ตัวแบบสโทแคสติกสำหรับดัชนีราคาเซต 50 และการหาราคาอนุพันธ์

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STOCHASTIC MODEL FOR SET50 INDEX AND DERIVATIVES PRICING

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ในวิทยานิพนธ์นี้ เราพัฒนาตัวแบบสโทแคสติกสำหรับดัชนีราคาหลักทรัพย์จากการเคลื่อนที่แบบบราวน์เรขาคณิตไปเป็นตัวแบบที่รวมผลของการปรับรายการหลักทรัพย์ นอกจากนี้เราได้หาสมการราคายุติธรรมของดัชนีราคาเซต 50 ล่วงหน้าและออปชันที่มีดัชนีราคาเซต 50 เป็นสินค้าอ้างอิง ภายใต้สมมติฐานที่ว่า ไม่มีโอกาสค้ากำไรโดยไม่มีความเสี่ยงเกิดขึ้นในตลาด

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We develop a model from the classic asset price model, geometric Brownian motion, for stock indices, and this model includes index reconstitution effect which is a normal characteristic for some stock indices. For example, SET50 Index revises its list every six months. Also, we derive closed-form solutions for no-arbitrage prices of SET50 Index futures and options.

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CONTENTS

CHAPTER I
INTRODUCTION

To accommodate the issuing of index futures and options in the future, and to provide a benchmark of investment in the Stock Exchange of Thailand, the SET50 Index was launched. This index is calculated, respectively, from the stock prices of the top 50 companies on SET in terms of large market capitalization, high liquidity and compliance with requirements regarding the distribution of shares to minor shareholders

The component stocks in the SET50 Index are reviewed every six months in order to adjust for any changes that have occurred in the stock market, such as new listings or public offerings. After review, stocks that meet the necessary qualifications are selected to become part of the SET50 Index and others are removed. This procedure is called stock revision.

A stochastic process $S(t)$ is called a Geometric Brownian motion if and only if $S(t)$ satisfies a stochastic differential equation $dS(t) = \mu S(t)dt + \sigma S(t)dW(t)$ where $W(t)$ is a Wiener process and μ, σ are constant. Modeling the price of SET50 Index by geometric Brownian motion would not include the effect of stock revision. We will develop the more realistic model including the effect of stock revision by changing μ, σ from constant functions to piecewise-constant functions according to stock revision period. Then we will find arbitrage prices of futures, European call and put options on this index.

In chapter 2, we give some background to the readers who are still not familiar with probability theory and stochastic calculus which are topics neccessary to understand the rest of the thesis. For those who knew the topics, this chapter is very useful for a review.

In chapter 3, we give explanation about stock index in overview, and SET50 Index is also described. In addition, the stochastic model of normal asset price are explained from its beginning along with the physical meaning of the model. Later, the geometric Brownian motion is explained. Finally, it is culminating at the end by our developed model with its simulation for some parameters.

In chapter 4, the futures contract is introduced; how it benefits, how it is used, who uses it. Necessary mathematical theorems are also explained. Ultimately, we use these theorems to find closed-form solution of arbitrage price of futures. Moreover, simulation of the futures price is discussed along with its consistence to our understanding about futures contract.

In chapter 5, we give some basic concepts for European option contracts. We then later explain mathematical theorems that are necessary for finding options prices. After finding SET50 Index options prices, we illustrate the simulation of SET50 Index options prices and discuss how it is consistent with real world situation.

In chapter 6, we conclude the result of the thesis.

CHAPTER II

Preliminaries

2.1 Probability Theory

2.1.1 Probability Space

Definition 2.1. A collection $\mathcal{F}$ of subsets of a non-empty Ω is called a $\sigma-algebra$ or $\sigma-field$ if

1. $\Omega \in \mathcal{F}$
2. if $A \in \mathcal{F}$, then $A^c \in \mathcal{F}$
3. if $A_n \in \mathcal{F}$ for $n = 1, 2, ...$, then $\bigcup_{n=1}^{\infty} A_n \in \mathcal{F}$

The sets in $\mathcal{F}$ are called *measurable sets* and $(\Omega, \mathcal{F})$ is called a *measurable space*.

Definition 2.2. Let Ω be a non empty set and $\mathcal{F}$ be a $\sigma-field$. Let $P : \mathcal{F} \to [0, 1]$ be a measure such that $P(\Omega) = 1$. Then $(\Omega, \mathcal{F}, P)$ is called a *probability space* and P, a *probability measure*. The set Ω is the *sure event*, and the element of $\mathcal{F}$ are called *events*. Elements of Ω are denoted ω.

2.1.2 Random Variables

Definition 2.3. If $\mathcal{F}$ is a $\sigma-field$ on Ω, then a function $\xi : \Omega \to \mathbb{R}$ is said to be $\mathcal{F}-measurable$ if

$$\{\xi \in B\} \in \mathcal{F}$$

for every Borel set $B \in \mathcal{B}(\mathbb{R})$. If $(\Omega, \mathcal{F}, P)$ is a probability space, then such a function ξ is called a *random variable*.

A notation for events such as $\{\xi \in B\}$ will be used to substitute

$$\{\omega \in \Omega : \xi(\omega) \in B\}$$

Incidentally, $\{\xi \in B\}$ is the inverse image $\xi^{-1}(B)$ of a set.

2.1.3 Distribution

Theorem 2.1. *A random variable ξ defined on $(\Omega, \mathcal{F}, P)$ induces a probability measure P_ξ on $\mathcal{B}$ defined by*

$$P_\xi(B) = P\{\xi \in B\}.$$

Definition 2.4. The measure P_ξ in Theorem 2.1 is called the *probability distribution* or the *distribution* or the *law* of ξ.

Definition 2.5. Let $F_\xi : \mathbb{R} \to [0,1]$ be defined by

$$F_\xi(x) = P_\xi((-\infty, x]) = P\{\xi \leq x\}$$

The function F_ξ is called the *distribution function* of ξ.

Definition 2.6. If there is a Borel function[1] $f_\xi : \mathbb{R} \to \mathbb{R}$ such that for any Borel set $B \subset \mathbb{R}$

$$P\{\xi \in B\} = \int_B f_\xi(x)\,dx,$$

then ξ is said to be a random variable with *absolute continuous distribution* and f_ξ is called the *density* of ξ.

Definition 2.7. If there is a sequence of distinct real numbers $x_1, x_2, \ldots$ such that for any Borel set $B \subset \mathbb{R}$

$$P\{\xi \in B\} = \sum_{x_i \in B} P\{\xi = x_i\}$$

then ξ is said to have *discrete distribution* with values $x_1, x_2, \ldots$ and *mass* $P\{\xi = x_i\}$ at x_i.

[1] $f : \mathbb{R} \to \mathbb{R}$ is called a *Borel function* if $f^{-1}(B) \in \mathcal{B}(\mathbb{R})$ for any Borel set B

2.1.4 Expectation

Definition 2.8. A random variable $\xi : \Omega \to \mathbb{R}$ is in $L^1(\Omega, \mathcal{F}, P)$ if

$$\int_\Omega |\xi| \; dP < \infty$$

Then

$$E(\xi) = \int_\Omega \xi \; dP$$

exists and is called the *expectation* of ξ.

Definition 2.9. A random variable $\xi : \Omega \to \mathbb{R}$ is in $L^2(\Omega, \mathcal{F}, P)$ if

$$\int_\Omega |\xi|^2 \; dP < \infty$$

Then the *variance* of ξ can be defined by

$$Var(\xi) = \int_\Omega (\xi - E(\xi))^2 \; dP$$

2.2 Stochastic Processes

Given a probability space $(\Omega, \mathcal{F}, P)$, a *stochastic process* with state space S is a collection of S-valued random variables indexed by a set I, i.e. a stochastic process X is a collection $\{X(t) : t \in I\}$ where each $X(t)$ is a S-valued random variable. For a fixed $\omega \in \Omega$, a function $X(\omega) : I \to S$, $X(t)(\omega)$, is called a realization, a trajectory, or a *sample path* of the process X. Usually, the state space S is $\mathbb{R}$ which comes with the Borel σ-algebra $\mathcal{B}(\mathbb{R})$, and the index set I is an interval $[0, T]$ or $[0, \infty)$ on $\mathbb{R}$. Two stochastic processes $U = \{U(t) : t \in I\}$ and $V = \{V(t) : t \in I\}$ on the same probability space are *independent* if $U(s)$ and $V(t)$ are independent for all $s, t \in I$.

For a random variable Y, the σ-algebra generated by Y, denoted by $\sigma(Y)$, is the smallest σ-algebra which makes Y measurable. For a stochastic process $X = \{X(t) : t \in I\}$, the σ-algebra generated by X, denoted by $\sigma(X)$, is the smallest σ-algebra which makes $X(t)$ measurable for all $t \in I$. A collection $\{\mathcal{F}_t : t \in I\}$ of σ-algebras on Ω is called a *filtration* if $\mathcal{F}_s \subseteq \mathcal{F}_t$ for all $s \leq t$. A stochastic process $X = \{X(t) : t \in I\}$ is said to be *adapted to the filtration* $\{\mathcal{F}_t : t \in I\}$ if $\sigma(X(t)) \subseteq \mathcal{F}_t$ for all $t \in I$ and we will call X an *adapted process* $\{X(t), \mathcal{F}_t : t \in I\}$. Every stochastic process $X = \{X(t) : t \in I\}$ is always adapted to the natural filtration generated by X: $\{\mathcal{F}_t = \sigma(\{X(s) : s \leq t\}) : t \in I\}$. If a stochastic process U is adapted to the natural filtration generated by a stochastic process V, we say that U is adapted to the stochastic process V.

2.3 Brownian Motion

2.3.1 Definition and Basic Properties

Definition 2.10. A *Wiener Process* (or *Brownian Motion*) is a stochastic process $W(t)$ with values in $\mathbb{R}$ defined for $t \in [0, \infty)$ such that

1. $W(0) = 0$ a.s.;
2. the sample paths $t \mapsto W(t)$ are a.s. continuous;
3. for any finite sequence of times $0 < t_1 < \cdots < t_n$ and Borel sets $A_1, \ldots, A_n \subset \mathbb{R}$

   $$P\{W(t_1) \in A_1, \ldots, W(t_n) \in A_n\} =$$
   $$\int_{A_1} \cdots \int_{A_n} p(t_1, 0, x_1)\, p(t_2 - t_1, x_1, x_2) \cdots p(t_n - t_{n-1}, x_{n-1}, x_n)\, dx_1 \cdots dx_n$$

   where
   $$p(t, x, y) = \frac{1}{\sqrt{2\pi t}} e^{-\frac{(x-y)^2}{2t}}$$
   defined for any $x, y \in \mathbb{R}$ and $t > 0$ is called *a transition density function.*

From the definition 2.10 we can show that

$$f_{W(t)}(x) = \frac{1}{\sqrt{2\pi t}} e^{-\frac{x^2}{2t}}$$

is the probability density of $W(t)$ which is the probability density of normal random variable with mean 0 and variance t.

