CHAPTER II

SUPERCONDUCTIVITY

II.1 One Band BCS Theory

The BCS[6] theory evolved from the observation by Cooper that if one considered two electrons excited slightly from the Fermi sea, they could form a real bound state(localized wave function and an energy below Fermi sea ground state) provided there was a weak attractive potential. This bound state had the lowest energy if its net momentum was zero,i.e.,if the wave function was composed of a superposition of state in which the two electrons had equal and opposite momentum.

The step taken by BCS is to assume that the ground state can be expressed wholly in terms of paired electrons, such that if the state $k\uparrow$ is vacant, $-k\downarrow$ is also vacant.

Bogoliubov originally(1947)developed a transformation of this sort for boson system, and he applied it in his theory of superconductivity(1958)shortly after the appearance of the BCS theory. Valatin[7] independently pointed out the convenience of the method for clarifying some aspects of BCS.

II.1.1 Mechanisms of Superconductivity

In 1956,Cooper showed that the Fermi sea of fermion system was unstable if an attractive interaction existed between particles in the fermion sea. This instability was due to the formation of bound pairs of particles. Since no external forces are being applied, the total(or net)momentum and spin of the system must remain constant. This implies that the electrons which are interacting with each other must have opposite momentum and spins. An interaction which might give rise to an attractive interaction is the exchange of phonons with energies below the Debye frequency ω_D.

The Hamiltonian which could(possibly)describe the superconducting state is

$$H = \sum_{k,\sigma} \epsilon_k c^+_{k\sigma} c_{k\sigma} + \sum_{k,k'} V_{kk'} c^+_{k\sigma} c^+_{-k-\sigma} c_{-k'-\sigma} c_{k'\sigma} , \qquad (2.1)$$

where $c^+_{k\sigma}$ and $c_{k\sigma}$ are creation and annihilation operators respectively, which obey the following anticommutation relations

$$\{c_{k\sigma}, c^+_{k'\sigma'}\} = \delta_{kk'}\delta_{\sigma\sigma'} ,$$

$$\{c_{k\sigma}, c_{k\sigma}\} = \{c^+_{k\sigma}, c^+_{k\sigma}\} = 0 \qquad (2.2)$$

$$V_{kk'} = \begin{cases} -V, & |\epsilon_k|, |\epsilon_{k'}| < \hbar\omega_D , \\ 0, & \text{otherwise.} \end{cases}$$

Because the model Hamiltonian does not commute with the total number of particles it is not possible to work in a subspace with a definite total number of particles. In considering the ensemble averages of the operators we have then to use a grand canonical ensemble with a definite chemical potential μ. Formally this is easily included by measuring the single-particle energies relative to μ. For the free electron model of the metal this means that

$$\epsilon_k = \frac{\hbar^2 k^2}{2m} - \mu .$$

The chemical potential has to be determined from the condition that the average number of particles present is a given number N :

$$\langle \sum_{k\sigma} c^{+}_{k\sigma} c_{k\sigma} \rangle = N \, .$$

Alternatively,we can regard μ as given and then this equation determines the average number of particles present.

By using Hartree-Fock approximation

$$\begin{aligned} c^{+}_{k\sigma} c^{+}_{-k-\sigma} c_{-k-\sigma} c_{k\sigma} \longrightarrow & \langle c^{+}_{k\sigma} c^{+}_{-k-\sigma} \rangle c_{-k-\sigma} c_{k\sigma} \\ & + \langle c_{-k-\sigma} c_{k\sigma} \rangle c^{+}_{k\sigma} c^{+}_{-k-\sigma} \\ & - 2 \langle c^{+}_{-k-\sigma} c_{-k-\sigma} \rangle c^{+}_{k\sigma} c_{k\sigma} \, . \end{aligned}$$

The last term $2\langle c^{+}_{-k-\sigma} c_{-k-\sigma} \rangle c^{+}_{k\sigma} c_{k\sigma}$ only leads to a shift of the Fermi energy. The Fermi energy comes in because the electrons have energies relative to the Fermi energy. Because of the Pauli's exclusion principle, only electrons with energies greater than ϵ_F can move.

