



บารมีนกร

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ภาคผนวก ก.

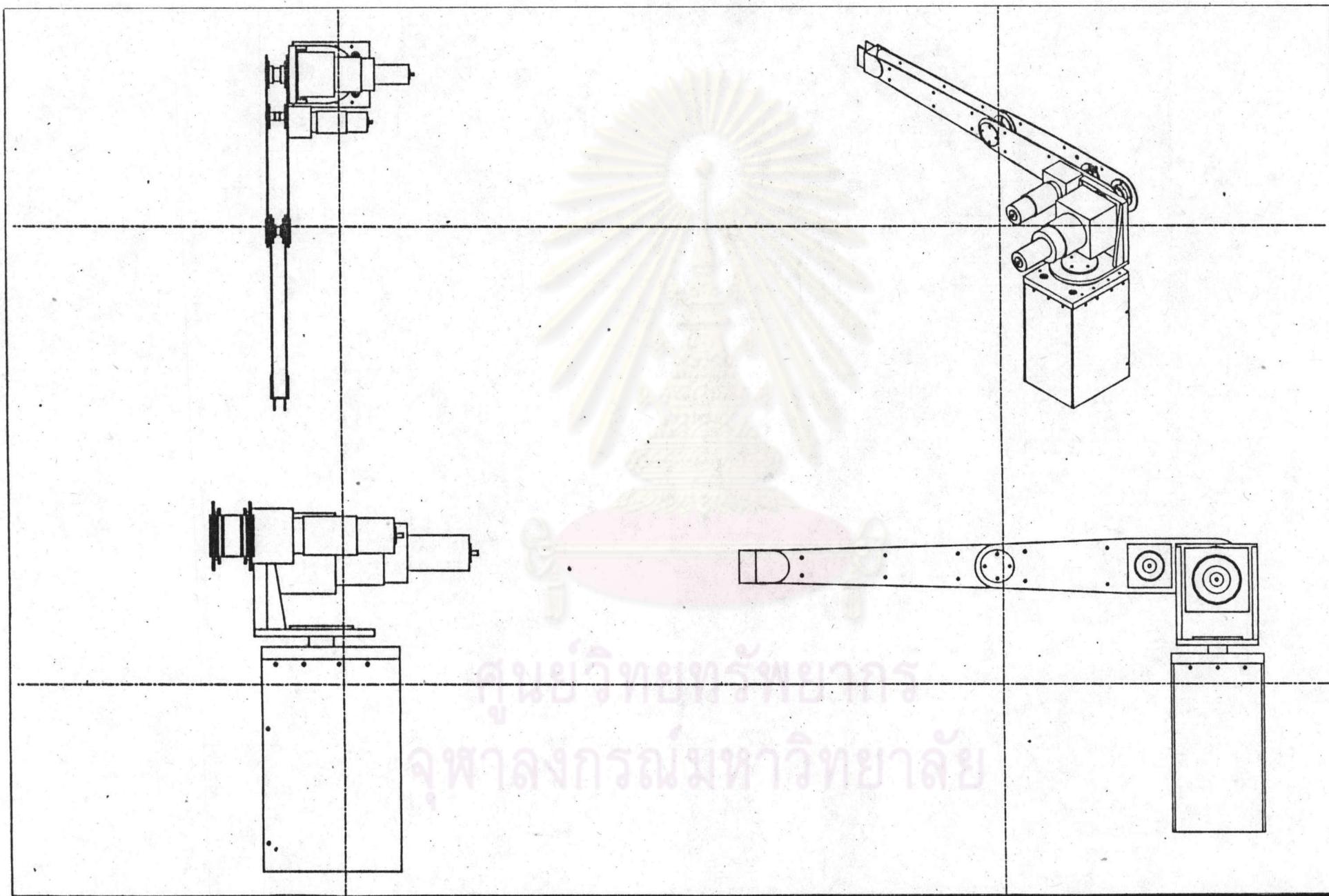
รายละเอียดของมอเตอร์และแบบโครงสร้างของแขนกล

มอเตอร์ที่ใช้ในโครงการวิทยานิพนธ์นี้ เป็นมอเตอร์กระแสตรงของบริษัท Electro Craft Model 586-022-113 และ Model 652-02-112 ต้านทานของมอเตอร์มีท้าวความต่อ
ด้านเพลามอเตอร์ชุดเดียวกันที่มีอัตราทด 100:1 และ 90:1 สำหรับรายละเอียดต่างๆ ของ
มอเตอร์ แสดงดังตารางต่อไปนี้

SPECIFICATIONS

MODELS

		E652-MG 0652-02-XXX	E586-MG
Rated Voltage	V _r	V	120
NO-LOAD SPEED at V _r	N _o	rpm	3750
MAX. RATED I at STALL	I _r	A	4.4
STALL TORQUE at I _r	T _o	oz-in	100
MAX. PULSE CURRENT	I _{pk}	A	25.0
TORQUE CONSTANT	K _t	oz-in/A	27
VOLTAGE CONSTANT	K _e	volts/krpm	20
TERMINAL RESISTANCE	R _t	Ω at 25°C	1.8
Armature Mom. Inertia	J _m	oz-in-s ²	3.2x10 ⁻³
Rotational Loss Constant	K _d	oz-in/krpm	0.68
Static Friction Torque	T _f	oz-in	7.0
Thermal Resistance Arm./Amb.	R _{th}	°C/W	2.8
ARMATURE INDUCTANCE	L _a	mH	6.7
ELECTRICAL Time Constant	τ _e	ms	3.7
MECHANICAL Time Constant	τ _m	ms	10.0
TACH. Voltage Gradient	K _g	V/krpm	21.0
TACH. Terminal Resistance	R _g	Ω at 25°C	800
TACH. Armature Inductance	L _g	mH	255
TACH. Load Resistance (optimum)	R _l	Ω	10,000
Ripple Amplitude	%pk-pk		5.0
LINEARITY	%		0.2
Temperature Coefficient	%/°C		-0.05



ภาคพนวก ช.

Q-Matrix Lagrangian formulation

การหาความล้มเหลวไดนามิกตามวิธีลากရานจ์ มีการหาอนุพันธ์ของกรานฟอร์ เมตริก และหารานฟอร์ของเมตริกบ่ออย่า เพื่อลดความยุ่งยากในการหาความล้มเหลว และลดเวลาต่อการคำนวณหาคำตอบด้วยคอมพิวเตอร์จึงได้มีการสร้างเมตริก Q ขึ้น โดยมีรายละเอียดและขั้นตอน ทั้งหมดตามวิธีการ Q เมตริก รายละเอียดในตารางที่ 1-3 และมีวิธีการนำไปใช้แสดงดังต่อไปนี้

TABLE I
MANIPULATOR KINEMATICS

		Coordinates
N	Number of degrees-of-freedom (DOF).	
X_i, Y_i, Z_i	Right cartesian axes defining the i th coordinate frame (at the i th link).	
r_i	Position four-vector of a point on line i , in the i th coordinate frame: $r_i = [r_{ix} r_{iy} r_{iz} 1]^T$.	
		Kinematic Link Parameters
θ_i	Angle ^a between links $(i - 1)$ and i ; θ_i is the joint coordinate if joint i is revolute.	
Δ_i	Distance ^a between links $(i - 1)$ and i ; Δ_i is the joint coordinate if joint i is prismatic.	
a_i	Length ^a of link i ; $a_i = 0$ if joint i is prismatic.	
α_i	Twist ^a of link i .	
		Homogeneous Transformations
A_i	Homogeneous (4×4) coordinate transformation matrix, from the i th coordinate frame to the $(i - 1)$ st coordinate frame; A_i , which is defined completely by the four kinematic link parameters $(\theta_i, \Delta_i, a_i$ and $\alpha_i)$, is	
	$A_i = \begin{bmatrix} \cos(\theta_i) & -\cos(\alpha_i)\sin(\theta_i) & \sin(\alpha_i)\sin(\theta_i) & a_i \cos(\theta_i) \\ \sin(\theta_i) & \cos(\alpha_i)\cos(\theta_i) & -\sin(\alpha_i)\cos(\theta_i) & a_i \sin(\theta_i) \\ 0 & \sin(\alpha_i) & \cos(\alpha_i) & \Delta_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$	
${}^p T_k$	Homogeneous (4×4) coordinate transformation matrix from the k th coordinate frame to the p th coordinate frame; ${}^p T_k$ is calculated according to	
	${}^p T_k = A_{p+1} A_{p+2} \cdots A_{k-1} A_k, \quad \text{for } 0 \leq p < k \leq N \quad (2)$	
	and	
	${}^p T_p \triangleq I.$	
${}^0 T_N$	Forward solution (i.e., homogeneous (4×4) coordinate transformation matrix) from the end-effector (N th) coordinate frame to the base (zeroth) coordinate frame.	

^aAngle, distance, length, and twist follow the definitions in [21].

TABLE II
MANIPULATOR DYNAMICS

Dynamic Link Parameters	
J_i	Pseudo-inertia (4×4) matrix of link i : $J_i = \int_{\text{link } i} [\bar{r}_i \bar{r}_i^T] dm_i$.
\bar{r}_i	Center of mass four-vector of link i , in the i th coordinate frame: $\bar{r}_i = [\bar{r}_{ix} \bar{r}_{iy} \bar{r}_{iz} 1]^T$.
m_i	Mass of link i .
Gravity Vector	
g	Gravity four-vector in the base coordinate frame: $g = [g_x g_y g_z 0]^T$.
Differential Kinematics	
${}^p U_{ki}$	First partial-derivative (4×4) matrix of ${}^p T_k$ with respect to q_i ; ${}^p U_{ki}$ is calculated according to
	${}^p U_{ki} = \frac{\partial {}^p T_k}{\partial q_i} = {}^p T_{(i-1)} Q_i^{(i-1)} T_k, \quad \text{for } 0 \leq p < i \leq k \leq N. \quad (3)$
${}^p \dot{U}_{kjm}$	Second partial-derivative (4×4) matrix of ${}^p T_k$ with respect to q_j and q_m ; ${}^p \dot{U}_{kjm}$ (which equals ${}^p U_{kmj}$) is calculated according to
	${}^p \dot{U}_{kjm} = \frac{\partial^2 {}^p T_k}{\partial q_j \partial q_m} = {}^p T_{(j-1)} Q_j^{(j-1)} T_{(m-1)} Q_m^{(m-1)} T_k, \quad \text{for } 0 \leq p < j \leq m \leq k \leq N. \quad (4)$
Q_i	Bejczy constant (4×4) Q -matrix for joint i :
	$Q_i = Q_R = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{for a revolute } i\text{th joint, and}$
	$Q_i = Q_T = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{for a prismatic } i\text{th joint.}$

TABLE III
DYNAMIC ROBOT MODEL

	$D(q)\ddot{q} + C(q, \dot{q}) + G(q) = F(t)$	(5a)
	$\sum_{j=1}^N d_{ij} \ddot{q}_j + \sum_{j=1}^N \sum_{k=1}^N \dot{q}_j c_{jk}(i) \dot{q}_k + G_i = F_i(t), \quad \text{for } i = 1, 2, \dots, N.$	(5b)
Joint Coordinates		
$q_i(q, \dot{q}_i)$	Generalized joint coordinate (velocity, acceleration) of joint i : $q_i = \theta_i$ if joint i is revolute; and $q_i = d_i$ if joint i is prismatic.	
$q(\dot{q}, \ddot{q})$	Generalized joint coordinate (velocity, acceleration) N -vector.	
Parameters		
$C(q, \dot{q})$	Centrifugal and Coriolis force N -vector with elements	
	$C_i = \dot{q}^T C(i) \dot{q}. \quad (6)$	
$C(i)$	Coupling ($N \times N$) matrix for joint i with elements	
	$c_{jk}(i) = \sum_{m=\max(i, j, k)}^N \text{Tr}\{{}^p U_{mjk} J_m {}^p U_{mi}^T\}, \quad \text{where } p \triangleq \min(i, j, k) - 1. \quad (7)$	
$D(q)$	Inertial coefficient ($N \times N$) matrix with elements	
	$d_{ij} = \sum_{k=\max(i, j)}^N \text{Tr}\{{}^p U_{kj} J_k {}^p U_{ki}^T\}, \quad \text{where } p \triangleq \min(i, j) - 1. \quad (8)$	
$G(q)$	Gravitational force N -vector with elements	
	$G_i = - \sum_{k=i}^N m_k g^T {}^0 U_{ki} \bar{r}_k. \quad (9)$	
$F(t)$	Generalized external joint force N -vector with elements $F_i(t)$	