2.3.2 Increments of Brownian Motion

Proposition 2.2. *For any $0 \leq s < t$ the increment $W(t) - W(s)$ has the normal distribution with mean 0 and variance $t - s$.*

Corollary 2.3. *Proposition 2.2 implies that $W(t)$ has stationary increments.*

Proposition 2.4. *For any $0 = t_0 \leq t_1 \leq \cdots \leq t_n$ the increments*

$$W(t_1) - W(t_0), \ldots, W(t_n) - W(t_{n-1})$$

are independent.

Theorem 2.5. *A stochastic process $W(t), t \geq 0$ is a Wiener process if and only if the following condition hold:*

1. *$W(0) = 0$ a.s.;*
2. *the sample path $t \mapsto W(t)$ are continuous a.s.;*
3. *$W(t)$ has stationary independent increments;*
4. *the increment $W(t) - W(s)$ has the normal distribution with mean 0 and variance $t - s$ for any $0 \leq s < t$.*

Theorem 2.6. *Let $W(t), t \geq 0$, be a stochastic process and let $\mathcal{F} = \sigma(W_s, s \leq t)$ be the filtration generated by it. Then $W(t)$ is a Wiener process if and only if the following conditions hold:*

1. *$W(0) = 0$ a.s.;*
2. *the sample path $t \mapsto W(t)$ are continuous a.s.;*
3. *$W(t)$ is a martingale with respect to the filtration $\mathcal{F}_t$;*
4. *$|W(t)|^2 - t$ is a martingale with respect to $\mathcal{F}_t$.*

Theorem 2.7. *With probability 1 the Wiener process $W(t)$ is non-differentiable at any $t \geq 0$.*

2.4 Itô Stochastic Calculus

2.4.1 Itô Stochastic Integral: Definition

Definition 2.11. We shall call $f(t), t \geq 0$ a *random step process* if there is a finite sequence of numbers $0 = t_0 < t_1 < \cdots < t_n$ and square integrable random variables $\eta_0, \eta_1, \ldots, \eta_{n-1}$ such that

$$f(t) = \sum_{j=0}^{n-1} \eta_j 1_{[t_j, t_{j+1})} \tag{2.1}$$

where η_j is $\mathcal{F}_{t_j}$-measurable for $j = 0, \ldots, n-1$. The set of random step processes will be denoted by M^2_{step}.

Definition 2.12. The *stochastic integral* of a random step process $f \in M^2_{step}$ of the form (2.1) is defined by

$$I(f) = \sum_{j=0}^{n-1} \eta_j (W(t_{j+1}) - W(t_j))$$

Proposition 2.8. *For any random step process $f \in M^2_{step}$ the stochastic integral $I(f)$ is a square integrable random variable, i.e. $I(f) \in L^2$, such that*

$$E\left(|I(f)|^2\right) = E\left(\int_0^\infty |f(t)|^2 \, dt\right)$$

The stochastic integral $I(f)$ has been defined for any random step process $f \in M^2_{step}$. We will extend I to a larger class of processes which can be defined as the following.

Definition 2.13. We denote by M^2 the class of stochastic processes $f(t), t \geq 0$ such that

$$E\left(\int_0^\infty |f(t)|^2 \, dt\right) < \infty$$

and there is a sequence $f_1, f_2, \ldots \in M^2_{step}$ of random step processes such that

$$\lim_{n \to \infty} E\left(\int_0^\infty |f(t) - f_n(t)|^2 \, dt\right) = 0 \tag{2.2}$$

Also we will say that the sequence of random step processes $f_1, f_2, \ldots$ *approximates f in M^2.*

Definition 2.14. We call $I(f) \in L^2$ the *Itô stochastic integral* (from 0 to ∞) of $f \in M^2$ if

$$\lim_{n\to\infty} E\left(|I(f) - I(f_n)|^2\right) = 0,$$

for any sequence $f_1, f_2, \ldots \in M^2_{step}$ of random step processes that approximates f in M^2, i.e. such that (2.2) is satisfied. We will also write

$$\int_0^\infty f(t)dW(t)$$

in place of $I(f)$.

Proposition 2.9. *For any $f \in M^2$ the stochastic integral $I(f) \in L^2$ exists, is unique (as an element of L^2, i.e. to within equality a.s.) and satisfies*

$$E\left(|I(f)|^2\right) = E\left(\int_0^\infty |f(t)|^2 \, dt\right).$$

We defined the Itô Stochastic Integral from 0 to ∞. We will define the Itô Stochastic Integral over finite time interval $[0, T]$.

Definition 2.15. For any $T > 0$ we will denote by M_T^2 the space of all stochastic processes $f(t), t \geq 0$ such that

$$1_{[0,T)} f \in M^2$$

The Itô Stochastic Integral (from 0 to T) of $f \in M_T^2$ is defined by

$$I_T(f) = I\left(1_{[0,T)} f\right)$$

We will also write

$$\int_0^T f(t)dW(t)$$

in place of $I_T(f)$.

2.4.2 Stochastic Differential and Itô Formula

Definition 2.16. A stochastic process $\xi(t),\ t \geq 0$ is called an *Itô process* if it has a.s. continuous paths and can be represented as

$$\xi(T) = \xi(0) + \int_0^T a(t)dt + \int_0^T b(t)dW(t) \tag{2.3}$$

where $b(t)$ is a process belonging to M_T^2 for all $T > 0$ and $a(t)$ is a process adapted to the filtration $\mathcal{F}_t$ such that

$$\int_0^T |a(t)|\, dt < \infty \ a.s.$$

for all $T \geq 0$. For an Itô Process ξ it is customary to write (2.3) as

$$d\xi(t) = a(t)dt + b(t)dW(t)$$

and to call $d\xi(t)$ and the *stochastic differential* of $\xi(t)$.

Theorem 2.10 (Itô formula). *Let $\xi(t)$ be an Itô process.Suppose that F(t,x) is a real-valued function with continuous partial derivatives $F_t(t,x), F_x(t,x)$, and $F_{xx}(t,x)$ for all $t \geq 0$ and $x \in \mathbb{R}$. We also assume that the process $b(t)F_x(t,x)$ belongs to M_2^T for all $T \geq 0$. Then $F(t,\xi(t))$ is an Itô process such that*

$$dF(t,\xi(t)) = \left(F_t(t,\xi(t)) + F_x(t,\xi(t))a(t) + \frac{1}{2}F_{xx}(t,\xi(t))b(t)^2) \right) dt + F_x(t,\xi(t))b(t)dW(t)$$

Theorem 2.11 (Existence and uniqueness theorem for stochastic differential equation). *Let $T > 0$ and $b(\cdot,\cdot) : [0,T] \times \mathbb{R}^n \to \mathbb{R}^n, c(\cdot,\cdot) : [0,T] \times \mathbb{R}^n \to \mathbb{R}^{n\times m}$ be measurable function satisfying*

$$|b(t,x)| + |c(t,x)| \leq C(1 + |x|); x \in \mathbb{R}^n, t \in [0,T] \tag{2.4}$$

for some constant C, (where $|c|^2 = \Sigma\, |c_{ij}|^2$) and such that

$$|b(t,x) - b(t,y)| + |c(t,x) - c(t,y)| \leq D\,|x - y|\,; x, y \in \mathbb{R}, t \in [0,T] \tag{2.5}$$

for some constant D. Let Z be a random variable which is independent of the $\sigma - algebra$ $\mathcal{F}_\infty^{(m)}$ generated by $W(s)(\cdot), s \geq 0$ and such that

$$E[|Z|^2] < \infty.$$

Then the stochastic differential equation

$$dX(t) = b(t,X(t))dt + c(t,X(t))dW(t), t \in [0,T], X(0) = Z \tag{2.6}$$

has a unique t-continuous solution $X(t)(\omega)$ with the property that $X(t)(\omega)$ is adapted to the filtration $\mathcal{F}_t^Z$ generated by Z and $W(s)(\cdot); s \leq t$ and

$$E\left[\int_0^T |X(t)|^2\, dt\right] < \infty.$$

CHAPTER III
Stochastic Model

The topic in this chapter will begin with overview of the stock index. Then, the characteristics of SET50 Index which is a stock index will be introduced, and later because we will develop our SET50 Index model from a geometric Brownian motion, a geometric Brownian motion will be described. Ultimately, we will developed SET50 Index model from geometric Brownian motion.

3.1 Stock Index

A stock index is a method of measuring a section of the stock market. Many indices are cited by news or financial services firms and are used as benchmarks, to measure the performance of portfolios such as mutual funds[1].

3.1.1 Types of indices

Stock market indices may be classed in many ways. A *world* or *global* stock index includes (typically large) companies without regard for where they are domiciled or traded. Two examples are MSCI World and SP Global 100. A *national* index represents the performance of the stock market of a given nation-and by proxy, reflects investor sentiment on the state of its economy. The most regularly quoted market indices are national indices composed of the stocks of large companies listed on a nation's largest stock exchanges, such as the American SP 500, the Japanese Nikkei 225, and the British FTSE 100. The concept may be extended well beyond an exchange. The Wilshire 5000 Index, the original total market

[1]A mutual fund is a professionally-managed type of collective investment scheme that pools money from many investors to buy stocks, bonds, short-term money market instruments, and/or other securities.

index, represents the stocks of nearly every publicly traded company in the United States, including all U.S. stocks traded on the New York Stock Exchange (but not ADRs or limited partnerships), NASDAQ and American Stock Exchange. Russell Investment Group added to the family of indices by launching the Russel Global Index. More specialised indices exist tracking the performance of specific sectors of the market. Some examples include the Wilshire US REIT which tracks more than 80 American real estate investment trusts and the Morgan Stanley Biotech Index which consists of 36 American firms in the biotechnology industry. Other indices may track companies of a certain size, a certain type of management, or even more specialized criteria one index published by Linux Weekly News tracks stocks of companies that sell products and services based on the Linux operating environment.

3.2 SET50 Index

In 1995, The Stock Exchange of Thailand (SET) launched SET50 Index, the first large-cap index of Thailand to provide a benchmark of investment in The Stock Exchange of Thailand. It is calculated from the stockprices of the top 50 listed companies on SET in terms of large market capitalization, high liquidity and compliance with requirements regarding the distribution of shares to minor shareholders (Free Float). The index was initially designed to be an underlying index for the derivative instruments.

3.2.1 Calculation Methodology

SET50 Index is market capitalization-weighted price index which compares the current market value of all listed common stocks with their market values on the base date which is 16th August 1995. The SET50 Index had set at 1000 points on

Index Portfolio Characteristics	
Number of constituents	50
Market Cap (Mil.Baht)	6,552,125
Company Size by Market Cap (Mil.Baht)	
Average	131,042
Largest	939,119
Smallest	19,119
Median	58,189

Table 3.1: Index Portfolio Characteristics on 4th Jan 2011

the base date.

$$SET50\ Index = \frac{Current\ market\ value\ (CMV)}{Base\ market\ value\ (BMV)} \times 1000 = \frac{\sum_{i=0}^{50} P_{it} Q_{it}}{\sum_{i=0}^{50} P_{i0} Q_{i0}} \times 1000$$

where

P_{it} = the price of the i^{th} listed stock at time t.

