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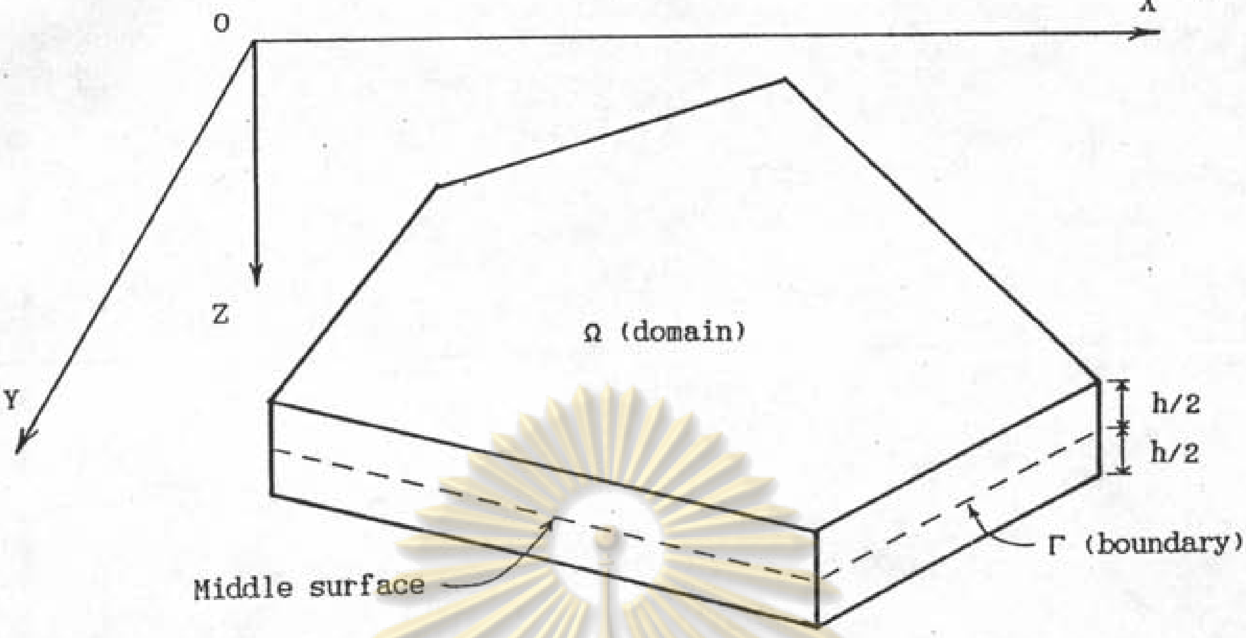


