumer Price Index for All Urban Consumers: Food
62	Consumer Price Index for All Urban Consumers: All Items Less Food

Table 56: The United State of America's Macroeconomic Time Series Data (continue)

Price Index (continue)	
63	Producer Price Index: Finished Consumer Goods Excluding Foods
64	Producer Price Index: All Commodities
65	Producer Price Index: Crude Energy Materials
66	Producer Price Index: Crude Foodstuffs & Feedstuffs
67	Producer Price Index: Finished Goods: Capital Equipment
68	Producer Price Index: Crude Materials for Further Processing
69	Producer Price Index: Fuels & Related Products & Power
70	Producer Price Index: Finished Consumer Foods
71	Producer Price Index: Finished Consumer Goods
72	Producer Price Index: Finished Energy Goods
73	Producer Price Index: Finished Goods
74	Producer Price Index: Finished Goods Less Energy
75	Producer Price Index: Finished Goods Excluding Foods
76	Producer Price Index: Industrial Commodities
77	Producer Price Index: Intermediate Energy Goods
78	Producer Price Index: Intermediate Foods & Feeds
79	Producer Price Index: Intermediate Materials: Supplies & Components
80	Producer Price Index: Finished Goods Less Food & Energy
Stock Return Indices	
81	DAX Price
82	DAX Performance
83	CDAX Price
84	CDAX Performance
85	REX Price
86	REX Performance
87	INDIA Price
88	CHINA Price
89	FRANCE Price
90	US(S&P500) Price
91	NYSE Composite
92	UK Index
93	JAPAN(Nikkei)
Employment	
94	Unemployment rate
95	Civilian labor force level
96	Employment level
97	Unemployment level

Table 56: The United State of America's Macroeconomic Time Series Data (continue)

Employment (continue)	
98	Total nonfarm
99	All Employees: Durable Goods Manufacturing
100	Employees on Nonfarm Payrolls: Manufacturing
101	All Employees: Nondurable Goods Manufacturing
102	Total Nonfarm Payrolls: All Employees
103	All Employees: Service-Providing Industries
104	Professional and Business Services: Temporary Help Services
105	All Employees: Construction
106	All Employees: Education & Health Services
107	All Employees: Financial Activities
108	All Employees: Goods-Producing Industries
109	All Employees: Government
110	All Employees: Information Services
111	All Employees: Leisure & Hospitality
112	All Employees: Natural Resources & Mining
113	All Employees: Professional & Business Services
114	All Employees: Total Private Industries
115	All Employees: Other Services
116	All Employees: Trade, Transportation & Utilities
117	All Employees: Retail Trade
118	All Employees: Wholesale Trade
Industrial Production	
119	Manufacturing (SIC); s.a.
120	Total index; s.a.
121	Crude processing (capacity); s.a.
122	Primary & semi-finished processing (capacity); s.a.
123	Finished processing (capacity); s.a.
124	Mining (NAICS = 21); s.a.
125	Electric power generation, transmission, and distribution (NAICS = 2211); s.a.
126	Electric and gas utilities (NAICS = 2211,2); s.a.
127	Natural gas distribution (NAICS = 2212); s.a.
128	Food, beverage, and tobacco (NAICS = 311,2); s.a.
129	Textiles and products (NAICS = 313,4); s.a.
130	Apparel and leather goods (NAICS = 315,6); s.a.
131	Wood product (NAICS = 321); s.a.
132	Paper (NAICS = 322); s.a.

Table 56: The United State of America's Macroeconomic Time Series Data (continue)