Let us now define gap parameter as

$$\Delta_k = -\sum_{k'} V_{k,k'} c_{-k-\sigma} c_{k\sigma} \, , \quad \Delta^{*}_k = -\sum_{k'} V_{kk'} c^{+}_{k\sigma} c^{+}_{-k-\sigma} \, , \tag{2.3}$$

so that the Hamiltonian (2.1)becomes

$$H = \sum_{k,\sigma} \epsilon_k c^{+}_{k\sigma} c_{k\sigma} - \sum_{k} (\Delta_k c^{+}_{k\sigma} c^{+}_{-k-\sigma} + \Delta^{*}_k c_{-k-\sigma} c_{k\sigma}) \, . \tag{2.4}$$

II.1.2 Bogoliubov Formulation

Since the above Hamiltonian is bilinear in the creation and annihilation operators,it is non-diagonal. It can be diagonalized by means of a linear canonical transformation of these operators. The canonical transformation which does this was introduced by Bogoliubov[8] and is called Bogoliubov transformation. It transforms the c's to the quasi-particle operators γ's, i.e.,

$$
\begin{aligned}
c_{k\sigma} &= u_k \gamma_{k_0} + v_k^* \gamma_{k_1}^* , \\
c_{-k-\sigma}^* &= -v_k \gamma_{k_0} + u_k^* \gamma_{k_1}^* ,
\end{aligned}
\qquad (2.5)
$$

where the coefficients u_k, v_k, are chosen to make the Hamiltonian diagonal; that is, they are chosen to make the coefficients of $\gamma_{k_0}^* \gamma_{k_1}^*$ and $\gamma_{k_1} \gamma_{k_0}$ in the Hamiltonian vanish. Since the $c_{k\sigma}$ and $c^+_{k\sigma}$ satisfy the anticommutation relation the γ_{k_0} and γ_{k_1} must also satisfy the anticommutation relation

$$
\begin{aligned}
\{\gamma_{k_0}, \gamma_{k_0}^+\} &= \{\gamma_{k_1}, \gamma_{k_1}^+\} = 1 , \\
\{\gamma_{k_0}, \gamma_{k_1}^+\} &= \{\gamma_{k_0}, \gamma_{k_0}\} = \{\gamma_{k_1}, \gamma_{k_1}\} = 0 .
\end{aligned}
\qquad (2.6)
$$

The condition that makes the Hamiltonian diagonal is

$$2 \epsilon_k u_k v_k + \Delta_k v_k^2 - \Delta_k^* u_k^2 = 0 . \qquad (2.7)$$

The constraints on the u_k and v_k is

$$|u_k|^2 + |v_k|^2 = 1 . \qquad (2.8)$$

Equations (2.7) and (2.8) are sufficient to determine u_k and v_k in terms of Δ. From the imaginary part of Eq.(2.7) we find that $\Delta v/_u$ is real, and hence Eq.(2.7) can be written

$$2 \epsilon_k |u_k v_k| + |\Delta_k| \left(|u_k|^2 - |v_k|^2 \right) = 0 .$$

From this and Eq.(2.8) we then find

$$
\begin{aligned}
|u_k|^2 &= \frac{1}{2} \left(1 + \frac{\epsilon_k}{E_k} \right) , \\
|v_k|^2 &= \frac{1}{2} \left(1 - \frac{\epsilon_k}{E_k} \right) ,
\end{aligned}
\qquad (2.9)
$$

where $E_k = (\epsilon_k^2 + |\Delta_k|^2)^{1/2}$. (2.10)

In terms of the new operators γ_{k_0} and γ_{k_1} , the Hamiltonian becomes

$$H = \sum_k E_k (\gamma_{k_0}^* \gamma_{k_0} + \gamma_{k_1}^* \gamma_{k_1}) + \text{constants} . \qquad (2.11)$$

Now we must remember that the physical observed quantities are not operators but the statistical average given by

$$\langle A \rangle = \mathrm{Tr}\left[\exp(-\beta H)A\right] \Big/ \mathrm{Tr}\left[\exp(-\beta H)\right] .$$

Therefore the energy gap parameter Δ_k is given by

$$\Delta_k = -\sum_{k'} V_{kk'} C_{-k-6} C_{k6}$$

$$= -\sum_{k'} V_{kk'} \frac{\mathrm{Tr}\left[\exp(-\beta H)(-v_k^* u_k \gamma_{k_0}^* \gamma_{k_0} + u_k v_k^* \gamma_{k_1} \gamma_{k_1}^*)\right]}{\mathrm{Tr}\left[\exp(-\beta H)\right]}$$

$$= -\sum_{k'} V_{kk'} u_{k'} v_{k'}^* \left[1 - 2 f(E_{k'})\right]$$

$$= -\sum_{k'} V_{kk'} \frac{\Delta_{k'}}{2E_{k'}} \left[1 - 2 f(E_{k'})\right] \qquad (2.12)$$

where $$f(E) = \left[\exp(\beta E) + 1\right]^{-1} .$$

This equation is a highly nonlinear equation for the gap parameter Δ_k . Given the interaction potential the equation can be solved.