FIGURE 1 Element of plate

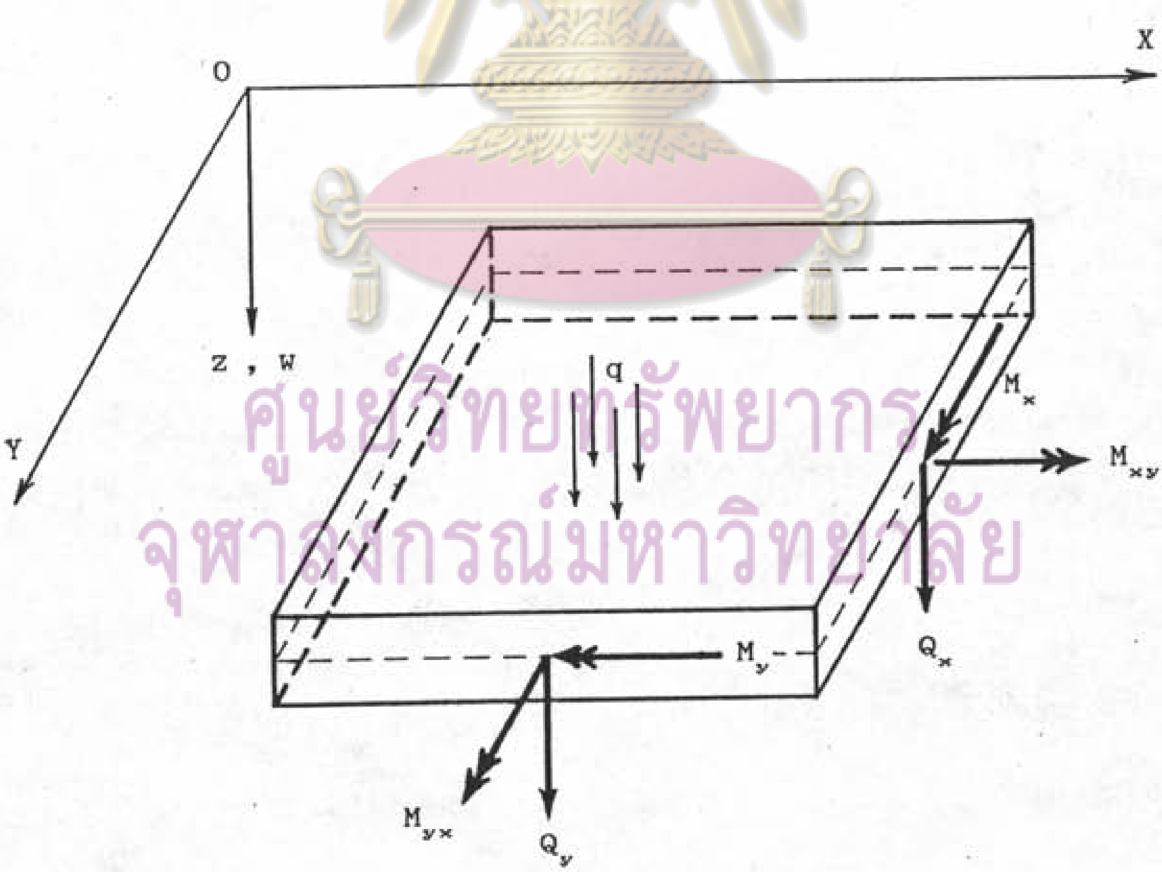


FIGURE 2 Sign convention of stress resultants

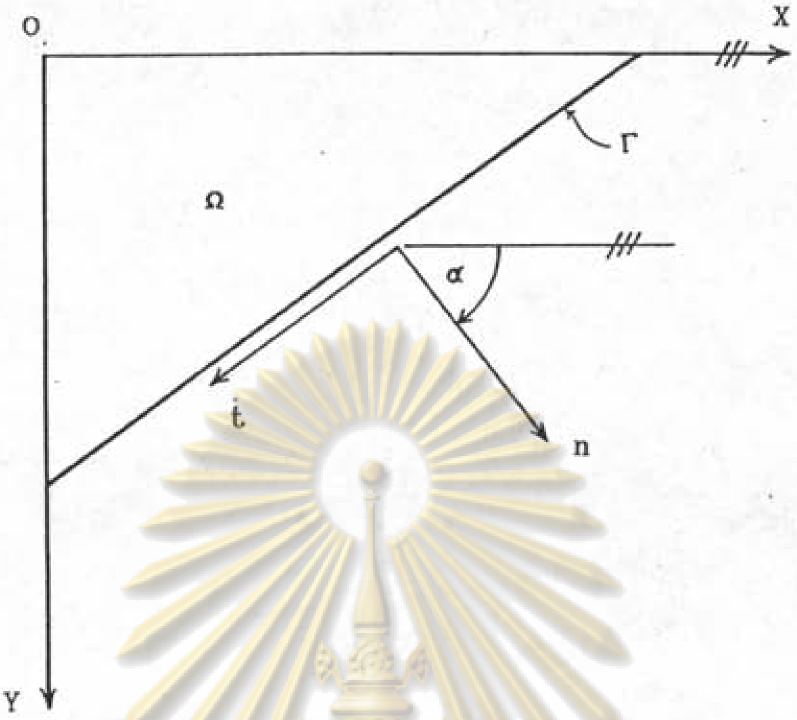


FIGURE 3 Normal co-ordinates

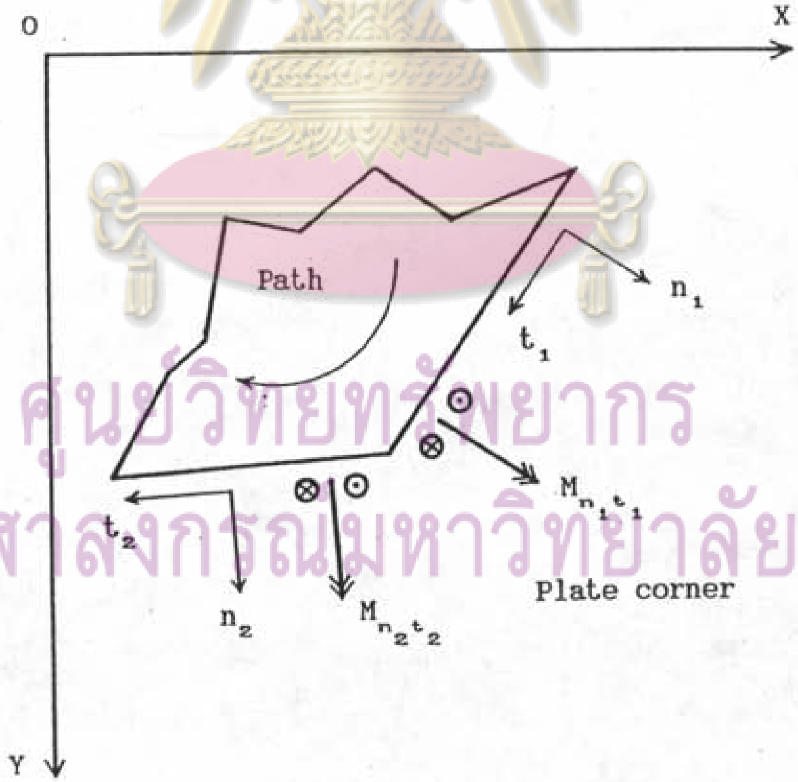


FIGURE 4 Representation of corner forces

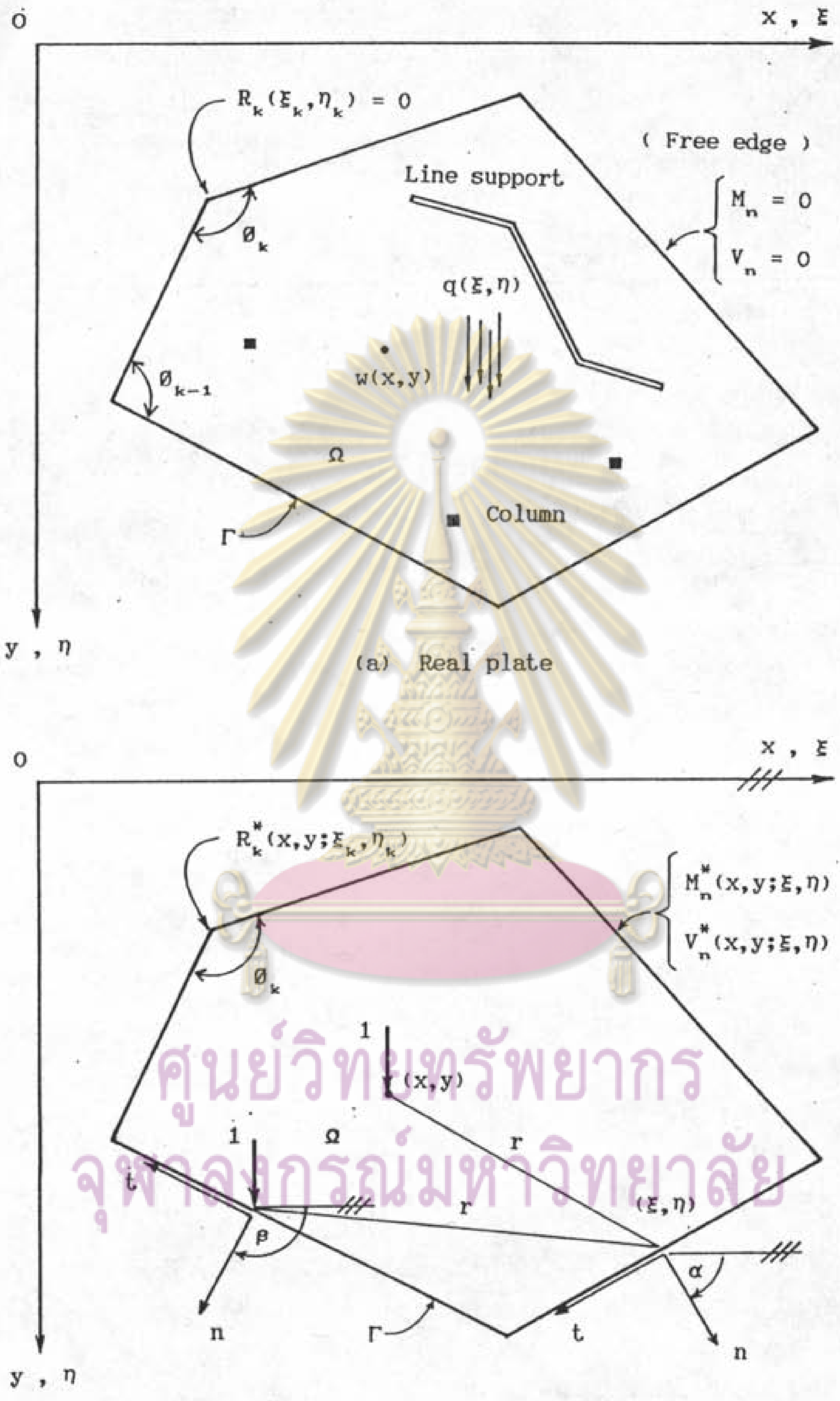


FIGURE 5 Force and displacement systems in Betti's theorem

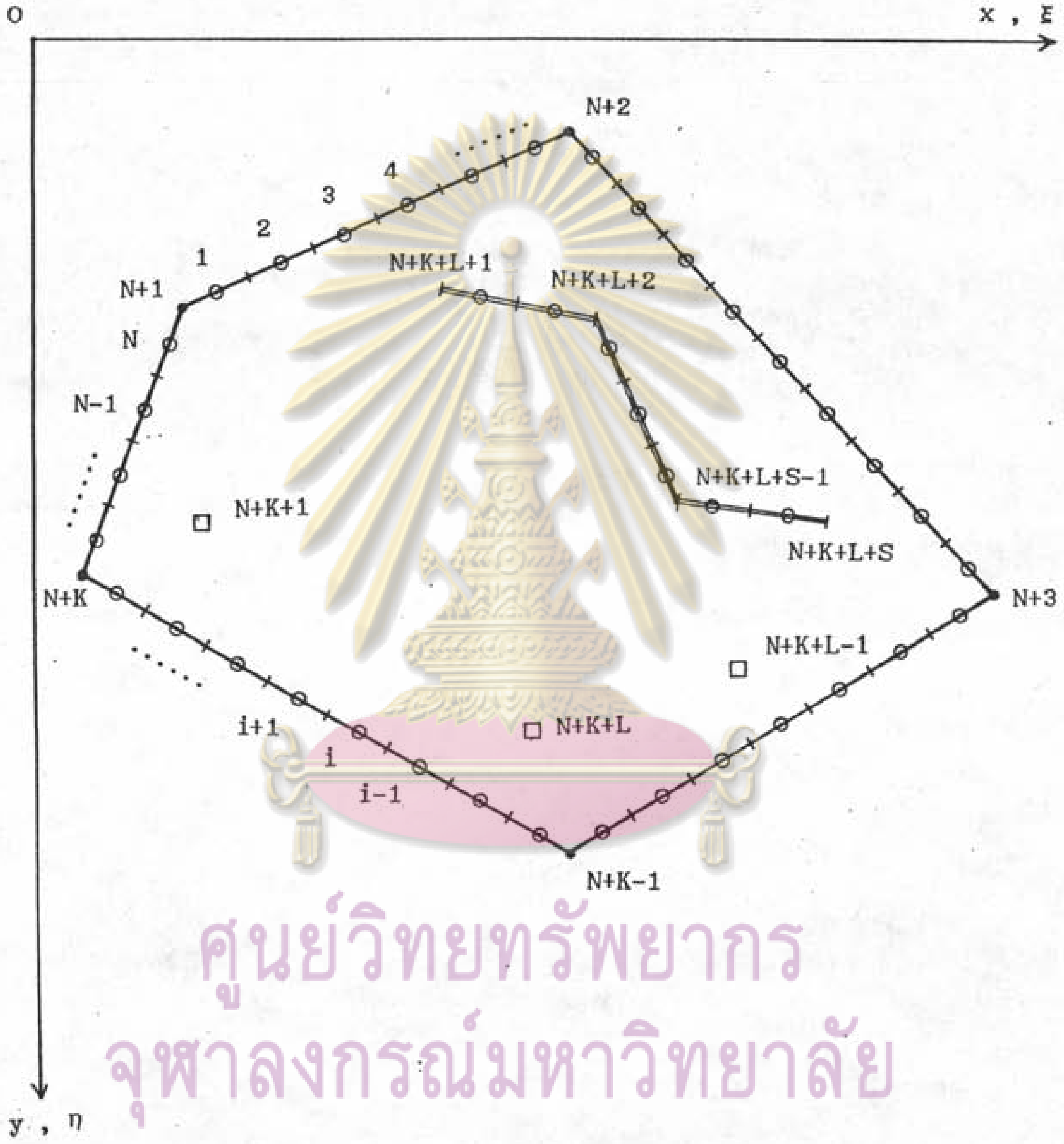


FIGURE 6 Subdivision of boundary and line support

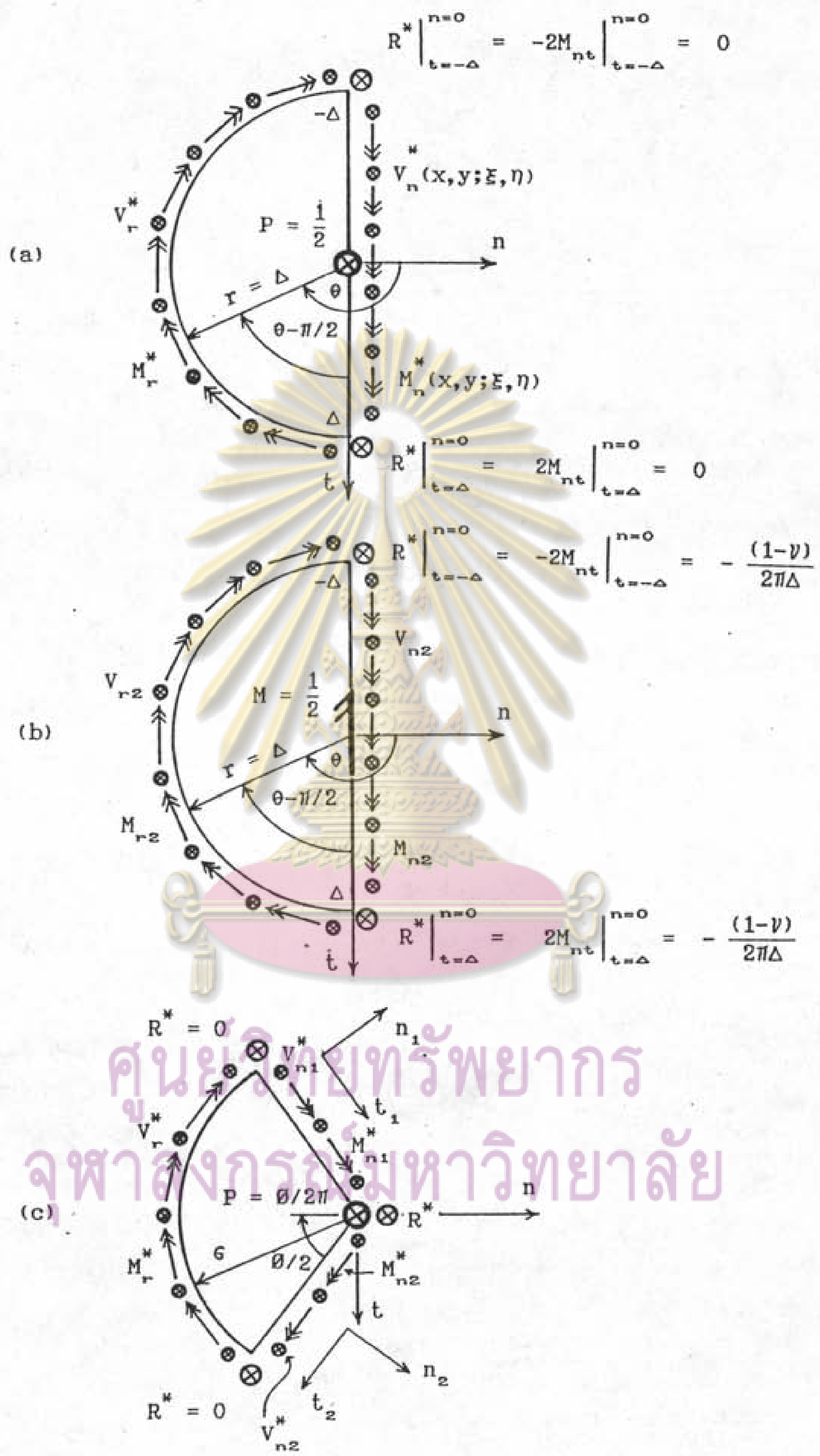


FIGURE 7 Force system of the free-body circular sector element

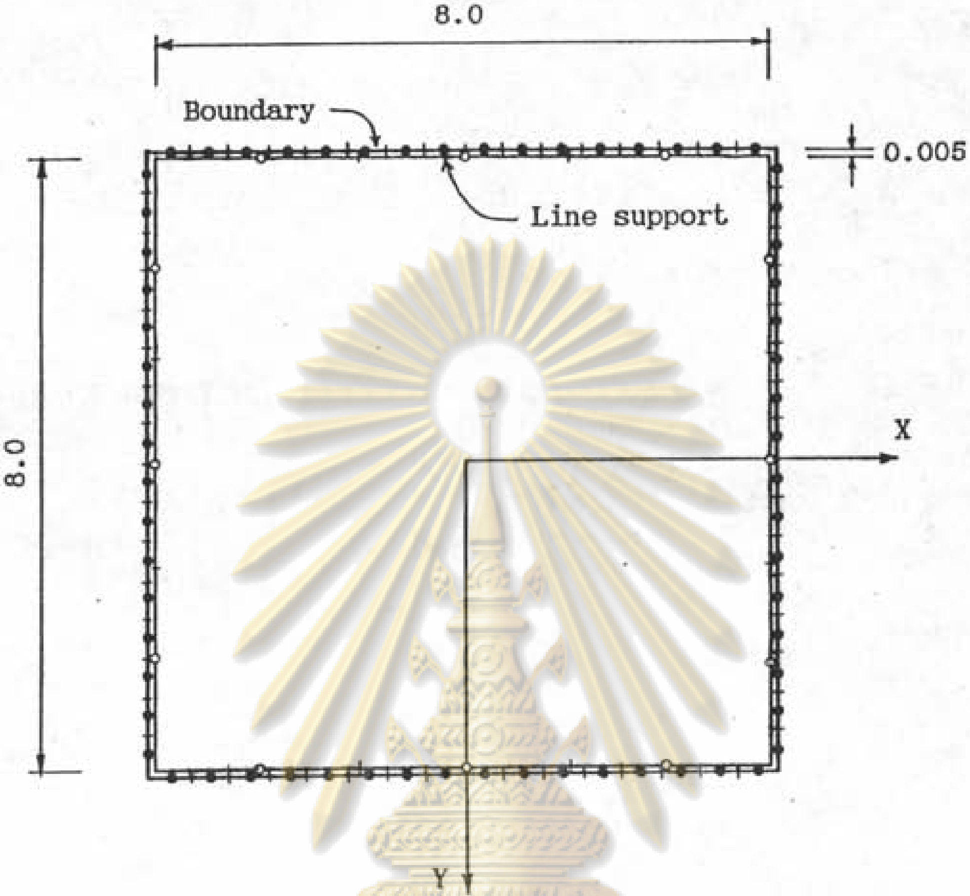


Plate width		=	8.01 x 8.01	m.x m.
Span length		=	8.00 x 8.00	m.x m.
Modulus of elasticity ;	E	=	2.204×10^{9}	kg / m^{2}
Plate thichness ;	h	=	0.30	m.
Poisson ratio ;	ν	=	0.15	
Uniformly distributed load ;	q	=	1,000	kg / m^{2}
Number of boundary subinterval		=	16	(per side)
Number of line support subinterval		=	3	(per line)