Industrial Production (continue)	
133	Printing and related support activities (NAICS = 323); s.a.
134	Petroleum and coal products (NAICS = 324); s.a.
135	Chemical (NAICS = 325); s.a.
136	Plastics and rubber products (NAICS = 326); s.a.
137	Nonmetallic mineral product (NAICS = 327); s.a.
138	Primary metal (NAICS = 331); s.a.
139	Fabricated metal product (NAICS = 332); s.a.
140	Machinery (NAICS = 333); s.a.
141	Computer and electronic product (NAICS = 334); s.a.
142	Electrical equipment, appliance, and component (NAICS = 335); s.a
143	Motor vehicles and parts (NAICS = 3361-3); s.a.
144	Aerospace and miscellaneous transportation eq. (NAICS = 3364-9); s.a.
145	Furniture and related product (NAICS = 337); s.a.
146	Miscellaneous (NAICS = 339); s.a.
147	Manufacturing (NAICS); s.a.
148	Durable manufacturing (NAICS); s.a.
149	Nondurable manufacturing (NAICS); s.a.
150	Other manufacturing; s.a.
Capital Utilization	
151	Manufacturing (SIC); s.a.
152	Total index; s.a.
153	Crude processing (capacity); s.a.
154	Primary & semi-finished processing (capacity); s.a.
155	Finished processing (capacity); s.a.
156	Mining (NAICS = 21); s.a.
157	Oil and gas extraction (NAICS = 211); s.a.
158	Mining (except oil and gas) (NAICS = 212); s.a.
159	Metal ore mining (NAICS = 2122); s.a.
160	Nonmetallic mineral mining and quarrying (NAICS = 2123); s.a.
161	Support activities for mining (NAICS = 213); s.a.
162	Electric power generation, transmission, and distribution (NAICS = 2211)
163	Electric and gas utilities (NAICS = 2211,2); s.a.
164	Natural gas distribution (NAICS = 2212); s.a.
165	Food (NAICS = 311); s.a.
166	Food, beverage, and tobacco (NAICS = 311,2); s.a.
167	Beverage and tobacco product (NAICS = 312); s.a.
168	Textile mills (NAICS = 313); s.a.

Table 56: The United State of America's Macroeconomic Time Series Data (continue)

Capital Utilization (continue)	
169	Textiles and products (NAICS = 313,4); s.a.
170	Textile product mills (NAICS = 314); s.a.
171	Apparel (NAICS = 315); s.a.
172	Apparel and leather goods (NAICS = 315,6); s.a.
173	Leather and allied product (NAICS = 316); s.a.
174	Wood product (NAICS = 321); s.a.
175	Paper (NAICS = 322); s.a.
176	Printing and related support activities (NAICS = 323); s.a.
177	Petroleum and coal products (NAICS = 324); s.a.
178	Chemical (NAICS = 325); s.a.
179	Synthetic rubber (NAICS = 325212); s.a.
180	Plastics and rubber products (NAICS = 326); s.a.
181	Nonmetallic mineral product (NAICS = 327); s.a.
182	Primary metal (NAICS = 331); s.a.
183	Iron and steel products (NAICS = 3311,2); s.a.
184	Fabricated metal product (NAICS = 332); s.a.
185	Machinery (NAICS = 333); s.a.
186	Computer and electronic product (NAICS = 334); s.a.
187	Computer and peripheral equipment (NAICS = 3341); s.a.
188	Communications equipment (NAICS = 3342); s.a.
189	Semiconductors and related equipment; s.a.
190	Electrical equipment, appliance, and component (NAICS = 335); s.a.
191	Transportation equipment (NAICS = 336); s.a.
192	Automobile and light duty motor vehicle (NAICS = 33611); s.a.
193	Motor vehicles and parts (NAICS = 3361-3); s.a.
194	Aerospace and miscellaneous transportation eq. (NAICS = 3364-9); s.a.
195	Furniture and related product (NAICS = 337); s.a.
196	Miscellaneous (NAICS = 339); s.a.
197	Manufacturing (NAICS); s.a.
198	Durable manufacturing (NAICS); s.a.
199	Nondurable manufacturing (NAICS); s.a.
200	Other manufacturing; s.a.
201	Computers, communications eq., and semiconductors (NAICS = 3341,3342,334412-9); s.a.
202	Coal mining (NAICS = 2121); s.a.
203	Plastics material and resin (NAICS = 325211); s.a.
204	Artificial and synthetic fibers and filaments (NAICS = 32522); s.a.

