CHAPTER IV

THE FEYNMAN AVERAGED PROPAGATOR

Introduction

It is well known that when a high intensity magnetic field is applied to a two-dimensional electron gas, the electron energy spectrum becomes completely discrete and the density of states becomes highly singular. The scattering of the electron by impurities, surface roughness, etc. leads to a broadening of these singularities. Theoretical results for the DOS of this system in a high intensity magnetic field were first obtained by Ando and Uemura [1] on the basis of a self consistent Born approximation (SCBA). The DOS versus energy curve obtained by them consists of half ellipses centered at the Landau energies separated by regions of vanishing DOS, shown as a dashed line in Fig. 10. However, because of the sharp edges of the elliptically shaped spectral lines which constitute the DOS, the theory of Ando and Uemura breaks down in the case of overlapping levels.

Theoretical predictions concerning the broadening of the Landau levels (LL's) were then proposed by Gerhardts [26]. He expressed the Green function of an electron in the presence of random impurities and a magnetic field in terms of the Feynman path integrals and obtained an approximate formula for the propagator at the lowest order cumulant expansion. He formulated the cumulant expansion in the position representation in which both the free propagator and the average propagator are diagonal. He obtained a Gaussian distribution of the DOS without the shortcomings

which lead to the breakdown of the theory of Ando and Uemura. Both the theories of Ando and Uemura and of Gerhardts essentially predict the DOS for sufficiently strong magnetic fields, the LL's are energetically well separated and the DOS is zero exponentially small in the gap between the Landau levels.

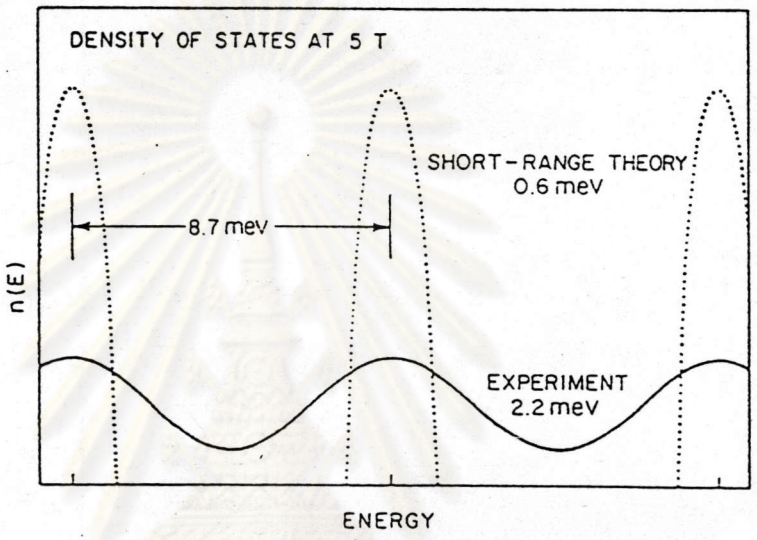

Fig. 10 Comparison of the model DOS (solid line) which fits 2DES data for B = 5 T (ref. 14) and the SCBA short - range interaction theory (dashed line). At B = 5 T, $\hbar\omega$ = 8.7 meV. (Figure from Ref. 14.)

A number of recent experiments [7, 10, 12], however, indicate strong evidence for an unexpectedly large DOS in the Landau gaps, due to disorder in the samples, and that there is a substantial DOS lying between Landau levels not obtained in existing calculations [1, 18, 19, 37]. This effect is seen in both GaAs - $Ga_{1-x}Al_xAs$ heterostructures and MOSFET's.

Recent theoretical investigations of the Landau broadening have been made by several groups. Wegner [21] succeeded in calculating exactly the DOS for the case of a

Gaussian white-noise distribution for the random impurity potential corresponding to zero-range impurities. Most theories consider the electrons interacting with disorder having an interaction in the zero-range. Typically they use the white-noise model in which the variance $W(\vec{r}-\vec{r}\,') = \langle V(\vec{r})V(\vec{r}\,')\rangle - \langle V(\vec{r})\rangle^2$ of the fluctuating potential $V(\vec{r})$ has zero correlation length,

$$W_{WN}(\vec{r}-\vec{r}\,') = W(0)\,\delta(\vec{r}-\vec{r}\,') \qquad (4.1)$$

None of these theories predicts a significant DOS lying between the LL's.