Q_{it} = the amount of the i^{th} listed stock at time t.

P_{i0} = the price of the i^{th} listed stock at base date.

Q_{i0} = the amount of the i^{th} listed stock at base date.

In addition, the index will be adjusted when the market value of the component stocks changes, e.g., due to conversion of convertible bonds, exercising of warrants, or issuing of new shares for capital increase of the constituent stocks.

Information	
Index Universe	Common stocks on SET's main board
Index Launch	16th August 1995
Base Date	16th August 1995
Base Value	1000 points

Table 3.2: SET50 Index's Fact

3.2.2 Base Adjustment Methodology

In the event of any increase or decrease in the current market value due to reasons other than fluctuations in the stock market such as public offering, changes in the number of component stocks, or stock revision, The SET50 Index will make necessary adjustments to the Base Market Value in order to eliminate all effects other than price movements from the index. The principle to do this is that

$$Index\ after\ Adjustment = Index\ before\ Adjustment.$$
$$\frac{CMV_N}{BMV_N} = \frac{CMV_0}{BMV_0}$$
$$BMV_N = \frac{CMV_N}{CMV_0} \times BMV_0$$
$$BMV_N = \frac{CMV_N}{CMV_N - Adjusted\ Value} \times BMV_0$$

where

BMV_0 = Base Market Value before adjustment

CMV_0 = Current Market Value before adjustment

BMV_N = Base Market value after adjustment

CMV_N = Current Market Value after adjustment.

We will explain how this formular is used by the following example.

Day 1 Suppose that this is base date.

Assumption: There are initially 3 common stocks listed in SET50 Index. However there should be 50 stocks but for the simplicity of explanation we will neglect the number of listed stocks in SET50 Index.

Stock A 100,000 shares, par = THB100, market price = THB110

Stock B 300,000 shares, par = THB100, market price = THB160

Stock C 200,000 shares, par = THB100, market price = THB120

$$\begin{aligned}
SET50\ Index &= \frac{Current\ market\ value\ (CMV)}{Base\ market\ value\ (BMV)} \times 1000 \\
&= \frac{\sum_{i=0}^{3} P_{it}Q_{it}}{\sum_{i=0}^{3} P_{i0}Q_{i0}} \times 1000 \\
&= \frac{(110 \times 100,000) + (160 \times 300,000) + (120 \times 200,000)}{(110 \times 100,000) + (160 \times 300,000) + (120 \times 200,000)} \times 1,000 \\
&= \frac{83,000,000}{83,000,000} \times 1,000 \\
&= 1,000
\end{aligned}$$

Day 2 Market price change

The market prices of stocks A, B and C change to THB120, THB170, and THB110, respectively. The second days Index then becomes

$$\begin{aligned}
SET50\ Index &= \frac{Current\ market\ value\ (CMV)}{Base\ market\ value\ (BMV)} \times 1,000 \\
&= \frac{(120 \times 100,000) + (170 \times 300,000) + (110 \times 200,000)}{83,000,00} \times 1,000 \\
&= \frac{85,000,000}{83,000,000} \times 1,000 \\
&= 1,024
\end{aligned}$$

Day 3 Stock Revision (stock D will be listed and C will be delisted on Day 4) The market prices of stocks A, B and C change to THB 110, THB 170 and THB 120, respectively. In addition, stock D is a newly listed stock this day, with 150,000 shares and closing price of THB140. When new stock is listed on the SET50 Index. The SET will adjust the Base Market Value according to that changes on the end-of-day process by employing the closing price of that stock on that trading day.

The index then becomes

$$
\begin{aligned}
SET50\ Index &= \frac{Current\ market\ value\ (CMV)}{Base\ market\ value\ (BMV)} \times 1,000 \\
&= \frac{(110 \times 100,000) + (170 \times 300,000) + (120 \times 200,000)}{83,000,00} \times 1,000 \\
&= \frac{86,000,000}{83,000,000} \times 1,000 \\
&= 1,036.1
\end{aligned}
$$

Then, the base Market Value for index calculation on Day 4 will be

$$
\begin{aligned}
BMV_N &= BMV_0 \times \frac{CMV_N}{CMV_0} \\
&= 83,000,000 \times \frac{(110 \times 100,000) + (170 \times 300,000) + (140 \times 150,000)}{(110 \times 100,000) + (170 \times 300,000) + (120 \times 200,000)} \\
&= 80,104,650
\end{aligned}
$$

Day 3 Market price change

The market prices of stocks A, B and D change to THB120, THB170, and THB130, respectively. The second days Index then becomes

$$
\begin{aligned}
SET50\ Index &= \frac{Current\ market\ value\ (CMV)}{Base\ market\ value\ (BMV)} \times 1,000 \\
&= \frac{(120 \times 100,000) + (170 \times 300,000) + (130 \times 150,000)}{80,104,650} \times 1,000 \\
&= \frac{82,500,000}{80,104,650} \times 1,000 \\
&= 1,030
\end{aligned}
$$

However, by supposing that the market prices of stocks A, B, and D remain the same as on the Day 3, we can see that base adjustment eliminates all effects other than price movements from the index. In other words, even if we change the stocks in the list of SET50 Index but if their price do not move, with base adjustment index value remains the same.

$$
\begin{aligned}
SET50\ Index &= \frac{Current\ market\ value\ (CMV)}{Base\ market\ value\ (BMV)} \times 1,000 \\
&= \frac{(110 \times 100,000) + (170 \times 300,000) + (140 \times 150,000)}{80,104,650} \times 1,000 \\
&= \frac{83,000,000}{80,104,650} \times 1,000 \\
&= 1,036.1
\end{aligned}
$$

Day 4's SET50 Index is equal to the value on Day 3.

3.3 Main Result: Stochastic Model

Any variable whose value changes over time in an uncertain way is said to follow a stochastic process. Stochastic processes can be classified as discrete time or continuous time. A discrete-time stochastic process is one where the value of the variable can change only at certain fixed points in time, whereas a continuous-time stochastic process is one where changes can take place at any time. Stochastic processes can also be classified as continuous variable or discrete variable. In a continuous-variable process, the underlying variable can take any value within a certain range, whereas in a discrete-variable process, only certain discrete values are possible.

3.3.1 The Markov Property

A *Markov process* is a particular type of stochastic process where only the present value of a variable is relevant for predicting the future. The past history of the variable and the way that the present has emerged from the past are irrelevant. Stock prices are usually assumed to follow a Markov process. Suppose that the price of IBM stock is $100 now. If the stock price follows a Markov process, our predictions for the future should be unaffected by the price one week ago, one month ago, or one year ago. The only relevant piece of information is that the price is now \$100. Predictions for the future are uncertain and must be expressed in terms of probability distributions. The Markov property implies that the probability distribution of the price at any particular future time is not dependent on the particular path followed by the price in the past.

The Markov property of stock prices is consistent with the weak form of market efficiency. This states that the present price of a stock impounds all the information contained in a record of past prices. If the weak form of market efficiency were not true, technical analysts could make above-average returns by interpreting charts of the past history of stock prices. There is very little evidence that they are in fact able to do this.

It is competition in the marketplace that tends to ensure that weak-form market efficiency holds! There are many investors watching the stock market closely. Trying to make a profit from it leads to a situation where a stock price, at any given time, reflects the information in past prices. Suppose that it was discovered that a particular pattern in stock prices always gave a 65% chance of subsequent steep price rises. Investors would attempt to buy a stock as soon as the pattern was observed, and demand for the stock would immediately rise. This would lead to an immediate rise in its price and the observed effect would be eliminated, as would any profitable trading opportunities.

3.3.2 Continuous-Time Stochastic Processes

Consider a variable that follows a Markov stochastic process. Suppose that its current value is 10 and that the change in its value during 1 year is $N(0,1)$, where $N(\mu,\sigma^2)$ denotes a probability distribution that is normally distributed with mean μ and variance σ^2. What is the probability distribution of the change in the value of the variable during 2 years?

The change in 2 years is the sum of two normal distributions, each of which has a mean of zero and variance of 1. Because the variable is Markov, the two probability distributions are independent. When we add two independent normal distributions, the result is a normal distribution where the mean is the sum of the means and the variance is the sum of the variances. The mean of the change during 2 years in the variable we are considering is, therefore, zero and the variance of this change is 2. Hence, the change in the variable over 2 years has the distribution $N(0, 2)$. The standard deviation of the distribution is $\sqrt{2}$.

Consider next the change in the variable during 6 months. The variance of the change in the value of the variable during 1 year equals the variance of the change during the first 6 months plus the variance of the change during the second 6 months. We assume these are the same. It follows that the variance of the change during a 6-month period must be 0.5. Equivalency, the standard deviation of the change is $\sqrt{0.5}$. The probability distribution for the change in the value of the variable during 6 months is $N(0, 0.5)$.

A similar argument shows that the probability distribution for the change in the value of the variable during 3 months is $N(0, 0.25)$. More generally, the change during any time period of length T is $N(0, T)$.

3.3.3 Brownian Motion

The process followed by the variable we have been considering is known as a *Wiener process* or *Brownian motion*. It is a particular type of Markov stochastic process with a mean change of zero and a variance rate of 1.0 per year. It has been used in physics to describe the motion of a particle that is subject to a large number of small molecular shocks and is sometimes referred to as Brownian motion.

With formal definition 2.10 on page 6, a Wiener process $W(t)$ can be proved to satisfies Markov property[2], and according to proposition 2.2, we can see that Wiener process $W(T) - W(0)$ is normally distributed, with mean of $W(T) - W(0) = 0$ and variance of $W(T) - W(0) = T$. Hence standard deviation of $W(T) - W(0) = \sqrt{T}$.These are consistent with the earlier discussion in this section. In fact, the process followed by the variable we have been considering is a Wiener process.

[2]Proof can be found in http://financelab.nctu.edu.tw/FinMath/%5BSC%5D3.5.ppt

3.3.4 Generalized Wiener Process

The mean change per unit time for a stochastic process is known as the *drift rate* and the variance per unit time is known as the *variance rate*. The basic Wiener process, $W(t)$, that has been developed so far has a drift rate of zero and a variance rate of 1.0 with year as a unit of time. The drift rate of zero means that the expected value of $W(t)$ at any future time t is equal to 0. The variance rate of 1.0 means that the variance of the change in z in a time interval of length T equals T. A generalized Wiener process for a variable $X(t)$ can be defined in terms of $dW(t)$ as

$$dX(t) = a\ dt + b\ dW(t) \tag{3.1}$$

To understand equation (3.1), it is useful to consider the two components on the right-hand side separately. The $a\ dt$ term implies that $X(t)$ has an expected drift rate of a per unit of time. Without the $b\ dW(t)$ term, the equation is $dX(t) = a\ dt$, which implies that $\frac{dX(t)}{dt} = a$. Integrating with respect to time, we get

$$X(t) = X(0) + at$$

Then in a period of time of length T, the variable X increases by an amount aT.

The $b\ dW(t)$ term on the right-hand side of equation (3.1) can be regarded as adding noise or variability to the path followed by X. The amount of this noise or variability is b times a Wiener process. A Wiener process has a standard deviation of 1.0. It follows that b times a Wiener process has a standard deviation of b.