ตัวอย่างการหาไดนามิกโนเมลของแขนกลดังรูปที่ 4.3 บทที่ 4 ข้างต้น โดยวิธี

Q-matrix Lagrangian formulation

จากสมการที่ 1 และตารางในรูปที่ 4.3 ได้

$$_0 T_1 = A_1 = \begin{vmatrix} C_1 & 0 & -S_1 & 0 \\ S_1 & 0 & C_1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \dots 10$$

$$_1 T_2 = A_2 = \begin{vmatrix} C_2 & -S_2 & 0 & a_2 C_2 \\ S_2 & C_2 & 0 & a_2 S_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \dots 11$$

$$_2 T_3 = A_3 = \begin{vmatrix} C_3 & 0 & S_3 & a_3 C_3 \\ S_3 & 0 & -C_3 & a_3 S_3 \\ 0 & 1 & 0 & d_3 \\ 0 & 0 & 0 & 1 \end{vmatrix} \dots 12$$

$$\text{เมื่อ } S_i = \sin \theta_i \quad S_{ij} = \sin(\theta_i + \theta_j)$$

$$C_i = \cos \theta_i \quad C_{ij} = \cos(\theta_i + \theta_j)$$

จากตารางที่ 2 Pseudo-inertia matrix คือ

$$J_1 = \begin{vmatrix} -I_{1xx} + I_{1yy} + I_{1zz} & I_{1xy} & I_{1xz} & m_i x_i \\ I_{1xy} & I_{1xx} - I_{1yy} + I_{1zz} & I_{1yz} & m_i y_i \\ I_{1xz} & I_{1yz} & I_{1xx} + I_{1yy} - I_{1zz} & m_i z_i \\ m_i x_i & m_i y_i & m_i z_i & m_i \end{vmatrix}$$

$$\text{หรือ } J_1 = \begin{vmatrix} J_{1xx} & I_{1xy} & I_{1xz} & m_i x_i \\ I_{1xy} & J_{1yy} & I_{1yz} & m_i y_i \\ I_{1xz} & I_{1yz} & J_{1zz} & m_i z_i \\ m_i x_i & m_i y_i & m_i z_i & m_i \end{vmatrix} \dots 13$$



และ Q-Matrix คือ

$$Q_1 = \begin{vmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{vmatrix} \quad \dots 14$$

จากสมการที่ 8 หาสัมประสิทธิ์แรงดึงดูดได้

$$\begin{aligned} D_{13} &= \Sigma \text{Tr}\{^p U_{k,j} J_k ^p U_{k,i}^T\} \quad p = \min(i,j)-1 \\ D_{11} &= \text{Tr}\{^0 U_{11} J_1 ^0 U_{11}^T\} + \text{Tr}\{^0 U_{21} J_2 ^0 U_{21}^T\} + \text{Tr}\{^0 U_{31} J_3 ^0 U_{31}^T\} \\ D_{12} &= \text{Tr}\{^0 U_{22} J_2 ^0 U_{21}^T\} + \text{Tr}\{^0 U_{32} J_3 ^0 U_{31}^T\} \\ D_{13} &= \text{Tr}\{^0 U_{33} J_3 ^0 U_{31}^T\} \\ D_{33} &= \text{Tr}\{^2 U_{33} J_3 ^2 U_{33}^T\} \\ D_{21} &= \text{Tr}\{^0 U_{21} J_2 ^0 U_{22}^T\} + \text{Tr}\{^0 U_{31} J_3 ^0 U_{32}^T\} \\ &= D_{12} \\ D_{22} &= \text{Tr}\{^1 U_{22} J_2 ^1 U_{22}^T\} + \text{Tr}\{^1 U_{32} J_3 ^1 U_{32}^T\} \\ D_{23} &= \text{Tr}\{^1 U_{33} J_3 ^1 U_{32}^T\} \\ D_{31} &= \text{Tr}\{^0 U_{31} J_3 ^0 U_{33}^T\} \\ &= D_{13} \\ D_{32} &= \text{Tr}\{^1 U_{32} J_3 ^1 U_{33}^T\} \\ &= D_{23} \\ D_{33} &= \text{Tr}\{^2 U_{33} J_3 ^2 U_{33}^T\} \end{aligned}$$

การหา D_{13}

จากสมการที่ 2 หา ${}^p T_k$ ได้

$${}^0 T_2 = A_1 A_2 = \begin{vmatrix} C_1 C_2 & -C_1 S_2 & -S_1 & a_2 C_1 C_2 \\ S_1 C_2 & -S_1 S_2 & C_1 & a_2 S_1 C_2 \\ -S_2 & -C_2 & 0 & -a_2 S_2 \\ 0 & 0 & 0 & 1 \end{vmatrix} \quad \dots 15$$

$${}^0 T_3 = A_1 A_2 A_3 = \begin{vmatrix} C_1 C_{23} & -S_1 & C_1 S_{23} & a_3 C_1 C_{23} + a_2 C_1 C_2 - d_3 S_1 \\ S_1 C_{23} & C_1 & S_1 S_{23} & a_3 S_1 C_{23} + a_2 S_1 C_2 + d_3 C_1 \\ -S_{23} & 0 & C_{23} & -a_3 S_{23} - a_2 S_2 \\ 0 & 0 & 0 & 1 \end{vmatrix} \quad \dots 16$$

จากสมการที่ 3 หา ${}^0U_{31}$ ได้

$${}^0U_{31} = {}^0T_0 Q_1 {}^0T_3$$

$${}^0U_{31} = \begin{vmatrix} -S_1 C_{23} & -C_1 & -S_1 S_{23} & -a_3 S_1 C_{23} - a_2 S_1 C_2 - d_3 C_1 \\ C_1 C_{23} & -S_1 & C_1 S_{23} & a_3 C_1 C_{23} + a_2 C_1 C_2 - d_3 S_1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{vmatrix} \dots 17$$

$${}^0U_{31}^T = \begin{vmatrix} -S_1 C_{23} & C_1 C_{23} & 0 & 0 \\ -C_1 & -S_1 & 0 & 0 \\ -S_1 S_{23} & C_1 S_{23} & 0 & 0 \\ -a_3 S_1 C_{23} - a_2 S_1 C_2 - d_3 C_1 & a_3 C_1 C_{23} + a_2 C_1 C_2 - d_3 S_1 & 0 & 0 \end{vmatrix}$$

$${}^0U_{33} = {}^0T_2 Q_1 {}^2T_3$$

$${}^0U_{33} = \begin{vmatrix} -C_1 S_{23} & 0 & C_1 C_{23} & -a_3 C_1 S_{23} \\ -S_1 S_{23} & 0 & S_1 C_{23} & -a_3 S_1 S_{23} \\ -C_{23} & 0 & -S_{23} & -a_3 C_{23} \\ 0 & 0 & 0 & 0 \end{vmatrix} \dots 18$$

$$J_3 {}^0U_{31}^T =$$

$$\begin{aligned} & [-J_{3xx} S_1 C_{23} - I_{3xy} C_1 - I_{3xz} S_1 S_{23} - m_3 x_3 (a_3 S_1 C_{23} + a_2 S_1 C_2 + d_3 C_1)] \\ & [-I_{3xy} S_1 C_{23} - J_{3yy} C_1 - I_{3yz} S_1 S_{23} - m_3 y_3 (a_3 S_1 C_{23} + a_2 S_1 C_2 + d_3 C_1)] \\ & [-I_{3xz} S_1 C_{23} - I_{3yz} C_1 - J_{3zz} S_1 S_{23} - m_3 z_3 (a_3 S_1 C_{23} + a_2 S_1 C_2 + d_3 C_1)] \\ & [-m_{3x3} S_1 C_{23} - m_{3y3} C_1 - m_{3z3} S_1 S_{23} - m_3 (a_3 S_1 C_{23} + a_2 S_1 C_2 + d_3 C_1)] \\ & [-J_{3xx} C_1 C_{23} - I_{3xy} S_1 - I_{3xz} C_1 S_{23} - m_3 x_3 (a_3 C_1 C_{23} + a_2 C_1 C_2 + d_3 S_1)] 0 0 \\ & [-I_{3xy} C_1 C_{23} - J_{3yy} S_1 - I_{3yz} C_1 S_{23} - m_3 y_3 (a_3 C_1 C_{23} + a_2 C_1 C_2 + d_3 S_1)] 0 0 \\ & [-I_{3xz} C_1 C_{23} - I_{3yz} S_1 - J_{3zz} C_1 S_{23} - m_3 z_3 (a_3 C_1 C_{23} + a_2 C_1 C_2 + d_3 S_1)] 0 0 \\ & [-m_{3x3} C_1 C_{23} - m_{3y3} S_1 - m_{3z3} C_1 S_{23} - m_3 (a_3 C_1 C_{23} + a_2 C_1 C_2 + d_3 S_1)] 0 0 \end{aligned}$$

$$\text{Tr}\{{}^0U_{33} J_3 {}^0U_{31}^T\} = I_{3xy} S_{23} + m_3 x_3 d_3 S_{23} - I_{3yz} C_{23} - m_3 z_3 d_3 C_{23} + m_3 y_3 a_3 S_{23} + m_3 d_3 a_3 S_{23} \dots 19$$

$$D_{13} = I_{3xy} S_{23} - I_{3yz} C_{23} - m_3 d_3 (x_3 S_{23} - z_3 C_{23}) + m_3 a_3 S_{23} (y_3 + d_3)$$

ในการองเดียวกันหา $D_{11} - D_{33}$ ได้

$$D_{11} = I_{1yy} + I_{2xx} S_2 S_2 + I_{2yy} C_2 C_2 + m_2 a_2 C_2 (2x_2 C_2 - 2y_2 S_2 + a_2 C_2) + I_{3xx} S_{23} S_{23} + I_{3zz} C_{23} C_{23} + m_3 (a_3 C_{23} + a_2 C_2) (a_3 C_{23} + a_2 C_2 + 2x_3 C_{23} + 2z_3 S_{23}) + m_3 d_3 (2y_3 + d_3)$$

$$\begin{aligned}
 D_{12} &= m_2 z_2 a_2 S_2 + m_3 d_3 (x_3 S_{23} - z_3 C_{23}) + m_3 (y_3 + d_3) (a_3 S_{23} + a_2 S_2) \\
 D_{13} &= I_{3xy} S_{23} - I_{3yz} C_{23} - m_3 d_3 (x_3 S_{23} - z_3 C_{23}) + m_3 a_3 S_{23} (y_3 + d_3) \\
 D_{21} &= D_{12} \\
 D_{22} &= I_{2zz} + m_2 a_2 (a_2 + 2x_2) + I_{3yy} + 2m_3 (a_2 C_3 + a_3) (a_3 + x_3) + \\
 &\quad 2m_3 z_3 a_2 S_3 \\
 D_{23} &= I_{3yy} + m_3 a_2 (x_3 C_3 + a_3 C_3 + z_3 S_3) + m_3 a_3 (2x_3 + a_3) \\
 D_{31} &= D_{13} \\
 D_{32} &= D_{23} \\
 D_{33} &= I_{3yy} + m_3 a_3 (a_3 + 2x_3)
 \end{aligned}$$