Table 56: The United State of America's Macroeconomic Time Series Data (continue)

Capital Utilization (continue)	
205	Manufacturing ex. computers, communications eq., and semiconductors; s.a.
206	Manufacturing ex. hi-tech and motor vehicles & pts.; s.a.
207	Total ex. computers, communications eq., and semiconductors; s.a.
Pay Rate	
208	Total private weekly hrs
209	Total private hourly earn
210	Average Hourly Earnings: Construction
211	Average Hourly Earnings: Manufacturing
212	Average Hourly Earnings: Total Private Industries
213	Aggregate Weekly Hours Index: Total Private Industries
214	Average Weekly Hours of Production and Nonsupervisory Employees: Manufacturing
215	Average Weekly Hours: Production and Nonsupervisory Employees: Total Private Industries
216	Average Weekly Hours: Overtime: Manufacturing
Export-Import	
217	Exports of Goods, Services and Income
218	Exports of Goods and Services
219	Exports of Merchandise: Adjusted, Excluding Military
220	Exports of Goods, Services and Income
221	Exports of Services
222	Exports of Services: U.S. Government Miscellaneous
223	Exports of Services: Transfers Under U.S. Military Agency Contracts
224	Exports of Other Private Services
225	Exports of Other Transportation Services
226	Exports of Services: Passenger Fares
227	Exports of Services: Royalties and Licensing Fees
228	Exports of Services: Travel
229	Imports of Goods, Services, and Income
230	Imports of Goods and Services
231	Imports of Merchandise: Adjusted, Excluding Military
232	Imports of Services
233	Imports of U.S. Government Miscellaneous Services
234	Imports of Services: Direct Defense Expenditures
235	Imports of Other Private Services
236	Imports of Other Transportation Services

Table 56: The United State of America's Macroeconomic Time Series Data (continue)

Export-Import (continue)	
237	Imports of Services: Passenger Fares
238	Imports of Services: Royalties and Licensing Fees
239	Imports of Services: Travel
240	U.S. Imports from Canada, Customs Basis
241	U.S. Imports from China, Mainland, Customs Basis
242	U.S. Imports from France, Customs Basis
243	U.S. Imports from Germany, Customs Basis
244	U.S. Imports from Japan, Customs Basis
245	U.S. Imports from Mexico, Customs Basis
246	U.S. Imports from the United Kingdom, Customs Basis
247	U.S. Exports to Canada, f.a.s. basis
248	U.S. Exports to China, Mainland, f.a.s. basis
249	U.S. Exports to France, f.a.s. basis
250	U.S. Exports to Germany, f.a.s. basis
251	U.S. Exports to Japan, f.a.s. basis
252	U.S. Exports to Mexico, f.a.s. basis
253	U.S. Exports to the United Kingdom, f.a.s. basis
Assets Liabilities Commercial Bank	
254	Bank credit, all commercial banks, s.a.
255	Securities in bank credit, all commercial banks, s.a.
256	Treasury and agency securities, all commercial banks, s.a.
257	Other securities, all commercial banks, s.a.
258	Loans and leases in bank credit, all commercial banks, s.a.
259	Commercial and industrial loans, all commercial banks, s.a.
260	Real estate loans, all commercial banks, s.a.
261	Real estate loans: Revolving home equity loans, all commercial banks, s.a.
262	Consumer loans, all commercial banks, seasonally adjusted
263	Interbank loans, all commercial banks, seasonally adjusted
264	Fed funds and reverse RPs with banks, all commercial banks, s.a.
265	Loans to commercial banks, all commercial banks, s.a.
266	Cash assets, all commercial banks, s.a.
267	Other assets, all commercial banks, s.a.
268	Other loans and leases, all commercial banks, s.a.
269	Total assets, all commercial banks, s.a.
270	Other loans and leases: Fed funds and reverse RPs with nonbanks , all commercial banks, s.a.

Table 56: The United State of America's Macroeconomic Time Series Data (continue)

