CHAPTER III

NUMERICAL SCHEME

3.1 Discretization of Boundary and Line Support

Equations (19) through (26) in the previous chapter are expressed in the form of integral equations formulated for the plate problem. To solve these equations numerically, the boundary of plate and line support have to be divided into a finite number of intervals. The center point of each interval is considered to be the nodal point in which the unknown functions are discretely assumed to be of constant values. Consequently, the integrals in equations (19) through (26) can be replaced approximately by equivalent summations.

According to the discretization of line support, stiffnesses of the whole line support are subdivided. Variation of element length may cause inconvenient in computation of each element stiffnesses. Generally for a flexural member, axial stiffness factor is $\frac{EA}{L}$ and rotational stiffness factor is $\frac{4EI}{L}$ or other factors as the case may be. For a line support, axial and rotational stiffness in normal direction are proportion to element length, only rotational stiffness in tangential direction is vary with power three of element length. Thus, stiffness per unit length and length of element are used to avoid the variation of element length. In the following equations, notations of K_a^w , K_r^n and K_r^t will represent the element stiffnesses of line support.

As mentioned above, boundary of plate $(\bar{\xi},\bar{\eta})$ and the line supports (ξ_w,η_w) may be devided into N and S intervals respectively. The two unknown functions at boundary, deflection $w(\bar{\xi},\bar{\eta})$ and normal slope $\frac{\partial w(\bar{\xi},\bar{\eta})}{\partial n}$, and three unknown functions at line support, deflection $w_w(\xi_w,\eta_w)$, normal slope $\frac{\partial w_w(\xi_w,\eta_w)}{\partial n_w}$ and tangential slope $\frac{\partial w_w(\xi_w,\eta_w)}{\partial t_w}$, are defined at each center point of intervals (ξ_j,η_j) and (ξ_t,η_t) for $j = 1,2,\ldots,N$ and $t = 1,2,\ldots,S$, respectively.

Thus, we define the replaced algebralic equations of (19) and (20) at the center of the above intervals, $(\bar{x}_i,\bar{y}_i)$, $i = 1,2,\ldots,N$ and at each corner of plate where $i = N+1,N+2,\ldots,N+K$. From equations (21), (22) and (23), we define at each column supports (x_i,y_i) where $i = N+K+1,N+K+2,\ldots,N+K+L$. From equations (24), (25) and (26), we also define at each center of intervals, (x_{wi},y_{wi}), of line supports inside the plate domain where $i = N+K+L+1,N+K+L+2,\ldots,N+K+L+S$.

Therefore, equations (19) through (26) may be replaced numerically by a set of (2N+K+3L+3S) algebraic equations in (2N+K+3L+3S) unknowns which are w_j, $j = 1,2,\ldots,N+K$, $\frac{\partial w_j}{\partial n}$, $j = 1,2,\ldots,N$ and w_j, $\frac{\partial w_j}{\partial \xi}$, $\frac{\partial w_j}{\partial \eta}$, $j = N+K+1,\ldots,N+K+L$, and w_{wj}, $\frac{\partial w_{wj}}{\partial n_w}$, $\frac{\partial w_{wj}}{\partial t_w}$, $j = N+K+L+1,\ldots,N+K+L+S$.

Rewriting equations (19), (20), (21), (22), (23), (24), (25), and (26)

$$\frac{\theta_i}{2\pi} w_i - \sum_{j=1,2}^{N} \int_{\Gamma j} M_n^*(\bar{x}_i,\bar{y}_i;\bar{\xi},\bar{\eta})\, d\Gamma(\bar{\xi},\bar{\eta}) \quad \frac{\partial w_j}{\partial n}$$

$$+ \sum_{j=1,2}^{N} \int_{\Gamma j} V_n^*(\bar{x}_i,\bar{y}_i;\bar{\xi},\bar{\eta})\, d\Gamma(\bar{\xi},\bar{\eta}) \quad w_j$$