FIGURE 8 Simply supported square plate , Example 1

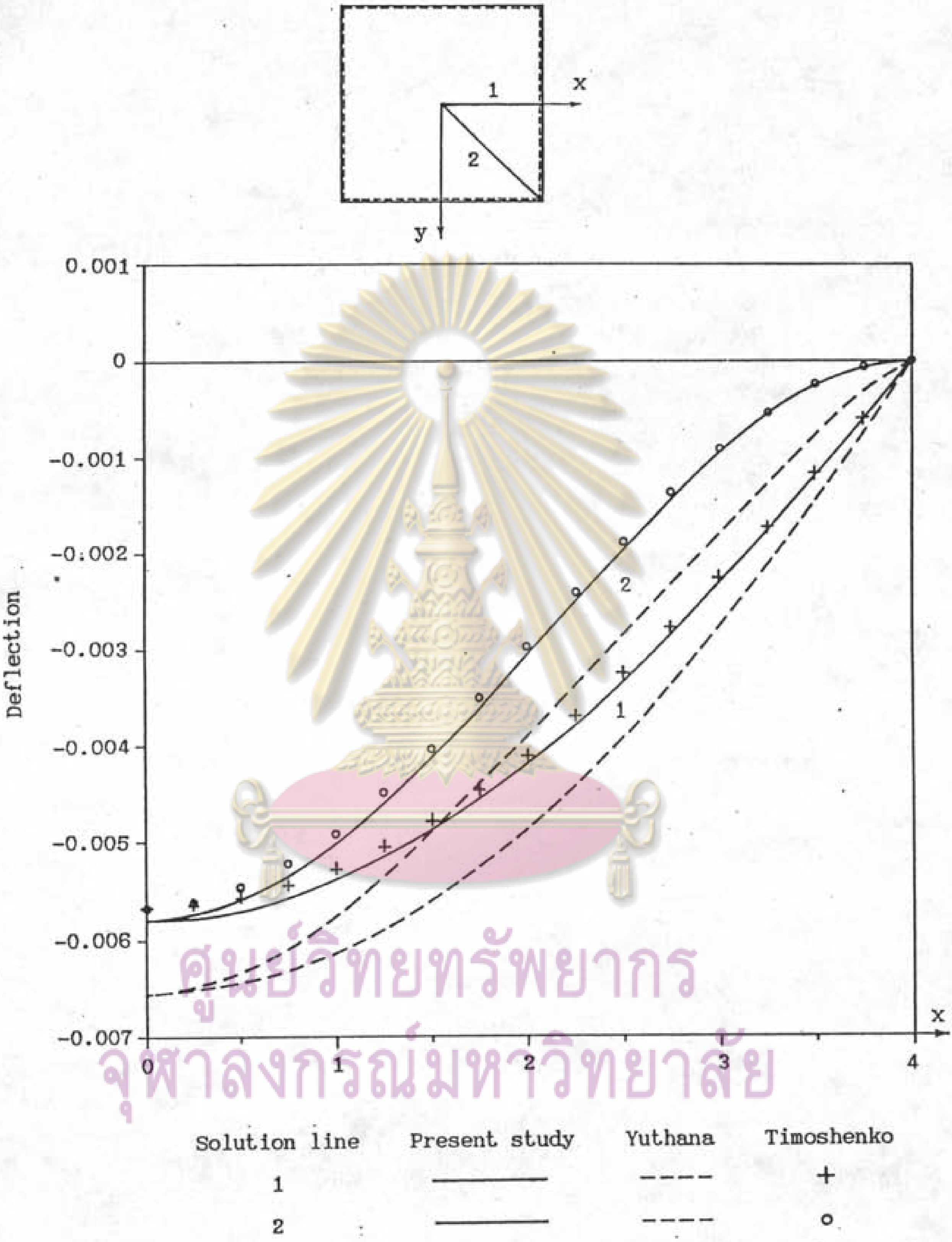
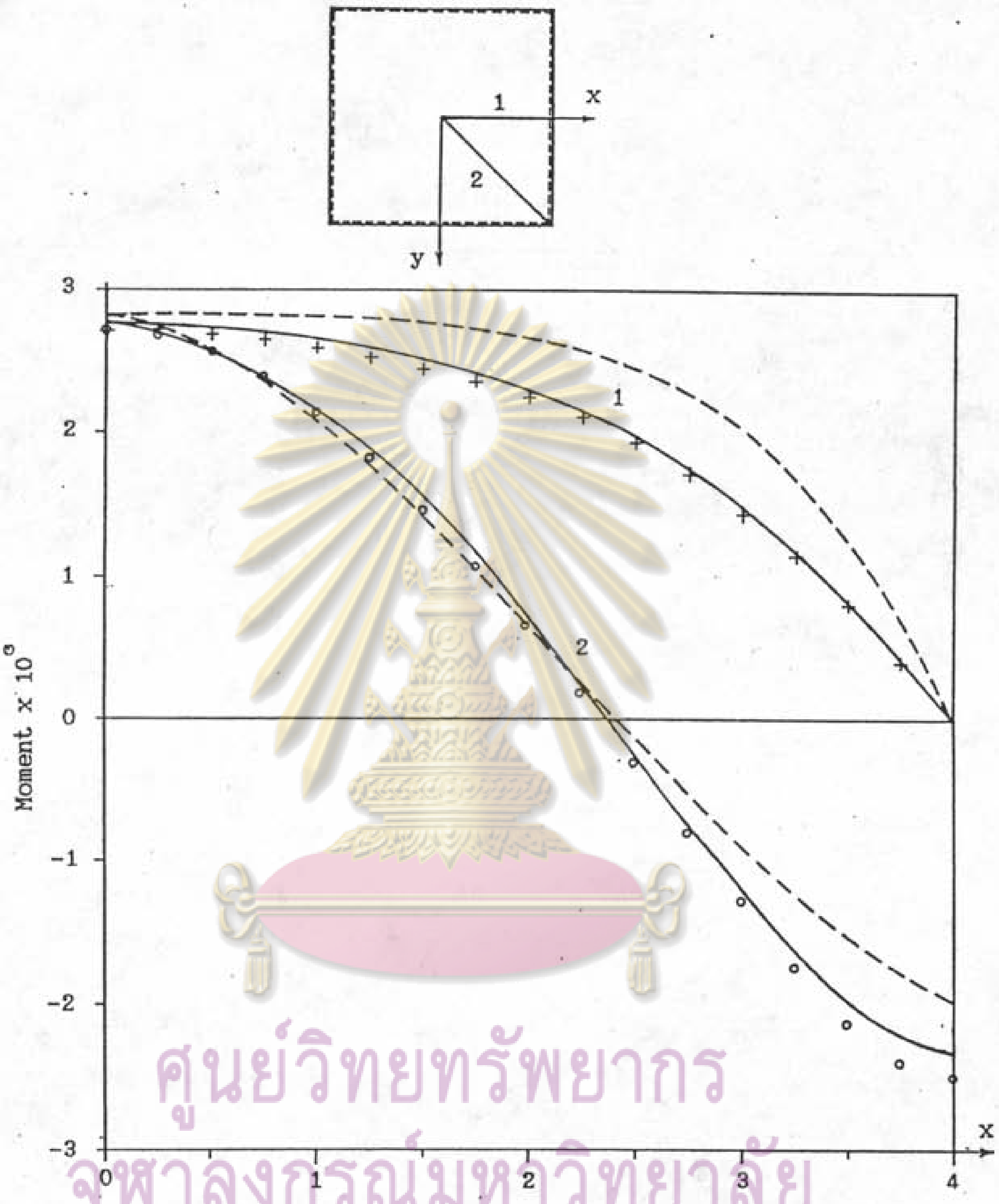
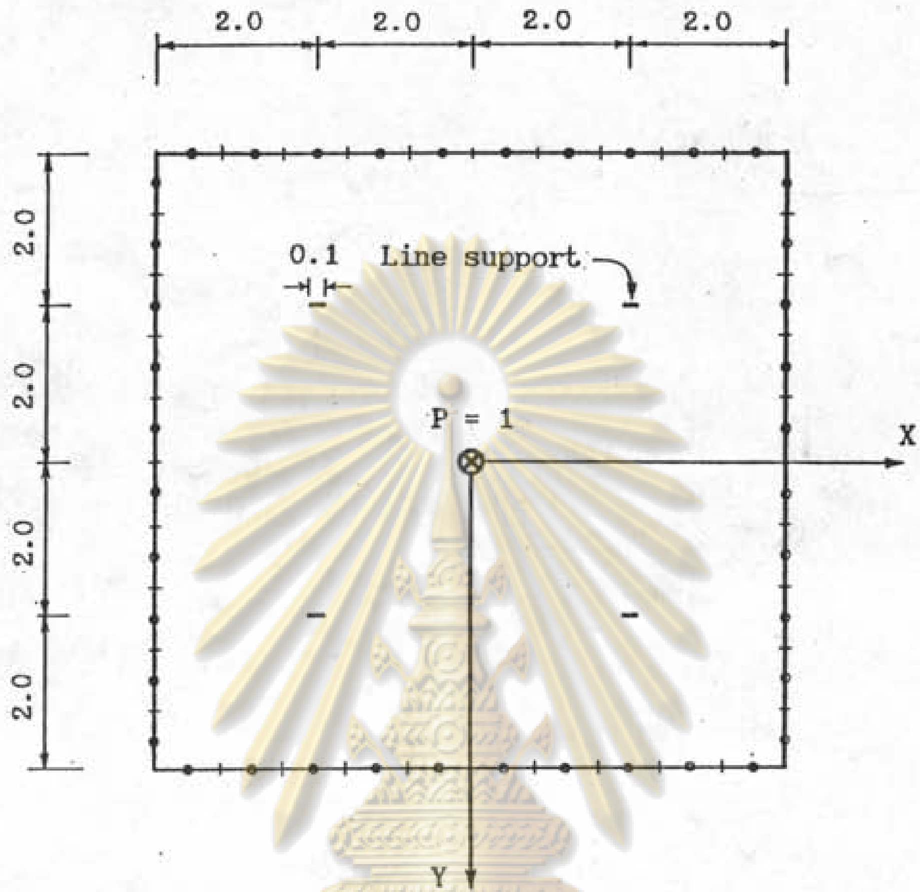


FIGURE 9 Deflection along line of symmetry and diagonal



Solution line	Present study	Yuthana	Timoshenko
1	———	- - - -	+
2	———	- - - -	o

FIGURE 10 Normal bending moment along line of symmetry and diagonal



Poisson ratio ; ν = 0.30

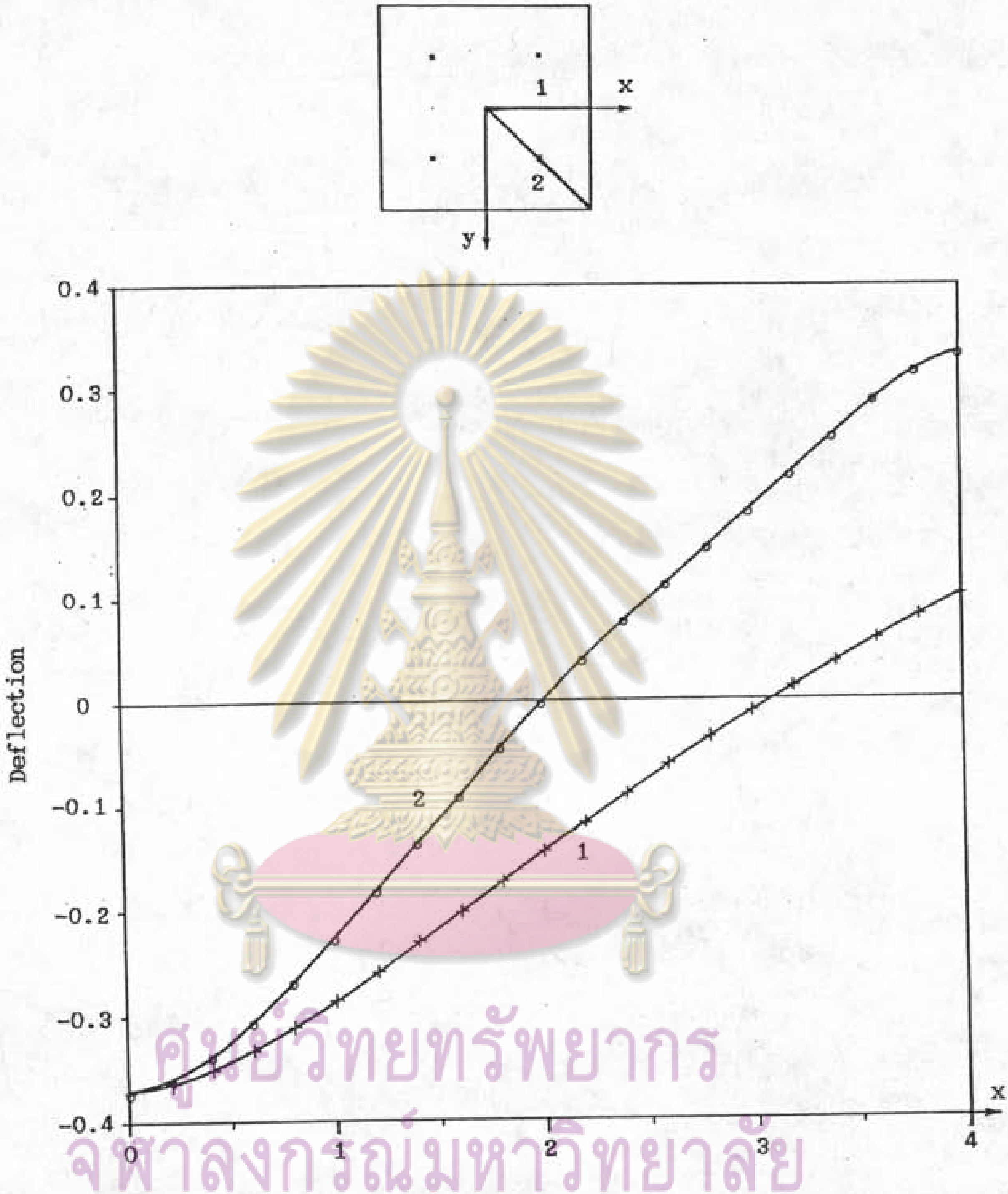
Plate rigidity ; D = 1.0

Concentrated load ; p = 1

Number of boundary subinterval = 10 (per side)