จากสมการที่ 7 หาล้มประลิทธีแรงคอริโอลิสและเซนติพิวเกิลได้

$$\begin{aligned}
 D_{ijk} &= C_{jk}(i) = \sum \text{Tr}\{^p U_{mjk} J_m ^p U_{mi} {}^T\} : p = \min(ijk)-1 \\
 ^p U_{mjk} &= {}^p T_{(j-1)} Q_j {}^{(j-1)} T_{(m-1)} Q_m {}^{(m-1)} T_k \quad 0 < p < j < k < m < N \\
 D_{111} &= \text{Tr}\{^0 U_{111} J_1 {}^0 U_{11} {}^T\} + \text{Tr}\{^0 U_{211} J_2 {}^0 U_{21} {}^T\} + \text{Tr}\{^0 U_{311} J_3 {}^0 U_{31} {}^T\} \\
 D_{112} &= \text{Tr}\{^0 U_{212} J_2 {}^0 U_{21} {}^T\} + \text{Tr}\{^0 U_{312} J_3 {}^0 U_{31} {}^T\} \\
 D_{113} &= \text{Tr}\{^0 U_{313} J_3 {}^0 U_{31} {}^T\} \\
 D_{121} &= \text{Tr}\{^0 U_{221} J_2 {}^0 U_{21} {}^T\} + \text{Tr}\{^0 U_{321} J_3 {}^0 U_{31} {}^T\} = 0 \\
 D_{122} &= \text{Tr}\{^0 U_{222} J_2 {}^0 U_{21} {}^T\} + \text{Tr}\{^0 U_{322} J_3 {}^0 U_{31} {}^T\} \\
 D_{123} &= \text{Tr}\{^0 U_{323} J_3 {}^0 U_{31} {}^T\} \\
 D_{131} &= \text{Tr}\{^0 U_{331} J_3 {}^0 U_{31} {}^T\} = 0 \\
 D_{132} &= \text{Tr}\{^0 U_{332} J_3 {}^0 U_{31} {}^T\} = 0 \\
 D_{133} &= \text{Tr}\{^0 U_{333} J_3 {}^0 U_{31} {}^T\} \\
 D_{211} &= \text{Tr}\{^0 U_{211} J_2 {}^0 U_{22} {}^T\} + \text{Tr}\{^0 U_{311} J_3 {}^0 U_{32} {}^T\} \\
 D_{212} &= \text{Tr}\{^0 U_{212} J_2 {}^0 U_{22} {}^T\} + \text{Tr}\{^0 U_{312} J_3 {}^0 U_{32} {}^T\} \\
 D_{213} &= \text{Tr}\{^0 U_{313} J_3 {}^0 U_{32} {}^T\} \\
 D_{221} &= \text{Tr}\{^0 U_{221} J_2 {}^0 U_{22} {}^T\} + \text{Tr}\{^0 U_{321} J_3 {}^0 U_{32} {}^T\} = 0 \\
 D_{222} &= \text{Tr}\{^1 U_{222} J_2 {}^1 U_{22} {}^T\} + \text{Tr}\{^1 U_{322} J_3 {}^1 U_{32} {}^T\} \\
 D_{223} &= \text{Tr}\{^1 U_{323} J_3 {}^1 U_{32} {}^T\} \\
 D_{231} &= \text{Tr}\{^0 U_{331} J_3 {}^0 U_{32} {}^T\} = 0 \\
 D_{232} &= \text{Tr}\{^1 U_{332} J_3 {}^1 U_{32} {}^T\} = 0 \\
 D_{233} &= \text{Tr}\{^1 U_{333} J_3 {}^1 U_{32} {}^T\} \\
 D_{311} &= \text{Tr}\{^0 U_{311} J_3 {}^0 U_{33} {}^T\}
 \end{aligned}$$

$$\begin{aligned}
 D_{312} &= \text{Tr}\{ {}^0U_{312} J_3 {}^0U_{33} {}^T \} \\
 D_{313} &= \text{Tr}\{ {}^0U_{313} J_3 {}^0U_{33} {}^T \} \\
 D_{321} &= \text{Tr}\{ {}^0U_{321} J_3 {}^0U_{33} {}^T \} = 0 \\
 D_{322} &= \text{Tr}\{ {}^1U_{322} J_3 {}^1U_{33} {}^T \} \\
 D_{323} &= \text{Tr}\{ {}^1U_{323} J_3 {}^1U_{33} {}^T \} \\
 D_{331} &= \text{Tr}\{ {}^0U_{331} J_3 {}^0U_{33} {}^T \} = 0 \\
 D_{332} &= \text{Tr}\{ {}^1U_{332} J_3 {}^1U_{33} {}^T \} = 0 \\
 D_{333} &= \text{Tr}\{ {}^2U_{333} J_3 {}^2U_{33} {}^T \}
 \end{aligned}$$

การหา D_{113}

จากสมการที่ 4 หา ${}^0U_{kjm}$ ได้

$$\begin{aligned}
 {}^0U_{313} &= {}^0T_0 Q_1 {}^0T_2 Q_2 {}^2T_3 \\
 &= \begin{bmatrix} S_1 S_{23} & 0 & -S_1 C_{23} & -a_3 S_1 S_{23} \\ -C_1 S_{23} & 0 & C_1 C_{23} & a_3 C_1 S_{23} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \dots 20
 \end{aligned}$$

$$\begin{aligned}
 \text{Tr}\{ {}^0U_{313} J_3 {}^0U_{31} {}^T \} &= (I_{3xx} - I_{3zz}) S_{23} C_{23} - I_{3xx} (S_{23}^2 - C_{23}^2) + \\
 &\quad m_3 (a_3 C_{23} + a_2 C_{23}) (z_3 C_{23} - x_3 S_{23} - a_3 S_{23}) - \\
 &\quad m_3 a_3 S_{23} (x_3 C_{23} + z_3 S_{23}) \\
 &= D_{113}
 \end{aligned}$$

ในทำนองเดียวกันหา $D_{111} - D_{333}$ ได้

$$D_{111} = 0$$

$$\begin{aligned}
 D_{112} &= (I_{2xx} - I_{2yy}) S_2 C_2 + (m_2 y_2 a_2) (S_2^2 - C_2^2) - m_2 a_2 S_2 C_2 (2x_2 + a_2) \\
 &\quad + (I_{3xx} - I_{3zz}) S_{23} C_{23} - m_3 (x_3 C_{23} + z_3 S_{23}) (a_2 S_2 + a_3 S_{23}) \\
 &\quad + m_3 (a_2 C_2 + a_3 C_{23}) (z_3 C_{23} - x_3 S_{23} - a_3 S_{23} - a_2 S_2)
 \end{aligned}$$

$$\begin{aligned}
 D_{113} &= (I_{3xx} - I_{3zz}) S_{23} C_{23} - m_3 a_3 S_{23} (x_3 C_{23} + z_3 S_{23}) \\
 &\quad + m_3 (a_2 C_2 + a_3 C_{23}) (z_3 C_{23} - x_3 S_{23} - a_3 S_{23})
 \end{aligned}$$

$$D_{122} = m_2 z_2 a_2 C_2 + m_3 d_3 (x_3 C_{23} + z_3 S_{23}) + m_3 (y_3 + d_3) (a_2 C_2 + a_3 C_{23})$$

$$D_{123} = m_3 d_3 (x_3 C_{23} + z_3 S_{23}) + m_3 a_3 C_{23} (y_3 + d_3)$$

$$D_{133} = D_{123}$$

$$D_{211} = -D_{112}$$

$$D_{223} = -m_3 a_2 (a_3 S_3 + x_3 S_{23} - z_3 C_3)$$

$$D_{233} = D_{223}$$



$$D_{311} = -D_{113}$$

$$D_{322} = -D_{223}$$

$$D_{212} = D_{213} = D_{222} = D_{312} = D_{313} = D_{323} = D_{333} = 0$$

จากสมการที่ 9 หาแรงโน้มถ่วงที่ทำกับลิงค์ได้

$$D_1 = G_1 = -\sum m_k g^{TO} U_{k1} r_k$$

$$D_1 = m_1 g^{TO} U_{11} r_1 + m_2 g^{TO} U_{21} r_2 + m_3 g^{TO} U_{31} r_3$$

$$D_2 = m_2 g^{TO} U_{22} r_2 + m_3 g^{TO} U_{32} r_3$$

$$D_3 = m_3 g^{TO} U_{33} r_3$$

การหา D_3

$$g^T = [0 \ 0 \ -g \ 0]$$

$$g^{TO} U_{33} = [g C_{23} \ 0 \ g S_{23} \ g a_3 C_{23}]$$

$$D_3 = -m_3 g(C_{23}(x_3 + a_3) + z_3 S_{23})$$

ในกรณีของเดียวกันหา $D_1 - D_2$ ได้

$$D_2 = m_2 g(C_2(-x_2 - a_2) + y_2 S_2) - m_3 g((x_3 + a_3)C_{23} + z_3 S_{23} + a_2 C_2)$$

$$D_1 = 0$$

จาก D_i, D_{ij}, D_{ijk} นำมาหาแรงรวมที่ทำให้ลิงค์ด้วยความเร็ว θ
ความเร่ง α ณ ตำแหน่ง θ ได้

$$F_1 = D_{11}\theta_1 + D_{21}\theta_2 + D_{31}\theta_3 + Ia_1\theta_1 + D_1 + D_{112}\theta_1\theta_2 + D_{113}\theta_1\theta_3 + D_{122}\theta_2^2 + D_{123}\theta_2\theta_3 + D_{133}\theta_3^2 \quad \dots 21$$

$$F_2 = D_{21}\theta_1 + D_{22}\theta_2 + D_{32}\theta_3 + Ia_2\theta_2 + D_2 + D_{223}\theta_2\theta_3 + D_{233}\theta_3^2 \quad \dots 22$$

$$F_3 = D_{31}\theta_1 + D_{32}\theta_2 + D_{33}\theta_3 + Ia_3\theta_3 + D_3 + D_{311}\theta_1^2 + D_{322}\theta_2^2 \quad \dots 23$$

จุฬาลงกรณ์มหาวิทยาลัย

ภาคผนวก ค
การจำลองแบบของแซนกลด้วยแบบจำลองไดนามิก

จากสมการของลาการานจ์ในภาคผนวก ช. มีรูปแบบทั่วไปเป็น

$$F_1 = \sum D_{i,j} \theta_i + Ia_i \theta_i + \sum \sum D_{ijk} \theta_j \theta_k + D_i \quad \dots \text{ค-1}$$

$$\text{กำหนดให้ } x_1 = \theta_1 \quad x_3 = \theta_2 \quad x_5 = \theta_3 \quad \dots \text{ค-2}$$

$$x_2 = x_1 \quad x_4 = x_3 \quad x_6 = x_5 \quad \dots \text{ค-3}$$

แทนสมการ(ค-2),(ค-3)ลงในสมการ ข.21-23 ได้

$$F_1 = D_{11}x_2 + D_{21}x_4 + D_{31}x_6 + Ia_1x_2 + D_1 + D_{112}x_2x_4 + D_{113}x_2x_6 + D_{122}x_{42} + \\ D_{123}x_{46} + D_{133}x_{62} \quad \dots \text{ค-4}$$

$$F_2 = D_{21}x_2 + D_{22}x_4 + D_{32}x_6 + Ia_2x_4 + D_2 + D_{223}x_{46} + D_{233}x_{62} \quad \dots \text{ค-5}$$

$$F_3 = D_{31}x_2 + D_{32}x_4 + D_{33}x_6 + Ia_3x_6 + D_3 + D_{311}x_{22} + D_{322}x_{42} \quad \dots \text{ค-6}$$

การจำลองแบบต้องการให้สมการมีรูปแบบเป็น $x = f(x, u, t)$ หรือ $X = A(t)X + B(t)u$
โดยที่ u เป็นฟังก์ชันของแรงที่กระทำต่อระบบจึงสามารถจัดรูปสมการได้ดังนี้

(ค-4) $x \cdot D_{32}$

$$D_{32}F_1 = (D_{32}D_{11} + D_{32}Ia_1)x_2 + D_{32}D_{21}x_4 + D_{32}D_{31}x_6 + D_{32}D_1 + D_{32}D_{112}x_2x_4 + \\ D_{32}D_{113}x_2x_6 + D_{32}D_{122}x_{42} + D_{32}D_{123}x_{46} + D_{32}D_{133}x_{62} \quad \dots \text{ค-7}$$

(ค-6) $x \cdot D_{21}$

$$D_{21}F_3 = D_{31}D_{21}x_2 + D_{32}D_{21}x_4 + (D_{33}D_{21} + D_{21}Ia_3)x_6 + D_3D_{21} + D_{21}D_{311}x_{22} + \\ D_{21}D_{322}x_{42} \quad \dots \text{ค-8}$$

(ค-7)-(ค-8)

$$D_{32}F_1 - D_{21}F_3 = (D_{32}D_{11} - D_{32}Ia_1 - D_{31}D_{21})x_2 + (D_{32}D_{31} - D_{33}D_{21} - D_{21}Ia_3)x_6 + \\ + D_{32}D_1 - D_{21}D_3 - D_{21}D_{311}x_{22} + D_{32}D_{112}x_2x_4 + D_{32}D_{113}x_2x_6 + \\ (D_{32}D_{122}D_{21}D_{322})x_{42} + D_{32}D_{123}x_{46} + D_{32}D_{133}x_{62} \quad \dots \text{ค-9}$$

หรือเขียนใหม่ได้

$$D_{32}F_1 - D_{21}F_3 = -Bx_2 + Mx_6 + D_{32}D_1 - D_{21}D_3 - D_{21}D_{311}x_{22} + D_{32}D_{112}x_2x_4 + \\ D_{32}D_{113}x_2x_6 + Sx_{42} + D_{32}D_{123}x_{46} + D_{32}D_{133}x_{62} \quad \dots \text{ค-10}$$

$$B = D_{31}D_{21} - D_{32}D_{11} - D_{32}Ia_1$$

$$M = D_{32}D_{31} - D_{33}D_{21} - D_{21}Ia_3$$

$$S = D_{32}D_{122} - D_{21}D_{322}$$

(ค-5) $\times D_{32}$

$$D_{32} F_2 = D_{32} D_{21} x_2 + (D_{32} D_{22} + D_{32} Ia_2) x_4 + D_{32} D_{32} x_6 + D_{32} D_{2} + D_{32} D_{311} x_{22} + \\ D_{32} D_{223} x_4 x_6 + D_{32} D_{233} x_{62} \quad \dots \text{ค-11}$$