$$+ \sum_{j=N+1}^{N+K} R^*(\bar{x}_i,\bar{y}_i;\bar{\xi}_j,\bar{\eta}_j) \quad w_j$$

$$+ \sum_{j=N+K+1}^{N+K+L} \left\{ K^c_{mj}\, w^*(\bar{x}_i,\bar{y}_i;\xi_j,\eta_j) \right\} \quad w_j$$

$$+ \sum_{j=N+K+1}^{N+K+L} \left\{ K^{y}_{rj}\, \frac{\partial w^*(\bar{x}_i,\bar{y}_i;\xi_j,\eta_j)}{\partial \xi} \right\} \quad \frac{\partial w_j}{\partial \xi}$$

$$+ \sum_{j=N+K+1}^{N+K+L} \left\{ K^{x}_{rj}\, \frac{\partial w^*(\bar{x}_i,\bar{y}_i;\xi_j,\eta_j)}{\partial \eta} \right\} \quad \frac{\partial w_j}{\partial \eta}$$

$$+ \sum_{j=N+K+L+1}^{N+K+L+S} \left\{ K^w_{mj} \int_{\psi j} w^*(\bar{x}_i,\bar{y}_i;\xi,\eta)\, d\psi(\xi,\eta) \right\} \quad w_{wj}$$

$$+ \sum_{j=N+K+L+1}^{N+K+L+S} \left\{ K^t_{rj} \int_{\psi j} \frac{\partial w^*(\bar{x}_i,\bar{y}_i;\xi,\eta)}{\partial n_w}\, d\psi(\xi,\eta) \right\} \quad \frac{\partial w_{wj}}{\partial n_w}$$

$$+ \sum_{j=N+K+L+1}^{N+K+L+S} \left\{ K^n_{rj} \int_{\psi j} \frac{\partial w^*(\bar{x}_i,\bar{y}_i;\xi,\eta)}{\partial t_w}\, d\psi(\xi,\eta) \right\} \quad \frac{\partial w_{wj}}{\partial t_w}$$

$$= \int_{\Omega} q(\xi,\eta) \quad w^*(\bar{x}_i,\bar{y}_i;\xi,\eta)\, d\Omega(\xi,\eta)$$

$$, \quad i = 1,2,\ldots,N+K \qquad (27)$$

$$\frac{\theta_i}{2\pi}\frac{\partial w_i}{\partial n(\bar{x},\bar{y})} - \sum_{j=1,2}^{N}\int_{\Gamma j}\frac{\partial M_n^*(\bar{x}_i,\bar{y}_i;\bar{\xi},\bar{\eta})}{\partial n(\bar{x},\bar{y})}\,d\Gamma(\bar{\xi},\bar{\eta})\;\frac{\partial w_j}{\partial n}$$

$$+ \sum_{j=1,2}^{N}\int_{\Gamma j}\frac{\partial V_n^*(\bar{x}_i,\bar{y}_i;\bar{\xi},\bar{\eta})}{\partial n(\bar{x},\bar{y})}\,d\Gamma(\bar{\xi},\bar{\eta})\;w_j$$

$$+ \sum_{j=N+1}^{N+K}\frac{\partial R^*(\bar{x}_i,\bar{y}_i;\bar{\xi}_j,\bar{\eta}_j)}{\partial n(\bar{x},\bar{y})}\;w_j$$

$$+ \sum_{j=N+K+1}^{N+K+L}\left\{K^c_{aj}\frac{\partial w^*(\bar{x}_i,\bar{y}_i;\xi_j,\eta_j)}{\partial n(\bar{x},\bar{y})}\right\}\;w_j$$

$$+ \sum_{j=N+K+1}^{N+K+L}\left\{K^y_{rj}\frac{\partial^2 w^*(\bar{x}_i,\bar{y}_i;\xi_j,\eta_j)}{\partial\xi\;\partial n(\bar{x},\bar{y})}\right\}\;\frac{\partial w_j}{\partial\xi}$$