The integral equation(2.12)always possesses a solution

$$\Delta_k = 0 , \qquad \text{for all } k.$$

For this solution the operators γ_k become equal to the operators c_k and we have just the solution for a normal Fermi liquid. The system will possess new properties only if Δ_k is not zero for a macroscpic number of values of k. We shall see that when Δ_k is not zero, the system does possess the properties of a superconductor, and that at low temperatures the superconductive state is the stable one. The criterion for superconductivity is, therefore, that Eq.(2.12) should possess a nontrivial solution.

II.2 Transport Properties of One Band Superconductors

II.2.1 Thermal Conductivity

The general Boltzmann equation for the distribution function $f_k(r)$ is

$$\frac{\partial f_k}{\partial t} + \frac{\partial k}{\partial t} \cdot \frac{\partial f_k}{\partial k} + \vec{v} \cdot \frac{\partial f_k}{\partial r} = \left.\frac{\partial f_k}{\partial t}\right|_{coll} . \qquad (2.13)$$

In steady state conditions with no mechanical or electromagnetic forces the first two terms on the left side are zero. The last term on the left side involves the velocity of a quasiparticle of momentum $\hbar k$. Since the energy of this particle is E_k, we expect the velocity to be

$$v = \frac{\partial E_k}{\hbar \partial k} .$$

This is borne out by more detailed consideration of the motion of wave packets.[9] Hence

$$v = \frac{\epsilon}{E}\frac{\hbar k}{m} . \tag{2.14}$$

The last term on the left side also involves the spatial derivative of the distribution function. Since a spatially dependent gap parameter will not by itself produce a thermal current, we can write

$$\frac{\partial f_k}{\partial r} = \left.\frac{\partial f_k}{\partial r}\right|_{\Delta\ \mathrm{const}} \approx -v \cdot \nabla T \, \frac{E}{T} \frac{\partial f_{eq}}{\partial T} . \tag{2.15}$$

The interaction between electrons and a single impurity is given by

$$H_i = \sum_{k,q,\sigma} U(q)\ c^{+}_{k+q\sigma} c_{k\sigma} \tag{2.16}$$

where U(q) is the Fourier transform of the potential due to the impurity. In terms of the operators for quasi-particles this can be rewritten

$$H_i = \sum_{k,q} U(q) \Big[(u_{k+q}v_k + v_{k+q}u_k)(\gamma^{*}_{k+qo}\gamma^{*}_{k1} + \gamma_{k+q1}\gamma_{ko}) + (u_k u_{k+q} - v_k v_{k+q})(\gamma^{*}_{k+qo}\gamma_{ko} + \gamma^{*}_{k1}\gamma_{k+q1}) \Big] .$$

Hence the probability per unit time that a quasi-particle of type o is scattered from k to k+q is

$$P(k,k+q) = \frac{2\pi}{\hbar}\left|U(q)\right|^2 (u_k u_{k+q} - v_k v_{k+q})^2 f_k (1-f_{k+q}) \delta(E_k - E_{k+q}) . \tag{2.17}$$

It follows that the rate of increase of quasi-particles in the state(k,o) is

$$\left.\frac{\partial f_k}{\partial t}\right|_{coll} = \sum_q \Big[P(k+q,k) - P(k,k+q)\Big]$$
$$= \frac{2\pi}{\hbar}\sum_q (u_k u_{k+q} - v_k v_{k+q})^2 (f_{k+q} - f_k)\delta(E_k - E_{k+q})\left|U(q)\right|^2 . \tag{2.18}$$