(ค-6) $\times (D_{22} + Ia_2)$

$$(D_{22} + Ia_2) F_3 = D_{31} (D_{22} + Ia_2) x_2 + D_{32} (D_{22} + Ia_2) x_4 + (D_{33} D_{22} + D_{22} Ia_3 + \\ D_{33} Ia_2 + Ia_3 Ia_2) x_6 + (D_{22} + Ia_2) D_3 + (D_{22} D_{311} + Ia_2 D_{311}) x_{22} + \\ (D_{22} D_{322} + Ia_2 D_{322}) x_{42} \quad \dots \text{ค-12}$$

(ค-11)-(ค-12)

$$D_{32} F_2 - (D_{22} + Ia_2) F_3 = -Ax_2 + Cx_6 + D_{32} D_2 - (D_{22} + Ia_2) D_3 + Ex_{22} + Lx_{42} + \\ D_{32} D_{223} x_4 x_6 + D_{32} D_{233} x_{62} \quad \dots \text{ค-13}$$

$$A = D_{31} D_{22} + D_{31} Ia_2 - D_{32} D_{21}$$

$$C = D_{32} D_{32} - D_{33} D_{22} - D_{22} Ia_3 - D_{33} Ia_2 - Ia_3 Ia_2$$

$$E = D_{32} D_{211} - D_{22} D_{311} - Ia_2 D_{311}$$

$$L = -D_{22} D_{322} - Ia_2 D_{322}$$

(ค-10) $\times C$

$$D_{32} CF_1 - D_{21} CF_3 = -BCx_2 + MCx_6 + D_{32} CD_1 - D_{21} CD_3 - D_{21} D_{311} Cx_{22} + \\ D_{32} D_{112} Cx_2 x_4 + D_{32} D_{113} Cx_2 x_6 + SCx_{42} + \\ D_{32} D_{123} Cx_4 x_6 + D_{32} D_{133} Cx_{62} \quad \dots \text{ค-14}$$

(ค-13) $\times M$

$$D_{32} MF_2 - (D_{22} + Ia_2) MF_3 = -AMx_{22} + MCx_6 + D_{32} MD_2 - (D_{22} + Ia_2) MD_3 + EMx_{22} + \\ LMx_{42} + D_{32} D_{223} Mx_4 x_6 + D_{32} D_{233} Mx_{62} \quad \dots \text{ค-15}$$

(ค-14)-(ค-15)

$$D_{32} CF_1 - D_{32} MF_2 + [(D_{22} + Ia_2) M - D_{21} C] F_3 = (AM - BC)x_2 + D_{32} CD_1 - D_{32} MD_3 + \\ [(D_{22} + Ia_2) M - D_{21} C] D_3 - (D_{21} D_{311} C - EM)x_{22} + D_{32} D_{112} Cx_2 x_4 + \\ D_{32} D_{113} Cx_2 x_6 + (SC - LM)x_{42} + (D_{32} D_{123} C - D_{32} D_{223} M)x_4 x_6 + \\ (D_{32} D_{133} C - D_{32} D_{233} M)x_{62} \quad \dots \text{ค.16}$$

จัดรูปใหม่ได้

$$x_2 = VA_2 x_2^2 + VB_2 x_2 x_4 + VC_2 x_2 x_6 + VD_2 x_4^2 + VE_2 x_4 x_6 + VF_2 x_6^2 + UA_2 (F_1 - D_1) + \\ UB_2 (F_2 - D_2) + UC_2 (F_3 - D_3) \quad \dots \text{ค.17}$$

$$VH_2 = AM - BC$$

$$UA_2 = D_{32} C / VH_2$$

$$VA_2 = (D_{21} D_{311} C - EM) / VH_2$$

$$UB_2 = -D_{32} M / VH_2$$

$$VB_2 = -D_{32} D_{112} C / VH_2$$

$$UC_2 = -[(D_{22} + Ia_2) M - D_{21} C] / VH_2$$



$$VC_2 = -D_{32} D_{113} C / VH_2$$

$$VD_2 = -(SC-LM) / VH_2$$

$$VE_2 = -(D_{32} D_{123} C - D_{32} D_{223} M) / VH_2$$

$$VF_2 = -(D_{32} D_{133} C - D_{32} D_{233} M) / VH_2$$

(ค-5) $\times D_{31}$

$$\begin{aligned} D_{31} F_2 &= D_{21} D_{31} x_2 + (D_{22} D_{31} + D_{31} Ia_3) x_4 + D_{32} D_{31} x_6 + D_{31} D_2 + D_{31} D_{211} x_{22} + \\ &\quad D_{31} D_{223} x_4 x_6 + D_{31} D_{233} x_{62} \end{aligned} \quad \dots \text{ค-18}$$

(ค-7)-(ค-18)

$$\begin{aligned} D_{32} F_1 - D_{31} F_2 &= -Bx_2 - Ax_4 + D_{32} D_1 - D_{31} D_2 - D_{31} D_{211} x_{22} + D_{32} D_{112} x_2 x_4 + \\ &\quad D_{32} D_{113} x_2 x_6 + D_{32} D_{122} x_{42} + Px_4 x_6 + Qx_{62} \end{aligned} \quad \dots \text{ค-19}$$

$$P = D_{32} D_{123} - D_{31} D_{223}$$

$$Q = D_{32} D_{133} - D_{31} D_{233}$$

(ค-5) $\times (D_{33} + Ia_3)$

$$\begin{aligned} (D_{33} + Ia_3) F_2 &= (D_{33} + Ia_3) D_{21} x_2 + (D_{33} D_{22} + D_{22} Ia_3 + D_{33} Ia_2 + Ia_2 Ia_3) x_4 + \\ &\quad (D_{33} + Ia_3) D_{32} x_6 + (D_{33} Ia_3) D_2 + (D_{33} + Ia_3) D_{211} x_{22} + \\ &\quad (D_{33} + Ia_3) D_{223} x_4 x_6 + (D_{33} + Ia_3) D_{233} x_{62} \end{aligned} \quad \dots \text{ค-20}$$

(ค-6) $\times D_{32}$

$$\begin{aligned} D_{32} F_3 &= D_{32} D_{31} x_2 + D_{32} D_{32} x_4 + (D_{33} + Ia_3) D_{32} x_6 + D_{32} D_3 + D_{32} D_{311} x_{22} + \\ &\quad D_{32} D_{322} x_{42} \end{aligned} \quad \dots \text{ค-21}$$

(ค-20)-(ค-21)

$$\begin{aligned} (D_{33} + Ia_3) F_2 - D_{32} F_3 &= -Mx_2 - Cx_4 + (D_{33} + Ia_3) D_2 - D_{32} D_3 + Rx_{22} - D_{32} D_{322} x_{42} + \\ &\quad + Tx_4 x_6 + Ox_{62} \end{aligned} \quad \dots \text{ค-22}$$

$$R = D_{33} D_{211} + D_{211} Ia_3 - D_{32} D_{311}$$

$$T = D_{33} D_{223} + D_{223} Ia_3$$

$$O = D_{33} D_{233} + D_{233} Ia_3$$

(ค-19) $\times M$

$$\begin{aligned} D_{32} MF_1 - D_{31} MF_2 &= -BMx_2 - AMx_4 + D_{32} MD_1 - D_{31} MD_2 - D_{31} D_{211} Mx_{22} + \\ &\quad D_{32} D_{112} Mx_2 x_4 + D_{32} D_{113} Mx_2 x_6 + D_{32} D_{122} Mx_{42} + \\ &\quad PMx_4 x_6 + QMx_{62} \end{aligned} \quad \dots \text{ค-23}$$

(ค-22) $\times B$

$$\begin{aligned} (D_{33} + Ia_3) BF_2 - D_{32} BF_3 &= -BMx_2 - CBx_4 + (D_{33} + Ia_3) BD_2 - D_{32} BD_3 + RBx_{22} - \\ &\quad D_{32} D_{322} Bx_{42} + TBx_4 x_6 + OBx_{62} \end{aligned} \quad \dots \text{ค-24}$$

(ค-23)-(ค-24) แล้วจัดสมการในทำนองเดียวกับสมการ (ค-16), (ค-17)

$$\begin{aligned}
 x_4 &= VA_4 x_2^2 + VB_4 x_2 x_4 + VC_4 x_2 x_6 + VD_4 x_4^2 + VE_4 x_4 x_6 + VF_4 x_6^2 + UA_4 (F_1 - D_1) + \\
 &\quad UB_4 (F_2 - D_2) + UC_4 (F_3 - D_3) \quad \dots \text{ค-25} \\
 VH_4 &= BC - AM \quad UA_4 = D_{32} M / VH_4 \\
 VA_4 &= (D_{31} D_{211} M - RB) / VH_4 \quad UB_4 = -[(D_{33} + Ia_3) + D_{31} M] / VH_4 \\
 VB_4 &= -D_{32} D_{112} M / VH_4 \quad UC_4 = D_{32} B / VH_4 \\
 VC_4 &= -D_{32} D_{113} M / VH_4 \\
 VD_4 &= -(D_{32} D_{122} M + D_{32} D_{322} B) / VH_4 \\
 VE_4 &= -(PM - TB) / VH_4 \\
 VF_4 &= (OB - QM) / VH_4
 \end{aligned}$$

(ค-10) $x A$

$$\begin{aligned}
 D_{32} AF_1 - D_{21} AF_3 &= -ABx_2 + MAX_6 + D_{32} AD_1 - D_{21} AD_3 - D_{21} D_{311} Ax_{22} + \\
 &\quad D_{32} D_{112} Ax_2 x_4 + D_{32} D_{113} Ax_2 x_6 + SAX_{42} + \\
 &\quad D_{32} D_{123} Ax_4 x_6 + D_{32} D_{133} Ax_{52} \quad \dots \text{ค-26}
 \end{aligned}$$

(ค-13) $x B$

$$\begin{aligned}
 D_{32} BF_2 - (D_{22} + Ia_2) BF_3 &= -ABx_2 + CBx_6 + D_{32} BD_2 - (D_{22} + Ia_2) BD_3 + EBx_{22} + \\
 &\quad LBx_{42} + D_{32} D_{223} Bx_4 x_6 + D_{32} D_{233} Bx_{52} \quad \dots \text{ค-27}
 \end{aligned}$$

(ค-26)-(ค-27) แล้วจัดสมการในทำนองเดียวกับสมการ (ค-16), (ค-17)

$$\begin{aligned}
 x_6 &= VA_6 x_2^2 + VB_6 x_2 x_4 + VC_6 x_2 x_6 + VD_6 x_4^2 + VE_6 x_4 x_6 + VF_6 x_6^2 + UA_6 (F_1 - D_1) + \\
 &\quad UB_6 (F_2 - D_2) + UC_6 (F_3 - D_3) \quad \dots \text{ค-28} \\
 VH_6 &= MA - BC \quad UA_6 = D_{32} A / VH_6 \\
 VA_6 &= (D_{21} D_{311} A - EB) / VH_6 \quad UB_6 = -D_{32} B / VH_6 \\
 VB_6 &= -D_{32} D_{112} A / VH_6 \quad UC_6 = [(D_{22} + Ia_2) B - D_{21} A] / VH_6 \\
 VC_6 &= -D_{32} D_{113} A / VH_6 \\
 VD_6 &= -(SA + LB) / VH_6 \\
 VE_6 &= -(D_{32} D_{223} B - D_{32} D_{123} A) / VH_6 \\
 VF_6 &= -(D_{32} D_{233} B - D_{32} D_{133} A) / VH_6
 \end{aligned}$$

จากสมการ(ค-3),(ค-18),(ค-29) และ(ค-33) จะได้ความล้มเหลวของ dynamic Model เป็นสมการ non-linear ของอนุพันธ์อันดับที่ 1 ที่มีความซับซ้อน ตั้งนัยการหาคำตอบให้กับสมการเหล่านี้จะอาศัยคอมพิวเตอร์เข้าช่วยในการอินเทอร์กรทด้วยวิธีทาง numerical ซึ่งได้แสดงวิธีการของ Runge-Kutta ไว้ในภาคผนวก ง.