Assets Liabilities Commercial Bank (continue)	
271	Other loans and leases: All other loans and leases, all commercial banks, s.a.
272	Deposits, all commercial banks, s.a.
273	Large time deposits, all commercial banks, s.a.
274	Residual (assets less liabilities), all commercial banks, s.a.
275	Borrowings, all commercial banks, s.a.
276	Total liabilities, all commercial banks, s.a.
277	Other liabilities, all commercial banks, s.a.
Consumer Credit	
278	Securitized total consumer loans
279	Total consumer loans owned by commercial banks
280	Total consumer loans owned by finance companies
281	Total consumer loans owned by federal government
282	Total consumer loans owned by nonfinancial businesses
283	Total consumer loans owned by credit unions
284	Total consumer loans owned by savings institutions
285	Securitized consumer revolving credit
286	Consumer revolving credit owned by commercial banks
287	Consumer revolving credit owned by finance companies
288	Consumer revolving credit owned by nonfinancial businesses
289	Consumer revolving credit owned by credit unions
290	Consumer revolving credit owned by savings institutions
291	Securitized Consumer Non-revolving Credit
292	Non-revolving Consumer Loans owned by Commercial Banks
293	Non-revolving consumer loans owned by finance companies
294	Non-revolving consumer loans the federal government
295	Non-revolving Consumer Loans owned by Nonfinancial Businesses
296	Non-revolving Consumer Loans owned by Credit Unions
297	Non-revolving Consumer Loans owned by Savings Institutions
298	Financial obligations ratio, s.a.
299	Debt service ratio, s.a.
300	Financial obligations ratio of homeowners, s.a.
301	Consumer financial obligations ratio of homeowners, s.a.
302	Mortgage financial obligations ratio of homeowners, s.a.
303	Financial obligations ratio of renters, s.a.
Income Payment and Receipts	
304	U.S. Government Grants Excluding Military

Table 56: The United State of America's Macroeconomic Time Series Data (continue)

Income Payment and Receipts (continue)	
305	U.S. Government Pensions and Other Transfers
306	Private Remittances and Other Transfers
307	Income Payments - Compensation of Employees
308	Income Payments on Foreign Direct Investment in U.S.
309	U.S. Government Income Payments on Foreign Assets in U.S.
310	Income Payments on Foreign Assets in the U.S.
311	Income Payments
312	Other Private Income Payments on Foreign Assets in U.S.
313	Income Receipts - Compensation of Employees
314	Income Receipts on U.S. Direct Investment Abroad
315	U.S. Government Income Receipts on Assets Abroad
316	Other Private Income Receipts on U.S. Assets Abroad
317	Income Receipts on U.S. Assets Abroad
318	Income Receipts
Monetary Aggregate	
319	Currency Component of M1 Plus Demand Deposits
320	Currency Component of M1
321	Demand Deposits at Commercial Banks
322	M1 Money Stock
323	Other Checkable Deposits at Commercial Banks
324	Other Checkable Deposits
325	Other Checkable Deposits at Thrift Institutions
326	Total Checkable Deposits
327	Travelers Checks Outstanding
328	M2 Minus Own Rate
329	M2 Minus
330	Institutional Money Funds
Gross Domestic Product Component	
331	Change in Private Inventories
332	Real Change in Private Inventories,
333	Final Sales of Domestic Product
334	Real Final Sales of Domestic Product,
335	Final Sales to Domestic Purchasers
336	Gross Domestic Purchases
337	Gross Domestic Product,
338	Real Gross Domestic Product,
339	Real Potential Gross Domestic Product

Table 56: The United State of America's Macroeconomic Time Series Data (continue)

Gross Domestic Product Component (continue)	
340	Gross National Product
341	Real Gross National Product

Table 57: German’s Macroeconomic Time Series Data

Monetary Aggregates	
1	M1
2	M2
3	M3
4	Money stock M3
Foreign Exchange Rates	
5	€/USD (United State of America)
6	€/JYP (Japan)
7	€/CHF (Switzerland)
8	€/GBP (United Kingdom)
9	€/CAD (Canada)
10	€/DKK (Denmark)
11	€/NOK (Norway)
12	€/SEK (Sweden)
Stock Return Indices	
13	DAX Price
14	DAX Performance
15	CDAX Price
16	CDAX Performance
17	REX Price
18	REX Performance
19	INDIA Stock Index (Price)
20	CHINA Stock Index (Price)
21	FRANCE Stock Index (Price)
22	US(S&P500) Stock Index (Price)
23	NYSE Composite
24	UK Stock Index (Price)
25	JAPAN Stock Index (Nikkei)(Price)
Price Indices	
26	Other PI (total raw material)
27	Other PI (producer price industry)
28	Other PI (producer price agriculture)
29	Other PI (export price)
30	Other PI (import price)
31	Other PI (raw energy material price)
32	PPI (total)
33	CPI (total)
34	CPI (food)
35	CPI (energy)