Number of line support subinterval = 1 (per line)

FIGURE 11 Square plate with four line supports , Example 2



—— Result of present study

+ } Yuthana
o }

FIGURE 12 Deflection along horizontal line of symmetry and diagonal

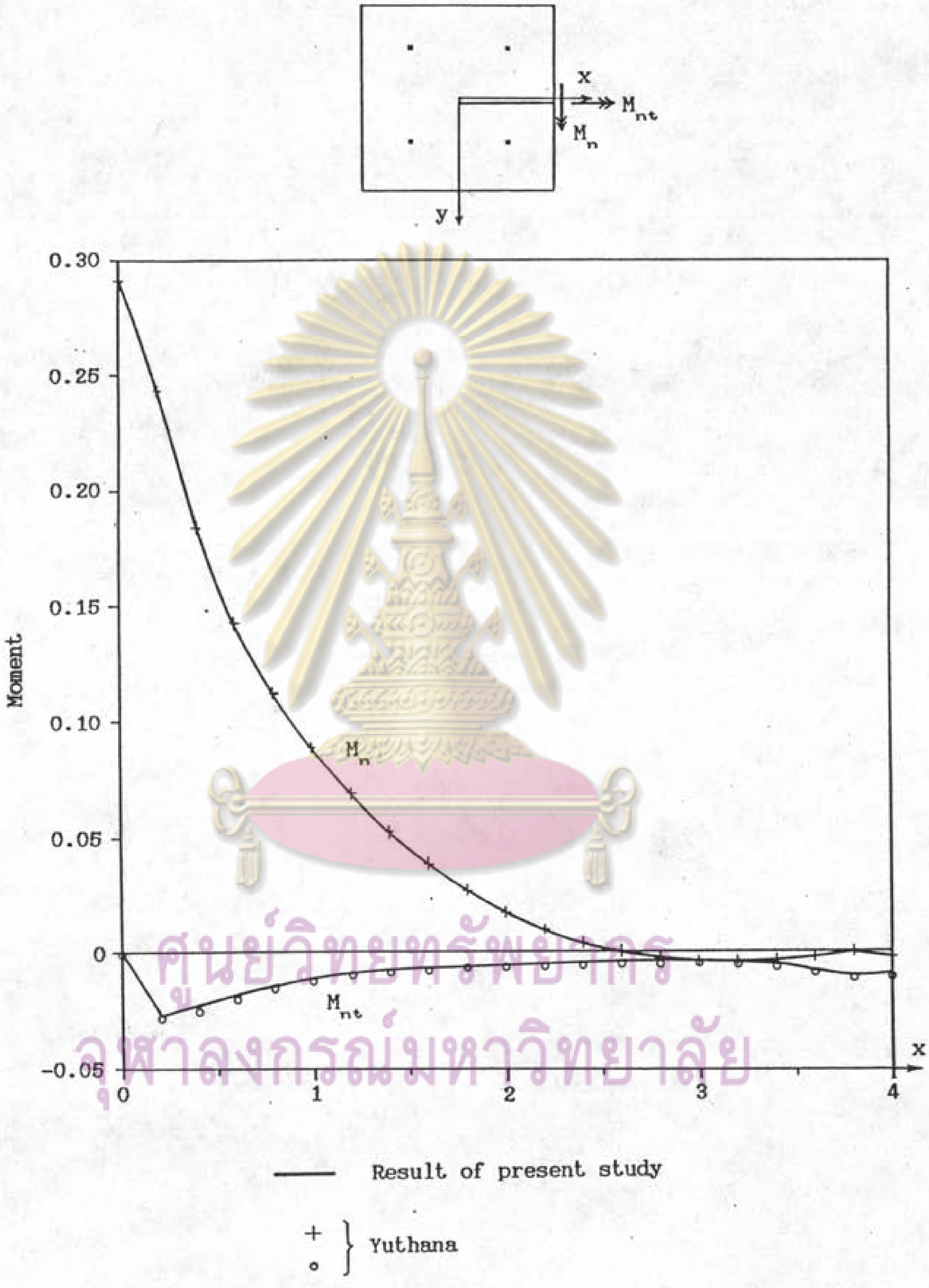


FIGURE 13 Normal bending moment and twisting moment along horizontal line (y = 0.25)

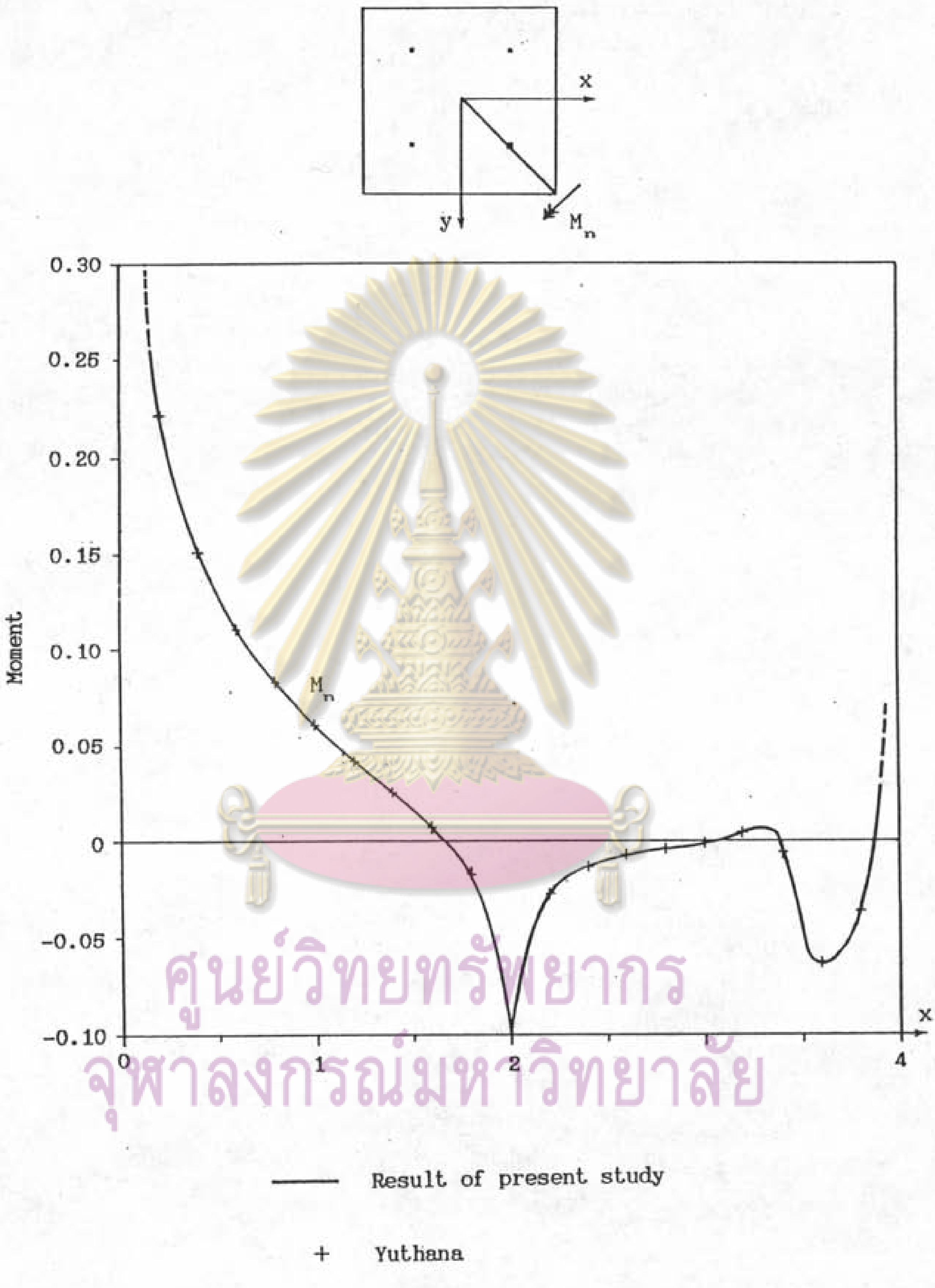
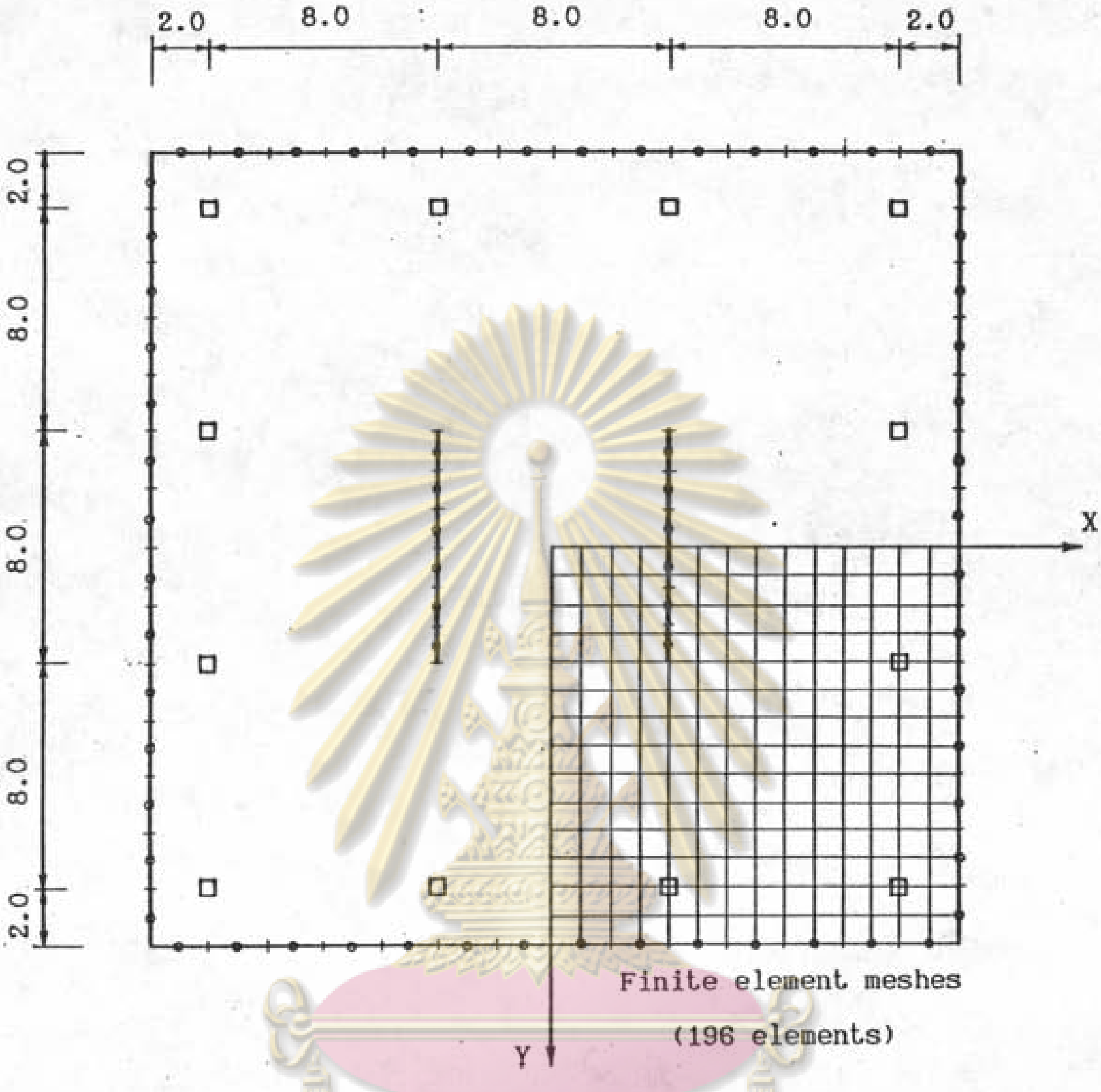
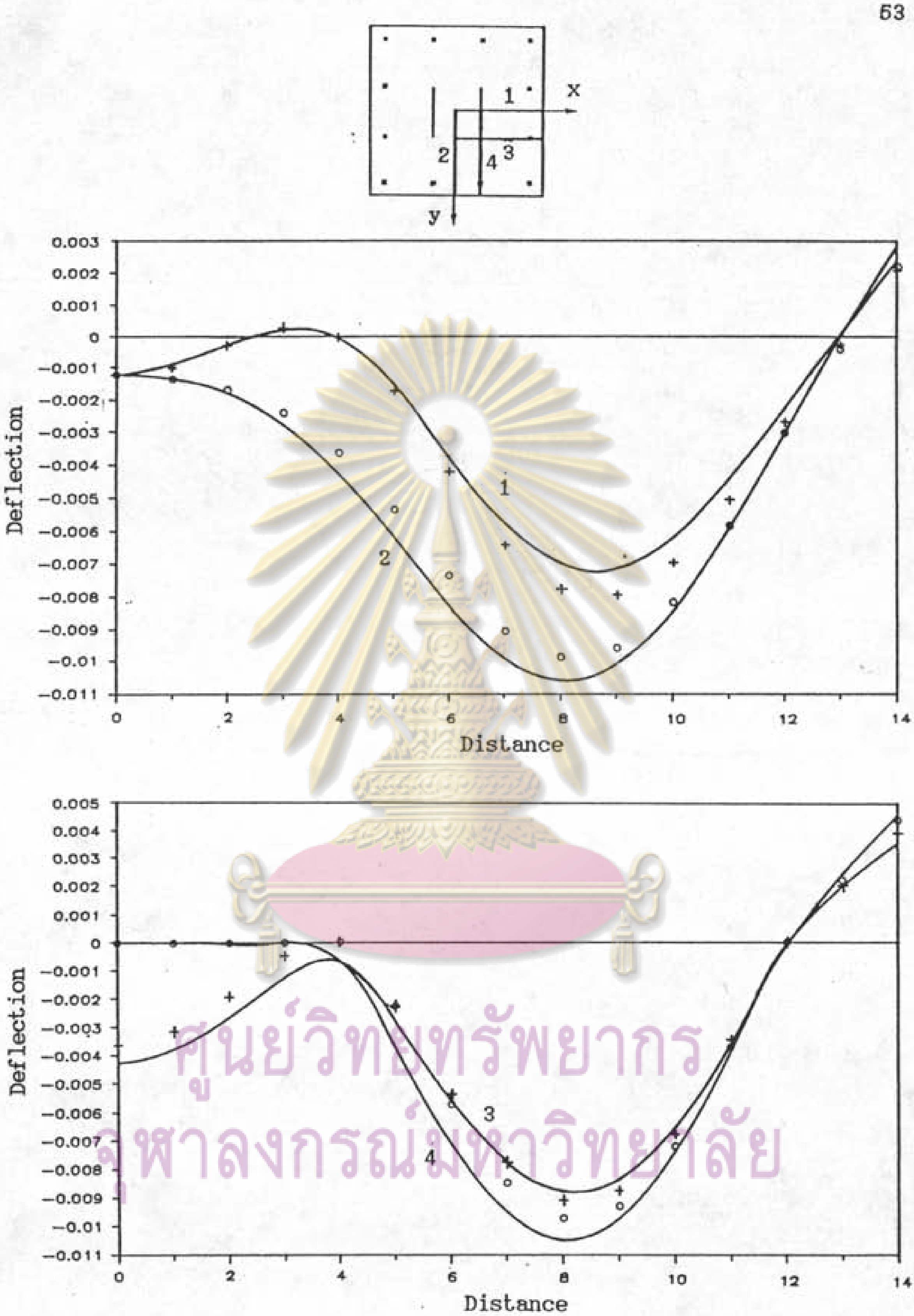