The aim here is to show that broad LL's and a significant DOS between LL's can be obtained in a simple and consistent manner from a simple model of disorder. The essential point is to keep the correlation length, L, of disorder finite from the outset [27, 28] and to combine it with a theory that is valid for broad LL's which can overlap. We represent here the disorder by a Gaussian variance

$$W(\vec{r}-\vec{r}\,') = \frac{W(L)}{\pi L^2}\exp\left\{-(\vec{r}-\vec{r}\,')^2/L^2\right\} \qquad (4.2)$$

The disorder is then characterized by a strength W(L). We determine the DOS in the presence of the disorder (4.2) and obtain a substantial DOS between the LL's for appropriate, physical choices of W and L. The method is very similar to the procedure that Sa-yakanit et al. [32, 33] used in three dimensions to explain the origin of Urbach tails in optical absorption near band edges. As before we solve for the DOS using the path integral method, which at present appears to be the best method for obtaining arbitrary L disorder. A model that we use to calculate the DOS will be introduced in next section.

Model of a Random System

We now consider a single electron confined to the xy plane, moving among a set of rigid impurities and in the presence of an arbitrary strong magnetic field $\vec{B} = \vec{\nabla} \times \vec{A}$ along the z axis. $\vec{A}$ is the vector potential. The electron sees a fluctuating potential $V(\vec{r})$ so that the Hamiltonian is $H = H_0 + V(\vec{r})$ where $H_0 = (\vec{p} + e\vec{A}/c)^2/2m$. The variance of $V(\vec{r})$ is modeled by the Gaussian function (4.2). For example, if the disorder is due to impurities located at random points $\vec{R}_i$ in the plane, so we can write the fluctuating potential as $V(\vec{r}) = \sum_{i=1}^{N} v(\vec{r} - \vec{R}_i)$. Such a system is described by the Hamiltonian

$$H = \frac{1}{2m}\left(\vec{p} - \frac{e\vec{A}}{c}\right)^2 + \sum_{i=1}^{N} v(\vec{r} - \vec{R}_i) \qquad (4.3)$$

where $v(\vec{r} - \vec{R}_i)$ represents the potential of a single impurity at position $\vec{R}_i$ and N is the number of electrons in an area A. The density of electrons is, $\rho = N/A$. The propagator describing the motion of the electron in this system satisfies Schrödinger's equation

$$\left[i\hbar\frac{\partial}{\partial t} - H(\{\vec{R}\})\right] K(\vec{r}', \vec{r}; t) = i\hbar\delta(\vec{r}' - \vec{r})\delta(t) \qquad (4.4)$$

The propagator, which we choose the symmetric gauge $\vec{A} = (-By, 0, 0)$ can be expressed in terms of a Feynman path integral as

$$K\left(\vec{r}',\vec{r};t,\{\vec{R}\}\right) = \int \mathcal{D}(\vec{r}(\tau))\exp\left[\frac{i}{\hbar}\int_0^t d\tau\left\{\frac{m}{2}\left(\dot{x}^2(\tau)+\dot{y}^2(\tau)\right.\right.\right.$$

$$\left.\left.\left.+\omega\left(x(\tau)\dot{y}(\tau)-y(\tau)\dot{x}(\tau)\right)\right)-\sum_{i=1}^{N} v\left(\vec{r}-\vec{R}_i\right)\right\}\right] \tag{4.5}$$

where $D(\vec{r}(t))$ denotes the path integral to be carried out with the boundary conditions $\vec{r}(0)=\vec{r}$ and $\vec{r}(t)=\vec{r}'$. The probability distribution of the scattering potentials is assumed to be

$$P(\{\vec{R}\})d(\{\vec{R}\}) = \prod_{N,V\to\infty} \frac{d\vec{R},\ldots,d\vec{R}_N}{V^N}. \tag{4.6}$$

As point out first by Edwards and Gulyaev [30], the average of (4.5) over all configurations can be performed exactly and the result is

$$\overline{K}\left(\vec{r}',\vec{r};t,\right) = \left\langle K\left(\vec{r}',\vec{r};t,\{\vec{R}\}\right)\right\rangle$$

$$= \int \mathcal{D}(\vec{r}(\tau))\exp\left[\frac{i}{\hbar}\int_0^t d\tau\frac{m}{2}\left\{\dot{x}^2(\tau)+\dot{y}^2(\tau)\right.\right.$$

$$\left.\left.+\omega\left(x(\tau)\dot{y}(\tau)-y(\tau)\dot{x}(\tau)\right)\right\}\right.$$

$$\left.+\rho\int d\vec{r}\left\{\exp\left(-\frac{i}{\hbar}\int_0^t d\tau\, v\left(\vec{r}(\tau)-\vec{R}\right)\right)-1\right\}\right] \tag{4.7}$$