In a unit time interval, the change in the value of $X(t)$ is given by the proposition 2.2 and the equation (3.1) as

$$\begin{aligned}
dX(s) &= a\ ds + b\ dW(s) \\
\int_t^{t+1} dX(s) &= \int_t^{t+1} ads + \int_t^{t+1} bdW(s) \\
X(t+1) - X(t) &= a + b(W(t+1) - W(t)) \\
X(t+1) - X(t) &= a + b(W(1) - W(0)) \\
X(t+1) - X(t) &= a + bW(1)
\end{aligned}$$

Thus the change in the value of X in a unit time interval, $X(t+1) - X(t)$, has a normal distribution with

$$\text{mean of } X(t+1) - X(t) = a$$
$$\text{standard deviation of } X(t+1) - X(t) = b$$
$$\text{variance of } X(t+1) - X(t) = b^2$$

Consequently, the generalized Wiener process given in equation (3.1) has an expected drift rate (i.e., average drift per unit of time) of a and a variance rate (i.e., variance per unit of time) of b^2. A sample path is illustrated in Figure 3.1.

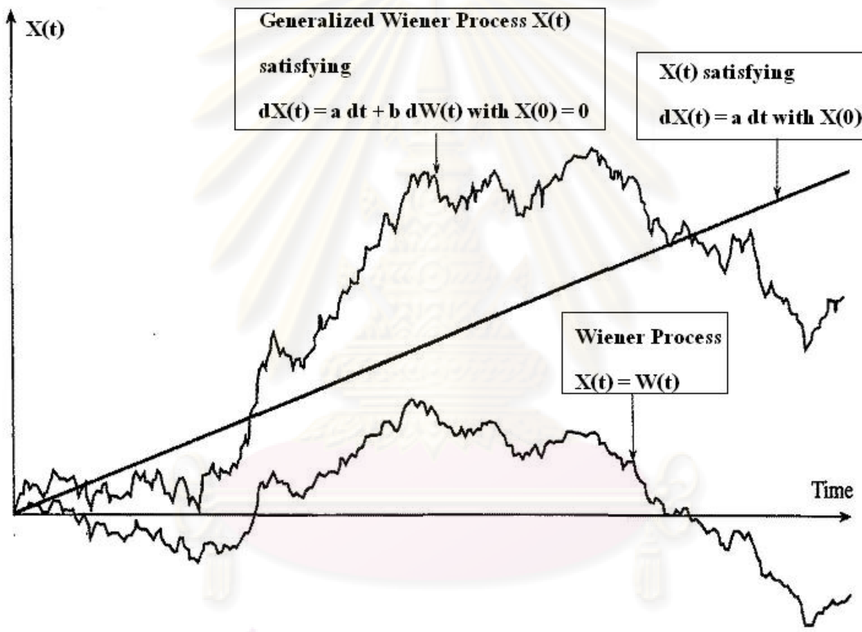

Figure 3.1: Generalized Wiener process with a = 0.3 and b = 1.5.

3.3.5 Geometric Brownian Motion

It is tempting to suggest that investment asset[3] price follows a generalized Wiener process; that is it has a constant expected drift rate and a constant variance rate.

[3]An investment asset is an asset that is held for investment purposes by significant numbers of investors. Stocks and bonds are clearly investment assets. Gold and silver are also examples of investment assets.

However, this model fails to capture a key aspect of the investment asset prices. This is that the expected percentage return required by investors from the asset is independent of the asset's price. If investors require a 14% per annum expected return when the asset price is \$10, then, ceteris paribus, they will also require a 14% per annum expected return when it is \$50.

Clearly, the assumption of constant expected drift rate is inappropriate and needs to be replaced by the assumption that the expected return (i.e., expected drift divided by the asset price) is constant. If $S(t)$ is the investment asset price at time t, then the expected drift rate in $S(t)$ should be assumed to be $\mu S(t)$ for some constant parameter μ. The parameter μ is the expected rate of return on the investment asset.

If the volatility of the asset price is always zero, then this model implies that

$$dS(t) = \mu S(t)dt$$

Integrating between time 0 and time T,

$$\frac{dS(t)}{S(t)} = \mu dt$$

$$S(T) = S(0)e^{\mu T}$$

This shows that when the variance rate is zero, the stock price grows at a continuously compounded rate of μ per unit of time.

In practice, of course, the asset price does exhibit volatility. An investor is just as uncertain of the percentage return when the stock price is \$50 as when it is \$10. This suggests that the standard deviation of the change in a short period of time Δt should be proportional to the stock price and leads to the model

$$dS(t) = \mu S(t)\ dt + \sigma S(t)\ dW(t).$$

This model is the most widely used model of the asset price behavior and is known as *geometric Brownian motion*. The variable σ is the volatility of the asset price. The variable μ is its expected rate of return.

3.3.6 Main Result

As we explained, some specialised stock indices choose a specific group of stocks to calculate these indices. For example, Russell 3000 Index considers stocks of 3,000 publicly held US companies or SET50 Index considers stocks of the top 50 listed companies on The Stock Exchange of Thailand. Some of these stock indices have the regulations that considers for a new list of stocks for every time period. This is called *reconstitution* or *stock revision*. For instance, Russell 3000 Index rebalances its indices once each year in June. Also, SET50 Index revises its list every six months.

From now on we will only consider SET50 Index. However, stochastic model, futures , and European options prices of other indices that have similar properties can be obtained by similar methods we use for SET50 Index in this chapter and the following chapters. Due to stock revision which happens every six months in the case of SET50 Index, expected rate of return, μ, must vary according to this time period. As well as the expected rate of return, μ, volatility of the stock index, σ, must be varied according to this time period. Then we propose the more realistic model including the effect of stock revision by changing μ, σ from constant functions to piecewise-constant functions according to stock revision period. However, the discontinuity leads to the problem when we solved stochastic differential equation. Then, we interpolate this two different constant values with a linear function and propose the model

$$dS(t) = \mu(t)S(t)dt + \sigma(t)S(t)dW(t) \tag{3.2}$$

where

$$\mu(t) = \begin{cases} \mu_1 & , 0 \le t \le 0.5 - \epsilon \\ \mu_i & , 0.5(i-1) + \epsilon \le t \le 0.5i - \epsilon \\ \frac{\mu_{i+1} - \mu_i}{2\epsilon}(t - 0.5i + \epsilon) + \mu_i & , 0.5i - \epsilon \le t \le 0.5i + \epsilon \end{cases}$$

Similarly,

$$\sigma(t) = \begin{cases} \sigma_1 & ,0 \leq t \leq 0.5 - \epsilon \\ \sigma_i & ,0.5(i-1) + \epsilon \leq t \leq 0.5i - \epsilon \\ \frac{\sigma_{i+1} - \sigma_i}{2\epsilon}(t - 0.5i + \epsilon) + \sigma_i & ,0.5i - \epsilon \leq t \leq 0.5i + \epsilon \end{cases}$$

and μ_i, σ_i are constant for all $i \in \mathbb{N}$.

From what we explained in the previous subsection μ_i is the expected rate of return of the stock index after the $i-1^{th}$ stock revision and before the i^{th} stock revision. Likewise, σ_i is the volatility of the stock index after the $i-1^{th}$ stock revision and before the i^{th} stock revision. The function $\frac{\mu_{i+1}-\mu_i}{2\epsilon}(t - 0.5i + \epsilon) + \mu_i$ is what we use to interpolate between two possibly different values, μ_i and μ_{i+1}. Similarly, The function $\frac{\sigma_{i+1}-\sigma_i}{2\epsilon}(t - 0.5i + \epsilon) + \sigma_i$ is used to interpolate between two possibly different values, σ_i and σ_{i+1}.

Theorem 3.1. *The unique solution of SDE (3.2) with initial condition* $S(0) = 1000$ *is*

$$S(t) = 1000e^{\int_0^t \mu(s) - \frac{\sigma^2(s)}{2} ds + \int_0^t \sigma(s) dW(s)}$$

Proof. According to the theorem 2.10 (Itô formula), we let $F(t, x) = 1000e^x$ and $\xi(t)$ be Itô process such that $\xi(t)$ satisfies

$$d\xi(t) = (\mu(t) - \frac{\sigma^2(t)}{2})dt + \sigma(t)dW(t) \quad \text{with } \xi(0) = 0. \tag{3.3}$$

Then, according to the definition 2.16 of Itô process on page 9, $a(t)$ and $b(t)$ in the definition are $\mu(t) - \frac{\sigma^2(t)}{2}$ and $\sigma(t)$ respectively. Somehow equation (3.3) can be rewritten as

$$\xi(t) = \xi(0) + \int_0^t \mu(s) - \frac{\sigma^2(s)}{2} ds + \int_0^t \sigma(s) dW(s)$$

$$\xi(t) = \int_0^t \mu(s) - \frac{\sigma^2(s)}{2} ds + \int_0^t \sigma(s) dW(s) \qquad \text{since } \xi(0) = 0.$$

To use Itô formula, we find $F_t(t,x)$, $F_x(t,x)$, and $F_{xx}(t,x)$.

$$\begin{aligned}
F_t(t,x) &= 0\\
F_x(t,x) &= 1000e^{x}\\
F_{xx}(t,x) &= 1000e^{x}
\end{aligned}$$

Then, by Itô formula $F(t,\xi(t))$ is Itô process satisfying

$$\begin{aligned}
dF(t,\xi(t)) &= \left(F_t(t,\xi(t)) + F_x(t,\xi(t))a(t) + \frac{1}{2}F_{xx}(t,\xi(t))b(t)^2\right)dt + F_x(t,\xi(t))b(t)dW(t)\\
&= \left(1000e^{\xi(t)}(\mu(t) - \frac{\sigma^2(t)}{2}) + \frac{1000}{2}e^{\xi(t)}\sigma^2(t)\right)dt + 1000e^{\xi(t)}\sigma(t)dW(t)\\
&= \mu(t)1000e^{\xi(t)}dt + \sigma(t)1000e^{\xi(t)}dW(t)\\
&= \mu(t)F(t,\xi(t))dt + \sigma(t)F(t,\xi(t))dW(t)
\end{aligned}$$

Thus $F(t,\xi(t))$ is a solution to

$$dS(t) = \mu(t)S(t)dt + \sigma(t)S(t)dW(t)$$

and satisfies initial condition $S(0) = 1000$.

$$\begin{aligned}
F(0,\xi(0)) &= 1000e^{\xi(0)}\\
&= 1000 \qquad \text{because } \xi(0) = 0.
\end{aligned}$$

Next, we will show that

$$\begin{aligned}
F(t,\xi(t)) &= 1000e^{\xi(t)}\\
&= 1000e^{\int_0^t \mu(t) - \frac{\sigma^2(t)}{2}dt + \int_0^t \sigma(t)dW(t)}
\end{aligned}$$

is the only solution to the equation (3.2) by using the theorem 2.11. Compare the equation (3.2) to the equation (2.6).