$$+ \sum_{j=N+K+1}^{N+K+L}\left\{K^x_{rj}\frac{\partial^2 w^*(\bar{x}_i,\bar{y}_i;\xi_j,\eta_j)}{\partial\eta\;\partial n(\bar{x},\bar{y})}\right\}\;\frac{\partial w_j}{\partial\eta}$$

$$+ \sum_{j=N+K+L+1}^{N+K+L+S}\left\{K^w_{aj}\int_{\psi j}\frac{\partial w^*(\bar{x}_i,\bar{y}_i;\xi,\eta)}{\partial n(\bar{x},\bar{y})}\,d\psi(\xi,\eta)\right\}\;w_{wj}$$

$$+ \sum_{j=N+K+L+1}^{N+K+L+S}\left\{K^t_{rj}\int_{\psi j}\frac{\partial^2 w^*(\bar{x}_i,\bar{y}_i;\xi,\eta)}{\partial n_w\;\partial n(\bar{x},\bar{y})}\,d\psi(\xi,\eta)\right\}\;\frac{\partial w_{wj}}{\partial n_w}$$

$$+ \sum_{j=N+K+L+1}^{N+K+L+S}\left\{K^n_{rj}\int_{\psi j}\frac{\partial^2 w^*(\bar{x}_i,\bar{y}_i;\xi,\eta)}{\partial t_w\;\partial n(\bar{x},\bar{y})}\,d\psi(\xi,\eta)\right\}\;\frac{\partial w_{wj}}{\partial t_w}$$

$$= \int_\Omega q(\xi,\eta)\;\frac{\partial w^*(\bar{x}_i,\bar{y}_i;\xi,\eta)}{\partial n(\bar{x},\bar{y})}\,d\Omega(\xi,\eta)$$

$$, \quad i = 1,2,\ldots,N \qquad (28)$$

$$
\begin{aligned}
w_i \quad & - \sum_{j=1,2}^{N} \int_{\Gamma j} M_n^{*}(x_i,y_i;\bar{\xi},\bar{\eta})\, d\Gamma(\bar{\xi},\bar{\eta}) \quad \frac{\partial w_j}{\partial n} \\
& + \sum_{j=1,2}^{N} \int_{\Gamma j} V_n^{*}(x_i,y_i;\bar{\xi},\bar{\eta})\, d\Gamma(\bar{\xi},\bar{\eta}) \quad w_j \\
& + \sum_{j=N+1}^{N+K} R^{*}(x_i,y_i;\bar{\xi}_j,\bar{\eta}_j) \quad w_j \\
& + \sum_{j=N+K+1}^{N+K+L} \left\{ K^{c}_{aj}\, w^{*}(x_i,y_i;\xi_j,\eta_j) \right\} \quad w_j \\
& + \sum_{j=N+K+1}^{N+K+L} \left\{ K^{y}_{rj}\, \frac{\partial w^{*}(x_i,y_i;\xi_j,\eta_j)}{\partial \xi} \right\} \quad \frac{\partial w_j}{\partial \xi} \\
& + \sum_{j=N+K+1}^{N+K+L} \left\{ K^{x}_{rj}\, \frac{\partial w^{*}(x_i,y_i;\xi_j,\eta_j)}{\partial \eta} \right\} \quad \frac{\partial w_j}{\partial \eta} \\
& + \sum_{j=N+K+L+1}^{N+K+L+S} \left\{ K^{\omega}_{aj} \int_{\psi j} w^{*}(x_i,y_i;\xi,\eta)\, d\psi(\xi,\eta) \right\} \quad w_{\omega j} \\
& + \sum_{j=N+K+L+1}^{N+K+L+S} \left\{ K^{t}_{rj} \int_{\psi j} \frac{\partial w^{*}(x_i,y_i;\xi,\eta)}{\partial n_{\omega}}\, d\psi(\xi,\eta) \right\} \quad \frac{\partial w_{\omega j}}{\partial n_{\omega}} \\
& + \sum_{j=N+K+L+1}^{N+K+L+S} \left\{ K^{n}_{rj} \int_{\psi j} \frac{\partial w^{*}(x_i,y_i;\xi,\eta)}{\partial t_{\omega}}\, d\psi(\xi,\eta) \right\} \quad \frac{\partial w_{\omega j}}{\partial t_{\omega}} \\
= \quad & \int_{\Omega} q(\xi,\eta) \quad w^{*}(x_i,y_i;\xi,\eta)\, d\Omega(\xi,\eta)
\end{aligned}
$$