If the impurities scatter independently, this is multiplied by the density of impurities N_i for the effect of all the impurities. It can be shown that to first order in the temperature gradient the equation has a solution of the form

$$f_k = f_{eq,k} + k.\nabla T\ g(E_k)\ , \qquad (2.19)$$

where f_{eq} is the equilibrium distribution of quasi-particles. With the form (2.19) for f_k the collision term is

$$\left.\frac{\partial f_k}{\partial T}\right|_{coll} = \frac{2\pi}{\hbar}\left(\frac{\epsilon}{E}\right)^2 g(E)N_i \sum_{k} \left|U(k-k')\right|^2 (k'-k).\nabla T\ \delta(E_k - E_k)$$

$$= -\frac{2\pi}{\hbar}\left(\frac{\epsilon}{E}\right)^2 g(E)N_i N(o)k.\nabla T \int d\epsilon'\ \frac{d\mu}{2}\left|U(k-k')\right|^2 (1-\mu)\ \delta(E'-E),$$

where k, k' can be put equal to k_F except in the terms involving ϵ, ϵ'. For the normal state the corresponding term is

$$\left.\frac{\partial f_k}{\partial T}\right|_{coll} = -\frac{2\pi}{\hbar}\left|\frac{\epsilon}{E}\right| g(E)N_i N(O)k.\nabla T \int d\mu \left|U(k-k')\right|^2 (1-\mu)$$

$$= -k.\nabla T g(E)/\tau$$

where τ is the mean free time in the normal state. Hence in the superconductive state one can write

$$\left.\frac{\partial f_k}{\partial T}\right|_{coll} = -\left|\frac{\epsilon}{E}\right|\ k.\nabla T\ \frac{g(E)}{\tau},$$

where τ is still the mean free time in the normal state.

The Boltzmann equation can now be written

$$\frac{\epsilon}{E}\ \frac{\hbar k}{m}.\nabla T\ \frac{E}{T}\ \frac{\partial f}{\partial E} = \left|\frac{\epsilon}{E}\right|\ g(E)\ \frac{k.\nabla T}{\tau},$$

and has the solution

$$g(E) = \frac{\epsilon}{|\epsilon|}\frac{\hbar\tau}{m}\frac{E}{T}\frac{\partial f}{\partial E}eq \quad . \tag{2.20}$$

The measured quantity is the thermal conductivity, defined by

$$K = \frac{\text{heat current}}{\text{thermal gradient}} \quad .$$

Since the energy of a quasi-particle is E_k and its velocity is v, the heat current carried by one kind of quasi-particle is

$$W_o = \sum_k E_k v f_k \quad .$$

Since the two kinds of quasi-particle carry the same amount of heat, the total heat current is

$$\begin{aligned} W &= 2\sum_k E_k vk.\nabla T \frac{\epsilon}{|\epsilon|}\frac{\hbar}{m}\frac{\tau E}{T} \cdot \frac{\partial f}{\partial E}eq \\ &= \frac{2}{3}\frac{\tau}{T} N(0) v_F^2 \nabla T \int_{-\infty}^{\infty} d\epsilon|\epsilon| E \frac{\partial f}{\partial E} \quad . \end{aligned} \tag{2.21}$$

Hence

$$K = \frac{4}{3}\frac{\tau}{T} N(0) v_F^2 \int_{\Delta}^{\infty} dE\, E^2 \frac{\partial f}{\partial E} \quad , \tag{2.22}$$

and for the normal state the integration is from 0 to ∞.

II.2.2 Ultrasonic Attenuation

The interaction can be written as

$$H_i = \frac{A}{V^{1/2}} \sum_{k,q,\sigma} \omega_q^{1/2} c^*_{k+q\sigma} c_{k\sigma}(b^*_{-q}+b_q) \quad , \tag{2.23}$$

where A is a constant and b_q^* and b_q are, respectively, creation and destruction operators for phonons.

The probability per unit time for a transition from any possible initial state $|m\rangle$ to any state $|n\rangle$ is

$$P = \frac{2\pi}{\hbar} \sum_{m,n} \frac{\exp(-\beta E_m) |\langle n|H_i|m\rangle|^2 \delta(E_m - E_n)}{\sum_m \exp(-\beta E_m)} .$$

We take the states $|m\rangle$ and $|n\rangle$ to be states with definite numbers of phonons and quasi-particles, and in terms of the quasi-particle operators the interaction H_i is

$$H_i = \frac{A}{V^{1/2}} \sum_{k,q} \omega_q^{1/2} (b^*_{-q} + b_q) \Big[(u_{k+q} v_k + v_{k+q} u_k)(\gamma^*_{k+q\,0} \gamma^*_{k1} + \gamma_{k+q,1} \gamma_{k0})$$

$$+ (u_k u_{k+q} - v_k v_{k+q})(\gamma^*_{k+q\,0} \gamma_{k0} + \gamma^*_{k1} \gamma_{k+q\,1}) \Big] . \qquad (2.24)$$