For $t \in [0,T]$,

$$\begin{aligned}
|\mu(t)x| + |\sigma(t)x| &\le (|\mu(t)| + |\sigma(t)|)(|x|+1)\\
&\le \left(\max_{1\le k\le 2T}(|\mu_k|) + \max_{1\le k\le 2T}(|\sigma_k|)\right)(|x|+1)
\end{aligned}$$

Then we get the inequality (2.4).

Next we will prove the inequality (2.5).

Let $t \in [0, T]$.

$$|\mu(t)x - \mu(t)y| = |\mu(t)|\,|x-y| \leq \max_{1\leq k\leq 2T}(|\mu_k|)\,|x-y| \tag{3.4}$$

$$|\sigma(t)x - \sigma(t)y| = |\sigma(t)|\,|x-y| \leq \max_{1\leq k\leq 2T}(|\sigma_k|)\,|x-y| \tag{3.5}$$

Then from the inequality (3.4) and (3.5),

$$|\mu(t)x - \mu(t)y| + |\sigma(t)x - \sigma(t)y| \leq \left(\max_{1\leq k\leq 2T}(|\mu_k|) + \max_{1\leq k\leq 2T}(|\sigma_k|)\right)|x-y|$$

□

However, under risk neutral measure $\mathbb{Q}$, SDE (3.2) is changed to [4]

$$dS(t) = (\mu(t) + r)S(t)dt + \sigma(t)S(t)dW(t) \tag{3.6}$$

where r is risk-free interest rate[5].

Theorem 3.2. *The unique solution of SDE (3.6) with initial condition* $S(0) = 1000$ *is*

$$S(t) = 1000e^{\int_0^t r+\mu(s)-\frac{\sigma^2(s)}{2}ds+\int_0^t \sigma(s)dW(s)} \tag{3.7}$$

where r is risk-free interest rate.

Proof of the theorem 3.2 can be done by the similar step in the proof of the theorem 3.1.

[4]The proof of this can be done by Girsanov Theorem which tells how stochastic processes change under changes in measure.

[5]Risk-free interest rate is the theoretical rate of return of an investment with zero risk, including default risk. The risk-free rate represents the interest that an investor would expect from an absolutely risk-free investment over a given period of time. Therefore, a rational investor will reject all the investments yielding sub-risk-free returns.

Simulation

According to [3], one of the simplest time discrete approximations of an Itô process is the *Euler approximation*, or the $Euler - Maruyama$ *approximation*. We use this method to simulate sample paths of the SDE (3.2). Here, we consider the time discretization

$$t = t_0 < t_1 < \ldots < t_n < \ldots < t_N = T$$

on the time interval $[t, T]$; we also consider equidistant discretization time. Then,

$$t_i = t_0 + i\frac{T-t}{N} \qquad \text{for } i \in \mathbb{N} \text{ and } 0 < i < N$$

for some integer N large enough so that $\frac{T-t}{N} \in (0, 1)$.

The Euler approximation, $s = s(t), t_0 \leq t \leq t_N$, satisfies the iterative scheme

$$s(t_{n+1}) = s(t_n) + \mu(t_n)(t_{n+1} - t_n) + \sigma(t_n)\left(W(t_{n+1}) - W(t_n)\right) \tag{3.8}$$

for $n = 0, 1, 2, \ldots, N-1$ with initial value $s(t_0) = S(t_0)$.

By the iterative scheme (3.8), we show 1,000 approximations of sample paths of the SDE (3.2) during the time $[0, 1]$ when $S(t_0) = 0, N = 500, \epsilon = 0.05$

$$\mu(t) = \begin{cases} 0.15 & , 0 \leq t \leq 0.45 \\ 0.5t - 0.075 & , 0.45 \leq t \leq 0.55 \\ 0.20 & , 0.55 \leq t \leq 0.95 \end{cases}$$

, and

$$\sigma(t) = \begin{cases} 0.3 & , 0 \leq t \leq 0.45 \\ 0.5t + 0.075 & , 0.45 \leq t \leq 0.55 \\ 0.35 & , 0.55 \leq t \leq 0.95 \end{cases}$$

SET50 Index starts at 1,000 points. Here, we simulate the movements of SET50 Index in 1 year (Our unit time is a year.) and the 1 year interval is discretized into 500 intervals. We consider the values of SET50 Index at the terminal time (or at the end of 1 year.) from the figure 3.2. Then, we obtain histogram and it

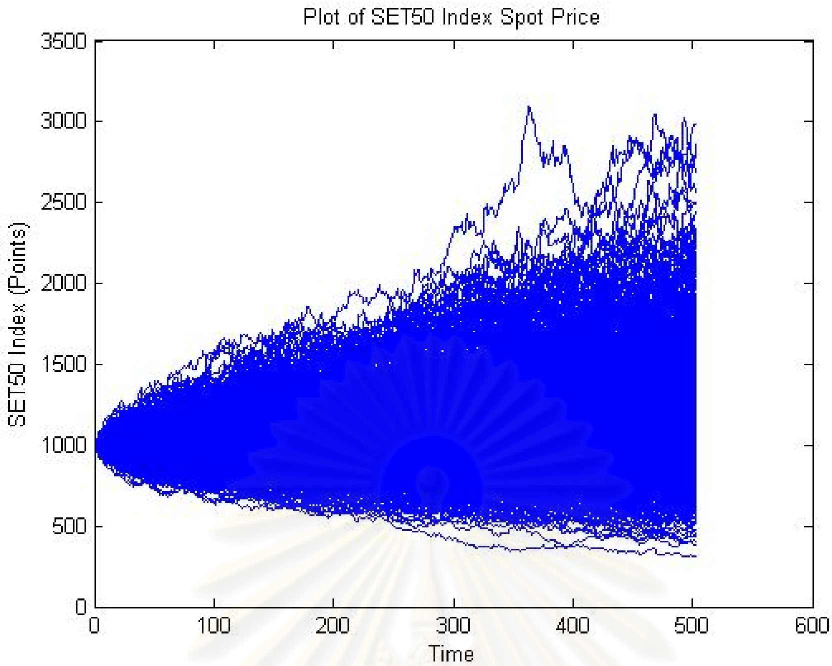

Figure 3.2: Simulation of the 1000 sample paths of SET50 Index in 1 year

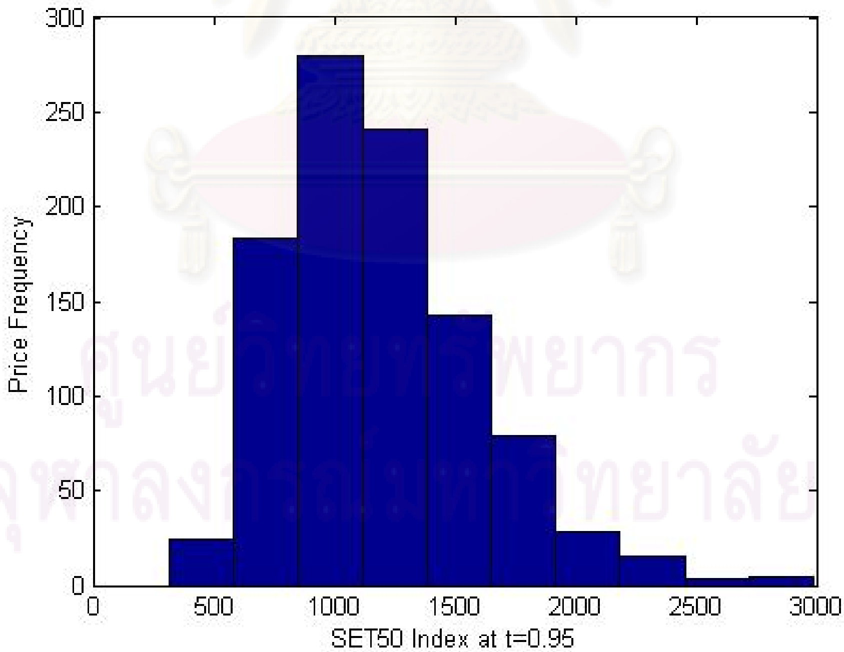

Figure 3.3: Histogram of the terminal values

is shown in the figure 3.3. From the histogram we see that SET50 Index values at the end of 1 year tend to be slightly higher than its initial value, 1,000, since the expected rate of return, μ, is slightly above zero in this case.

CHAPTER IV

Futures

A *derivative* can be defined as a financial instrument whose value depends on (or derives from) the value of other, more basic, underlying variables. Very often the variables underlying derivatives are the prices traded assets. A stock options, for example, is a derivative whose value is dependent on the price of a stock. However, derivatives can be dependent on almost any variable, from the price of hogs to the amount of snow falling at a certain ski resort.

In the last 30 years derivatives have become increasingly important in finance. Futures and options are now traded actively on many exchanges through out the world. For Thailand, derivatives market increasingly takes more role in the business sector and continuously lauched many types of derivatives such as agricultural futures contract for Jasmine rice, Cassava, and Para rubber and SET50 Index futures and options.

In this chapter we take the first look at futures markets and provide overview of how they are used by hedgers, speculators, and arbitrageurs. Finally, we will consider some mathematical theorems used as tools to calculate futures price over the developed stochastic model in the past chapter.

4.1 Futures Contracts

A futures contract, or simply futures, (but not future or future contract) is a standardized contract between two parties to buy or sell a specified asset (eg.oranges, oil, gold) of standardized quantity and quality at a specified future date at a price agreed today (the *futures price*). This is in contrast with the *spot price* which is the price that is quoted for immediate (spot) settlement (payment and delivery).

The contracts are traded on a futures exchange. Futures contracts are not *direct* securities like stocks, bonds, rights or warrants. They are still securities, however, though they are a type of derivative contract. The party agreeing to buy the underlying asset in the future assumes a long position, and the party agreeing to sell the asset in the future assumes a short position.

The price is determined by the instantaneous equilibrium between the forces of supply and demand among competing buy and sell orders on the exchange at the time of the purchase or sale of the contract.

In many cases, the underlying asset to a futures contract may not be traditional commodities at all that is, for financial futures, the underlying asset or item can be currencies, securities or financial instruments and intangible assets or referenced items such as stock indices and interest rates.

The future date is called the delivery date or final settlement date. The official price of the futures contract at the end of a day's trading session on the exchange is called the settlement price for that day of business on the exchange.

A futures contract gives the holder the obligation to make or take delivery under the terms of the contract. In other words, both parties of a futures contract must fulfill the contract on the settlement date. The seller delivers the underlying asset to the buyer, or, if it is a cash-settled futures contract, then cash is transferred from the futures trader who sustained a loss to the one who made a profit. To exit the commitment prior to the settlement date, the holder of a futures position has to offset his/her position by either selling a long position or buying back (covering) a short position, effectively closing out the futures position and its contract obligations.

Futures contracts are exchange-traded derivatives. The exchange's clearing house acts as counterparty on all contracts, sets margin requirements, and crucially also provides a mechanism for settlement.

4.2 Type of Futures Traders

Derivatives markets have been outstandingly successful. The main reason is that they have attracted many different types of traders and have a great deal of liquidity. When an investor wants to take one side of a contract, there is usually no problem in finding someone that is prepared to take the other side.