$$, \ i = N+K+1, N+K+2, \ldots, N+K+L \qquad (29)$$

$$
\begin{aligned}
\frac{\partial w_i}{\partial x} \quad & - \sum_{j=1,2}^{N} \int_{\Gamma j} \frac{\partial M_n^*(x_i,y_i;\bar{\xi},\bar{\eta})}{\partial x} \, d\Gamma(\bar{\xi},\bar{\eta}) \quad \frac{\partial w_j}{\partial n} \\
& + \sum_{j=1,2}^{N} \int_{\Gamma j} \frac{\partial V_n^*(x_i,y_i;\bar{\xi},\bar{\eta})}{\partial x} \, d\Gamma(\bar{\xi},\bar{\eta}) \quad w_j \\
& + \sum_{j=N+1}^{N+K} \frac{\partial R^*(x_i,y_i;\bar{\xi}_j,\bar{\eta}_j)}{\partial x} \quad w_j \\
& + \sum_{j=N+K+1}^{N+K+L} \left\{ K^c_{mj} \frac{\partial w^*(x_i,y_i;\xi_j,\eta_j)}{\partial x} \right\} \quad w_j \\
& + \sum_{j=N+K+1}^{N+K+L} \left\{ K^y_{rj} \frac{\partial^2 w^*(x_i,y_i;\xi_j,\eta_j)}{\partial \xi \, \partial x} \right\} \quad \frac{\partial w_j}{\partial x} \\
& + \sum_{j=N+K+1}^{N+K+L} \left\{ K^x_{rj} \frac{\partial^2 w^*(x_i,y_i;\xi_j,\eta_j)}{\partial \eta \, \partial x} \right\} \quad \frac{\partial w_j}{\partial y} \\
& + \sum_{j=N+K+L+1}^{N+K+L+S} \left\{ K^w_{mj} \int_{\psi j} \frac{\partial w^*(x_i,y_i;\xi,\eta)}{\partial x} \, d\psi(\xi,\eta) \right\} \quad w_{wj} \\
& + \sum_{j=N+K+L+1}^{N+K+L+S} \left\{ K^t_{rj} \int_{\psi j} \frac{\partial^2 w^*(x_i,y_i;\xi,\eta)}{\partial n_w \, \partial x} \, d\psi(\xi,\eta) \right\} \quad \frac{\partial w_{wj}}{\partial n_w} \\
& + \sum_{j=N+K+L+1}^{N+K+L+S} \left\{ K^n_{rj} \int_{\psi j} \frac{\partial^2 w^*(x_i,y_i;\xi,\eta)}{\partial t_w \, \partial x} \, d\psi(\xi,\eta) \right\} \quad \frac{\partial w_{wj}}{\partial t_w} \\
& = \int_{\Omega} q(\xi,\eta) \quad \frac{\partial w^*(x_i,y_i;\xi,\eta)}{\partial x} \, d\Omega(\xi,\eta)
\end{aligned}
$$