Table 57: German's Macroeconomic Time Series Data (continue)

Export-Import	
36	Import (total)
37	Export (total)
38	Balance Foreign Trade
39	External Trade in Good Export
40	External Trade in Good Import
41	External Trade in Good Balance
42	Total Value Foreign Trade Balance
43	Trade in Good Supplement Trade Balance
44	Total Service Transaction Receive
45	Total Service Transaction Expenditure
46	Total Service Transaction Balance
47	Total Income Receive
48	Total Income Expenditure
49	Total Income Balance
50	Total Current Transfer
51	Balance on Current Account
52	National (Export)
53	National (Import)
Employment	
54	Employment
55	Unemployment
56	Vacancies
57	Participant
58	Total construction all enter
59	Short-Time Worker
60	National (Labor cast per employee)
Output	
61	Ming and Manufacturing
62	Main Construction Industry
63	Intermediate Goods
64	Capital Goods
65	Consumer Goods
66	Durable Goods
67	Nondurable Goods
68	Construction
69	General construction Work
70	Civil Engineer
71	Main Grouping Energy

Table 57: German's Macroeconomic Time Series Data (continue)

Output (continue)	
72	Industry Goods
73	Production include construction
74	Production exclude construction
75	National (Domestic Used)
76	National (GDP)
Order Receives	
77	Order Receive (construction)
78	Order Receive (housing construction)
79	Order Receive (industrial clients)
80	Order Receive (public sector)
81	Order Receive (total industry)
82	Order Receive (total intermediary)
83	Order Receive (total capital)
84	Order Receive (total consumer)
85	Order Receive (total durable)
86	Order Receive (total non-durable)
87	Order Receive (domestic total)
88	Order Receive by Industry (volume manufacture sector)
89	Order Receive (production in construction)
90	Order Receive (production in industry)
91	Order Receive (retail turnover)
Pay Rates	
92	Pay rate overall economy (hr)
93	Pay rate overall economy (mth)
94	Pay rate production sector (incl. construction) (hr)
95	Pay rate production sector (incl. instruction) (mth)
96	Pay rate overall economy all items excluding one-off payment (hr)
97	Pay rate overall economy all items excluding one-off payment (mth)
98	Pay rate product sector (incl. construction) excluding one-off payment (hr)
99	Pay rate product sector (incl. construction) excluding one-off payment (mth)
100	Basic pay rate overall economy excluding ancillary benefit excluding one-off payment (hr)
101	Basic pay rate overall economy excluding ancillary benefit excluding one-off payment (mth)
102	Pay rate production sector (incl. construction) excluding ancillary benefit excluding one-off payment (hr)
103	Pay rate production sector (incl. construction) excluding ancillary benefit excluding one-off payment (mth)

Table 57: German's Macroeconomic Time Series Data (continue)

Retail Trade Turnover	
104	Retail trade turnover (total value)
105	Retail trade turnover (total volume)
106	Retail trade turnover (motor vehicle, petrol station)
107	Retail trade turnover (volume)
108	Retail trade turnover (motor vehicle)
109	Value Retail Turnover
Factor Income & Services	
110	Factor income total receive
111	Factor income total expenditure
112	Factor income investment income receive
113	Factor income investment income expenditure
114	Service total receive
115	Service total expenditure
116	Service travel receive
117	Service travel expenditure
118	Total Capital Transfer & Acquisition
119	Financial Transaction Direct Investment Balance
120	Financial Transaction portfolio investment & derivative balance
121	Financial Transaction Other Investment
122	Financial Transaction LT Credit Transaction Financial Investment
123	Financial ST Credit Monetary Financial Instrument
124	Change Reserve Assets Bundes
125	Balance on financial Account
126	Balance of Unclassifiable Transaction
127	National (Gross Fixed Capital Formation)
Private Household Sector	
128	Private household transaction acquisition financing (currency & deposit)
129	Private household transaction acquisition financing (time deposit)
130	Private household transaction acquisition financing (saving deposit)
131	Private household transaction acquisition financing (saving certificate)
132	Private household transaction acquisition financing (money market paper)
133	Private household transaction acquisition financing (bond)
134	Private household transaction acquisition financing (share)
135	Private household transaction acquisition financing (other equity)
136	Private household transaction acquisition financing (mutual fund share)
137	Private household transaction acquisition financing (claim on insurance corporation)
138	Private household transaction acquisition financing (s-t claim insurance)