Futures traders are traditionally placed in one of two groups: *hedgers*, who use derivatives to reduce the risk that they face from potential future movements in a market variable and *speculators*, who seek to make a profit by predicting market moves and opening a derivative contract related to the asset on paper, while they have no practical use for or intent to actually take or make delivery of the underlying asset. In other words, the investor is seeking exposure to the asset in a long futures or the opposite effect via a short futures contract.

Hedgers typically include producers and consumers of a commodity or the owner of an asset or assets subject to certain influences such as an interest rate.

For example, in traditional commodity markets, farmers often sell futures contracts for the crops and livestock they produce to guarantee a certain price, making it easier for them to plan. Similarly, livestock producers often purchase futures to cover their feed costs, so that they can plan on a fixed cost for feed.

An example that has both hedge and speculative notions involves a mutual fund or separately managed account whose investment objective is to track the performance of a stock index such as the SET50 Index. The Portfolio manager often equitizes cash inflows in an easy and cost effective manner by investing in (opening long) SET50 Index futures. This gains the portfolio exposure to the index which is consistent with the fund or account investment objective without having to buy an appropriate proportion of each of the individual 50 stocks just yet. This also preserves balanced diversification, maintains a higher degree of the percent of assets invested in the market and helps reduce tracking error in the performance of the fund/account. When it is economically feasible (an efficient amount of shares of every individual position within the fund or account can be purchased), the portfolio manager can close the contract and make purchases of each individual stock.

The social utility of futures markets is considered to be mainly in the transfer of risk, and increased liquidity between traders with different risk and time preferences, from a hedger to a speculator, for example.

4.3 Main Result: Pricing SET50 Index Futures

A fundamental implication of asset pricing theory is that under the no-arbitrage assumptions[1] the fair price of a derivatives security (futures or options contract) at a current time can be represented by the expected value of its discounted payoff function at the maturity date under a risk-neutral probability measure. In fact, valuing derivatives reduces to computing the expectation with respect to the probability measure. In terms of pricing futures contracts the following theorems are neccessary.

Theorem 4.1 (S.Rujiva [5]). *Under the no-arbitrage assumptions in a futures market, the no-arbitrage futures price on day t with maturity date T, denoted by $F^T(t, S(t))$, must satisfy*

$$F^T(t, S(t)) = E_{\mathbb{Q}}\left[S(T) \middle| \mathcal{F}_t\right] \tag{4.1}$$

where the expectation is taken under a risk-neutral probability measure $\mathbb{Q}$ conditioned on the information $\mathcal{F}_t$.

Relation (4.1) tells that the no-arbitrage futures price today is an unbiased estimator of the spot price at the maturity date of the contract where we consider under the risk-neutral probability measure and the information available today.

According to the relation (4.1), we can show that F^T is the solution of the partial differential equation

$$\frac{\partial F^T}{\partial t} + \frac{1}{2}\sigma^2(t)x^2\frac{\partial^2 F^T}{\partial x^2} + (r + \mu(t))x\frac{\partial F^T}{\partial x} = 0 \tag{4.2}$$

with terminal condition $F^T(T, x) = x$.

An explanation will be given but it is necessary to introduce Feynman-Kac formula.

---

[1] If a portfolio requires a null investment and is riskless, then its terminal value at time T has to be zero. In other words it basically states that it is not possible to get something for nothing.

Theorem 4.2 (H.Jin [6], Feynman–Kac Formula). *Let $X(t)$ be the n-dimensional Itô process satisfying the stochastic differential equation*

$$dX(t) = a(t, X(t))dt + b(t, X(t))dW(t), \quad X(t) = x \tag{4.3}$$

Then

$$f(t, x) = E\left[e^{-\int_t^T V(X(\tau))d\tau}\psi(X(T))|X(t) = x\right] \tag{4.4}$$

where V, ψ, a, b are known functions is a solution to the partial differential equation.

$$\frac{\partial f}{\partial t} + a(t, x)\frac{\partial f}{\partial x} + \frac{1}{2}b^2(t, x)\frac{\partial^2 f}{\partial x^2} = V(t, x)f \tag{4.5}$$

defined for all real x and t in the interval $[0, T]$, subject to the terminal condition $f(T, x) = \psi(x)$.

We then apply Feynman–Kac formula to the equations (4.1) and (3.7) to find SET50 Index futures price.

Theorem 4.3 (Determination of Futures Prices). *For given and fixed maturity date T,the no-arbitrage futures price on day t with maturity date T, which is denoted by $F^T(t, S(t))$ and satisfies the equation (4.1), has the closed-form solution*

$$F^T(t, S(t)) = S(t)e^{B(T-t)}$$

where $B(T - t) = r(T - t) + \int_t^T \mu(s)ds^2$.

Proof. From the relation (4.1) which under risk neutral measure, $\mathbb{Q}$, its stochastic process, $S(t)$, follows (3.7), we will find SET50 Index futures price, $E_{\mathbb{Q}}[S(T)|\mathcal{F}_t]$, which in this prove we will omit subscript $\mathbb{Q}$ by applying the theorem 4.2 (Feynman- –Kac formula). Comparing equation (3.7) to (4.3), we get $a(t, S(t)) = \mu(t)S(t)$ and $b(t, S(t)) = \sigma(t)S(t)$. Also, we can show that $E[S(T)|\mathcal{F}_t] = f(t, S(t))$ where $f(t, x) = E\left[e^{-\int_t^T V(S(\tau))d\tau}\psi(S(T))|S(t) = x\right]$ is in the form as in the equation (4.4) with $V = 0$ and $\psi(x) = x$.

$$\begin{aligned}
f(t, S(t)) &= E\left[e^{-\int_t^T V(S(\tau))d\tau}\psi(X(T))|S(t) = S(t)\right] \\
&= E[S(T)|S(t)] \qquad (\sigma(S(t)) = \sigma(\{S(t) = S(t)\}).) \\
&= E[S(T)|\mathcal{F}_t] \qquad (S(t) \text{ has markov property from [7].})
\end{aligned}$$

[2] r is risk-free interest rate.

Note that $\sigma(S(t)) = \sigma(\{S(t) = S(t)\})$ means that $S(t)$ can be anything as long as it is measurable. In summary, from this equality we can find the futures price , $E[S(T)|\mathcal{F}_t]$, by applying Feynman–Kac formula to find the value of $f(t,x) = E[S(T)|S(t) = x]$. Then, solve partial differential equation that follows and the futures price , $E[S(T)|\mathcal{F}_t]$, will be equal to $f(t, S(t))$.

$$\frac{\partial f}{\partial t} + (r+\mu(t))x\frac{\partial f}{\partial x} + \frac{1}{2}\sigma^2(t)x^2\frac{\partial^2 f}{\partial x^2} = 0 \quad 0 \le t \le T \tag{4.6}$$

with terminal condition $f(T,x) = x$.

Let $\tau = T - t$ and $f(t,x) = xe^{B(T-t)}$.

$$\frac{\partial f}{\partial t} = xe^{B(T-t)}B'(\tau)\frac{\partial \tau}{\partial t}$$
$$\frac{\partial f}{\partial x} = e^{B(T-t)}$$

Then, substitute these values to the equation (4.6). We get

$$-xB'(\tau)e^{B(\tau)} + (r+\mu(t)))xe^{B(\tau)} = 0$$
$$-B'(\tau) + (r+\mu(t)) = 0$$
$$B'(\tau) = r + \mu(t).$$

From its terminal condition $f(T,x) = x$, we get $B(0) = 0$. We get ordinary differential equation

$$\begin{cases} B'(\tau) = r + \mu(t) \\ B(0) = 0 \end{cases}$$

According to fundamental theorem of calculus,

$$\int_0^{\tau_0} B'(\tau)d\tau = \int_0^{\tau_0} r + \mu(t)\ d\tau$$
$$B(\tau_0) - B(0) = \int_0^{\tau_0} r + \mu(T-\tau)\ d\tau$$
$$B(\tau_0) - 0 = r\tau_0 + \int_0^{\tau_0} \mu(T-\tau)d\tau$$
$$B(\tau_0) = r\tau_0 - \int_T^{T-\tau_0} \mu(T-\tau)d(T-\tau)$$
$$B(\tau_0) = r\tau_0 + \int_{T-\tau_0}^{T} \mu(s)ds.$$

Substitute $B(\tau) = B(T-t)$ into $f(t,x) = xe^{B(T-t)}$. We get

$$f(t,x) = xe^{r(T-t)+\int_t^T \mu(s)ds}.$$

$\square$

We will give the explicit form of the no-arbitrage futures price on day $t = 0$ with maturity date T, $F^T(0, S(0))$.

Example 4.4.

$$\begin{aligned} F^T(0,S(0)) &= S(0)e^{B(T-t)} \\ &= 1000e^{B(T-0)} \\ &= 1000e^{B(T)} \\ &= 1000e^{rT+\int_0^T \mu(s)ds} \end{aligned}$$

1. *For $t = 0$ and $T \in [0.5(i_T - 1) + \epsilon, 0.5i_T - \epsilon]$ for some $i_T \in \mathbb{N}$. Explicit form of $\int_0^T \mu(s)ds$ can be found from the area under the curve $\mu(t)$ plotted on the graph between $\mu(t)$ and t.*

$$\int_0^T \mu(s)ds = \begin{cases} \mu_1 T \quad ; \textit{if } i_T = 1 \\ \mu_1(0.5-\epsilon) + \frac{1}{2}2\epsilon(\mu_2+\mu_1) + \sum_{n=2}^{i_T-1}(\mu_n(0.5-2\epsilon) \\ +\frac{1}{2}(2\epsilon)(\mu_n+\mu_{n+1}) + \mu_{i_T}(T-(0.5(i_T-1)+\epsilon)) \quad ; \textit{otherwise} \end{cases}$$

2. *For $t = 0$ and $T \in [0.5i_T - \epsilon, 0.5i_T + \epsilon]$ for some $i_T \in \mathbb{N}$.*

$$\begin{aligned} \int_0^T \mu(s)ds =& \mu_1(0.5-\epsilon) + \sum_{n=2}^{i_T}(\mu_n(0.5-2\epsilon) \\ &+ \epsilon(\mu_n+\mu_{n-1}) + \frac{1}{2}(T-0.5i_T-\epsilon)(\mu_{i_T}+\mu(T)) \end{aligned}$$

4.3.1 Simulation

We now show the relationship between the function $B(T-t)$ in the theorem 4.3 and time, t, in the figure 4.1 when $\mu(t)$, and T are known. Here we assume that $T = 1.45,\ \epsilon = 0.05,\ \mu_1 = 0.15,\ \mu_2 = 0.2,\ \mu_3 = 0.1$. Then,

$$\mu(t) = \begin{cases} 0.15 & 0 \le t \le 0.45 \\ 0.5t - 0.075 & 0.45 \le t \le 0.55 \\ 0.2 & 0.55 \le t \le 0.95 \\ -t + 1.15 & 0.95 \le t \le 1.05 \\ 0.1 & 1.05 \le t \le 1.45 \end{cases}$$