$$, \; i = N+K+1, N+K+2, \ldots, N+K+L \qquad (30)$$

$$
\begin{aligned}
&\frac{\partial w_i}{\partial y} - \sum_{j=1,2}^{N} \int_{\Gamma j} \frac{\partial M_n^*(x_i,y_i;\bar{\xi},\bar{\eta})}{\partial y}\, d\Gamma(\bar{\xi},\bar{\eta}) \; \frac{\partial w_j}{\partial n} \\
&+ \sum_{j=1,2}^{N} \int_{\Gamma j} \frac{\partial V_n^*(x_i,y_i;\bar{\xi},\bar{\eta})}{\partial y}\, d\Gamma(\bar{\xi},\bar{\eta}) \; w_j \\
&+ \sum_{j=N+1}^{N+K} \frac{\partial R^*(x_i,y_i;\bar{\xi}_j,\bar{\eta}_j)}{\partial y} \; w_j \\
&+ \sum_{j=N+K+1}^{N+K+L} \left\{ K^c_{aj} \frac{\partial w^*(x_i,y_i;\xi_j,\eta_j)}{\partial y} \right\} w_j \\
&+ \sum_{j=N+K+1}^{N+K+L} \left\{ K^y_{rj} \frac{\partial^2 w^*(x_i,y_i;\xi_j,\eta_j)}{\partial \xi \, \partial y} \right\} \frac{\partial w_j}{\partial x} \\
&+ \sum_{j=N+K+1}^{N+K+L} \left\{ K^x_{rj} \frac{\partial^2 w^*(x_i,y_i;\xi_j,\eta_j)}{\partial \eta \, \partial y} \right\} \frac{\partial w_j}{\partial y} \\
&+ \sum_{j=N+K+L+1}^{N+K+L+S} \left\{ K^\omega_{aj} \int_{\psi j} \frac{\partial w^*(x_i,y_i;\xi,\eta)}{\partial y}\, d\psi(\xi,\eta) \right\} w_{\omega j} \\
&+ \sum_{j=N+K+L+1}^{N+K+L+S} \left\{ K^t_{rj} \int_{\psi j} \frac{\partial^2 w^*(x_i,y_i;\xi,\eta)}{\partial n_\omega \, \partial y}\, d\psi(\xi,\eta) \right\} \frac{\partial w_{\omega j}}{\partial n_\omega} \\
&+ \sum_{j=N+K+L+1}^{N+K+L+S} \left\{ K^n_{rj} \int_{\psi j} \frac{\partial^2 w^*(x_i,y_i;\xi,\eta)}{\partial t_\omega \, \partial y}\, d\psi(\xi,\eta) \right\} \frac{\partial w_{\omega j}}{\partial t_\omega} \\
&= \int_{\Omega} q(\xi,\eta) \; \frac{\partial w^*(x_i,y_i;\xi,\eta)}{\partial y}\, d\Omega(\xi,\eta)
\end{aligned}
$$

$, \; i = N+K+1, N+K+2, \ldots, N+K+L$ (31)

