CHAPTER II

SUPERFLUIDS

2.1 Basic Properties of Liquid ^4He

The two isotopes of helium have the lowest normal boiling points of all known substances 4.12 K for ^4He and 3.19 K for ^3He. When the temperature is reduced further, both ^3He and ^4He remain liquid under the saturated vapour pressure, and would apparently remain as such right down to absolute zero. To produce the solid phase application of a rather high pressure 25 atmospheres or more (Fig.2.1 and Fig.2.2) are required.

At a given instant of time, one particular atom in liquid ^4He occupies a certain volume bounded by the atoms immediately surrounding it. Owing to the motion of the atoms, this volume varies, but we can say that, on average, the atom is contained within a sphere of volume equal to the atomic volume V, and that the sphere has radius $R \sim V^{1/3}$. From the quantum mechanical uncertainly relation, it can be infered that a particle inside such a cavity has uncertainly in its momentum $\Delta P \sim h/R$, and consequently, that it processes kinetic energy of localization or zero point, $E_0 \sim (\Delta P)^2/2m_4 \sim h^2/2m_4R^2$, when m_4 is the mass of a ^4He atom. In terms of the atomic volume $E_0 \sim h^2/2mV^{2/3}$ and this dependent of E_0 upon V is shown schematically in (Fig.2.3). Calculation of the potential energy of the liquid is not easy; it depends upon the choice of model interaction between two atoms.

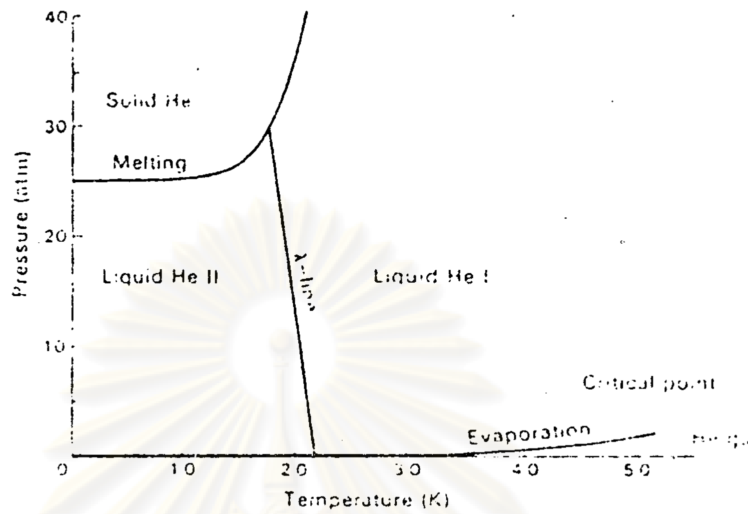

Fig 2.1 Phase diagram of He^4 (after London 1954).

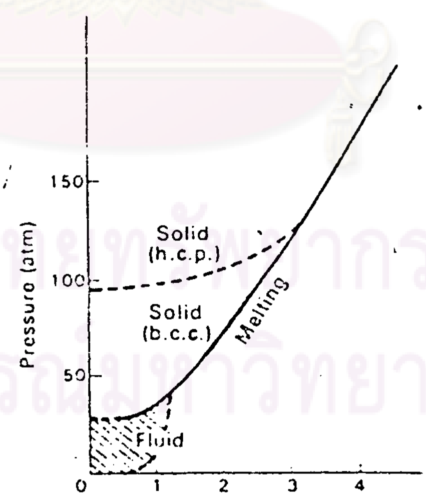

Fig 2.2 Phase diagram of He^3 (after Grilly and Mills 1959). Hatched area shows region of negative expansion coefficient.

It will have general form of the lowest curve in Fig.2.3. Since m_4 is small, the zero point energy is comparable in magnitude to the minimum in the potential curve. Thus the total energy of the liquid reaches a minimum at a considerable greater atomic volume than the potential energy minimum. The interatomic forces are strong enough to produce the liquid phase at a low enough temperature, but the high zero point energy keeps the density of the liquid rather small.