Table 57: German's Macroeconomic Time Series Data (continue)

Private Household Sector (continue)	
139	Private household transaction acquisition financing (l-t claim total)
140	Private household transaction acquisition financing (other claim total)
141	Private household transaction acquisition financing (acquisition of financial assets)
142	private household external financing (total loan)
143	private household external financing (total s-t loan)
144	private household external financing (total l-t loan)
145	private household external financing (total other liability)
146	private household external financing (total external financing)
147	Private household stock financial assets (currency & deposit)
148	Private household stock financial assets (current & transfer deposit)
149	Private household stock financial assets (time deposit)
150	Private household stock financial assets (saving deposit)
151	Private household stock financial assets (saving certificate)
152	Private household stock financial assets (money market paper)
153	Private household stock financial assets (bond)
154	Private household stock financial assets (share)
155	Private household stock financial assets (other equity)
156	Private household stock financial assets (claim on insurance corporation)
157	Private household stock financial assets (s-t claim insurance)
158	Private household stock financial assets (claim for company pension commitment)
159	Private household stock financial assets (total claim on pension commitment)
160	Private household stock financial assets (total other claim)
161	Private household stock financial assets (total financial assets)
162	Private household stock liability (total loan)
163	Private household stock liability (total s-t loan)
164	Private household stock liability (total l-t loan)
165	Private household stock liability (total other liability)
166	Private household stock liability (total liability)
167	National (Private Consumption)
168	Household Income (Gross wage)
169	Household Income (Net wage)
170	Household Income (Money Social Benefit)
171	Household Income (Mass income)
172	Household Income (Disable Income)
173	Household Income (Saving)

Table 57: German's Macroeconomic Time Series Data (continue)

Private Household Sector (continue)	
174	Household Income (Saving ratio)
General Government Sector	
175	Government transaction acquisition financing (currency & deposit)
176	Government transaction acquisition financing (time deposit)
177	Government transaction acquisition financing (saving deposit)
178	Government transaction acquisition financing (saving certificate)
179	Government transaction acquisition financing (money market paper)
180	Government transaction acquisition financing (bond)
181	Government transaction acquisition financing (financial derivative)
182	Government transaction acquisition financing (share)
183	Government transaction acquisition financing(other equity)
184	Government transaction acquisition financing (mutual fund share)
185	Government transaction acquisition financing (loan)
186	Government transaction acquisition financing (s-t loan)
187	Government transaction acquisition financing (l-t loan)
188	Government transaction acquisition financing (claim on insurance corporation)
189	Government transaction acquisition financing (s-t claim)
190	Government transaction acquisition financing (other claim)
191	Government transaction acquisition financing (acquisition of financial assets)
192	Government transaction external financing (currency & deposit)
193	Government transaction external financing (money market paper)
194	Government transaction external financing (bond)
195	Government transaction external financing (loan)
196	Government transaction external financing (s-t loan)
197	Government transaction external financing (l-t loan)
198	Government transaction external financing (other liability)
199	Government transaction external financing (external financing)
200	Government stock financial assets (currency & deposit)
201	Government stock financial assets (current & transfer deposit)
202	Government stock financial assets (time deposit)
203	Government stock financial assets (saving deposit)
204	Government stock financial assets (saving certificate)
205	Government stock financial assets (money market paper)
206	Government stock financial assets (bond)
207	Government stock financial assets (financial derivative)
208	Government stock financial assets (share)
209	Government stock financial assets (other equity)

Table 57: German's Macroeconomic Time Series Data (continue)