$$
\begin{aligned}
w_i \quad & - \sum_{j=1,2}^{N} \int_{\Gamma j} M_n^*(x_i,y_i;\bar{\xi},\bar{\eta})\, d\Gamma(\bar{\xi},\bar{\eta}) \quad \frac{\partial w_j}{\partial n} \\
& + \sum_{j=1,2}^{N} \int_{\Gamma j} V_n^*(x_i,y_i;\bar{\xi},\bar{\eta})\, d\Gamma(\bar{\xi},\bar{\eta}) \quad w_j \\
& + \sum_{j=N+1}^{N+K} R^*(x_i,y_i;\bar{\xi}_j,\bar{\eta}_j) \quad w_j \\
& + \sum_{j=N+K+1}^{N+K+L} \left\{ K^c_{mj}\, w^*(x_i,y_i;\xi_j,\eta_j) \right\} \quad w_j \\
& + \sum_{j=N+K+1}^{N+K+L} \left\{ K^y_{rj}\, \frac{\partial w^*(x_i,y_i;\xi_j,\eta_j)}{\partial \xi} \right\} \quad \frac{\partial w_j}{\partial \xi} \\
& + \sum_{j=N+K+1}^{N+K+L} \left\{ K^x_{rj}\, \frac{\partial w^*(x_i,y_i;\xi_j,\eta_j)}{\partial \eta} \right\} \quad \frac{\partial w_j}{\partial \eta} \\
& + \sum_{j=N+K+L+1}^{N+K+L+S} \left\{ K^w_{mj} \int_{\psi j} w^*(x_i,y_i;\xi,\eta)\, d\psi(\xi,\eta) \right\} \quad w_{wj} \\
& + \sum_{j=N+K+L+1}^{N+K+L+S} \left\{ K^t_{rj} \int_{\psi j} \frac{\partial w^*(x_i,y_i;\xi,\eta)}{\partial n_w}\, d\psi(\xi,\eta) \right\} \quad \frac{\partial w_{wj}}{\partial n_w} \\
& + \sum_{j=N+K+L+1}^{N+K+L+S} \left\{ K^n_{rj} \int_{\psi j} \frac{\partial w^*(x_i,y_i;\xi,\eta)}{\partial t_w}\, d\psi(\xi,\eta) \right\} \quad \frac{\partial w_{wj}}{\partial t_w} \\
= \quad & \int_{\Omega} q(\xi,\eta) \quad w^*(x_i,y_i;\xi,\eta)\, d\Omega(\xi,\eta)
\end{aligned}
$$

$$, \; i = N+K+L+1, N+K+L+2, \ldots, N+K+L+S \qquad (32)$$

$$
\begin{aligned}
&\frac{\partial w_i}{\partial n_{wi}} - \sum_{j=1,2}^{N} \int_{\Gamma j} \frac{\partial M_n^*(x_i,y_i;\bar{\xi},\bar{\eta})}{\partial n_{wi}}\, d\Gamma(\bar{\xi},\bar{\eta})\ \frac{\partial w_j}{\partial n} \\
&+ \sum_{j=1,2}^{N} \int_{\Gamma j} \frac{\partial V_n^*(x_i,y_i;\bar{\xi},\bar{\eta})}{\partial n_{wi}}\, d\Gamma(\bar{\xi},\bar{\eta})\ w_j \\
&+ \sum_{j=N+1}^{N+K} \frac{\partial R^*(x_i,y_i;\bar{\xi}_j,\bar{\eta}_j)}{\partial n_{wi}}\ w_j \\
&+ \sum_{j=N+K+1}^{N+K+L} \left\{ K^c_{aj} \frac{\partial w^*(x_i,y_i;\xi_j,\eta_j)}{\partial n_{wi}} \right\} w_j \\
&+ \sum_{j=N+K+1}^{N+K+L} \left\{ K^y_{rj} \frac{\partial^2 w^*(x_i,y_i;\xi_j,\eta_j)}{\partial \xi\ \partial n_{wi}} \right\} \frac{\partial w_j}{\partial x} \\
&+ \sum_{j=N+K+1}^{N+K+L} \left\{ K^x_{rj} \frac{\partial^2 w^*(x_i,y_i;\xi_j,\eta_j)}{\partial \eta\ \partial n_{wi}} \right\} \frac{\partial w_j}{\partial y} \\
&+ \sum_{j=N+K+L+1}^{N+K+L+S} \left\{ K^w_{aj} \int_{\psi j} \frac{\partial w^*(x_i,y_i;\xi,\eta)}{\partial n_{wi}}\, d\psi(\xi,\eta) \right\} w_{wj} \\
&+ \sum_{j=N+K+L+1}^{N+K+L+S} \left\{ K^t_{rj} \int_{\psi j} \frac{\partial^2 w^*(x_i,y_i;\xi,\eta)}{\partial n_w\ \partial n_{wi}}\, d\psi(\xi,\eta) \right\} \frac{\partial w_{wj}}{\partial n_w} \\
&+ \sum_{j=N+K+L+1}^{N+K+L+S} \left\{ K^n_{rj} \int_{\psi j} \frac{\partial^2 w^*(x_i,y_i;\xi,\eta)}{\partial t_w\ \partial n_{wi}}\, d\psi(\xi,\eta) \right\} \frac{\partial w_{wj}}{\partial t_w} \\
&= \int_{\Omega} q(\xi,\eta)\ \frac{\partial w^*(x_i,y_i;\xi,\eta)}{\partial n_{wi}}\, d\Omega(\xi,\eta)
\end{aligned}
$$