In the figure 4.2 we show the evolution of the futures prices obtained from the

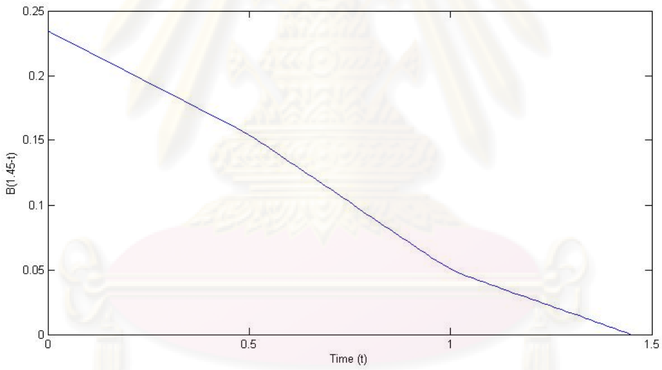

Figure 4.1: The relationship between $B(1.45-t)$ and t when $T = 1.45,\ \epsilon = 0.05,\ \mu_1 = 0.15,\ \mu_2 = 0.2,$ and $\mu_3 = 0.1$.

closed-form solution in the theorem 4.3 with the parameters $T = 1.45,\ \epsilon = 0.05,\ \mu_1 = 0.15,\ \mu_2 = 0.2,$ and $\mu_3 = 0.1$ like before when $S(t)$ is varied from 800 to 900. In this figure, we can see that before the futures contract expires ($T < 1.45$), the futures price is higher than the spot price since $B(T-t) > 0$ (notice in the figure 4.1), and from the theorem 4.3

$$F^T(t, S(t)) = S(t)e^{B(T-t)}$$

if $B(T-t) > 0$, $e^{B(T-t)} > 1$. Then, $F^T(t, S(t)) > S(t)$. This is consistent with our intuition; suppose that we expect the SET50 Index during the time t to T to perform well (In other words, the rate of return is positive ($\mu(t) > 0$).). We must expect its value in the future time, T, to be higher than its value at time t. On the other hand if we expect the SET50 Index during the time t to T not to perform well enough ($\mu(t)$ makes $B(T-t) < 0$.), we expect SET50 Index value in the future time, T, to be lower than its value at time t ($F^T(t, S(t)) < S(t)$).

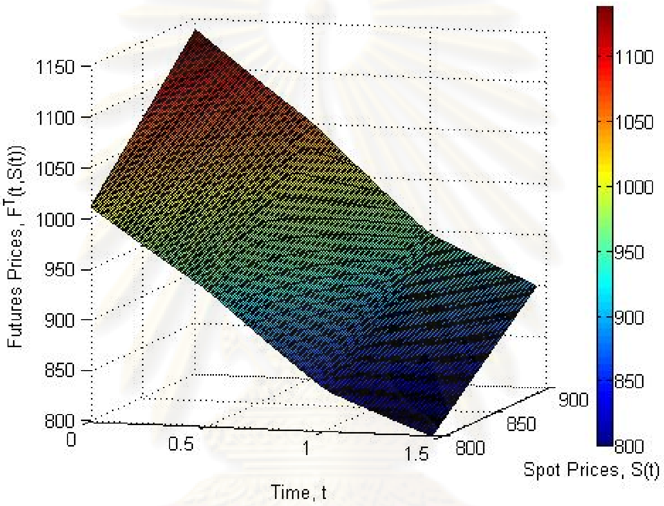

Figure 4.2: Evolution of the futures prices with the parameters $T = 1.45$, $\epsilon = 0.05$, $\mu_1 = 0.15$, $\mu_2 = 0.2$, and $\mu_3 = 0.1$.

CHAPTER V
Options

Options are fundamentally different from futures contracts. An option gives the holder of the option the right to do something, but the holder does not have to exercise this right. By contrast, in a futures contract, the two parties have committed themselves to some action. It costs a trader nothing to enter into a forward or futures contract, whereas the purchase of an option requires an up-front payment.

In this chapter we take the first look at options markets, what terminology is used. Then, we will consider some mathematical theorems used as tools to calculate options price over the developed stochastic model in the past chapter. Finally, we find prices of both SET50 Index call and put options.

5.1 Option Contracts

In finance, an option is a derivative financial instrument that establishes a contract between two parties concerning the buying or selling of an asset at a reference price. The buyer of the option gains the right, but not the obligation, to engage in some specific transaction on the asset, while the seller incurs the obligation to fulfill the transaction if so requested by the buyer.

5.1.1 Types of Options

There are two basic types of options. A *call option* gives the holder of the option the right to buy an asset by a certain date for a certain price. A *put option* gives the holder the right to sell an asset by a certain date for a certain price. The date specified in the contract is known as the *expiration date* or the *maturity date*. The price specified in the contract is known as the *exercise price* or the *strike price*.

Options can be either American or European, a distinction that has nothing to do with geographical location. *American options* can be exercised at any time up to the expiration date, whereas *European options* can be exercised only on the expiration date itself. Most of the options that are traded on exchanges are American. However, European options are generally easier to analyze than American options, and some of the properties of an American option are frequently deduced from those of its European counterpart.

Call Options

Consider the situation of an investor who buys a European call option with a strike price of \$100 to purchase 100 shares of a certain stock. Suppose that the current stock price is \$98, the expiration date of the option is in 4 months, and the price of an option to purchase one share is \$5. The initial investment is \$500. Because the option is European, the investor can exercise only on the expiration date. If the stock price on this date is less than \$100, the investor will clearly choose not to exercise. (There is no point in buying for \$100 a share that has a market value of less than \$100.) In these circumstances, the investor loses the whole of the initial investment of \$500. If the stock price is above \$100 on the expiration date, the option will be exercised. Suppose, for example, that the stock price is \$115. By exercising the option, the investor is able to buy 100 shares for \$100 per share. If the shares are sold immediately, the investor makes a gain of \$15 per share, or \$1,500, ignoring transactions costs. When the initial cost of the option is taken into account, the net profit to the investor is \$1,000.

Figure 5.1 shows how the investor's net profit or loss on an option to purchase one share varies with the final stock price in the example. It is important to realize that an investor sometimes exercises an option and makes a loss overall. Suppose that, in the example, the stock price is \$102 at the expiration of the option. The investor would exercise the option contract for a gain of $100 \times (\$102 - \$100) = \$200$ and realize a loss overall of \$300 when the initial cost of the option is taken into account. It is tempting to argue that the investor should not exercise the option in these circumstances. However, not exercising would lead to an overall loss of \$500, which is worse than the \$300 loss when the investor exercises. In general, call options should always be exercised at the expiration date if the stock price is above the strike price.

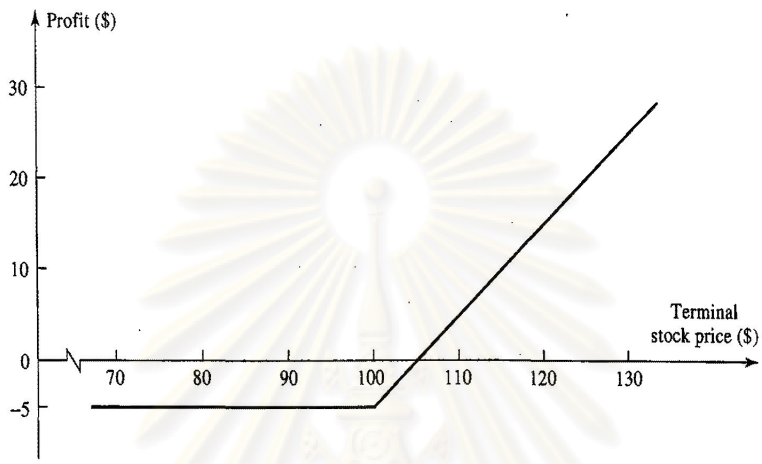

Figure 5.1: Profit from buying a European call option on one share of a stock. Option price = \$5; strike price = \$100.

Put Options

Whereas the purchaser of a call option is hoping that the stock price will increase, the purchaser of a put option is hoping that it will decrease. Consider an investor who buys a European put option with a strike price of \$70 to sell 100 shares of a certain stock. Suppose that the current stock price is \$65, the expiration date of the option is in 3 months, and the price of an option to sell one share is \$7. The initial investment is \$700. Because the option is European, it will be exercised only if the stock price is below \$70 on the expiration date. Suppose that the stock price is \$55 on this date. The investor can buy 100 shares for \$55 per share and, under the terms of the put option, sell the same shares for \$70 to realize a gain of \$15 per share, or \$1,500. (Again, transactions costs are ignored.) When the \$700 initial cost of the option is taken into account, the investor's net profit is \$800.

There is no guarantee that the investor will make a gain. If the final stock price is above \$70, the put option expires worthless, and the investor loses \$700. Figure 5.2 shows the way in which the investor's profit or loss on an option to sell one share varies with the terminal stock price in this example.

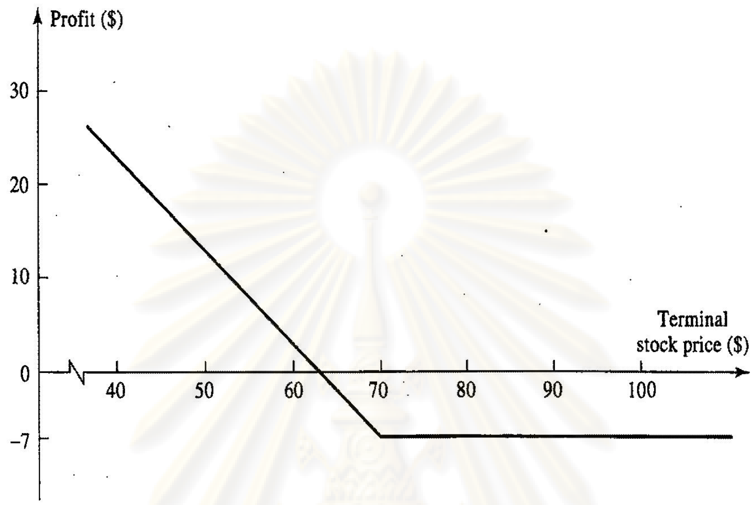

Figure 5.2: Profit from buying a European put option on one share of a stock. Option price = \$7; strike price = \$70.

5.1.2 Option Positions

There are two sides to every option contract. On one side is the investor who has taken the long position (i.e., has bought the option). On the other side is the investor who has taken a short position (i.e., has sold or written the option). The writer of an option receives cash up front, but has potential liabilities later. The writer's profit or loss is the reverse of that for the purchaser of the option. Figures 5.3 and 5.4 show the variation of the profit or loss with the final stock price for writers of the options considered in Figures 5.1 and 5.2.