In the limit of high density $\rho \to \infty$ and weak scattering $v \to 0$ so that ρv^2 remains finite, expression(4.10) is simplified to

$$\overline{K}\left(\vec{r}',\vec{r};t,\right) = \int \mathcal{D}(\vec{r}(\tau))\exp\left[\frac{i}{\hbar}\int_0^t d\tau \frac{m}{2}\left\{\dot{x}^2(\tau)+\dot{y}^2(\tau)\right.\right.$$

$$\left.+\omega\left(x(\tau)\dot{y}(\tau)-y(\tau)\dot{x}(\tau)\right)\right\}$$

$$\left.-\frac{\rho\eta^2}{2\hbar^2}\int_0^t\int_0^t d\tau\, d\sigma W\left(\vec{r}(\tau)-\vec{r}(\sigma)\right)\right] \tag{4.8}$$

where the mean potential energy has been taken as zero and W denotes the correlation function, defined as

$$W\left(\vec{r}(\tau)-\vec{r}(\sigma)\right) = \int d\vec{R}\, v\left(\vec{r}(\tau)-\vec{R}\right)v\left(\vec{r}(\sigma)-\vec{R}\right) \tag{4.9}$$

The parameter η denoting the weakness of the scattering potential is explicitly written here to indicate the dimensions involved. Expression (4.8) can be expressed formally in terms of an action S as

$$\overline{K}\left(\vec{r}',\vec{r};t,\right) = \int \mathcal{D}(\vec{r}(\tau))\exp\left[\frac{i}{\hbar}S\right], \tag{4.10}$$

where S is defined by

$$S = \frac{m}{2}\int_0^t \left\{ \dot{x}^2(\tau) + \dot{y}^2(\tau) \right\} d\tau + \frac{m\omega}{2}\int_0^t \left\{ x(\tau)\dot{y}(\tau) - y(\tau)\dot{x}(\tau) \right\} d\tau$$

$$+ \frac{i\rho\eta^2}{2\hbar}\int_0^t \int_0^t d\tau\, d\sigma\, W\left(\vec{r}(\tau) - \vec{r}(\sigma) \right) d\tau\, d\sigma \qquad (4.11)$$

and $\omega = eB/mc$ is the cyclotron frequency. For an impurity potential having the Gaussian form,

$$V\left(\vec{r}(\tau) - \vec{R}\right) = \frac{u}{\pi \ell^2} \exp\left(- \frac{\left|\vec{r}(\tau) - \vec{R}\right|^2}{\ell^2} \right) \qquad (4.12)$$

where u is another parameter introduced in order to take care of the dimension of expression (4.12), the correlation function can be written as expression (4.2), L denoting the correlation length of the random system given by $L^2 = 2\ell^2$ and $W(L) = \rho\eta^2 u^2$. In (4.8) the statistical nature of the random scatterers is now contained in the correlation function W. This correlation may be interpreted as a two-body interaction. The path integral in (4.8) describes an electron propagating in free space under an interaction W. The averaged propagator (4.8) cannot be reduced to Schrödinger's equation or to any differential equation. Therefore the precise interpretation of this equation cannot be made. Nevertheless we can interpret the "averaged propagator" in (4.8) as an "averaged electron" moving in a physical system .

Approximate Averaged Propagator

In the previous section, the average propagator representing the average motion of an electron in a transverse magnetic field in random potential was expressed in terms of a path integral. In this section, an approximate method for obtaining the average propagator is presented. The technique used in the present investigation is similar to that developed by Feynman [38] for the theory of the polaron.

In the previous section, substituting the correlation into (4.3) the action can be written as

$$\bar{S} = \frac{m}{2}\int_0^t \left\{ \dot{x}^2(\tau) + \dot{y}^2(\tau) \right\} d\tau + \frac{m\omega}{2}\int_0^t \left\{ x(\tau)\dot{y}(\tau) - y(\tau)\dot{x}(\tau) \right\} d\tau$$

$$+ \frac{i\xi_L}{2\hbar}\int_0^t \int_0^t \exp\left(- \frac{|\vec{r}(\tau) - \vec{r}(\sigma)|^2}{L^2} \right) d\tau\, d\sigma \qquad (4.13)$$

where $\xi_L = W(L)/\pi\ell^2$ as the magnitude of the Gaussian variance, has the dimension of the energy squared. To obtain the averaged propagator, we have to find an approximate expression for K. We follow the work given in Ref. [35], by writing expression (4.10) as

$$\bar{K}(\vec{r}, \vec{r}'; t) = K_0(\vec{r}, \vec{r}'; t) \left\langle \exp\left\{ \frac{i}{\hbar}(S - S_0) \right\} \right\rangle_{S_0} \qquad (4.14)$$

where $$K_0(\vec{r}, \vec{r}'; t) = \int D(\vec{r}(\tau)) \exp\left[\frac{i}{\hbar} S_0 \right] \qquad (4.15)$$

is the propagator of a free electron moving in two dimensions in the transverse magnetic field $\vec{B}$ and S_0 is expressed as

$$S_0 = \frac{m}{2}\int_0^t \left\{ \dot{x}^2(\tau) + \dot{y}^2(\tau) \right\} d\tau + \frac{m\omega}{2}\int_0^t \left\{ x(\tau)\dot{y}(\tau) - y(\tau)\dot{x}(\tau) \right\} d\tau . \tag{4.16}$$