ภาคผนวก ๔.
Runge-Kutta Method

Fifth-order Runge-kutta method เป็นวิธีการทาง numerical ที่ใช้กันอย่างแพร่หลายในการหาคำตอบให้กับสมการอนุพันธ์ เพราะว่าวิธีนี้สามารถหาคำตอบได้โดยอาศัยข้อมูลที่คำแห่งนึงเริ่มต้นหรือคำแห่งนั้นเป็นจุดเดียว แต่หาคำตอบของคำแห่งต่อไป ไม่ต้องการข้อมูลของคำแห่งก่อนหน้า (Self-start) หรือหาค่า y_{i+1} ได้เมื่อทราบค่าของ y_i โดยหาคำตอบทีละลำดับชั้น (single step) ซึ่งทำให้ง่ายต่อการเขียนโปรแกรมคำนวณด้วยเครื่องคอมพิวเตอร์ แต่ก็มีข้อเสียที่ต้องหาคำตอบตามความล้มเหลวของฟังก์ชัน $f(x, y)$ ในทุกครั้ง เพื่อจะได้คำตอบของชั้นต่อๆ ไป ซึ่งทำให้ต้องใช้เวลาในการคำนวณมาก แต่ถ้าเวลาไม่ใช่สิ่งสำคัญมากแล้ววิธีนี้เป็นวิธีที่ดีมีรูปแบบดังสมการ 1 และ 2

$$y = f(x, y) \quad \dots 1$$

$$y_{i+1} = y_i + 1/90(7k_1 + 32k_3 + 12k_4 + 32k_5 + 7k_6) \quad \dots 2$$

เมื่อ $k_1 = (h)f(x_i, y_i)$

$$k_2 = (h)f(x_i + h/4, y_i + k_1/4)$$

$$k_3 = (h)f(x_i + h/4, y_i + k_1/8 + k_2/8)$$

$$k_4 = (h)f(x_i + h/2, y_i + k_2/2 + k_3)$$

$$k_5 = (h)f(x_i + 3h/4, y_i + 3k_1/16 + 9k_4/16)$$

$$k_6 = (h)f(x_i + h, y_i + (3k_1 + 2k_2 + 12k_3 + 12k_4 + 8k_5)/7)$$

h เป็นช่วงของการเพิ่มค่าของ x (step size)

การคำนวณด้วยเครื่องคอมพิวเตอร์สามารถเขียนเป็นโปรแกรมย่อยด้วยภาษา C ได้ดังรายละเอียดตามภาคผนวก ๔.

จุฬาลงกรณ์มหาวิทยาลัย

ภาคผนวก จ.

รายละเอียดโปรแกรม

โปรแกรมทั้งหมดเขียนด้วยภาษา C โดยใช้คอมไพลเลอร์ ของบริษัทคอมพิวเตอร์ อินโนเวชั่น จำกัด Optimizing C86 เวอร์ชัน 2.20J ซึ่งมีรายละเอียดของโปรแกรมดังต่อไปนี้

โปรแกรมการจำลองแบบระบบหุ่นยนต์อุตสาหกรรม

```

/* program simulation 14:50:15 11/14/1987 */

#include "stdio.h"

float smp,vmx,amx,Kp[3],Ki[3],Kd[3],data[500][16];

main()
{
int chk,i;

chk = 0;
do {
    edit_gain(&chk);
    ctrl_sim(&chk);
    graph_menu(chk);
    crt_mode(2);
    puts(" DO YOU WANT TEST CONTINUE ? ");
    do { i = key_getc();
        } while(i != 5497 && i != 12654);
    chk = 1;
} while(i != 12654);
crt_mode(2);
puts(" good bye ! ");
}

***** generate reference signal *****
refsnl (zs,zf,nl)

float zs[3],zf[3];
int *nl;

{
extern float smp,amx,vmx,sqrt(),fabs(),data[][16];
float ttovm,ztovm,dz[3],Tam[3],Tvm[3],tt[3],Tofm,tts[3],
      tmv[3],zvc[3],tvc[3],time,tme;
int i,l,md[3];

crt_srcp(1,64,0);
puts(" No. ref. ");
Tofm = 0;
ttovm = 0;

```



```

    /* find dz,dt start to vmax */
    while (amx*ttovm < vmx)
    {
        ztovm = amx*ttovm*ttovm/2.0;
        ttovm += 0.001;
    };
    for (i = 0; i < 3; i++)
    {
        dz[i] = fabs(zf[i]-zs[i]);
        Tam[i] = 4.0*dz[i]/amx;
        Tam[i] = sqrt(Tam[i]);
        Tvm[i] = 2.0*dz[i]/vmx;
        /**** set condition to mode of motion ****/
        if (Tvm[i] <= Tam[i])
        {
            md[i] = 1;
            tt[i] = Tam[i];
            tmv[i] = tt[i]/2.0;
            zvc[i] = tvc[i] = 0;
        }
        else
        {
            md[i] = 0;
            tt[i] = ttovm*2.0+(dz[i]-2.0*ztovm)/vmx;
            tmv[i] = vmx/amx;
            ztovm = amx*tmv[i]*tmv[i]/2.0;
            zvc[i] = dz[i]-2.0*ztovm;
            tvc[i] = zvc[i]/vmx;
        };
        /**** find time of motion ****/
        if (Tofm < tt[i])
            Tofm = tt[i];
    };

    /**** create input.ref file ****/
    time = 0;    l = 0;
    while (time <= Tofm+5.0*smp)
    {
        for(i = 0; i < 3; i++)
        {
            if (time <= tmv[i])
            {
                if (zs[i] <= zf[i])
                {
                    data[l][i+1] = zs[i]+amx*time*time/2.0;
                    data[l][i+7] = amx*time;
                }
                else
                {
                    data[l][i+1] = zs[i]-amx*time*time/2.0;
                    data[l][i+7] = -amx*time;
                };
            }
            else if (time <= tmv[i]+tvc[i] && md[i] == 0)
            {
                if (zs[i] <= zf[i])
                {
                    data[l][i+1] = zs[i]+((time-tmv[i])*vmx+ztovm);
                    data[l][i+7] = vmx;
                }
                else
            }
        }
        l++;
    }
}

```

```

    {
        data[1][i+1] = zs[i]-((time-tmv[i])*vmx+ztovm);
        data[1][i+7] = -vmx;
    };
}
else if (time <= tt[i])
{
    tts[i] = tt[i]-tvc[i];
    tme = time-tvc[i];
    if (zs[i] <= zf[i])
    {
        data[1][i+1] = zs[i]+(amx*(tts[i]*tme-
            tme*tme/2.0-tts[i]*tts[i]/4.0)+zvc[i]);
        data[1][i+7] = amx*(tts[i]-tme);
    }
    else
    {
        tts[i] = tt[i]-tvc[i];
        tme = time-tvc[i];
        data[1][i+1] = zs[i]-(amx*(tts[i]*tme-
            tme*tme/2.0-tts[i]*tts[i]/4.0)+zvc[i]);
        data[1][i+7] = -amx*(tts[i]-tme);
    };
}
else
{
    data[1][i+1] = zf[i];
    data[1][i+7] = 0.0;
};

/* end of for */
crt_srcp(1,74,0);
printf(" %4d ",l);
time += smp; l += 1;
}; /* end of while */
*n1 = l-1;
}

/* control loop with P I D controller */
float ca[4],sa[4],d[4],a[4],xb[4],yb[4];
float Ixx[4],Iyy[4],Izz[4],Ixxy[4],Ixz[4];
float zb[4],ms[4],Iyz[4],Jzz[4];
float m1,motr,Ia1,Ia2,Ia3;

ctrl_sim(chk)
int *chk;

{
extern float smp,vmx,amx,Kp[],Ki[],Kd[],data[][16];
float pi = 3.1415927;
int i,l,lnk,nl,n[3];
float zs[3],f[4],time,err[3],itgerr[3],Km[3],xact[7];

/* spec. of motor */
n[0] = 90; n[1] = 90; n[2] = 100;
Km[0] = 0.0741; Km[1] = 0.0741; Km[2] = 0.05; /*

nl = *chk;
itgerr[0]=itgerr[1]=itgerr[2]=0;
time = 0; l = 0;
}

```



```

for(l=0;l<3;l++)
{
    xact[2*l+1] = data[0][l+1];
    xact[2*l+2] = data[0][l+7];
}
/** control loop */
for(l=0;l<=n1-1;l++) /* No. of cycle */
{
    crt_srcp(1,74,0);
    printf(" %4d ",l);
    data[1][0] = time;
    for(lnk=0;lnk<3;lnk++)
    {
        err[lnk]=data[l+1][lnk+1]-data[l][lnk+4];
        itgerr[lnk] += smp*err[lnk];
        f[lnk+1] = Kp[lnk]*err[lnk]+Ki[lnk]*itgerr[lnk] +
                    Kd[lnk]*data[l][lnk];
        f[lnk+1] = Km[lnk]*f[lnk+1]*n[lnk]/2.0; /* assume
        data[l][lnk+3] = f[lnk+1];
    } /* end of each link */
    xact[0] =0; f[0] = 0;
    romech(6,&time,xact,smp,f);
    for(lnk=0;lnk<3;lnk++)
    {
        data[l+1][lnk+4] = xact[2*lnk+1] ;
        data[l+1][lnk+10] = xact[2*lnk+2];
    }
} /* end of control loop */
data[n1][0] = time;
for(lnk=0;lnk<3;lnk++)
{
    zs[lnk] = data[n1][lnk+4]*57.2957795;
    crt_srcp(18,31,0);
    printf("%7.2f",zs[0]);
    crt_srcp(18,46,0);
    printf("%7.2f",zs[1]);
    crt_srcp(18,61,0);
    printf("%7.2f",zs[2]);
    putchar('\007');

    crt_srcp(22,25,0);
    puts(" PRESS ANY KEY TO CONTINUE ");
    i = key_getc();
}

} /* **** Runge-Kutta sixth-order **** */
#define NofE 6
romech (nofe,x,y,h,f)

int nofe;
float *x,y[],f[],h;

{
    int i;
    float k1[NofE+1],k2[NofE+1],k3[NofE+1],k4[NofE+1],xfcn;
    float k5[NofE+1],k6[NofE+1],ydot[NofE+1],yfcn[NofE+1];

    k1[0] = k2[0] = k3[0] = 0;
    k4[0] = k5[0] = k6[0] = 0;
    ydot[0] = yfcn[0] = 0;
    for (i = 1; i < nofe+1; i++)
        yfcn[i]=y[i];
    xfcn=*x;
    fcn (nofe,&xfcn,yfcn,ydot,f);
}

```

```

for (i = 1; i < nofe+1; i++)
    k1[i]=h*ydot[i];
for (i = 1; i < nofe+1; i++)
    yfcn[i]=y[i]+k1[i]/4.0;
    xfcn=*x+h/4.0;
    fcn (nofe,&xfcn,yfcn,ydot,f);
for (i = 1; i < nofe+1; i++)
    k2[i] = h*ydot[i];
for (i = 1; i < nofe+1; i++)
    yfcn[i]=y[i]+k1[i]/8.0+k2[i]/8.0;
    xfcn=*x+h/4.0;
    fcn (nofe,&xfcn,yfcn,ydot,f);
for (i = 1; i < nofe+1; i++)
    k3[i]=h*ydot[i];
for (i = 1; i < nofe+1; i++)
    yfcn[i]=y[i]-k2[i]/2.0+k3[i];
    xfcn=*x+h/2.0;
    fcn (nofe,&xfcn,yfcn,ydot,f);
for (i = 1; i < nofe+1; i++)
    k4[i]=h*ydot[i];
for (i = 1; i < nofe+1; i++)
    yfcn[i]=y[i]+k1[i]*3.0/16.0+k4[i]*9.0/16.0;
    xfcn=*x+h*0.75;
    fcn (nofe,&xfcn,yfcn,ydot,f);
for (i = 1; i < nofe+1; i++)
    k5[i]=h*ydot[i];
for (i = 1; i < nofe+1; i++)
    yfcn[i]=y[i]-(k1[i]*3.0+k2[i])*2.0+k3[i]*12.0-
        k4[i]*12.0+k5[i]*8.0)/7.0;
    xfcn=*x+h;
    fcn (nofe,&xfcn,yfcn,ydot,f);
for (i = 1; i < nofe+1; i++)
    k6[i]=h*ydot[i];
for (i = 1; i < nofe+1; i++)
    y[i]=y[i]+(7.0*k1[i]+32.0*k3[i]+12.0*k4[i]+32.0*k5[i]
        +7.0*k6[i])/90.0;
    *x=*x+h;
}

/* complex form of fcn() */
#define ROW 4
#define CLM 4
#define N 3

fcn (nofe,time,xfcn,xdot,f)
int nofe;
float *time,xfcn[],xdot[],f[];

{
    extern int fclose();
    extern float sin(),cos();
    extern float ca[N+1],sa[N+1],d[N+1],a[N+1],xb[N+1],yb[N+1];
    extern float Ixx[N+1],Iyy[N+1],Izz[N+1],Ixxy[N+1],Ixz[N+1];
    extern float zb[N+1],ms[N+1],Iyz[N+1],Jzz[N+1];
    extern float ml,motr,Ia1,Ia2,Ia3;
    float A[N][ROW][CLM],T[N][N+1][ROW][CLM],g;
    float TQ[N][N][ROW][CLM],Q[ROW][CLM],sz[N+1],cz[N+1];
}