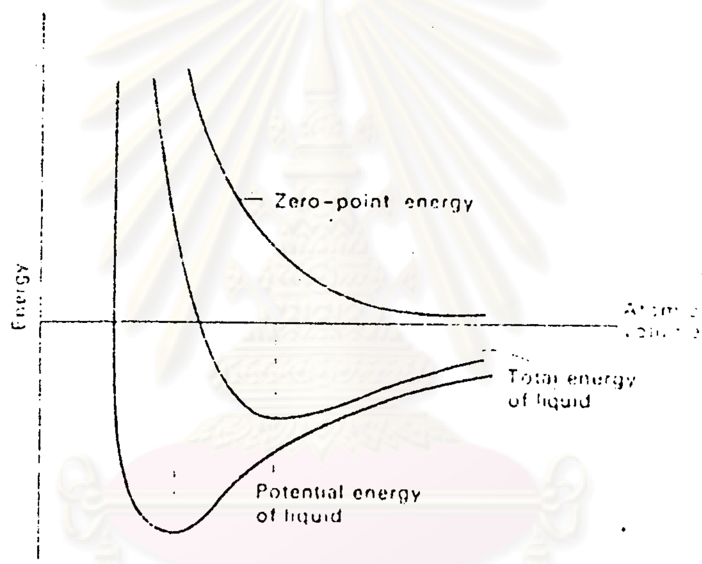

Fig 2.3 Energy of liquid helium. Total energy is sum of potential energy and zero-point energy

Immediately below their respective boiling points, both ^3He and ^4He behave like ordinary liquids with small viscosity. However, at 2.17 K liquid ^4He undergoes a change which is not shared by ^3He. This transition is signalled by specific heat anomaly, whose characteristic shape has lead to the name λ-point being given to temperature (T_λ) at which it occurs. Furthermore, observation of the liquid at the instant that its temperature, if when is reduce below T_λ reveals a remarkable alteration in its appearances. Liquid helium is maintained at temperature below 4.2 K, but by lowering the vapour above bath boiling can occur. Above T_λ, bubbles of vapour form within the bulk of the liquid **in** the customary way and the whole liquid is violently agitated as these rise to the free surface and escape. On the other hand, as soon as the transition point is reached the liquid becomes quite still and no more bubbles are formed. We infer that T_λ marks the transition between two different forms of liquid ^{4}He, known conventionally as Helium I above the λ-point and Helium II below it. London has proposed that the transition between liquid He I and He II is the result of the same process which cause the condensation of an ideal Bose-Einstein gas.

Below T_λ, liquid ^{4}He II is capable of two different motions at the same moment. Each of these has its own local velocity, respectively $\vec{v}_n$ and $\vec{v}_s$ for the normal fluid and the superfluid. Likewise each has its own effective mass density ρ_n and ρ_s. The total density ρ of the He II is given by

$$\rho = \rho_n + \rho_s \quad , \tag{2-1}$$

and the total current density by

$$\vec{J} = \rho_n\vec{v}_n + \rho_s\vec{v}_s \quad . \tag{2-2}$$

This approach in which the two fluid are treated independently is most useful when the velocities are small. At higher velocities the superfluid becomes dissipative, the normal fluid exhibits turbulence, and there is the possibility of interaction between the two. Tisza(15) has developed phenomenological theory based on this. Later Landau(16) has developed the microscopic theory using two-fluid model.

An example of the flow properties of liquid ^{4}He below the λ-point is provided by the behavior of film which covers an exposed surface of a body partially immered in He II. Normal adsorption on a surface in contact with any liquid or its saturated vapour is common enough, but in He II, the films are unusally thick. Optical measurement revealed that a typical thickness under saturated vapour is 30 nm or about 100 atomic layer, sufficiently wide to permit superfluid flow through the film. Owing to the presence of the film on its walls, an empty beaker lowered into a He II bath begins to fill with liquid, even though the rim is kept well above the bath surface (Fig.2-4). Filling continues until the inner level reaches the level of the bath, at which point its stops. The beaker is now raised, it empties itself again, and if it is raised clear of the bath, drops are seen to fall from the base of the beaker.