Since the angular frequency is less than $\Delta/\hbar$, only the terms of the interaction which scatter quasi-particles are important. Hence the probability per unit time that a phonon is absorbed is

$$P_a = \frac{4\pi A^2 \omega_q n_q}{\hbar V} \sum_k (u_k u_{k+q} - v_k v_{k+q})^2 f_k (1 - f_{k+q}) \delta(E_{k+q} - E_k - \hbar\omega_q),$$

where n_q is the number of phonons of wave vector q present initially, which for a macroscopic sound wave is much greater than unity. Similarly, the probability per unit time that a phonon is emitted is

$$P_e = \frac{4\pi A^2 \omega_q n_q}{\hbar V} \sum_k (u_k u_{k+q} - v_k v_{k+q})^2 f_{k+q} (1 - f_k) \delta(E_{k+q} - E_k - \hbar\omega_q).$$

It follows that

$$\frac{dn_q}{dt} = -\frac{4\pi A^2 \omega_q n_q}{\hbar V} \sum_k (u_k u_{k+q} - v_k v_{k+q})^2 (f_k - f_{k+q}) \delta(E_{k+q} - E_k - \hbar\omega_q),$$

and for small ω_q this can be written

$$\frac{dn_q}{dt} = \frac{4\pi A^2 \omega_q^2 n_q}{V} \sum_k (u_k^2 - v_k^2)^2 \frac{\partial f_k}{\partial E_k} \delta(E_{k+q} - E_k)$$

$$= \frac{4\pi A^2 \omega_q^2 n_q}{V} N(0) \int d\epsilon \left(\frac{\epsilon}{E}\right)^2 \frac{\partial f}{\partial E} \int \frac{d\mu}{2} \left|\frac{E}{\epsilon}\right| \delta(\epsilon_{k+q} - \epsilon_k).$$

The attenuation coefficient is proprotional to the coefficient of n_q. Since this coefficient is correctly given here both for Δ finite and for Δ equal to zero, we find that

$$\frac{\alpha_s}{\alpha_n} = \frac{\int_0^\infty d\epsilon \, \frac{\epsilon}{E} \frac{\partial f}{\partial E}}{\int_0^\infty d\epsilon \, \frac{\partial f}{\partial \epsilon}} = 2f(\Delta). \tag{2.25}$$

This very simple result means that ultrasonic attenuation is a useful tool for finding the gap parameter Δ.

II.2.3 Electrodynamic Properties

In the simple form of the theory normally used, the calculation of the electromagnetic response is carried out strictly in a transverse field, so that $\operatorname{div} \vec{A} = 0$. To cope with longitudinal fields, we would need a formalism capable of handling collective modes and backflow associated with the quasi-particles. This more general treatment has been given by Rickayzen.[10]

The effect of a given self-consistent field on the superconductor can be described through a classical vector potential $A(r,t)$. Since we are concerned only with linear effects we need only the linear term in the interaction Hamiltonian, and this is

$$H_i(t) = \frac{ie\hbar}{2mc} \int d^3r \, \psi^*(r) \left(\nabla . A + A . \nabla\right) \psi(r).$$

In terms of $a(q,t)$, the spatial Fourier transform of the vector potential, and of the quasi-particle operators, we have

$$H_i(t) = -\frac{e\hbar}{2mc}\sum_{k,q}\Big[a(-q,t)\cdot(2k+q)$$
$$\times\Big[(u_{k+q}v_k - v_{k+q}u_k)(\gamma^*_{k+q0}\gamma^*_{k1} - \gamma_{k1}\gamma_{k+q0})$$
$$+(u_k u_{k+q} + v_k v_{k+q})(\gamma^*_{k+q0}\gamma_{k0} - \gamma^*_{k1}\gamma_{k+q0})\Big]. \quad (2.26)$$

The response of the superconductor to the external field is usually given in terms of the current density. To first order in the field, the operator for the current density is

$$j_{op}(r) = \frac{e}{2m}\Big\{\psi^*(-i\hbar\nabla - \frac{e}{c}A)\psi - \big[(-i\hbar\nabla + \frac{e}{c}A)\psi^*\big]\psi\Big\}$$

$$= j_1(r) + j_2(r), \quad (2.27)$$

where $$j_2(r) = \frac{e^2}{2mc}A(r,t)\,\psi^*\psi, \quad (2.28)$$

and, in terms of quasi-particle operators,

$$j_1(r) = \frac{e\hbar}{2mV}\sum_{k,q}(2k+q)\exp(iq\cdot r)\Big[(u_k v_{k+q} - v_k u_{k+q})$$
$$\times(\gamma^*_{k0}\gamma^*_{k+q1} - \gamma_{k1}\gamma_{k+q0}) + (u_k u_{k+q} + v_k v_{k+q})(\gamma^*_{k0}\gamma_{k+q0} - \gamma^*_{k+q1}\gamma_{k1})\Big]. \quad (2.29)$$