```

```

float U[N][N+1][N+1][ROW][CLM], D[N+1][N+1];
float UC[N][N+1][N+1][N+1][ROW][CLM];
float J[N+1][ROW][CLM], JU[N][N+1][N+1][ROW][CLM], rtr;
float R[N+1][ROW], gUR, G[N+1], C[N+1][N+1][N+1], gU[ROW];
float xbn[N+1], ybn[N+1], zbn[N+1], Jxx[N+1], Jyy[N+1];
float ha[N+1][N+1], la[N+1][N+1], pa[N+1][N+1], qa[N+1][N+1];
float VF[N+1][N+1][N+1], UF[N+1][N+1], wa, wb, wc, wm;
int i, j, k, l, p, m, maxij, minij, maxijk, minijk;
FILE *spc;

    /* **** open file for read data ****/
spc = NULL;
if (*time == 0)
{ if ((spc=fopen("spec.dat","r"))==NULL)
    { printf("cannot open spec.dat \n");
      exit(1);
    }
for (i=1; i < N+1; i++)      /* *** read sin(alpha) ***/
    fscanf(spc, " %f ", &sai[i]);
for (i=1; i < N+1; i++)      /* *** read cos(alpha) ***/
    fscanf(spc, " %f ", &ca[i]);
for (i=1; i < N+1; i++)      /* *** read d(i) ***/
    fscanf(spc, " %f ", &d[i]);
for (i=1; i < N+1; i++)      /* *** read a(i) ***/
    fscanf(spc, " %f ", &a[i]);
for (i=1; i < N+1; i++)      /* *** read mass(i) ***/
    fscanf(spc, " %f ", &ms[i]);
for (i=1; i < N+1; i++)      /* *** read CG.of x-axis ***/
    fscanf(spc, " %f ", &xbs[i]);
for (i=1; i < N+1; i++)      /* *** read CG.of y-axis ***/
    fscanf(spc, " %f ", &ybs[i]);
for (i=1; i < N+1; i++)      /* *** read CG.of z-axis ***/
    fscanf(spc, " %f ", &zbs[i]);
for (i=1; i < N+1; i++)      /* *** read Ixx ***/
    fscanf(spc, " %f ", &Ix[i]);
for (i=1; i < N+1; i++)      /* *** read Iyy ***/
    fscanf(spc, " %f ", &Iy[i]);
for (i=1; i < N+1; i++)      /* *** read Izz ***/
    fscanf(spc, " %f ", &Iz[i]);
fscanf(spc, "%f %f %f", &Ia1, &Ia2, &Ia3); /* gear inertia */
fscanf(spc, "%f %f ", &motr, &m1); /* motor mass payload */

    /* *** move cg. for payload ***
xbn[3] = ms[3]*xb[3]/(ms[3]+m1);
ybn[3] = ms[3]*yb[3]/(ms[3]+m1);
zbn[3] = ms[3]*zb[3]/(ms[3]+m1);
Ix[3]=Ix[3]+ms[3]*((ybn[3]-yb[3])*(ybn[3]-yb[3])+(zbn[3]
-zb[3])*(zbn[3]-zb[3]))+m1*(ybn[3]*ybn[3]+zbn[3]*zbn[3]);
Iy[3]=Iy[3]+ms[3]*((xbn[3]-xb[3])*(xbn[3]-xb[3])+(zbn[3]
-zb[3])*(zbn[3]-zb[3]))+m1*(xbn[3]*xbn[3]+zbn[3]*zbn[3]);
Iz[3]=Iz[3]+ms[3]*((xbn[3]-xb[3])*(xbn[3]-xb[3])+(ybn[3]
-yb[3])*(ybn[3]-yb[3]))+m1*(xbn[3]*xbn[3]+ybn[3]*ybn[3]);
ms[3] = ms[3]+m1;
xb[3] = xbn[3];
yb[3] = ybn[3];
zb[3] = zbn[3];

```

```

***** move cg. for motor mass *****
xbn[2] = ms[2]*xb[2]/(ms[2]+motr);
ybn[2] = ms[2]*yb[2]/(ms[2]+motr);
zbn[2] = ms[2]*zb[2]/(ms[2]+motr);
Ix[2]=Ix[2]+ms[2]*((ybn[2]-yb[2])*(ybn[2]-yb[2])+(zbn[2]
-zb[2])*(zbn[2]-zb[2]))+motr*(ybn[2]*ybn[2]+zbn[2]*zbn[2]);
Iy[2]=Iy[2]+ms[2]*((xbn[2]-xb[2])*(xbn[2]-xb[2])+(zbn[2]
-zb[2])*(zbn[2]-zb[2]))+motr*(xbn[2]*xbn[2]+zbn[2]*zbn[2]);
Iz[2]=Iz[2]+ms[2]*((xbn[2]-xb[2])*(xbn[2]-xb[2])+(ybn[2]
-yb[2])*(ybn[2]-yb[2]))+motr*(xbn[2]*xbn[2]+ybn[2]*ybn[2]);
ms[2] = ms[2]+motr;
xb[2] = xbn[2];
yb[2] = ybn[2];
zb[2] = zbn[2];
for (i=1; i<N+1; i++)
{
    Jxx[i] = (-Ix[2]+Iy[2]+Iz[2])/2.0;
    Jyy[i] = (Ix[2]-Iy[2]+Iz[2])/2.0;
    Jzz[i] = (Ix[2]+Iy[2]-Iz[2])/2.0;
    Ixy[i] = Ixz[i] = Iyz[i] = 0;
}
} /* if time for open */
if(spc) fclose(spc);
***** assemble g(1,4) *****
g = 9.81;
***** assemble A-matrices *****
/* A[0] = A1 */
for(i = 1; i < N+1; i++)
{
    sz[i] = sin(xfcn[2*i-1]); cz[i] = cos(xfcn[2*i-1]);
    A[i-1][0][0] = cz[i]; A[i-1][0][1] = -ca[i]*sz[i];
    A[i-1][0][2] = sa[i]*sz[i]; A[i-1][0][3] = a[i]*cz[i];
    A[i-1][1][0] = sz[i]; A[i-1][1][1] = ca[i]*cz[i];
    A[i-1][1][2] = -sa[i]*cz[i]; A[i-1][1][3] = a[i]*sz[i];
    A[i-1][2][0] = 0; A[i-1][2][1] = sa[i];
    A[i-1][2][2] = ca[i]; A[i-1][2][3] = d[i];
    A[i-1][3][0] = A[i-1][3][1] = A[i-1][3][2] = 0;
    A[i-1][3][3] = 1;
}
***** assemble ri(4) *****
for (i = 1; i < N+1; i++)
{
    R[i][0] = xb[i]; R[i][1] = yb[i];
    R[i][2] = zb[i]; R[i][3] = 1;
}
***** assemble J-matrix *****
for (i = 1; i < N+1; i++)
{
    J[i][0][0] = Jxx[i]; J[i][0][1] = J[i][1][0] = Ixy[i];
    J[i][1][1] = Jyy[i]; J[i][0][2] = J[i][2][0] = Ixz[i];
    J[i][2][2] = Jzz[i]; J[i][1][2] = J[i][2][1] = Iyz[i];
    J[i][3][3] = ms[i];
    J[i][0][3] = J[i][3][0] = ms[i]*xb[i];
    J[i][1][3] = J[i][3][1] = ms[i]*yb[i];
    J[i][2][3] = J[i][3][2] = ms[i]*zb[i];
}
***** assemble I-matrix *****
for (k = 0; k < N; k++)
{
    for (i = 0; i < 4; i++)
        for (j = 0; j < 4; j++)
            if (i == j)
                T[k][k][i][j] = 1;
}

```

```

        else
            T[k][k][i][j] = 0;
    };
};

/****** assemble Q-matrix *****/
for (i = 0; i < 4; i++)
{
    for (j = 0; j < 4; j++)
        if (i == 0 && j == 1)
            Q[i][j] = -1;
        else { if (i == 1 && j == 0)
            Q[i][j] = 1;
            else
                Q[i][j] = 0;
        };
};

/****** compute T-matrices *****/
for (p = 0; p < N; p++)
{
    for (k = p; k < N; k++)
        Mtxmul (T[p][k],A[k],4,4,4,T[p][k+1]);
};

/****** compute Uij-matrices *****/
for (p = 0; p < N; p++)
{
    for (i = p; i < N; i++)
        Mtxmul (T[p][i],Q,4,4,4,TQ[p][i]);
};

for (p = 0; p < N; p++)
{
    for (i = p+1; i < N+1; i++)
        {
            for (k = i; k < N+1; k++)
                Mtxmul (TQ[p][i-1],T[i-1][k],4,4,4,U[p][k][i]);
        };
};

/****** compute Uijk-matrices *****/
for (p = 0; p < N; p++)
{
    for (j = p+1; j < N+1; j++)
        {
            for (m = j; m < N+1; m++)
                {
                    for (k = m; k < N+1; k++)
                        {
                            Mtxmul (TQ[p][j-1],U[j-1][k][m],4,4,4,UC[p][k][j][m]);
                            if (j != m)
                                for (i=0; i<ROW; i++)
                                    for (l=0; l<CLM; l++)
                                        UC[p][k][m][j][i][l] = UC[p][k][j][m][i][l];
                                };
                            };
                        };
        };
};

/****** compute Dij *****/
for (p = 0; p < N; p++)
{
    for (i = p+1; i < N+1; i++)
        {
            for (k = i; k < N+1; k++)
                {
                    Tsp (U[p][k][i]);
                    Mtxmul (J[k],U[p][k][i],4,4,4,JU[p][k][i]);
                    Tsp (U[p][k][i]);
                };
        };
};

for (i = 0; i < N+1; i++)
    for (j = 0; j < N+1; j++)
        D[i][j] = 0;

```



```

for (i = 1; i < N+1; i++)
{ for (j = i; j < N+1; j++)
    { rtr = 0;
        if (i < j) { maxij = j;
                      minij = i; }
        else { maxij = i;
                      minij = j; };
        for (k = maxij; k < N+1; k++)
        { p = minij-1;
          Trace (U[p][k][j],JU[p][k][i],4,4,&rtr);
            D[i][j] += rtr;
          };
        D[j][i] = D[i][j];
      };
    };
  /****** compute Dijk *****/
for (i = 0; i < N+1; i++)
  for (j = 0; j < N+1; j++)
    for (k = 0; k < N+1; k++)
      C[i][j][k] = 0;
for (i = 1; i < N+1; i++)
{ for (j = i; j < N+1; j++)
  {- for (k = j; k < N+1; k++)
    { rtr = 0;
        if (i < j) { maxij = j;
                      minij = i; }
        else { maxij = i;
                      minij = j; };
        if (maxij < k) maxijk = k;
        else maxijk = maxij;
        if (minij < k) minijk = minij;
        else minijk = k;
        for (m = maxijk; m < N+1; m++)
        { p = minijk-1;
          Trace (UC[p][m][j][k],JU[p][m][i],4,4,&rtr);
            C[i][j][k] += rtr;
          };
        C[i][k][j] = C[i][j][k];
      };
    };
  };
  /****** compute Gi *****/
for (i=0; i<N+1; i++)
  G[i] = 0;
  for (i = 1; i < N+1; i++)
  { gUR = 0;
    for (k = i; k < N+1; k++)
    { for (j = 0; j < ROW; j++)
      {
        gU[j] = g*U[0][k][i][2][j];
        gUR += gU[j]*R[k][j];
      };
    G[i] += -ms[k]*gUR;
  };
}
  /**** creat function xdot = f(xfcn,time) ****/
wa = D[3][1]*D[2][2]+D[3][1]*Ia2-D[3][2]*D[2][1];
wb = D[3][1]*D[2][1]-D[3][2]*D[1][1]-D[3][2]*Ia1;
wc = D[3][2]*D[3][2]-D[3][3]*D[2][2]-D[2][2]*Ia3
    -D[3][3]*Ia2-Ia2*Ia3;
wm = D[3][2]*D[3][1]-D[3][3]*D[2][1]-D[2][1]*Ia3;