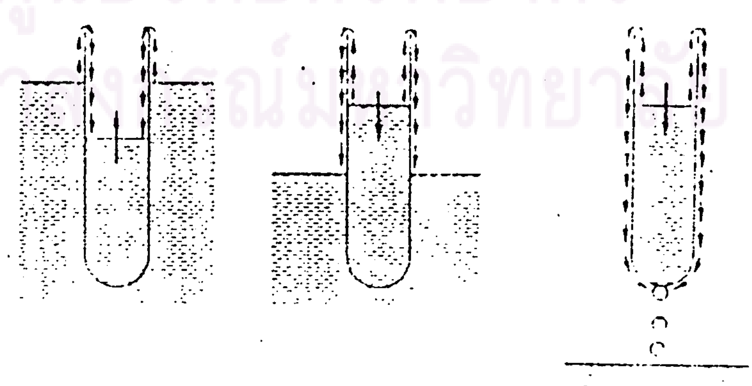

Fig.2.4 Film flow of He II over the walls of a beaker.

We conclude that the superfluid fraction flows through the film whenever there is a height difference between the two bulk liquid levels. In other words, the film acts like a siphon, the driving force for the superfluid being provided by the gravitational potential difference between the ends of the film. By observing the rate at which the beaker liquid level change,the superfluid velocity may be determined; (a typical value is 20 $cm.s^{-1}$). On the other hand, by virtue of its viscosity, the normal fluid fraction is almost stationary in the film.

Other properties of liquid ^{4}He below the λ-point is manifestation of a thermomechanical effect which clearly shows that heat transfer and mass transfer in He II are inseparable. The steady supply of heat to the bulk liquid, achieved for example by passing direct current through a resistor, and its removal elsewhere into a constant -temperature reservoir causes internal convection (Fig 2.5). Normal fluid flows from the source to the sink of heat whilst superfluid flows in the oppsite direction, under the constraint that the total density remains constant everywhere. Thus heat is not transferred in He II by the ordinary processes of conduction and simple convection of the whole fluid. Only the normal fluid fraction carries heat; superfluid flow by itself cannot transport heat.

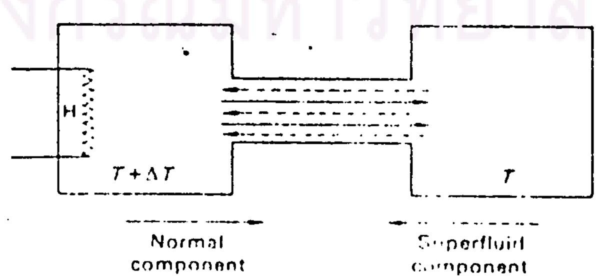

Fig 2.5 Internal convection in He II. Heat is supplied by heater H and temperatures are held constant.

The pure superfluid constitutes the ground state of He II. The ^4He atom has a resultant spin of zero, and so it is a boson. Thus an assembly of ^4He atoms is governed by Bose-Einstein statistics. As it well known, an ideal boson gas of particles with non-zero rest mass exhibits the phenomenon known as the Bose-Einstein condensation. At low temperature, the particles tend to crowd in to the same quantum state, the lowest single-particle energy level of the system, forming a condensate. The condensation begins at certain critical temperature and is complete at absolute zero. It seems certain that liquid ^4He behaves in a very similar way. The λ-point is the temperature which marks the onset of condensation, and the condensate is associate with the superfluid fraction of He II.