Hence the averaged $\langle \ ..... \ \rangle_{S_0}$ is defined by

$$\langle O \rangle_{S_0} = \frac{\int \mathcal{D}[\vec{r}(\tau)]\, O \exp\left[\frac{i}{\hbar} S_0\right]}{\int \mathcal{D}[\vec{r}(\tau)] \exp\left[\frac{i}{\hbar} S_0\right]} . \tag{4.17}$$

Because of the translational invariance of the actions S and S_0, the diagonal part of the propagator K, which is needed for the DOS, can be readily seen to be

$$\bar{K}(0,0;t) = K_0(0,0;t)\left\langle \exp\left\{ \frac{i}{\hbar}(S - S_0) \right\} \right\rangle_{S_0} . \tag{4.18}$$

From (4.18), we expand the average on the right hand side in a cumulant series [42],

$$\bar{K}(0,0;t) = K_0(0,0;t) \exp\left[\frac{i}{\hbar}\langle S - S_0 \rangle_{S_0} + \left(\frac{i}{\hbar}\right)^2 \frac{1}{2}\left\{ \langle (S - S_0)^2 \rangle_{S_0} - \langle S - S_0 \rangle_{S_0}^2 \right\} + \cdots \right] \tag{4.19}$$

$$K_1(0,0;t) = K_0(0,0;t)\exp\left[\frac{i}{\hbar}\langle S - S_0\rangle_{S_0}\right]$$

$$= K_0(0,0;t)\exp\left[-\frac{\eta^2}{2\hbar^2}\int_0^t\int_0^t \langle W(\vec{r}(\tau)-\vec{r}(\sigma))\rangle_{S_0}\,d\tau\,d\sigma\right]. \tag{4.20}$$

The average $\langle W(\vec{r}(\tau)-\vec{r}(\sigma))\rangle_{S_0}$ can be conveniently evaluated by making a Fourier decomposition of W. Then we can write

$$\langle W(\vec{r}(\tau)-\vec{r}(\sigma))\rangle_{S_0} = \int \frac{1}{2\pi} W(L)\left\langle \exp\left\{ i\vec{k}\cdot(\vec{r}(\tau)-\vec{r}(\sigma))\right\}\right\rangle_{S_0} d^2k \tag{4.21}$$

where $$W(L) = \rho u^2 \exp\left(-\frac{k^2L^2}{4}\right). \tag{4.22}$$

The average on the right side of expression (4.21) can be done by using a characteristic functional(see Feynman and Hibbs(1965) p. 184). After getting the average of $\exp\left(i\vec{k}\cdot(\vec{r}-\vec{r}')\right)$ in (4.21) and performing the $\vec{k}$ - integration, expression (4.20) becomes

$$K_1(0,0;t) = K_0(0,0;t)$$

$$\cdot\exp\left[-\frac{W(L)}{2\hbar^2\pi L^2}\int_0^t\int_0^t\left\{1+\frac{4i\hbar}{L^2}\frac{\sin\left(\frac{\omega}{2}(t-|\tau-\sigma|)\right)\sin\left(\frac{\omega}{2}|\tau-\sigma|\right)}{m\omega\sin\frac{\omega t}{2}}\right\}^{-1} d\tau\,d\sigma\right] \tag{4.23}$$

(4.23)

where $K_0(0, 0; t) = \frac{m}{2\pi i\hbar t}\left(\frac{\omega t}{2\sin\frac{\omega t}{2}}\right)$. (4.24)

Let $E_\omega = \hbar\omega$ and $E_L = \hbar^2/2mL^2$ be the energy associated with localizing an electron within the correlation length. Then, using the property

$$A(T, \tau, \sigma) = A(t, t-(\tau-\sigma)),$$

the double integral in (4.23) can be reduced to a single integration. We obtain

$$K_1(0, 0; t) = \frac{m}{2\pi i\hbar t}\left(\frac{\omega t}{2\sin\frac{\omega t}{2}}\right)\cdot\exp\left[-\frac{\xi_L t}{2\hbar^2}\int_0^t\left\{1+8i\frac{E_L}{E_\omega}g(t, y)\right\}^{-1}dy\right] \tag{4.25}$$

where $g(t, y) = \sin\left(\frac{\omega}{2}(\tau-y)\right)\frac{\sin\frac{\omega y}{2}}{\sin\frac{\omega t}{2}}$. (4.26)

Finally, we obtain an approximate averaged propagator which arise from the use of the Gaussian potential. In the next chapter, we will make use of this result to obtain the density of states.