General Government Sector (continue)	
210	Government stock financial assets (loan)
211	Government stock financial assets (s-t loan)
212	Government stock financial assets (l-t loan)
213	Government stock financial assets (claim on insurance corporation)
214	Government stock financial assets (s-t claim)
215	Government stock financial assets (other claim)
216	Government stock financial assets (financial assets)
217	Government stock liability (currency & deposit)
218	Government stock liability (current & transfer deposit)
219	Government stock liability (money market paper)
220	Government stock liability (bond)
221	Government stock liability (loan)
222	Government stock liability (s-t loan)
223	Government stock liability (l-t loan)
224	Government stock liability (other liability)
225	Government stock liability (liability)
226	National (Government Consumption)
Monetary Financial Institution	
227	Monetary financial institution transaction acquisition (currency gold & special drawing)
228	Monetary financial institution transaction acquisition (currency & deposit)
229	Monetary financial institution transaction acquisition (current & transfer deposit)
230	Monetary financial institution transaction acquisition (time deposit)
231	Monetary financial institution transaction acquisition (money market paper)
232	Monetary financial institution transaction acquisition (bond)
233	Monetary financial institution transaction acquisition (financial derivative)
234	Monetary financial institution transaction acquisition (loan)
235	Monetary financial institution transaction acquisition (s-t loan)
236	Monetary financial institution transaction acquisition (l-t loan)
237	Monetary financial institution transaction acquisition (share)
238	Monetary financial institution transaction acquisition (other equity)
239	Monetary financial institution transaction acquisition (mutual fund share)
240	Monetary financial institution transaction acquisition (other claim)
241	Monetary financial institution transaction acquisition (acquisition financial assets)

Table 57: German’s Macroeconomic Time Series Data (continue)

Monetary Financial Institution (continue)	
242	Monetary financial institution transaction external financing (currency & deposit)
243	Monetary financial institution transaction external financing (current & transfer deposit)
244	Monetary financial institution transaction external financing (time deposit)
245	Monetary financial institution transaction external financing (saving certificate)
246	Monetary financial institution transaction external financing (saving deposit)
247	Monetary financial institution transaction external financing (money market paper)
248	Monetary financial institution transaction external financing (bond)
249	Monetary financial institution transaction external financing (share)
250	Monetary financial institution transaction external financing (other equity)
251	Monetary financial institution transaction external financing (claim on company pension commitment)
252	Monetary financial institution transaction external financing (other liability)
253	Monetary financial institution transaction external financing (external financing)
254	Monetary financial institution stock financial assets (currency gold & special drawing)
255	Monetary financial institution stock financial assets (currency & deposit)
256	Monetary financial institution stock financial assets (current & transfer deposit)
257	Monetary financial institution stock financial assets (time deposit)
258	Monetary financial institution stock financial assets (money market paper)
259	Monetary financial institution stock financial assets (bond)
260	Monetary financial institution stock financial assets (loan)
261	Monetary financial institution stock financial assets (s-t loan)
262	Monetary financial institution stock financial assets (l-t loan)
263	Monetary financial institution stock financial assets (share)
264	Monetary financial institution stock financial assets (other equity)
265	Monetary financial institution stock financial assets (mutual fund share)
266	Monetary financial institution stock financial assets (other claim)
267	Monetary financial institution stock financial assets (financial assets)
268	Monetary financial institution stock liability (currency & deposit)
269	Monetary financial institution stock liability (current & transfer deposit)
270	Monetary financial institution stock liability (time deposit)

Table 57: German's Macroeconomic Time Series Data (continue)

Monetary Financial Institution (continue)	
271	Monetary financial institution stock liability (saving certificate)
272	Monetary financial institution stock liability (saving deposit)
272	Monetary financial institution stock liability (saving deposit)
273	Monetary financial institution stock liability (money market paper)
274	Monetary financial institution stock liability (bond)
275	Monetary financial institution stock liability (share)
276	Monetary financial institution stock liability (other equity)
277	Monetary financial institution stock liability (insurance technical reserve)
278	Monetary financial institution stock liability (other liability)
279	Monetary financial institution stock liability (liability)
Non-Financial Corporation	
280	Non financial corporation transaction acquisition (currency & deposit)
281	Non financial corporation transaction acquisition (current & transfer deposit)
282	Non financial corporation transaction acquisition (time deposit)
283	Non financial corporation transaction acquisition (saving deposit)
284	Non financial corporation transaction acquisition (saving certificate)
285	Non financial corporation transaction acquisition (money market paper)
286	Non financial corporation transaction acquisition (bond)
287	Non financial corporation transaction acquisition (financial derivative)
288	Non financial corporation transaction acquisition (share)
289	Non financial corporation transaction acquisition (other equity)
290	Non financial corporation transaction acquisition (mutual fund share)
291	Non financial corporation transaction acquisition (loan)
292	Non financial corporation transaction acquisition (s-t loan)
293	Non financial corporation transaction acquisition (l-t loan)
294	Non financial corporation transaction acquisition (claim on insurance corporation)
295	Non financial corporation transaction acquisition (s-t claim)
296	Non financial corporation transaction acquisition (acquisition financial assets)
297	Non financial corporation transaction external financing (money market paper)
298	Non financial corporation transaction external financing (bond)
299	Non financial corporation transaction external financing (share)
300	Non financial corporation transaction external financing (other equity)
301	Non financial corporation transaction external financing (loan)
302	Non financial corporation transaction external financing (s-t loan)
303	Non financial corporation transaction external financing (l-t loan)