```

```

for (i=1; i<N+1; i++)
    for (j=i; j<N+1; j++)
        {ha[i][j] = D[3][2]*C[1][i][j]-D[2][1]*C[3][i][j];
         if (i != j)
             ha[i][j] = 2.0*ha[i][j];
        };

for (i=1; i<N+1; i++)
    for (j=i; j<N+1; j++)
    {
        la[i][j] = D[3][2]*C[2][i][j]-(D[2][2]+Ia2)*C[3][i][j];
        if (i != j)
            la[i][j] = 2.0*la[i][j];
    };

for (i=1; i<N+1; i++)
    for (j=i; j<N+1; j++)
        {pa[i][j] = D[3][2]*C[1][i][j]-D[3][1]*C[2][i][j];
         if (i != j)
             pa[i][j] = 2.0*pa[i][j];
        };

for (i=1; i<N+1; i++)
    for (j=i; j<N+1; j++)
    {
        qa[i][j] = (D[3][3]+Ia3)*C[2][i][j]-D[3][2]*C[3][i][j];
        if (i != j)
            qa[i][j] = 2.0*qa[i][j];
    };

        VF[1][0][0] = wa*wm-wb*wc;
for(i=1; i<N+1; i++)
    for(j=i; j<N+1; j++)
        VF[1][i][j] = (ha[i][j]*wc-la[i][j]*wm)/VF[1][0][0];
        UF[1][1] = D[3][2]*wc/VF[1][0][0];
        UF[1][2] = -D[3][2]*wm/VF[1][0][0];
        UF[1][3] = ((D[2][2]+Ia2)*wm-D[2][1]*wc)/VF[1][0][0];

        VF[2][0][0] = wb*wc-wa*wm;
for (i=1; i<N+1; i++)
    for (j=i; j<N+1; j++)
        VF[2][i][j] = (pa[i][j]*wm-qa[i][j]*wb)/VF[2][0][0];
        UF[2][1] = D[3][2]*wm/VF[2][0][0];
        UF[2][2] = -((D[3][3]+Ia3)*wb+D[3][1]*wm)/VF[2][0][0];
        UF[2][3] = D[3][2]*wb/VF[2][0][0];

        VF[3][0][0] = wa*wm-wb*wc;
for (i=1; i<N+1; i++)
    for (j=i; j<N+1; j++)
        VF[3][i][j] = (ha[i][j]*wa-la[i][j]*wb)/VF[3][0][0];
        UF[3][1] = D[3][2]*wa/VF[3][0][0];
        UF[3][2] = -D[3][2]*wb/VF[3][0][0];
        UF[3][3] = ((D[2][2]+Ia2)*wb-D[2][1]*wa)/VF[3][0][0];

for (k=1; k<N+1; k++)
{ xdot[2*k-1] = 0;
  xdot[2*k] = 0;
  for (i=1; i<N+1; i++)
    { xdot[2*k] += UF[k][i]*(f[i]-G[i]);
      for (j=i; j<N+1; j++)
    }
}

```

```

        xdot[2*k] += VF[k][i][j]*xfc[n[2*i]*xfc[n[2*j]];
    };
    xdot[2*k-1] = xfc[n[2*k]];
};

/*
***** Transpose a(m,n)t *****/
Tsp (a)

float a[][][CLM];

{
    int i,j;
    float temp[ROW][CLM];

    for (i = 0; i < CLM; i++)
    {
        for (j = 0; j < ROW; j++)
            temp[i][j] = a[i][j];
    };
    for (i = 0; i < CLM; i++)
    {
        for (j = 0; j < ROW; j++)
            a[i][j] = temp[j][i];
    };
}

/*
***** a(m,n)*b(n,j) = c(m,j) *****/
Mtxmul (a,b,m,n,j,c)

int m, n, j;
float a[][][CLM], b[][][CLM], c[][][CLM];

{
    int clu, row, l;
    float cij;

    for (clu = 0; clu < j ; clu++)
    {
        for (row = 0; row < m; row++)
        {
            cij = 0;
            for (l = 0; l < n; l++)
                cij += a[row][l]*b[l][clu];
            c[row][clu] = cij;
        };
    };
}

/*
***** Tr[a(m,n)*b(n,m)] = tr *****/
Trace (a,b,m,n,tr)

int m, n;
float a[][][CLM], b[][][CLM], *tr;

{
    int row, l;
    float cij;

    *tr = 0;
    for (row = 0; row < m; row++)
    {
        cij = 0;
        for (l = 0; l < n; l++)
            cij += a[row][l]*b[l][row];
        *tr += cij;
    };
}

```

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```

/* model control 11:10:07 11/9/1987 */

#include "stdio.h"

float smp,vmx,amx,Kp[3],Ki[3],Kd[3],data[500][19],zeta[6];

main()
{
int chk,i;
extern float smp,vmx,amx,Kp[],Ki[],Kd[],data[][19],zeta[];
float pos[3];

chk = 1;
smp = .01;      vmx = .0.5;      amx = 0.25;
Kp[0] = 315;    Kp[1] = 585;    Kp[2] = 275;
Ki[0] = 145;    Ki[1] = 275;    Ki[2] = 125;
Kd[0] = 3;      Kd[1] = 7;      Kd[2] = 2;
zeta[0] = zeta[3] = 90;  zeta[1] = zeta[4] = 85;
zeta[2] = zeta[5] = 70;
printf("Enter maxout[0..2] :");
scanf(" %f %f %f",&maxout[0],&maxout[1],&maxout[2]);
do { zero();
     edit_gain(&chk);
     crt_srcp(22,25,0);
     printf(" MOVE ARM TO START POSITION ");
     for(i=0;i<3;i++)
       pos[i] = data[0][i+1];
     go_desire(pos);
     crt_srcp(22,25,0);
     printf(" MOVE ARM WITH THE PROGRAM ");
     control(&chk);
     crt_srcp(22,25,0);
     printf(" MOVE ARM TO HOME POSITION ");
     pos[0] = 1.571;  pos[1] = 1.4835;  pos[2] = 1.2217;
     go_desire(pos);
     graph_menu(chk);
     zero();
     crt_mode(2);
     puts(" DO YOU WANT TEST CONTINUE ? ");
     do { i = key_getc();
           } while(i != 5497 && i != 12654);
     chk = 1;
   } while(i != 12654);
crt_mode(2);
puts(" good bye ! ");
}

```



```

edit_gain(chck)
int *chck;
{
extern float smp,vmx,amx,Kp[],Ki[],Kd[],zeta[];
float zs[3],zf[3],val[13];
unsigned int pvl;
int chk,nl,lnk,i,ii,iii;

chk = *chck;
crt_mode(2);
block2(0,0,23,79);
crt_srcp(1,20,0);
puts(" Test manipulator arm by PID controller ");
crt_srcp(4,10,0);
puts(" Moving data ");
crt_srcp(4,24,0);
puts(" vmx : ");
crt_srcp(4,31,0);
if(chk==0) scanf(" %f ",&vmx);
crt_srcp(4,31,0);
printf("%7.2f",vmx);
crt_srcp(4,39,0);
puts(" amx : ");
crt_srcp(4,46,0);
if(chk==0) scanf(" %f ",&amx);
crt_srcp(4,46,0);
printf("%7.2f",amx);
crt_srcp(4,54,0);
puts(" smp : ");
crt_srcp(4,61,0);
if(chk==0) scanf(" %f ",&smp);
crt_srcp(4,61,0);
if(chk!=0) smp = smp*1000;
printf("%7.2f",smp);
crt_srcp(7,25,0);
puts(" PID controller gain ");
crt_srcp(9,10,0);
puts(" Waist ");
crt_srcp(10,10,0);
puts(" Shoulder ");
crt_srcp(11,10,0);
puts(" Elbow ");
for(lnk=0;lnk<3;lnk++)
{
  crt_srcp(9+lnk,25,0);
  puts(" Kp : ");
  crt_srcp(9+lnk,31,0);
  if(chk==0) scanf(" %f ",&Kp[lnk]);
  crt_srcp(9+lnk,31,0);
  printf("%7.2f",Kp[lnk]);
  crt_srcp(9+lnk,40,0);
  puts(" Ki : ");
  crt_srcp(9+lnk,46,0);
  if(chk==0) scanf(" %f ",&Ki[lnk]);
  crt_srcp(9+lnk,46,0);
  printf("%7.2f",Ki[lnk]);
  crt_srcp(9+lnk,55,0);
}

```

```

    puts(" Kd : ");
    crt_srcp(9+lnk,61,0);
    if(chk==0) scanf(" %f ",&Kd[lnk]);
    crt_srcp(9+lnk,61,0);
    printf("%7.2f",Kd[lnk]);
}
crt_srcp(22,4,0);
printf(" Use      move cursor to change data          PRESS
crt_srcp(22,9,0);
printf("%c %c ",'\033','\032');
val[0] = 0;
for(lnk=0;lnk<3;lnk++)
{
    i = 4+lnk; ii = 5+lnk; iii = 6+lnk;
    val[i+2*lnk] = Kp[lnk]; val[ii+2*lnk] = Ki[lnk];
    val[iii+2*lnk] = Kd[lnk];
} val[1] = vmx; val[2] = amx; val[3] = smp;
adj_data(val,1);
for(lnk=0;lnk<3;lnk++)
{
    i = 4+lnk; ii = 5+lnk; iii = 6+lnk;
    Kp[lnk] = val[i+2*lnk]; Ki[lnk] = val[ii+2*lnk];
    Kd[lnk] = val[iii+2*lnk];
} vmx = val[1]; amx = val[2]; smp = val[3];
smp = smp/1000;
crt_srcp(22,4,0);
printf(""

crt_srcp(22,25,0);
puts(" INPUT POSITION TO MOVE ARM ");
putchar('\007');
crt_srcp(14,25,0);
puts(" Position of motion (deg) ");
crt_srcp(16,10,0);
puts(" Move from ");
crt_srcp(17,10,0);
puts("      To ");
crt_srcp(18,10,0);
puts(" Stop at ");
for(lnk=0;lnk<3;lnk++)
{
    crt_srcp(16+lnk,25,0);
    puts(" wa : ");
    crt_srcp(16+lnk,31,0);
    if(lnk==0) printf(" (>0) ");
    if(lnk==1) printf(" (<180) ");
    crt_srcp(16+lnk,40,0);
    puts(" sh : ");
    crt_srcp(16+lnk,46,0);
    if(lnk==0) printf(" (>0) ");
    if(lnk==1) printf(" (<165) ");
    crt_srcp(16+lnk,55,0);
    puts(" el : ");
    crt_srcp(16+lnk,61,0);
    if(lnk==0) printf(" (>0) ");
    if(lnk==1) printf(" (<145) ");
}

crt_srcp(16,31,0);
if(chk==0) scanf(" %f ",&zeta[0]);
if(zeta[0]<0 || zeta[0]>180) { putchar('\007'); zeta[0]
crt_srcp(16,31,0);
printf("%7.2f",zeta[0]);
crt_srcp(16,46,0);

```

```

if(chk==0) scanf(" %f ",&zeta[1]);
if(zeta[1]<0 || zeta[1]>165)
    { putchar('\007');    zeta[1] = 85; }
crt_srcp(16,46,0);
printf("%7.2f",zeta[1]);
crt_srcp(16,61,0);
if(chk==0) scanf(" %f ",&zeta[2]);
if(zeta[2]<0 || zeta[2]>145)
    { putchar('\007');    zeta[2] = 75; }
crt_srcp(16,61,0);
printf("%7.2f",zeta[2]);
crt_srcp(17,31,0);
if(chk==0) scanf(" %f ",&zeta[3]);
if(zeta[3]<0 || zeta[3]>180)
    { putchar('\007');    zeta[3] = zeta[0]; }
crt_srcp(17,31,0);
printf("%7.2f",zeta[3]);
crt_srcp(17,46,0);
if(chk==0) scanf(" %f ",&zeta[4]);
if(zeta[4]<0 || zeta[4]>165)
    { putchar('\007');    zeta[4] = zeta[1]; }
crt_srcp(17,46,0);
printf("%7.2f",zeta[4]);
crt_srcp(17,61,0);
if(chk==0) scanf(" %f ",&zeta[5]);
if(zeta[5]<0 || zeta[5]>165)
    { putchar('\007');    zeta[5] = zeta[2]; }
crt_srcp(17,61,0);
printf("%7.2f",zeta[5]);
crt_srcp(22,4,0);
printf(" Use      move cursor to change data      ");
crt_srcp(22,9,0);
printf("%c %c ",'\033','\032');
for(lnk=0;lnk<6;lnk++)
{
    val[1+lnk] = zeta[lnk];
    adj_data(val,2);
}
for(lnk=0;lnk<6;lnk++)
{
    zeta[lnk] = val[lnk+1];
    /* generate input singal */
}
for (lnk=0;lnk<3;lnk++)
{
    {
        zs[lnk] = zeta[lnk]*0.017453292; /* rad */
        zf[lnk] = zeta[lnk+3]*0.017453292;
    }
}
crt_srcp(22,4,0);
printf("      ");
crt_srcp(22,25,0);
puts("      Creating Reference      ");
refsnl (zs,zf,&n1);
*chck = n1;
putchar('\007');
crt_srcp(22,25,0);
puts(" PRESS ANY KEY TO MOVE ARM ");
i = key_getc();
putchar('\007');