$$, \quad i = N+K+L+1, N+K+L+2, \ldots, N+K+L+S \qquad (33)$$

$$
\begin{aligned}
&\frac{\partial w_i}{\partial t_{wi}} - \sum_{j=1,2}^{N} \int_{\Gamma j} \frac{\partial M_n^{*}(x_i,y_i;\bar{\xi},\bar{\eta})}{\partial t_{wi}} \, d\Gamma(\bar{\xi},\bar{\eta}) \; \frac{\partial w_j}{\partial n} \\
&+ \sum_{j=1,2}^{N} \int_{\Gamma j} \frac{\partial V_n^{*}(x_i,y_i;\bar{\xi},\bar{\eta})}{\partial t_{wi}} \, d\Gamma(\bar{\xi},\bar{\eta}) \; w_j \\
&+ \sum_{j=N+1}^{N+K} \frac{\partial R^{*}(x_i,y_i;\bar{\xi}_j,\bar{\eta}_j)}{\partial t_{wi}} \; w_j \\
&+ \sum_{j=N+K+1}^{N+K+L} \left\{ K^{c}_{mj} \frac{\partial w^{*}(x_i,y_i;\xi_j,\eta_j)}{\partial t_{wi}} \right\} w_j \\
&+ \sum_{j=N+K+1}^{N+K+L} \left\{ K^{y}_{rj} \frac{\partial^2 w^{*}(x_i,y_i;\xi_j,\eta_j)}{\partial \xi \, \partial t_{wi}} \right\} \frac{\partial w_j}{\partial x} \\
&+ \sum_{j=N+K+1}^{N+K+L} \left\{ K^{x}_{rj} \frac{\partial^2 w^{*}(x_i,y_i;\xi_j,\eta_j)}{\partial \eta \, \partial t_{wi}} \right\} \frac{\partial w_j}{\partial y} \\
&+ \sum_{j=N+K+L+1}^{N+K+L+S} \left\{ K^{w}_{mj} \int_{\psi j} \frac{\partial w^{*}(x_i,y_i;\xi,\eta)}{\partial t_{wi}} \, d\psi(\xi,\eta) \right\} w_{wj} \\
&+ \sum_{j=N+K+L+1}^{N+K+L+S} \left\{ K^{t}_{rj} \int_{\psi j} \frac{\partial^2 w^{*}(x_i,y_i;\xi,\eta)}{\partial n_w \, \partial t_{wi}} \, d\psi(\xi,\eta) \right\} \frac{\partial w_{wj}}{\partial n_w} \\
&+ \sum_{j=N+K+L+1}^{N+K+L+S} \left\{ K^{n}_{rj} \int_{\psi j} \frac{\partial^2 w^{*}(x_i,y_i;\xi,\eta)}{\partial t_w \, \partial t_{wi}} \, d\psi(\xi,\eta) \right\} \frac{\partial w_{wj}}{\partial t_w} \\
&= \int_{\Omega} q(\xi,\eta) \, \frac{\partial w^{*}(x_i,y_i;\xi,\eta)}{\partial t_{wi}} \, d\Omega(\xi,\eta)
\end{aligned}
$$

$$, \quad i = N+K+L+1, N+K+L+2, \ldots, N+K+L+S \qquad (34)$$

Equations (27) through (34) are boundary integral equations which formed by applied Betti's reciprocal theorem together with boundary integral technique. These equation are used to analyse plate problems by creating a system of simultaneous equations. All the functions of virtual plate, denoted by asterisks, in the left hand side of these equations will be calculated to form a coefficient matrix. For well-seen of coefficient matrix components, the asterisked functions are concluded in Appendix C according to its source and field points.