Table 57: German’s Macroeconomic Time Series Data (continue)

Non-Financial Corporation (continue)	
304	Non financial corporation transaction external financing (l-t claim)
305	Non financial corporation transaction external financing (other liability)
306	Non financial corporation transaction external financing (external financing)
307	Non financial corporation stock financial assets (currency & deposit)
308	Non financial corporation stock financial assets (current & transfer deposit)
309	Non financial corporation stock financial assets (time deposit)
310	Non financial corporation stock financial assets (saving deposit)
311	Non financial corporation stock financial assets (saving certificate)
312	Non financial corporation stock financial assets (money market paper)
313	Non financial corporation stock financial assets (bond)
314	Non financial corporation stock financial assets (share)
315	Non financial corporation stock financial assets (other equity)
316	Non financial corporation stock financial assets (mutual fund share)
317	Non financial corporation stock financial assets (loan)
318	Non financial corporation stock financial assets (s-t loan)
319	Non financial corporation stock financial assets (l-t loan)
320	Non financial corporation stock financial assets (claim on insurance corporation)
321	Non financial corporation stock financial assets (s-t claim)
322	Non financial corporation stock financial assets (other claim)
323	Non financial corporation stock financial assets (financial asset)
324	Non financial corporation stock liability (money market paper)
325	Non financial corporation stock liability (bond)
326	Non financial corporation stock liability (share)
327	Non financial corporation stock liability (other equity)
328	Non financial corporation stock liability (loan)
329	Non financial corporation stock liability (s-t loan)
330	Non financial corporation stock liability (l-t loan)
331	Non financial corporation stock liability (claim on company pension commitment)
332	Non financial corporation stock liability (other liability)
333	Non financial corporation stock liability (liability)
Other Financial Intermediary	
334	Other financial intermediary transaction acquisition (currency & deposit)
335	Other financial intermediary transaction acquisition (current & transfer deposit)
336	Other financial intermediary transaction acquisition (time deposit)
337	Other financial intermediary transaction acquisition (money market paper)

Table 57: German's Macroeconomic Time Series Data (continue)

Other Financial Intermediary (continue)	
338	Other financial intermediary transaction acquisition (bond)
339	Other financial intermediary transaction acquisition (share)
340	Other financial intermediary transaction acquisition (other equity)
341	Other financial intermediary transaction acquisition (loan)
342	Other financial intermediary transaction acquisition (l-t loan)
343	Other financial intermediary transaction acquisition (acquisition financial assets)
344	Other financial intermediary transaction external financing (mutual fund share)
345	Other financial intermediary transaction external financing (loan)
346	Other financial intermediary transaction external financing (s-t loan)
347	Other financial intermediary transaction external financing (l-t loan)
348	Other financial intermediary transaction external financing (external financing)
349	Other financial intermediary stock financial assets (currency & deposit)
350	Other financial intermediary stock financial assets (time deposit)
351	Other financial intermediary stock financial assets (money market paper)
352	Other financial intermediary stock financial assets (bond)
353	Other financial intermediary stock financial assets (share)
354	Other financial intermediary stock financial assets (other equity)
355	Other financial intermediary stock financial assets (loan)
356	Other financial intermediary stock financial assets (l-t loan)
357	Other financial intermediary stock financial assets (financial assets)
358	Other financial intermediary stock liability (mutual fund share)
359	Other financial intermediary stock liability (liability)

BIOGRAPHY

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