FIGURE 14 Normal bending moment along the diagonal



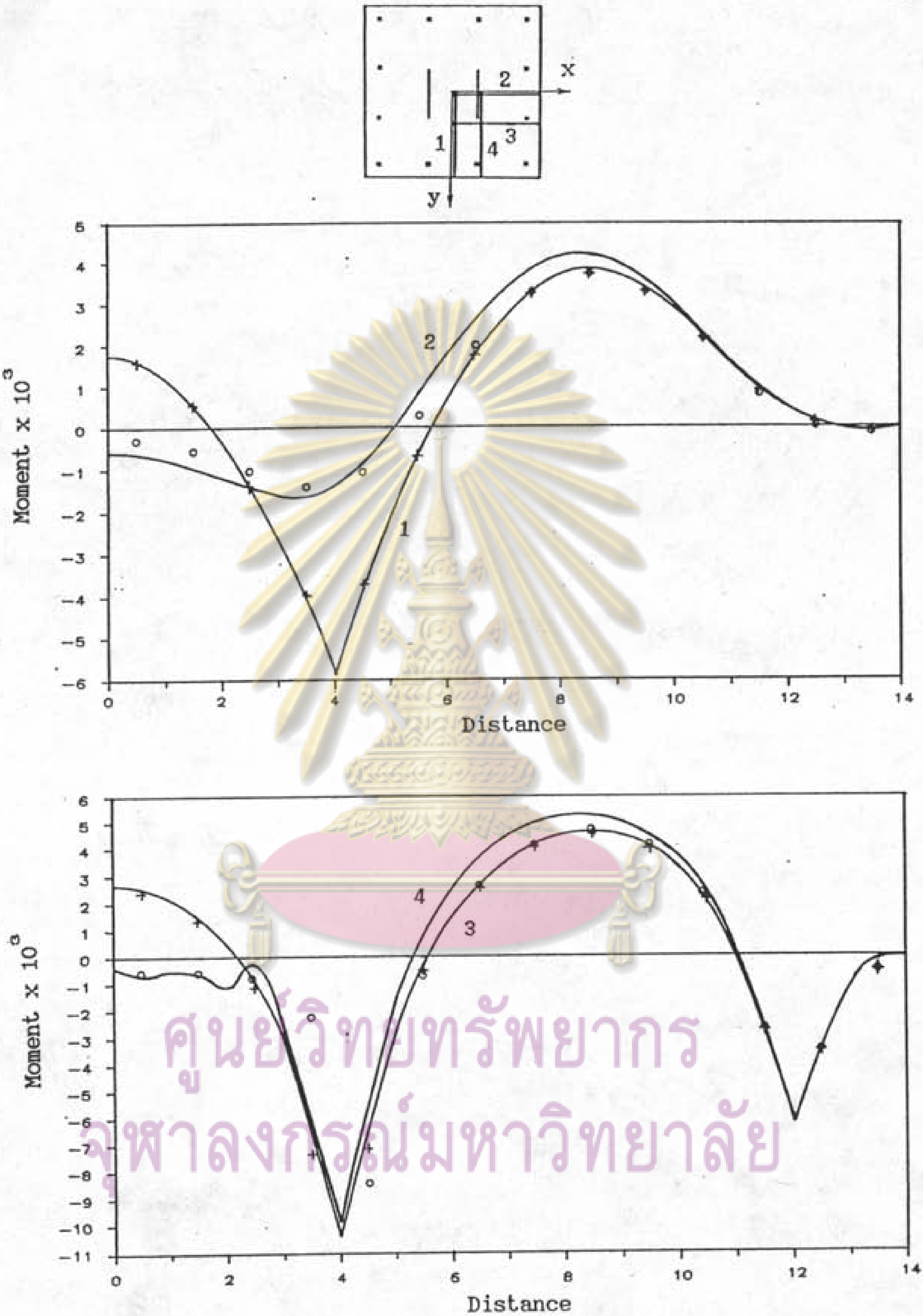
Modulus of elasticity ;	E	=	2.204×10^{9}	kg / m^2
Plate thichness ;	h	=	0.25	m.
Poisson ratio ;	ν	=	0.15	
Uniformly distributed load ;	q	=	1,500	kg / m^2
Number of boundary subinterval		=	14	(per side)
Number of line support subinterval		=	6	(per line)
Number of finite element		=	196	(per quarter)

FIGURE 15 Flat plate , Example 3



Solution line	Present study	Finite element
1,3	————	+
2,4	————	o

FIGURE 16 Deflection along x-axis , y-axis and line y = 4.0 , x = 4.0



Solution line	Present study	Finite element
1,3	————	+
2,4	————	o

FIGURE 17 Normal bending moment along line y = 0.5 , x = 0.5 and line y = 4.5 , x = 4.5

APPENDICES

APPENDIX A

TRANSFORMATION OF CO-ORDINATES

First order derivatives :

$$\begin{Bmatrix} \dfrac{\partial w}{\partial n} \\ \dfrac{\partial w}{\partial t} \end{Bmatrix} = \begin{bmatrix} \cos(\alpha) & \sin(\alpha) \\ -\sin(\alpha) & \cos(\alpha) \end{bmatrix} \begin{Bmatrix} \dfrac{\partial w}{\partial x} \\ \dfrac{\partial w}{\partial y} \end{Bmatrix} \qquad \text{(a1)}$$

Second order derivatives :

$$\begin{Bmatrix} \dfrac{\partial^2 w}{\partial n^2} \\ \dfrac{\partial^2 w}{\partial n\ \partial t} \\ \dfrac{\partial^2 w}{\partial t^2} \end{Bmatrix} = \begin{bmatrix} \cos^2(\alpha) & \sin(2\alpha) & \sin^2(\alpha) \\ -\dfrac{\sin(2\alpha)}{2} & \dfrac{\cos(2\alpha)}{2} & \dfrac{\sin(2\alpha)}{2} \\ \sin^2(\alpha) & -\sin(2\alpha) & \cos^2(\alpha) \end{bmatrix} \begin{Bmatrix} \dfrac{\partial^2 w}{\partial x^2} \\ \dfrac{\partial^2 w}{\partial x\ \partial y} \\ \dfrac{\partial^2 w}{\partial y^2} \end{Bmatrix} \qquad \text{(a2)}$$

Third order derivatives :

$$\begin{Bmatrix} \dfrac{\partial^3 w}{\partial n^3} \\ \dfrac{\partial^3 w}{\partial n^2 \partial t} \\ \dfrac{\partial^3 w}{\partial n \partial t^2} \\ \dfrac{\partial^3 w}{\partial t^3} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & A_{34} \\ A_{41} & A_{42} & A_{43} & A_{44} \end{bmatrix} \begin{Bmatrix} \dfrac{\partial^3 w}{\partial x^3} \\ \dfrac{\partial^3 w}{\partial x^2 \partial y} \\ \dfrac{\partial^3 w}{\partial x \partial y^2} \\ \dfrac{\partial^3 w}{\partial y^3} \end{Bmatrix} \qquad \text{(a3)}$$

where

$$A_{11} = \cos^3(\alpha) \qquad A_{31} = \frac{\sin(\alpha)\sin(2\alpha)}{2}$$

$$A_{12} = \frac{3\cos(\alpha)\sin(2\alpha)}{2} \qquad A_{32} = \sin^3(\alpha)-\cos(\alpha)\sin(2\alpha)$$

$$A_{13} = \frac{3\sin(\alpha)\sin(2\alpha)}{2} \qquad A_{33} = \cos^3(\alpha)-\sin(\alpha)\sin(2\alpha)$$

$$A_{14} = \sin^3(\alpha) \qquad A_{34} = \frac{\cos(\alpha)\sin(2\alpha)}{2}$$

$$A_{21} = -\frac{\cos(\alpha)\sin(2\alpha)}{2} \qquad A_{41} = -\sin^3(\alpha)$$

$$A_{22} = \cos^3(\alpha)-\sin(\alpha)\sin(2\alpha) \qquad A_{42} = \frac{3\sin(\alpha)\sin(2\alpha)}{2}$$

$$A_{23} = -\sin^3(\alpha)+\cos(\alpha)\sin(2\alpha) \qquad A_{43} = -\frac{3\cos(\alpha)\sin(2\alpha)}{2}$$

$$A_{24} = \frac{\sin(\alpha)\sin(2\alpha)}{2} \qquad A_{44} = \cos^3(\alpha)$$

APPENDIX B

INFLUENCE FUNCTION

$$\int_{t_1}^{t_2} w^* \, dt = \frac{1}{72\pi D} \left[3n^2 t(3\ln r - 2) + t^3(3\ln r - 1) + 6n^3 \tan^{-1}\left(\frac{t}{n}\right) \right]_{t_1}^{t_2} \quad \text{(b1)}$$

$$\int_{t_1}^{t_2} \frac{\partial w^*}{\partial n_w} \, dt = \frac{1}{8\pi D} \left[nt(2\ln r - 1) + 2n^2 \tan^{-1}\left(\frac{t}{n}\right) \right]_{t_1}^{t_2} \quad \text{(b2)}$$

$$\int_{t_1}^{t_2} \frac{\partial w^*}{\partial t_w} \, dt = \frac{1}{8\pi D} \left[r^2 \ln r \right]_{t_1}^{t_2} \quad \text{(b3)}$$