2.2 The Two-Fluid Model

The dynamics of He II can be understood in terms of the phenomenological theory as follows. At temperature other than zero, He behaves as if it were a mixture of two different liquids. One of which is a superfluid and moves with zero viscosity along a solid surface. The other is a normal viscous fluid. No friction occurs between these two parts of the liquid in the relative motion, ie, no momentum is transfered from one to the other. The total density of helium II is

$$\rho = \rho_s + \rho_n \qquad (2-3)$$

where ρ_s is superfluid density, and ρ_n is normal density. This theory of superfluid was developed by Landau(16), and are called tow-fluid model for liquid helium II. We shall denote by v_s and v_n the velocities of the superfluid and normal flow respectively. The total mass current density is given by

$$\vec{J} = \rho_s \vec{v}_s + \rho_n \vec{v}_n \qquad (2\text{-}4)$$

the theory of Landau is not concerned with the Bose-Einstein condensation. He treated the superfluid flow by expressing the macroscopic hydrodynamical variables in terms of quantum-mechanical operator. Initially he proposed that it should consist of two branches (Fig.2.6), one for phonons and another separated from the first by an energy gap. When he found that this spectrum did not yield the correct

Fig 2.6 Landau's (1941) initial suggestion for the form of He II excitation spectrum.

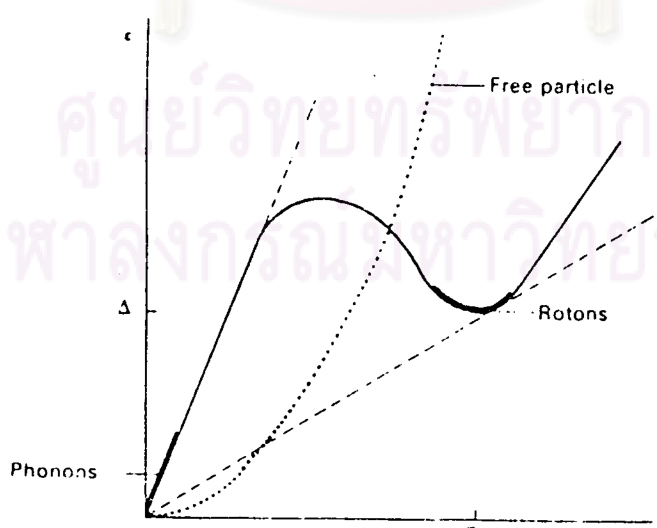

Fig 2.7 Phonon-roton spectrum suggested by Landau (1947) Broken lines indicate definitions of superfluid critical velocity . Dotted line shows free-particle parabola for comparison

thermodynamic behavior for ^{4}He II, he replaced it with the continuous spectrum shown in Fig.2.7. At low energies the curve is a straight line, corresponding to the phonon dispersion relation

$$\varepsilon = u_1 p, \tag{2-5}$$

where u_1 is the velocity of sound. At higher energies, the spectrum deviates from the straight line, passing first through a maximum and then a minimum. The excitations with energies near this minimum provide the only major contribution to the thermodynamic parameters besides that of the phonons, and their energy momentum relation can be expressed in the form

$$\varepsilon = \Delta + \frac{(p - p_0)^2}{2\mu_r} \tag{2-6}$$

where μ_r is an effective mass. We shall see shortly that the existence of the finite energy gap Δ for these excitation, called rotons, is crucial for the occurrence of superfluidity in He II. For the phonon gas, Bose-Einstein distribution function is appropriate, but the roton distribution function includes the factor $\exp(\Delta/k_B T)$ which is very large in the He II temperature range and this means that Maxwell-Boltmann statistics can be used for the rotons. The number densities of phonons and rotons are found to be

$$N_{ph} = 9.60\left(\frac{k_B T}{hu_1}\right)^3$$

$$N_r = \frac{2p_0^2(\mu_r k_B T)^{1/2}\exp(-\Delta/k_B T)}{(2\pi)^{3/2}\hbar^3} \tag{2-7}$$

Below 0.6 K the number of rotons excited is negligible and the phonons are the only significant excitations, but above 1 K the roton play the dominant role from the thermodynamic viewpoint.

Consider, firstly, He II at 0 K moving through a narrow tube. Superfluid flow is maintained as long as it is slow enough. However, above a certain critical velocity v_{sc}, the atoms move so fast that when they collide with the irregularities in the tube wall, and so they are removed from the ground state into the excited states. In terms of the quasi-particle model, thermal excitations are created in the liquid with equivalent loss of kinetic energy from the superfluid. On reaching the velocity v_{sc} the flow of He II ceases to be frictionless.