}

adj_data(value,sec)
    int sec;
    float value[];

```



PRESS

```

{

extern key_getc();
extern double atof();
int    j,check,dachk,i,index,point,sect;
char   s_data[15];
double temp,old_temp,val[13];

sect = sec;
for(j=1;j<13;j++)
    val[j] = value[j];
point = 0;
index = 1;
i = 0;
cursor_pos(index,sect);
s_data[i] = '\0';
old_temp = temp = val[index];
do
{
    check = key_getc();
    switch(check)
    {

        case 19200 : /* left arrow */
        if (i==0)
            val[index] = temp = old_temp ;
        cursor_pos(index,sect);
        printf("%7.2f",temp);
        if (index == 1 && sect == 1) index = 13;
        if (index == 1 && sect == 2) index = 7;
        index--;
        old_temp = temp = val[index];
        cursor_pos(index,sect);
        point = i = 0;
        break;
        case 19712 : /* right arrow */
        if (i==0)
            val[index] = temp = old_temp ;
        cursor_pos(index,sect);
        printf("%7.2f",temp);
        if (index == 12 && sect == 1) index = 0;
        if (index == 6 && sect == 2) index = 0;
        index++;
        old_temp = temp = val[index];
        cursor_pos(index,sect);
        point = i = 0;
        break;
        case 7181 : /* Return */
        if (i==0)
            val[index] = temp = old_temp ;
        cursor_pos(index,sect);
        printf("%7.2f",temp);
        cursor_pos(index,sect);
        old_temp = temp;
        point = i = 0;
        break;
    }
    dachk = check << 8;
    dachk = dachk >> 8;
    if (dachk <=57 && dachk >=48 || dachk == 46)
    {
        if (i == 0)

```

```

    {
        printf("          ");
        cursor_pos(index,sect);
    }
    s_data[i] = dachk;
    s_data[i+1] = '\0';
    printf("%c",s_data[i]);
    if (point > 0) point++;
    i++;
    temp = atof(s_data);
    val[index] = temp;
}

} while(check != 14624); /* Space */
for(j=1;j<13;j++)
value[j] = val[j];
}

cursor_pos(index,sect)
int index,sect;
{
    int r,c,i,j,chk,idex,sec;

if (sect == 1)
{
    if(index>0 && index<4) r = 4;
    if(index>3 && index<7) r = 9;
    if(index>6 && index<10) r = 10;
    if(index>9 && index<13) r = 11;
    for(j=1;j<=3;j++)
    {
        chk = j;
        for(i=1;i<=4;i++)
        {
            if(index == chk) c = 16+15*j;
            chk += 3;
        }
    }
}
else
{
    if(index>0 && index<4) r = 16;
    if(index>3 && index<7) r = 17;
    if(index == 1 || index == 4) c = 31;
    if(index == 2 || index == 5) c = 46;
    if(index == 3 || index == 6) c = 61;
}
crt_srcp(r,c,0);
}

control(chk)
int *chk;

{
extern float fabs(),smp,vmx,amx,Kp[3],Ki[3],Kd[3],data[];
float pi = 3.1415927;
unsigned int pvlu,vvlu,dtoa,selc;
int dl,dly,i,l,lnk,nl,n[3],reset;
float zs[3],zf[3],time,verr[3],err[3],itgerr[3],opvo,Km[3]
zero();
}

```

```

n1 = *chk; dly = (smp-8.355)/0.022 ;
itgerr[0]=itgerr[1]=itgerr[2]=0;
time = 0; l = 0; reset = 1;
/* *** control loop ***
for (l=0;l<n1;l++) /* No. of cycle */
{
    data[1][0] = time;
    for(d1=0;d1<=dly;d1++); /* for delay */
    for (lnk=0;lnk<3;lnk++)
    {
        in(13+lnk,&pvlu); /* read actual position (rad) */
        if(lnk == 0) data[1][4] = 0.001369*pvlu-1.110758 ;
        if(lnk == 1) data[1][5] = 4.140573-0.001367*pvlu ;
        if(lnk == 2) data[1][6] = 4.954108-0.001376*pvlu ;
        in(0+lnk,&vvlu); /* read actual velocity (rad/sec) */
        if(lnk == 0) data[1][10] = 0.002268*vvlu-4.766;
        if(lnk == 1) data[1][11] = 0.002268*vvlu-4.714;
        if(lnk == 2) data[1][12] = 0.002751*vvlu-6.000;
        verr[lnk]=data[1+1][lnk+7]-data[1][lnk+10];
        err[lnk]=data[1+1][lnk+1]-data[1][lnk+4];
        itgerr[lnk] += smp*err[lnk];
        if(data[1+1][lnk+1] == data[1][lnk+1] && reset == 1)
            { itgerr[lnk] = 0; reset = 0; }
        data[1][lnk+13]=Kp[lnk]*err[lnk]+Ki[lnk]*itgerr[lnk]
                        +Kd[lnk]*verr[lnk];
        opvo = data[1][lnk+13];
        if(lnk == 1) dtoa = opvo*57.365913+2057.54; /* amp
        if(lnk == 0) dtoa = opvo*69.209770+2053.23; /* amp
        if(lnk == 2) dtoa = opvo*69.209770+2053.23; /* amp
        if(dtoa < 0) dtoa = 5;
        if(dtoa > 4095) dtoa = 4090;
        selc = 4 >> 2-lnk ;
        out(selc,dtoa);
    } /* end of each link */
    time += smp;
} /* end of control loop */
zero();
putchar('\007');

/** read stop position (a/d ch13-15) **/
in(13,&pvlu); /* waist */
zs[0] = 0.078429*pvlu-63.641750 ;
in(14,&pvlu); /* shoulder */
zs[1] = 237.237335-0.078343*pvlu ;
in(15,&pvlu); /* elbow */
zs[2] = 283.849487-0.078855*pvlu ;
crt_srcp(18,31,0);
printf("%7.2f",zs[0]);
crt_srcp(18,46,0);
printf("%7.2f",zs[1]);
crt_srcp(18,61,0);
printf("%7.2f",zs[2]);
data[n1][0] = time;
data[n1][4] = zs[0]*pi/180;
data[n1][5] = zs[1]*pi/180;
data[n1][6] = zs[2]*pi/180;
data[n1][10] = data[n1][11] = data[n1][12] = 0;
hold_sh();
zero();
putchar('\007');
crt_srcp(22,25,0);
puts(" PRESS ANY KEY TO CONTINUE ");
i = key_getc();
}

```

```

/* a/d in routine */
in (chnl,value)
unsigned int chnl,*value;

{
unsigned int i,a,b,c;

    outportb(635,0); /* clear A/D register */
    outportb(632,chnl); /* select A/D channel */

    for (i=1;i<=7;i++)
        a = importb(636);
    for (i=1;i<=7;i++)
        a = importb(637);

    b = importb(634);
    c = importb(633);

    a = b << 12;
    b = a >> 4;
    *value = b+c;
}

/* d/a out routine */
out (lnkslc,value)
unsigned int lnkslc,value;

{
unsigned int i,a,b,c;

    b = value >> 8 ;
    a = value << 8 ;
    c = a >> 8 ;
    outportb(638,c); /* d/a out */
    outportb(639,b);
    outportb(638,c);
    outportb(639,b);
    outportb(888,lnkslc); /* link selected */
    for(i=0;i<=10;i++);
        outportb(888,0);
}
/* exit routine */
zero()

{
unsigned int lnk,dtoa,selc;

    for(lnk=0;lnk<3;lnk++)
    {
        if(lnk == 0) dtoa = 2053;          /* amp 40/2 w */
        if(lnk == 1) dtoa = 2057;          /* amp 150 w */
        if(lnk == 2) dtoa = 2053;          /* amp 40 w */
        selc = 4 >> 2-lnk ;
        out(selc,dtoa);
    }
    hold_sh();
}

```

```

hold_sh()
{
    extern float fabs(),cos();
    unsigned int pvlu,dtoa,selc;
    int i;
    float zs[3],opvo,c2,c23,d2,hv;
    /* read actual position (rad) */
    in(14,&pvlu);      zs[1] = 4.14057-0.001367*pvlu ;
    in(15,&pvlu);      zs[2] = 4.95411-0.001376*pvlu;
    c2 = cos(-0.1308997-zs[1]);
    c23 = cos(-zs[1]+1.05-zs[2]);
    d2 = 9.3187*c2+1.34456*c23;
    opvo = d2/2.5;
    dtoa = opvo*19.036495+2028.65; /* amp2 sw 120 */
    if(dtoa < 0 || dtoa > 4095) zero();
    selc = 2 ;
    out(selc,dtoa);
}

go_desire(zd)
float zd[];
{
    extern int key_scan();
    extern float fabs(),vmx,Kp[],Ki[],Kd[],maxout[];
    float zt[3],mvo[3],err[3],itgerr[3],opvo,vel[3],oldp[3],verr
    unsigned int pvlu,dtoa,selc;
    int lnk,chk;
    float smp = 0.015;

    itgerr[0] = itgerr[1] = itgerr[2] = 0;
    do {
        for (lnk=0;lnk<3;lnk++)
        {
            in(13+lnk,&pvlu); /* read actual position (rad) */
            if(lnk == 0) zt[0] = 0.001369*pvlu-1.110758 ;
            if(lnk == 1) zt[1] = 4.140573-0.001367*pvlu ;
            if(lnk == 2) zt[2] = 4.954108-0.001376*pvlu ;
            vel[lnk] = (zt[lnk]-oldp[lnk])/smp;
            oldp[lnk] = zt[lnk];
            verr[lnk] = 0-vel[lnk];
            err[lnk]=zd[lnk]-zt[lnk];
            if(fabs(err[lnk])>0.05) ^ itgerr[lnk] = 0;
            itgerr[lnk] += smp*err[lnk];
            mvo[lnk]=Kp[lnk]*err[lnk]+Ki[lnk]*itgerr[lnk]+Kd[lnk]*verr
            if(mvo[lnk]>0 && mvo[lnk]>maxout[lnk])
                mvo[lnk] = maxout[lnk];
            if(mvo[lnk]<0 && mvo[lnk]<-1.0*maxout[lnk])
                mvo[lnk] = -1.0*maxout[lnk];
            opvo = mvo[lnk];
            if(lnk == 0) dtoa = opvo*69.209770+2053.23; /* amp
            if(lnk == 1) dtoa = opvo*57.365913+2057.54; /* amp
            if(lnk == 2) dtoa = opvo*69.209770+2053.23; /* amp
            if(dtoa < 0 || dtoa > 4095) zero();
            selc = 4 >> 2-lnk ;
            out(selc,dtoa);
        } /* end of each link */
        chk = key_scan();
        if(chk != -1) { err[0] = err[1] = err[2] = 0; }
    }while(fabs(err[0])>0.001 || fabs(err[1])>0.001
          || fabs(err[2])>0.001);
}

```

จากรายละเอียดของโปรแกรมทั้งหมดที่แสดงข้างต้น ตัวแปรที่สำคัญและความหมายของตัวแปรมีดังต่อไปนี้

smp : sampling period
 vmx : maximum velocity
 amx : maximum acceleration
 Kp[0] : proportional gain of waist
 Ki[1] : integral gain of shoulder
 Kd[2] : derivative gain of elbow
 data[][i]
 i = 0 : time
 i = 1-3 : reference position
 i = 4-6 : actual position
 i = 7-9 : reference velocity
 i = 10-12 : actual velocity
 i = 13-15 : driven torque

ประวัติผู้เขียน

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ยูนิเอนบอควิปเม้นท์ จำกัด



คุณยุวทธพยากรณ์
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