$$\int_{t_1}^{t_2} \frac{\partial w^*}{\partial n_{w1}} \, dt = -\frac{1}{8\pi D} \left[\left\{ nt(2\ln r - 1) + 2n^2 \tan^{-1}\left(\frac{t}{n}\right) \right\} \cos(\beta - \alpha) + r^2 \ln r \sin(\beta - \alpha) \right]_{t_1}^{t_2} \quad \text{(b4)}$$

$$\int_{t_1}^{t_2} \frac{\partial w^*}{\partial t_{w1}} \, dt = -\frac{1}{8\pi D} \left[-\left\{ nt(2\ln r - 1) + 2n^2 \tan^{-1}\left(\frac{t}{n}\right) \right\} \sin(\beta - \alpha) + r^2 \ln r \cos(\beta - \alpha) \right]_{t_1}^{t_2} \quad \text{(b5)}$$

$$\int_{t_1}^{t_2} \frac{\partial w^*}{\partial n_w \partial n_{w1}} \, dt = -\frac{1}{8\pi D} \left[\left\{ t(2\ln r - 1) + 4n \tan^{-1}\left(\frac{t}{n}\right) \right\} \cos(\beta - \alpha) + 2n\ln r \sin(\beta - \alpha) \right]_{t_1}^{t_2} \quad \text{(b6)}$$

$$\int_{t_1}^{t_2} \frac{\partial w^*}{\partial n_w \partial t_{w1}} \, dt = -\frac{1}{8\pi D} \left[-\left\{ t(2\ln r - 1) + 4n \tan^{-1}\left(\frac{t}{n}\right) \right\} \sin(\beta - \alpha) + 2n\ln r \cos(\beta - \alpha) \right]_{t_1}^{t_2} \quad \text{(b7)}$$

$$\int_{t_1}^{t_2} \frac{\partial w^*}{\partial t_w \partial n_{w1}} \, dt = -\frac{1}{8\pi D} \left[2n\ln r \cos(\beta - \alpha) + t(2\ln r + 1) \sin(\beta - \alpha) \right]_{t_1}^{t_2} \quad \text{(b8)}$$

$$\int_{t_1}^{t_2} \frac{\partial w^*}{\partial t_w \partial t_{w1}} \, dt = -\frac{1}{8\pi D} \left[t(2\ln r + 1) \cos(\beta - \alpha) - 2n\ln r \sin(\beta - \alpha) \right]_{t_1}^{t_2} \quad \text{(b9)}$$

APPENDIX C

COEFFICIENT MATRIX COMPONENTS

SOURCE POINT	APPLIED FORCE	FUNCTIONS at FIELD POINT								
		BOUNDARY ELEMENTS		CORNERS	COLUMN SUPPORTS			LINE SUPPORTS		
BOUNDARY ELEMENTS	1-P	$\int M_n^* dt$	$\int V_n^* dt$	R^*	w^*	$\frac{\partial w^*}{\partial \xi}$	$\frac{\partial w^*}{\partial \eta}$	$\int w^* dt$	$\int \frac{\partial w^*}{\partial n_r} dt$	$\int \frac{\partial w^*}{\partial t_r} dt$
	1-Mn	$\int \frac{\partial M_n^*}{\partial n_s} dt$	$\int \frac{\partial V_n^*}{\partial n_s} dt$	$\frac{\partial R^*}{\partial n_s}$	$\frac{\partial w^*}{\partial n_s}$	$\frac{\partial^2 w^*}{\partial \xi \partial n_s}$	$\frac{\partial^2 w^*}{\partial \eta \partial n_s}$	$\int \frac{\partial w^*}{\partial n_s} dt$	$\int \frac{\partial^2 w^*}{\partial n_r \partial n_s} dt$	$\int \frac{\partial^2 w^*}{\partial t_r \partial n_s} dt$
CORNERS	1-P	$\int M_n^* dt$	$\int V_n^* dt$	R^*	w^*	$\frac{\partial w^*}{\partial \xi}$	$\frac{\partial w^*}{\partial \eta}$	$\int w^* dt$	$\int \frac{\partial w^*}{\partial n_r} dt$	$\int \frac{\partial w^*}{\partial t_r} dt$
COLUMN SUPPORTS	1-P	$\int M_n^* dt$	$\int V_n^* dt$	R^*	w^*	$\frac{\partial w^*}{\partial \xi}$	$\frac{\partial w^*}{\partial \eta}$	$\int w^* dt$	$\int \frac{\partial w^*}{\partial n_r} dt$	$\int \frac{\partial w^*}{\partial t_r} dt$
	1-Mx	$\int \frac{\partial M_n^*}{\partial X} dt$	$\int \frac{\partial V_n^*}{\partial X} dt$	$\frac{\partial R^*}{\partial X}$	$\frac{\partial w^*}{\partial X}$	$\frac{\partial^2 w^*}{\partial \xi \partial X}$	$\frac{\partial^2 w^*}{\partial \eta \partial X}$	$\int \frac{\partial w^*}{\partial X} dt$	$\int \frac{\partial^2 w^*}{\partial n_r \partial X} dt$	$\int \frac{\partial^2 w^*}{\partial t_r \partial X} dt$
	1-My	$\int \frac{\partial M_n^*}{\partial Y} dt$	$\int \frac{\partial V_n^*}{\partial Y} dt$	$\frac{\partial R^*}{\partial Y}$	$\frac{\partial w^*}{\partial Y}$	$\frac{\partial^2 w^*}{\partial \xi \partial Y}$	$\frac{\partial^2 w^*}{\partial \eta \partial Y}$	$\int \frac{\partial w^*}{\partial Y} dt$	$\int \frac{\partial^2 w^*}{\partial n_r \partial Y} dt$	$\int \frac{\partial^2 w^*}{\partial t_r \partial Y} dt$
LINE SUPPORTS	1-P	$\int M_n^* dt$	$\int V_n^* dt$	R^*	w^*	$\frac{\partial w^*}{\partial \xi}$	$\frac{\partial w^*}{\partial \eta}$	$\int w^* dt$	$\int \frac{\partial w^*}{\partial n_r} dt$	$\int \frac{\partial w^*}{\partial t_r} dt$
	1-Mn	$\int \frac{\partial M_n^*}{\partial n_s} dt$	$\int \frac{\partial V_n^*}{\partial n_s} dt$	$\frac{\partial R^*}{\partial n_s}$	$\frac{\partial w^*}{\partial n_s}$	$\frac{\partial^2 w^*}{\partial \xi \partial n_s}$	$\frac{\partial^2 w^*}{\partial \eta \partial n_s}$	$\int \frac{\partial w^*}{\partial n_s} dt$	$\int \frac{\partial^2 w^*}{\partial n_r \partial n_s} dt$	$\int \frac{\partial^2 w^*}{\partial t_r \partial n_s} dt$
	1-Mt	$\int \frac{\partial M_n^*}{\partial t_s} dt$	$\int \frac{\partial V_n^*}{\partial t_s} dt$	$\frac{\partial R^*}{\partial t_s}$	$\frac{\partial w^*}{\partial t_s}$	$\frac{\partial^2 w^*}{\partial \xi \partial t_s}$	$\frac{\partial^2 w^*}{\partial \eta \partial t_s}$	$\int \frac{\partial w^*}{\partial t_s} dt$	$\int \frac{\partial^2 w^*}{\partial n_r \partial t_s} dt$	$\int \frac{\partial^2 w^*}{\partial t_r \partial t_s} dt$

<u>REMARK</u> : Functions in the blocks will be replaced with Singularity Functions when $r = 0$

APPENDIX D

EVALUATION OF THE DOMAIN INTEGRALS

The domain integrals which appear in the right hand side of equation (27) through (34) take the general form :

$$I = \int_{\Omega} q(\xi,\eta)\ f^{*}(x,y;\xi,\eta)\ d\Omega(\xi,\eta) \quad . \qquad (c1)$$

In the case of singular load, P , acting at (ξ,η), this load can be merely replaced by a Dirac delta function , $\delta(\xi,\eta)$, for which

$$\int_{\Omega} \delta(\xi,\eta)\ f^{*}(x,y;\xi_{o},\eta_{o})\ d\Omega(\xi_{o},\eta_{o}) = f^{*}(x,y;\xi,\eta) \quad ,$$

and equation (35) becomes

$$I = P(\xi,\eta)\ f^{*}(x,y;\xi,\eta) \quad . \qquad (c2)$$

As mentioned above, the uniformly distributed load , $q(\xi,\eta)$, may be treated by dividing the loaded area into M finite strips, each with width $\Delta\eta$ and in which assume an the equivalent line load , p_{i},

$$p_{i}(\xi,\eta) = q_{i}(\xi,\eta)\ \Delta\eta_{i} \qquad (c3)$$

acting at the center of each strip. Therefore, equation (35) can be replaced approximately by

$$I = \sum_{i=1,2}^{T} \left\{ p_{i}(\xi,\eta) \int_{\xi i} f^{*}(x,y;\xi,\eta)\ d\Omega(\xi) \right\} \quad . \qquad (c4)$$

APPENDIX E

TREATMENT OF SINGULARITIES

Using the equation of equilibrium, Tottenham can avoid the problem of singularities of the integration of function M_n, V_n, $\frac{\partial V_n}{\partial n}$, $\frac{\partial M_n}{\partial n}$ and R^* on the plate boundary when $(\bar{x}_i, \bar{y}_i)$ and $(\bar{\xi}_j, \bar{\eta}_j)$ are coincident since these functions have term $\ln(r)$, $\frac{1}{r}$ and $\frac{1}{r^2}$ which are singular at $r = 0$.

FUNCTION V_n and M_n

From the fundamental solution (eq.(17)) , the expression of bending moment, twisting moment and shear in polar co-ordinates can be shown to be :

$$M_r^* = -D \left[\frac{\partial^2 w^*}{\partial r^2} + \nu \left\{ \frac{1}{r} \frac{\partial w^*}{\partial r} + \frac{1}{r^2} \frac{\partial^2 w^*}{\partial \theta^2} \right\} \right] \qquad \text{(d1a)}$$

$$M_{r\theta}^* = D\,(1-\nu) \frac{\partial \left(\frac{1}{r} \frac{\partial w^*}{\partial \theta} \right)}{\partial r} \qquad \text{(d1b)}$$

$$Q_r^* = -D \frac{\partial (\nabla^2 w^*)}{\partial r} \qquad \text{(d1c)}$$

$$V_r^* = Q_r^* + \frac{1}{r} \frac{\partial M_{\theta r}^*}{\partial \theta} \qquad \text{(d1d)}$$

$$\nabla^2 w^* = \frac{\partial^2 w^*}{\partial r^2} + \frac{1}{r} \frac{\partial w^*}{\partial r} + \frac{1}{r^2} \frac{\partial^2 w^*}{\partial \theta^2} \qquad \text{(d1e)}$$

Thus, from (17) and (d1),

$$M_r^* = -\frac{1}{8\pi}\left\{ 2(1+\nu)\ln(r) + 3 + \nu \right\}$$

$$Q_r^* = -\frac{1}{2\pi r}$$

$$M_{r\theta}^* = 0$$

$$V_r^* = -\frac{1}{2\pi r} = Q_r^*$$

Consider the equilibrium of a semi-circular disc (Fig.7a), center of circle at (0,0) of which the unit load is applied. The total vertical shear along the straight edge can be found from the condition of zero vertical force as

$$\int_{-\Delta}^{+\Delta} V_n^* \, dt + \frac{1}{2} + \int_{\pi/2}^{3\pi/2} (V_r^*)_{r=\Delta} \, \Delta \, d\theta + R_{t=\Delta}^* + R_{t=-\Delta}^* = 0$$

or $$\int_{-\Delta}^{+\Delta} V_n^* \, dt = -\frac{1}{2} - \int_{\pi/2}^{3\pi/2} \frac{(-1)}{2\pi\Delta} \, \Delta \, d\theta = 0 \quad . \qquad \text{(d2)}$$

The total normal bending moment along the same edge can be obtained by considering the moment equilibrium as

$$\int_{-\Delta}^{+\Delta} M_n^* \, dt + \int_{\pi/2}^{3\pi/2} (M_r^*)_{r=\Delta} \cos\theta \, \Delta \, d\theta$$

$$- \int_{\pi/2}^{3\pi/2} (V_r^*)_{r=\Delta} \, \Delta^2 \cos\theta \, d\theta = 0$$

thus $$\int_{-\Delta}^{+\Delta} M_n^* \, dt = \frac{\Delta}{4\pi}\left\{ 1 - \nu - 2(1+\nu)\ln(\Delta) \right\} \quad . \qquad \text{(d3)}$$

<u>FUNCTION</u> $\frac{\partial V_n^*}{\partial n}$ <u>and</u> $\frac{\partial M_n^*}{\partial n}$.