3.2 Evaluation of Domain Integrals

The domain integrals which appear in the right hand side of equations (27) through (34) are replaced by equivalent summations for the cases of singular load, P, and uniformly distributed load, $q(\xi,\eta)$. Appendix D shows the evaluation of domain integrals presented by Yuthana (8).

3.3 Treatment of Singularities

In equation (27) and (28), the integration of function Mn, Vn, $\frac{\partial M_n}{\partial n}$ and $\frac{\partial V_n}{\partial n}$ on the plate boundary when $(\bar{x}_i,\bar{y}_i)$ and $(\bar{\xi}_j,\bar{\eta}_j)$ are coincident introduces some problems since these functions have term $\ln(r)$, $\frac{1}{r}$, $\frac{1}{r^2}$ which are singular at $r = 0$. Appendix E shows the treatment of singularity problems introduced by Tottenham (6).

3.4 Domain Solution

The deflection function at any point, (x,y), as written in equation (18) can be computed numerically in the form :

$$
\begin{aligned}
w(x,y) \quad = \quad & \sum_{j=1,2}^{N} \int_{\Gamma j} M_n^{*}(x,y;\bar{\xi},\bar{\eta}) \; d\Gamma(\bar{\xi},\bar{\eta}) \; \frac{\partial w_j}{\partial n} \\
- \; & \sum_{j=1,2}^{N} \int_{\Gamma j} V_n^{*}(x,y;\bar{\xi},\bar{\eta}) \; d\Gamma(\bar{\xi},\bar{\eta}) \; w_j \\
- \; & \sum_{j=N+1}^{N+K} R^{*}(x,y;\bar{\xi}_j,\bar{\eta}_j) \; w_j \\
- \; & \sum_{j=N+K+1}^{N+K+L} \left\{ K^{c}_{sj} \, w^{*}(x,y;\xi_j,\eta_j) \right\} w_j \\
- \; & \sum_{j=N+K+1}^{N+K+L} \left\{ K^{y}_{rj} \frac{\partial w^{*}(x,y;\xi_j,\eta_j)}{\partial \xi} \right\} \frac{\partial w_j}{\partial \xi} \\
- \; & \sum_{j=N+K+1}^{N+K+L} \left\{ K^{x}_{rj} \frac{\partial w^{*}(x,y;\xi_j,\eta_j)}{\partial \eta} \right\} \frac{\partial w_j}{\partial \eta} \\
- \; & \sum_{j=N+K+L+1}^{N+K+L+S} \left\{ K^{w}_{sj} \int_{\psi j} w^{*}(x,y;\xi,\eta) \; d\psi(\xi,\eta) \right\} w_{\omega j} \\
- \; & \sum_{j=N+K+L+1}^{N+K+L+S} \left\{ K^{t}_{rj} \int_{\psi j} \frac{\partial w^{*}(x,y;\xi,\eta)}{\partial n_{\omega}} \; d\psi(\xi,\eta) \right\} \frac{\partial w_{\omega j}}{\partial n_{\omega}} \\
- \; & \sum_{j=N+K+L+1}^{N+K+L+S} \left\{ K^{n}_{rj} \int_{\psi j} \frac{\partial w^{*}(x,y;\xi,\eta)}{\partial t_{\omega}} \; d\psi(\xi,\eta) \right\} \frac{\partial w_{\omega j}}{\partial t_{\omega}} \\
+ \; & \int_{\Omega} q(\xi,\eta) \; w^{*}(x,y;\xi,\eta) \; d\Omega(\xi,\eta) \quad . \qquad (35)
\end{aligned}
$$

Finally, the desired stress resultants inside the domain can be obtained by appropriate differentiating of the influence functions in equation (35) as the case may be.