To estimate the value of v_{sc}, we imagine a body of large mass M moving at constant velocity $\vec{v}$ through the superfluid which is at rest in the laboratory system of coordinates. The critical velocity v_{sc} is equal to the minimum value of $\vec{v}$ at which an excitation can be created. Suppose that the appearence of an excitation with energy $\varepsilon(p)$ and momentum $\vec{p}$ causes the velocity of the body to change from $\vec{v}$ to $\vec{v}_1$. The process must conserve energy

$$\frac{1}{2}Mv^2 = \frac{1}{2}Mv_1^2 + \varepsilon(p) \qquad (2\text{-}8)$$

and momentum

$$M\vec{v} = M\vec{v}_1 + \vec{p}. \qquad (2\text{-}9)$$

If we eliminate $\vec{v}_1$ between eqs.(2-6) and (2-7), we obtain

$$\varepsilon(p) - \vec{p}.\vec{v} + \frac{p^2}{2M} = 0. \qquad (2\text{-}10)$$

We assume that M is so large that the last term in eq.(2-8) can be neglected. If θ is the angle between $\vec{p}$ and $\vec{v}$, we then have

$$pv \cos\theta = \varepsilon(p) \qquad (2\text{-}11)$$

and since $\cos\theta < 1$, the condition

$$\vec{v} \geqslant \frac{\varepsilon(p)}{p}$$

must be satisfied for excitations to be created. Thus the critical velocity is given by

$$v_{sc} = \left[\frac{\varepsilon(p)}{p}\right]_{min} \qquad (2\text{-}12)$$

Superfluidity can therefore occur if

$$v_{sc} > 0$$

a condition which is known as the Landau criterion for superfluid. Minimum values of $\varepsilon(p)/p$ are found where

$$\frac{d\varepsilon(p)}{dp} = \frac{\varepsilon(p)}{p} \qquad (2\text{-}13)$$

There are two solutions of eq. (2-11) on the He II excitation curve. One occurs at the origin, and indeed at all points of the linear part of the spectrum. In this region

$$v_{sc} = \frac{\varepsilon(p)}{p} = u_1 \qquad \text{(phonons)} \qquad (2\text{-}14)$$

which indicates that the critical velocity for the creation of phonons is the velocity of first sound.

To find the second solution to eq .(2-11), we draw the straight line which passes through the origin and touches the curve close to the roton minimum(Fig 2.5). From this we obtain

$$v_{sc} \simeq \frac{\Delta}{p_0} = 58 \quad ms^{-1} \qquad (2-15)$$

In Fig. 2.7, we show the free-particle parabola, corresponding to the dispersion relation $\varepsilon = p^2/2m_4$. We see immediately that condition eq .(2-11) can be satisfied on this curve only at the origin, giving

$$v_{sc} = 0 \qquad \text{(free particle)} \qquad (2-16)$$

A critical velocity of zero means that superfluidity is impossible in any system where free particle motion can take place. Thus it is the energy gap Δ, together with the lack of any other thermal excitations below the Landau curve, which ensures a finite value of superfluid critical velocity in He II.

London(10) first proposed a connection between the λ transition in liquid ^{4}He and the phenomena of Bose-Einstein condensation. Fröhlich(17) and Cummings et al(18) defined quantities such as condensate density (ρ_c) and depletion density (ρ_d) respectively, Total density of liquid Helium II at between temperature 0 K to 2.17 K is

$$\rho = \rho_c + \rho_d \qquad (2-17)$$

and it is a two fluid model. However, it has been developed further by using the macroscopic theory. Penrose(19) has used the reduced density matrices with a two fluid model. Use of reduced density matrices has been taken to the derivation of a closed set of thermo-hydrodynamic equation of ^{4}He II in the bulk system. We will present this in the chapter IV.