Similarly for the case which corresponds to a unit couple applied at the origin (Fig.7b), deflection, w_2 , and corresponding stress resultants may be expressed as

$$w_2 = \frac{\partial w^*}{\partial n} = -\frac{r\cos\theta}{8\pi D}\left\{ 1 + 2\ln(r) \right\}$$

$$M_{r2} = \frac{(1+\nu)\cos\theta}{4\pi r}$$

$$M_{r\theta 2} = \frac{(1-\nu)\sin\theta}{4\pi r}$$

$$Q_{r2} = -\frac{\cos\theta}{2\pi r^2}$$

$$V_{r2} = -\frac{(3-\nu)\cos\theta}{4\pi r^2}$$

Using the same procedure as before to evaluate these integrals, it can be shown that

$$\int_{-\Delta}^{+\Delta} V_{n2}\, dt + \int_{\pi/2}^{3\pi/2} (V_{r2})_{r=\Delta}\, \Delta\, d\theta + R_2\Big|_{t=\Delta}^{n=0} + R_2\Big|_{t=-\Delta}^{n=0} = 0$$

$$\text{or} \quad \int_{-\Delta}^{+\Delta} \frac{\partial V_n^*}{\partial n}\, dt = \int_{-\Delta}^{+\Delta} V_{n2}\, dt = -\frac{(1+\nu)}{2\pi\Delta} \qquad \text{(d4)}$$

$$\text{and} \quad \int_{-\Delta}^{+\Delta} \frac{\partial M_n^*}{\partial n}\, dt = \int_{-\Delta}^{+\Delta} M_{n2}\, dt = 0 \; . \qquad \text{(d5)}$$

FUNCTION R^*

Consider the corner force, R^*, in eq.(27) where $(\bar{x},\bar{y})$ and (ξ,η) coincide. The corner force, R^*, can be found to be zero by using equilibrium condition of vertical forces of the free-body circular sector element of the plate corner (Fig.7c). Let ϵ be the radius of the circular sector. The portion of applied unit force, resulting shears along edges and the corner force must be in equilibrium :

$$R^* + \int_{-\epsilon}^{0} V^*_{n1}\, dt_1 + \int_{0}^{\epsilon} V^*_{n2}\, dt_2 + \frac{\emptyset}{2\pi}$$

$$+ \int_{\pi-\emptyset/2}^{\pi+\emptyset/2} (V^*_r)_{r=\epsilon}\, \epsilon\, d\theta = 0$$

thus $$R^* + \int_{-\epsilon}^{0} V^*_{n1}\, dt_1 + \int_{0}^{\epsilon} V^*_{n2}\, dt_2 = 0 .$$

From symmetry it can be shown that the shearing forces that act on the diametral sections of the element must be zero (10).

Therefore, $$R^* = 0 . \qquad \text{(d6)}$$

APPENDIX F

USER'S GUIDE

I HEADING

	variable	entry
(1)	HEAD	Title of problem (80 alphabets)

II MASTER CONTROL PARAMETERS

	variable	entry
(1)	KSIDE	Total number of sides of the plate (maximum = 20)
(2)	NCOL	Total number of column supports (maximum = 200)
(3)	NLine	Total number of line supports (maximum = 40)
(4)	MODEX	Program execution mode : EQ. 0 problem solution EQ. 1 data check only

III BOUNDARY COORDINATES

	variable	entry
(1)	N	Vertex number
(2)	CV(N,1)	X -ordinate of N-th vertex
	CV(N,2)	Y -ordinate of N-th vertex
(3)	NELEM(N)	Number of intervals on side number N

IV INTERIOR SUPPORT DATA

A. Column supports

	variable	entry
(1)	N	Column support number
(2)	CC(N,1)	X -ordinate of N-th column
	CC(N,2)	Y -ordinate of N-th column
(3)	STA(N)	Axial stiffness of column support
	STR(N,1)	Rotational stiffness of column support about x-axis
	STR(N,2)	Rotational stiffness of column support about y-axis

B. Line supports

	variable	entry
(1)	N	Line support number
(2)	CVLI(N,1)	X -ordinate of starting point of N-th line support
	CVLI(N,2)	Y -ordinate of starting point of N-th line support
	CVLJ(N,1)	X -ordinate of ending point of N-th line support
	CVLJ(N,2)	Y -ordinate of ending point of N-th line support
(3)	NELEML(N)	Number of intervals of N-th line support

(4)	STA(N)	Axial stiffness per unit length of N-th line support
	STRL(N,1)	Rotational stiffness about n -axis of N-th line support
	STRL(N,2)	Rotational stiffness about t -axis of N-th line support

V GEOMETRIC AND MATERIAL PROPERTIES INFORMATION

	variable	entry
(1)	TH	Plate thickness
(2)	E	Modulus of elasticity
(3)	PR	Poisson's ratio

VI LOADING DATA

A. <u>Control parameters</u>

	variable	entry
(1)	NPL	Total number of concentrated loads
(2)	NZ	Total number of zones subjected to distributed load
(3)	NCM	Total number of concentrated moments

B. <u>Concentrated load</u>

	variable	entry
(1)	N	Concentrated load number
(2)	PL(N)	Magnitude of z-direction force
(3)	XP(N,1)	X -ordinate
	XP(N,2)	Y -ordinate

C. Distributed load

	variable	entry
(1)	N	Zone number
(2)	INCRM(N)	Dividing into strip on x or y-direction EQ. 0 x-direction EQ. 1 y-direction
(3)	NS(N)	Number of strips
(4)	U(1,N)	Intensity of distributed load at 1st vertex
	U(2,N)	Intensity of distributed load at 3rd vertex
(5)	CVL(1,1,N)	X -ordinate of 1st vertex
	CVL(1,2,N)	Y -ordinate of 1st vertex
	CVL(2,1,N)	X -ordinate of 2nd vertex
	CVL(2,2,N)	Y -ordinate of 2nd vertex
	CVL(3,1,N)	X -ordinate of 3rd vertex
	CVL(3,2,N)	Y -ordinate of 3rd vertex
	CVL(4,1,N)	X -ordinate of 4th vertex
	CVL(4,2,N)	Y -ordinate of 4th vertex

D. Concentrated moment

	variable	entry
(1)	N	Concentrated moment number
(2)	CM(N,1)	Magnitude of x-axis moment
(3)	CM(N,2)	Magnitude of y-axis moment
(4)	CCM(N,1)	X -ordinate
	CCM(N,2)	Y -ordinate

VII SOLUTION OUTPUT

A. Control parameter

	variable	entry
(1)	NDL	Number of solution line to compute deflection and stress resultants (maximum = 50)

B. Coordinates

	variable	entry
(1)	N	Solution line number
(2)	CDLI(N,1)	X -ordinate of starting point
	CDLI(N,2)	Y -ordinate of starting point
	CDLJ(N,1)	X -ordinate of ending point
	CDLJ(N,2)	Y -ordinate of ending point
(3)	NI(N)	Number of intervals of solution line

INPUT DATA

```
* * *  E X A M P L E   3  * * *  PLATE  WITH  MIXED  SUPPORT  * * *
4,12,2,0
1,-14.0,-14.0,14
2,14.0,-14.0,14
3,14.0,14.0,14
4,-14.0,14.0,14
1,-12.0,-12.0,1.0E+50,1.0E-50,1.0E-50
2,-4.0,-12.0,1.0E+50,1.0E-50,1.0E-50
3,4.0,-12.0,1.0E+50,1.0E-50,1.0E-50
4,12.0,-12.0,1.0E+50,1.0E-50,1.0E-50
5,12.0,-4.0,1.0E+50,1.0E-50,1.0E-50
6,12.0,4.0,1.0E+50,1.0E-50,1.0E-50
7,12.0,12.0,1.0E+50,1.0E-50,1.0E-50
8,4.0,12.0,1.0E+50,1.0E-50,1.0E-50
9,-4.0,12.0,1.0E+50,1.0E-50,1.0E-50
10,-12.0,12.0,1.0E+50,1.0E-50,1.0E-50
11,-12.0,4.0,1.0E+50,1.0E-50,1.0E-50
12,-12.0,-4.0,1.0E+50,1.0E-50,1.0E-50
1,-4.0,4.0,-4.0,-4.0,6,1.0E+50,1.0E+50,1.0E-50
2,4.0,-4.0,4.0,4.0,6,1.0E+50,1.0E+50,1.0E-50
0.25,2.204E+9,0.15
0,1,0
1,1,56,1000.0,1000.0,-14.0,-14.0,14.0,-14.0,14.0,14.0,-14.0,14.0
4
1,0.0,0.001,13.995,0.001,14
2,0.001,0.0,0.001,13.995,14
3,0.0,4.001,13.995,4.001,14
4,4.001,0.0,4.001,13.995,14
```

OUTPUT RESULTS

*** EXAMPLE 3 *** PLATE WITH MIXED SUPPORT ***

CONTROL PARAMETERS

NUMBER OF SIDES : 4 (MAX. = 20)
NUMBER OF COLUMN-SUPPORTS = 12 (MAX. = 200)
NUMBER OF LINE-SUPPORTS : 2 (MAX. = 40)
SOLUTION MODE : 0
EQ. 0 PROBLEM SOLUTION
EQ. 1 DATA CHECK

BOUNDARY DATA

VERTEX NUMBER	COORDINATES X	Y	NUMBER OF INTERVALS
1	-14.00000	-14.00000	14
2	14.00000	-14.00000	14
3	14.00000	14.00000	14
4	-14.00000	14.00000	14

INTERIOR SUPPORT DATA

COLUMN SUPPORT

SUPPORT NUMBER	COORDINATES X	Y	AXIAL STIFFNESS	X-ROTAT. STIFFNESS	Y-ROTAT. STIFFNESS
1	-12.0000	-12.0000	0.10000E+51	0.10000E-49	0.10000E-49
2	-4.0000	-12.0000	0.10000E+51	0.10000E-49	0.10000E-49
3	4.0000	-12.0000	0.10000E+51	0.10000E-49	0.10000E-49
4	12.0000	-12.0000	0.10000E+51	0.10000E-49	0.10000E-49
5	12.0000	-4.0000	0.10000E+51	0.10000E-49	0.10000E-49
6	12.0000	4.0000	0.10000E+51	0.10000E-49	0.10000E-49
7	12.0000	12.0000	0.10000E+51	0.10000E-49	0.10000E-49
8	4.0000	12.0000	0.10000E+51	0.10000E-49	0.10000E-49
9	-4.0000	12.0000	0.10000E+51	0.10000E-49	0.10000E-49
10	-12.0000	12.0000	0.10000E+51	0.10000E-49	0.10000E-49
11	-12.0000	4.0000	0.10000E+51	0.10000E-49	0.10000E-49
12	-12.0000	-4.0000	0.10000E+51	0.10000E-49	0.10000E-49

LINE - SUPPORT

SUPPORT NUMBER	COORDINATES I-X	I-Y	J-X	J-Y	NUMBER OF INTERVALS	AXIAL STIFFNESS	N-ROTAT. STIFFNESS	T-ROTAT. STIFFNESS
1	-4.0000	4.0000	-4.0000	-4.0000	6	0.10000E+51	0.10000E+51	0.10000E-49
2	4.0000	-4.0000	4.0000	4.0000	6	0.10000E+51	0.10000E+51	0.10000E-49

GEOMETRIC AND MATERIAL PROPERTIES

PLATE THICKNESS	YOUNG'S MODULUS	POISSON'S RATIO	PLATE RIGIDITY (D)
0.2500	0.22040E+10	0.1500	2935848.25234442

LOADING DATA

TOTAL NUMBER OF CONCENTRATED LOADS : 0
TOTAL NUMBER OF ZONES SUBJECTED TO DISTRIBUTED LOAD : 1
TOTAL NUMBER OF CONCENTRATED MOMENTS : 0

DISTRIBUTED LOAD

ZONE NUMBER	DIVIDING DIRECTION	NUMBER OF STRIPS	INTENSITY 1 st VERTEX	INTENSITY 3 rd VERTEX
1	1	56	0.10000E+04	0.10000E+04

COORDINATE	X	Y
1 st VERTEX	-14.0000	-14.0000
2 nd VERTEX	14.0000	-14.0000
3 rd VERTEX	14.0000	14.0000
4 th VERTEX	-14.0000	14.0000

SOLUTION OUTPUT

NUMBER OF SOLUTION LINES = 4

LINE NUMBER	I-X	I-Y	J-X	J-Y	NUMBER OF INTERVALS
1	0.0000	0.0010	13.9950	0.0010	14
2	0.0010	0.0000	0.0010	13.9950	14
3	0.0000	4.0010	13.9950	4.0010	14
4	4.0010	0.0000	4.0010	13.9950	14