The expectation value of the current density in the external field can be found by simple perturbation theory. We have to first order in the external field[11] that the q^{th} Fourier component of this expectation value is

$$J(q,t) = \mathrm{Tr}\,\rho_M\, j_{op}(q) - \frac{i}{\hbar}\int_0^t dt'\,\mathrm{Tr}\,\rho_M\big[\hat{j}_{op}(q,t), \hat{H}_i(t')\big], \quad (2.30)$$

where ρ_M is the density matrix for the model Hamiltonian, that is,

$$\rho_M = \exp(-\beta H_m) \big/ \mathrm{Tr}\exp(-\beta H_M) ,$$

and j_{op}, H_i are Heisenberg operators defined by

$$\hat{j}_{op}(t) = \exp(iH_M t/\hbar)\, j_{op} \exp(-iH_M t/\hbar) , \tag{2.31}$$

and a similar equation for $\hat{H}_i$. Itis assumed in Eq.(2.29)that the field is switched on at the time, $t = 0$.

The component j_2 of j_{op} will contribute only to the first term of Eq.(2.29) and will give a contribution

$$J_2(q,\omega) = -\frac{ne^2}{mc} a(q,\omega) , \tag{2.32}$$

where n is the density of electrons.

The component j_1 of j_{op} contributes only to the second term of Eq.(2.29) and yields a contribution

$$J_1(q,\omega) = \sum_m \exp(-\beta E_m) \left\{ \frac{\langle m|j_1(q)|n\rangle\langle n|H_i(\omega)|m\rangle}{E_m - E_n + \hbar\omega + i\hbar/\tau} \right.$$

$$\left. - \frac{\langle m|H_i(\omega)|n\rangle\langle n|j_1(q)|m\rangle}{E_n - E_m + \hbar\omega + i\hbar/\tau} \right\} \times \Big[\sum_m \exp(-\beta E_m)\Big]^{-1} , \tag{2.33}$$

where

$$H_i(\omega) = -\frac{V}{c}\sum_q a(-q,\omega)\cdot j_1(q) .$$

Hence

$$J(q,\omega) = \frac{e^2\hbar^2}{4m^2 V}\sum_k a(q,\omega)\cdot(2k+q)(2k+q)$$

$$\times \left\{ (u_k u_{k+q} + v_k v_{k+q})^2 \left[\frac{f_k - f_{k+q}}{E_{k+q} - E_k + \hbar\omega + i\hbar/\tau} + \frac{f_{k+q} - f_k}{E_k - E_{k+q} + \hbar\omega + i\hbar/\tau} \right] \right.$$

$$\left. + (u_k v_{k+q} - v_k u_{k+q})^2 \left[\frac{1 - f_k - f_{k+q}}{E_{k+q} + E_k + \hbar\omega + i\hbar/\tau} + \frac{1 - f_k - f_{k+q}}{E_{k+q} + E_k - \hbar\omega - i\hbar/\tau} \right] \right\}$$

$$- \frac{ne^2}{mc} a(q,\omega) . \tag{2.34}$$

For the weak dissipation we have assumed the limit of $\mathcal{T}$ tending to infinity.

The relation(2.34) between current and field can also be written in the form

$$J(q,\omega) = -\frac{c}{4\pi} K(q,\omega)\, a(q,\omega). \tag{2.35}$$

Since the vector potential is related to the electric field by

$$a(q,\omega) = c\, E(q,\omega)/i\omega,$$

the superconductor has a frequency-and wave vector-dependent complex conductivity $\sigma(q,\omega)$ defined by

$$\sigma(q,\omega) = \sigma_1 + i\sigma_2$$

$$= -c^2 K(q,\omega)/4\pi i\omega$$

or

$$K(q,\omega) = \{4\pi\omega/ic^2\}\, \sigma(q,\omega), \tag{2.36}$$

so that if $\sigma_2(q,\omega) \sim \frac{1}{\omega}$, $K(q,\omega)$ =constant.