NO. OF EQUATIONS = 188
REQUIRED STORAGES = 36038

*** GENERATING OF COEFFICIENT MATRIX COMPLETED *******

*** GENERATING OF LOAD VECTOR COMPLETED *************

*** SOLVING OF UNKNOWNS COMPLETED *******************

SOLUTION LINE NUMBER 1

	COORDINATES X	Y	INTERNAL FORCES W	Mn	Mnt	Qn	Vn
1	0.0000	0.0010	0.14071E-02	0.17352E+04	-0.52973E-03	-0.13478E-04	0.84360E-05
2	0.9996	0.0010	0.11053E-02	0.11967E+04	-0.19926E+00	-0.13003E+03	-0.64268E+03
3	1.9993	0.0010	0.38619E-03	-0.31732E+03	-0.28537E+00	-0.35487E+03	-0.12873E+04
4	2.9989	0.0010	-0.22769E-03	-0.25673E+04	-0.28093E+00	-0.70853E+03	-0.19341E+04
5	3.9986	0.0010	0.45801E-04	-0.17161E+04	-0.30136E+03	-0.70605E+06	-0.25799E+04
6	4.9982	0.0010	0.17640E-02	-0.21137E+04	0.15757E+00	-0.80113E+04	-0.32159E+04
7	5.9979	0.0010	0.42225E-02	0.60818E+03	0.14896E+00	-0.84613E+04	-0.38255E+04
8	6.9975	0.0010	0.64865E-02	0.26137E+04	0.41713E-01	-0.88144E+04	-0.43803E+04
9	7.9971	0.0010	0.78746E-02	0.36785E+04	-0.19234E+00	-0.90417E+04	-0.48387E+04
10	8.9968	0.0010	0.80315E-02	0.37240E+04	-0.45295E+00	-0.91251E+04	-0.51584E+04
11	9.9964	0.0010	0.69567E-02	0.29046E+04	-0.63453E+00	-0.90138E+04	-0.53397E+04
12	10.9961	0.0010	0.49493E-02	0.16581E+04	-0.57057E+00	-0.86548E+04	-0.54748E+04
13	11.9957	0.0010	0.24502E-02	0.63608E+03	-0.15043E+00	-0.81099E+04	-0.56076E+04
14	12.9954	0.0010	-0.18450E-03	0.26971E+03	0.46453E+00	-0.74874E+04	-0.54321E+04
15	13.9950	0.0010	-0.28250E-02	0.20809E+03	-0.40920E+03	0.66377E+05	0.22690E+06

SOLUTION LINE NUMBER 2

	COORDINATES X	Y	INTERNAL FORCES W	Mn	Mnt	Qn	Vn
1	0.0010	0.0000	0.14071E-02	-0.59951E+03	0.14982E-03	-0.24440E-02	-0.12399E-02
2	0.0010	0.9996	0.15603E-02	-0.76306E+03	0.21485E+00	-0.90852E+03	-0.65336E+03
3	0.0010	1.9993	0.20800E-02	-0.11805E+04	0.38256E+00	-0.19455E+04	-0.13086E+04
4	0.0010	2.9989	0.31175E-02	-0.15364E+04	0.31293E+00	-0.32433E+04	-0.19659E+04
5	0.0010	3.9986	0.47977E-02	-0.12503E+04	-0.11326E+00	-0.47915E+04	-0.26218E+04
6	0.0010	4.9982	0.70191E-02	0.19110E+01	-0.55684E+00	-0.63067E+04	-0.32677E+04
7	0.0010	5.9979	0.93485E-02	0.17933E+04	-0.63846E+00	-0.74791E+04	-0.38861E+04
8	0.0010	6.9975	0.11163E-01	0.33541E+04	-0.40774E+00	-0.82457E+04	-0.44481E+04
9	0.0010	7.9971	0.11916E-01	0.41502E+04	-0.63029E-01	-0.87011E+04	-0.49109E+04
10	0.0010	8.9968	0.11327E-01	0.40011E+04	0.27824E+00	-0.89176E+04	-0.52307E+04
11	0.0010	9.9964	0.94490E-02	0.30279E+04	0.51567E+00	-0.88851E+04	-0.54084E+04
12	0.0010	10.9961	0.66256E-02	0.16664E+04	0.48358E+00	-0.85694E+04	-0.55374E+04
13	0.0010	11.9957	0.33300E-02	0.58521E+03	0.63666E-01	-0.80414E+04	-0.56507E+04
14	0.0010	12.9954	-0.67655E-04	0.23278E+03	-0.58883E+00	-0.73914E+04	-0.53890E+04
15	0.0010	13.9950	-0.34464E-02	0.21717E+03	0.48162E+03	0.79520E+05	0.26816E+06

SOLUTION LINE NUMBER 3

	COORDINATES		INTERNAL FORCES				
	X	Y	W	Mn	Mnt	Qn	Vn
1	0.0000	4.0010	0.48025E-02	0.26943E+04	0.21615E-02	0.44506E-02	0.56910E-03
2	0.9996	4.0010	0.43137E-02	0.21874E+04	0.10007E+03	0.66865E+03	-0.58320E+03
3	1.9993	4.0010	0.30058E-02	0.55372E+03	0.13402E+03	0.19467E+04	-0.11780E+04
4	2.9989	4.0010	0.13907E-02	-0.29500E+04	0.12114E+03	0.63559E+04	-0.17941E+04
5	3.9986	4.0010	0.67158E-03	-0.36111E+05	0.14934E+04	0.38587E+07	-0.24387E+04
6	4.9982	4.0010	0.30535E-02	-0.24779E+04	-0.44396E+03	-0.14448E+05	-0.31197E+04
7	5.9979	4.0010	0.61717E-02	0.15235E+04	-0.47777E+03	-0.10204E+05	-0.38528E+04
8	6.9975	4.0010	0.86523E-02	0.36520E+04	-0.47392E+03	-0.92762E+04	-C.46731E+04
9	7.9971	4.0010	0.98148E-02	0.46545E+04	-0.40069E+03	-0.92268E+04	-0.56562E+04
10	8.9968	4.0010	0.93324E-02	0.46420E+04	-0.29970E+03	-0.97007E+04	-0.69782E+04
11	9.9964	4.0010	0.71967E-02	0.34956E+04	-0.19877E+03	-0.10913E+05	-0.91624E+04
12	10.9961	4.0010	0.37595E-02	0.55960E+03	-0.10678E+03	-0.14642E+05	-0.14843E+05
13	11.9957	4.0010	0.13917E-04	-0.23409E+05	0.71398E+03	-0.16552E+07	-0.22807E+07
14	12.9954	4.0010	-0.20823E-02	-0.12987E+04	0.83455E+03	-0.28749E+03	0.46093E+04
15	13.9950	4.0010	-0.37581E-02	-0.15710E+08	0.41276E+08	-0.73923E+10	-0.23400E+11

SOLUTION LINE NUMBER 4

	COORDINATES		INTERNAL FORCES				
	X	Y	W	Mn	Mnt	Qn	Vn
1	4.0010	0.0000	0.48255E-04	0.43844E+04	-0.16283E-02	-0.23073E+00	-0.60620E-03
2	4.0010	0.9996	0.44468E-04	-0.10708E+04	0.62843E+02	-0.54578E+04	-0.59427E+03
3	4.0010	1.9993	0.41002E-04	-0.21140E+04	0.11261E+03	-0.28858E+04	-0.12000E+04
4	4.0010	2.9989	0.49395E-04	-0.28947E+04	0.13697E+03	0.18340E+05	-0.18269E+04
5	4.0010	3.9986	0.66836E-03	-0.34713E+05	-0.14755E+04	0.38384E+07	-0.24824E+04
6	4.0010	4.9982	0.41173E-02	-0.13662E+04	0.14916E+03	-0.14562E+05	-0.31747E+04
7	4.0010	5.9979	0.78780E-02	0.25648E+04	0.11281E+03	-0.10352E+05	-0.39203E+04
8	4.0010	6.9975	0.10624E-01	0.45245E+04	0.61919E+02	-0.93526E+04	-0.47552E+04
9	4.0010	7.9971	0.11740E-01	0.53261E+04	0.41593E+01	-0.92707E+04	-0.57575E+04
10	4.0010	8.9968	0.10970E-01	0.51201E+04	-0.51337E+02	-0.97473E+04	-0.71084E+04
11	4.0010	9.9964	0.83759E-02	0.37909E+04	-0.97665E+02	-0.10995E+05	-0.93478E+04
12	4.0010	10.9961	0.43712E-02	0.65923E+03	-0.13568E+03	-0.14842E+05	-0.15194E+05
13	4.0010	11.9957	0.16184E-04	-0.24137E+05	-0.92414E+03	-0.17078E+07	-0.23534E+07
14	4.0010	12.9954	-0.26477E-02	-0.13479E+04	-0.88578E+03	-0.29397E+02	0.49069E+04
15	4.0010	13.9950	-0.48952E-02	-0.13696E+08	-0.35987E+08	-0.64435E+10	-0.20398E+11

*** CALCULATION OF INTERNAL FORCES COMPLETED **********

SUPPORT DISPLACEMENTS AND REACTIONS

COLUMN-SUPPORT

SUPPORT NO.	DEFLECTION	SLOPE W.R.T. X	SLOPE W.R.T. Y	REACTION	MOMENT ABOUT X-AXIS	MOMENT ABOUT Y-AXIS
1	0.30513E-45	0.27919E-02	0.30133E-02	0.30513E+05	0.30133E-52	0.27919E-52
2	0.48284E-45	-0.83668E-03	0.35612E-02	0.48284E+05	0.35612E-52	-0.83668E-53
3	0.48284E-45	0.83664E-03	0.35612E-02	0.48284E+05	0.35612E-52	0.83664E-53
4	0.30514E-45	-0.27917E-02	0.30131E-02	0.30514E+05	0.30131E-52	-0.27917E-52
5	0.46789E-45	-0.29683E-02	-0.11127E-02	0.46789E+05	-0.11127E-52	-0.29683E-52
6	0.46790E-45	-0.29684E-02	0.11128E-02	0.46790E+05	0.11128E-52	-0.29684E-52
7	0.30513E-45	-0.27919E-02	-0.30133E-02	0.30513E+05	-0.30133E-52	-0.27919E-52
8	0.48284E-45	0.83666E-03	-0.35612E-02	0.48284E+05	-0.35612E-52	0.83666E-53
9	0.48284E-45	-0.83662E-03	-0.35612E-02	0.48284E+05	-0.35612E-52	-0.83662E-53
10	0.30514E-45	0.27917E-02	-0.30132E-02	0.30514E+05	-0.30132E-52	0.27917E-52
11	0.46789E-45	0.29683E-02	0.11127E-02	0.46789E+05	0.11127E-52	0.29683E-52
12	0.46790E-45	0.29684E-02	-0.11128E-02	0.46790E+05	-0.11128E-52	0.29684E-52

LINE-SUPPORT No. 1

ELEMENT NO.	DEFLECTION	SLOPE W.R.T. N	SLOPE W.R.T. T	REACTION	MOMENT ABOUT N-AXIS	MOMENT ABOUT T-AXIS
1	0.18071E-45	0.89051E-03	-0.51624E-45	0.24095E+05	-0.68832E+05	0.11873E-52
2	0.20266E-45	0.96293E-03	-0.18774E-45	0.27022E+05	-0.25033E+05	0.12839E-52
3	0.14462E-45	0.10049E-02	-0.47408E-46	0.19282E+05	-0.63211E+04	0.13399E-52
4	0.14462E-45	0.10049E-02	0.47406E-46	0.19282E+05	0.63208E+04	0.13399E-52
5	0.20267E-45	0.96294E-03	0.18774E-45	0.27022E+05	0.25032E+05	0.12839E-52
6	0.18071E-45	0.89053E-03	0.51624E-45	0.24095E+05	0.68832E+05	0.11874E-52

LINE-SUPPORT No. 2

ELEMENT NO.	DEFLECTION	SLOPE W.R.T. N	SLOPE W.R.T. T	REACTION	MOMENT ABOUT N-AXIS	MOMENT ABOUT T-AXIS
1	0.18071E-45	0.89052E-03	-0.51624E-45	0.24095E+05	-0.68832E+05	0.11874E-52
2	0.20267E-45	0.96294E-03	-0.18774E-45	0.27022E+05	-0.25033E+05	0.12839E-52
3	0.14462E-45	0.10049E-02	-0.47407E-46	0.19282E+05	-0.63210E+04	0.13399E-52
4	0.14462E-45	0.10049E-02	0.47407E-46	0.19282E+05	0.63209E+04	0.13399E-52
5	0.20267E-45	0.96294E-03	0.18774E-45	0.27022E+05	0.25033E+05	0.12839E-52
6	0.18071E-45	0.89052E-03	0.51624E-45	0.24095E+05	0.68832E+05	0.11874E-52

VITA

Mr. Thanan Jarupoom was born in Bangkok in 1957. He graduated from Faculty of Engineering, Chulalongkorn University in 1979 then joined the Electricity Generating Authority of Thailand. He continued his education for Master Degree in Civil Engineering in Chulalongkorn University in 1987.