There are four types of option positions:

1. A long position in a call option

2. A long position in a put option

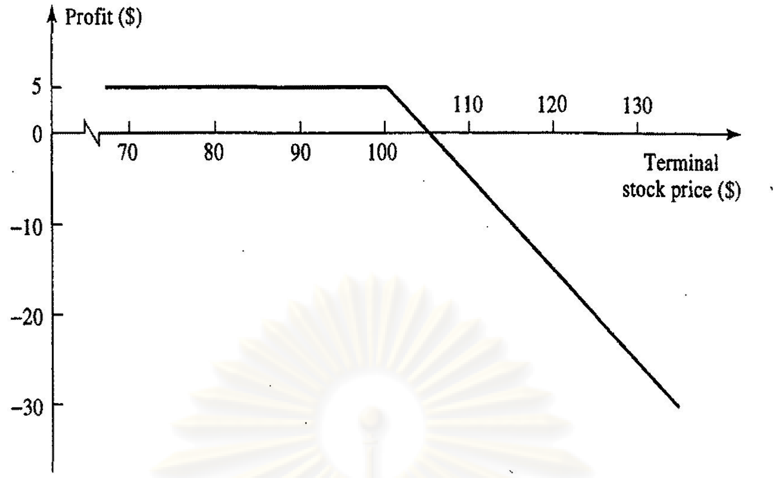

Figure 5.3: Profit from writing a European call option on one share of a stock. Option price = \$5; strike price = \$100.

3. A short position in a call option

4. A short position in a put option

It is often useful to characterize a European option in terms of its payoff to the purchaser of the option. The initial cost of the option is then not included in the calculation. If K is the strike price and $S(T)$ is the final price of the underlying asset, the payoff from a long position in a European call option is $max(S(T)-K,0)$

This reflects the fact that the option will be exercised if $S(T) > K$ and will not be exercised if $S(T) \leq K$. The payoff to the holder of a short position in the European call option is

$$-\max(S(T)-K,0) = \min(K-S(T),0)$$

The payoff to the holder of a long position in a European put option is $\max(K - S(T), 0)$ and the payoff from a short position in a European put option is

$$-\max(K-S(T),0) = \min(S(T)-K,0)$$

Figure 5.5 illustrates these payoffs.

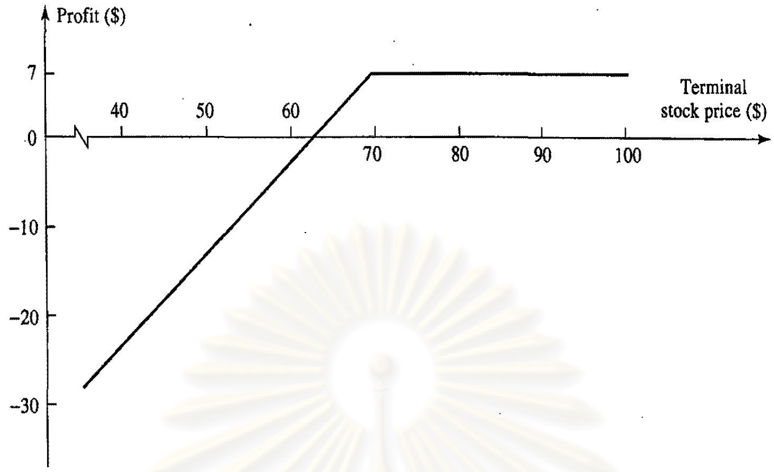

Figure 5.4: Profit from writing a European put option on one share of a stock. Option price = \$7; strike price = \$70.

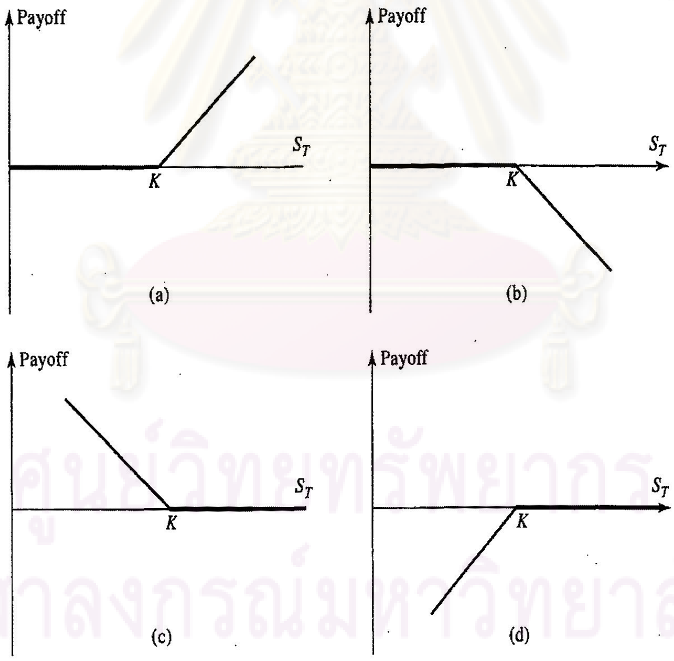

Figure 5.5: Payoffs from positions in European options: (a) long call; (b) short call; (c) long put; (d) short put. Strike price = K; price of asset at maturity = $S(T)$

5.1.3 Index Options

Many different index options currently trade throughout the world in both the over-the-counter market and the exchange-traded market. The most popular exchange-traded contracts in the United States are those on the S&P 500 Index (SPX), the S&P 100 Index (OEX), the Nasdaq 100 Index (NDX), and the Dow Jones Industrial Index (DJX). All of these trade on the Chicago Board Options Exchange. Most of the contracts are European. An exception is the OEX contract on the S&P 100, which is American. One contract is usually to buy or sell 100 times the index at the specified strike price. Settlement is always in cash, rather than by delivering the portfolio underlying the index. Consider, for example, one call contract on the S&P 100 with a strike price of 980. If it is exercised when the value of the index is 992, the writer of the contract pays the holder $(992 - 980) \times 100 = \$1,200$. This cash payment is based on the index value at the end of the day on which exercise instructions are issued. (Not surprisingly, investors usually wait until the end of a day before issuing these instructions.)

5.2 Main Result: Pricing SET50 Index Options

SET50 Index options is European options which we will use the following theorems to find call options price. As previously described in the section 4.3, the fair price of a derivatives security (futures or option contracts) at a current time can be represented by the expected value of its discounted payoff function at the maturity date under a risk-neutral probability measure.

Theorem 5.1 (S.Rujivan [5]). *Under the no-arbitrage assumptions in a futures market, the call option price must equal to the present value of the expected payoff of the call option under the risk neutral measure $\mathbb{Q}$. The no-arbitrage options price on day t with maturity date T and excercise price K, denoted by $C(T, t, S(t), K)$, must satisfy*

$$C(T, t, S(t), K) = e^{-r(T-t)} E_{\mathbb{Q}} \left[max(0, S(T) - K) | \mathcal{F}_t \right] \tag{5.1}$$

Lemma 5.2.

$$\int_{t=t_1}^{t=t_2} \sigma(t)dW(t) \stackrel{d}{=} \sqrt{\int_{t_1}^{t_2} \sigma^2(t)dt}\ Z$$

where Z is a standard normal random variable and $\sigma(t)$ is defined like in the equation 3.2.

Proof. Let $\Delta t = \frac{t_2 - t_1}{n}$

and $t_j^n = t_1 + j\Delta t$ where $j = 0, 1, \ldots, n$ be a partition on $[t_1, t_2]$.

$$\begin{aligned}
\int_{t_1}^{t_2} \sigma(t)dW(t) &= \lim_{n\to\infty} \sum_{j=0}^{n-1} \sigma(t_j^n)\left(W(t_{j+1}^n) - W(t_j^n)\right) \\
&= \lim_{n\to\infty} \sum_{j=0}^{n-1} \sigma(t_j^n) W(t_{j+1}^n - t_j^n) \qquad \text{(Proposition 2.2)} \\
&= \lim_{n\to\infty} \sum_{j=0}^{n-1} \sigma(t_j^n) \sqrt{t_{j+1}^n - t_j^n} Z_j \qquad \text{(Property of a normal random variable)} \\
&= \lim_{n\to\infty} \sum_{j=0}^{n-1} \sigma(t_j^n) \sqrt{\Delta t} Z_j \\
&= \lim_{n\to\infty} \sum_{j=0}^{n-1} Y_j \qquad (Y_j \sim N(0, \sigma^2(t_j^n)\Delta t.)) \\
&= \lim_{n\to\infty} Y_n \quad \text{where } Y_n \sim N(0, \sum_{j=0}^{n-1} \sigma^2(t_j^n)\Delta t).
\end{aligned}$$

By the sum of independent normal random variables $(\sum_{j=0}^{n-1} Y_j)$, the last equlity is true. Next, we will show that $Y_n \stackrel{dist}{\to} Y$ when $Y \sim N(0, \int_{t_1}^{t_2} \sigma^2(t)dt)$. Let $F_{Y_n}(x)$ be a cumulative distribution function of Y_n.

$$\begin{aligned}
F_{Y_n}(x) &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \frac{e^{-\frac{(s-\mu)^2}{2\sigma^2}}}{\sigma} ds \\
&= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \frac{e^{-\frac{(s-\mu)^2}{2\sum_{j=0}^{n-1} \sigma^2(t_j^n)\Delta t}}}{\sqrt{\sum_{j=0}^{n-1} \sigma^2(t_j^n)\Delta t}} ds
\end{aligned}$$

$$\lim_{n\to\infty} F_{Y_n}(x) = \lim_{n\to\infty} \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \frac{e^{-\frac{(s-\mu)^2}{2\sum_{j=0}^{n-1}\sigma^2(t_j^n)\Delta t}}}{\sqrt{\sum_{j=0}^{n-1}\sigma^2(t_j^n)\Delta t}} ds$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \lim_{n\to\infty} \frac{e^{-\frac{(s-\mu)^2}{2\sum_{j=0}^{n-1}\sigma^2(t_j^n)\Delta t}}}{\sqrt{\sum_{j=0}^{n-1}\sigma^2(t_j^n)\Delta t}} ds \quad \text{(Apply dominated convergence theorem.)}$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \frac{e^{-\frac{(s-\mu)^2}{2\lim_{n\to\infty}\sum_{j=0}^{n-1}\sigma^2(t_j^n)\Delta t}}}{\sqrt{\lim_{n\to\infty}\sum_{j=0}^{n-1}\sigma^2(t_j^n)\Delta t}} ds \quad \text{(Continuity)}$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \frac{e^{-\frac{(s-\mu)^2}{2\int_{t_1}^{t_2}\sigma^2(t)dt}}}{\sqrt{\int_{t_1}^{t_2}\sigma^2(t)dt}} ds$$

$$= F_Y(x)$$

□

Lemma 5.3. *$S(T) > K \Leftrightarrow Z > -d_1$*

where

$$d_1 := \frac{\int_t^T r + \mu(s) - \frac{\sigma^2(s)}{2} ds - \ln(\frac{K}{S(t)})}{\sqrt{\int_t^T \sigma^2(s)ds}}$$

K is a real constant and Z is a standard normal random variable.

Proof. Let Z be a standard normal variable.

$$S(T) > K \Leftrightarrow S(t)e^{\int_t^T r+\mu(s)-\frac{\sigma^2(s)}{2}ds + \int_t^T \sigma(s)dW(s)} > K \quad \text{(Use the equation (3.6).)}$$

$$\Leftrightarrow \int_t^T r + \mu(s) - \frac{\sigma^2(s)}{2} ds + \int_t^T \sigma(s)dW(s) > \ln(\frac{K}{S(t)}) \quad \text{(Use the lemma (5.2).)}$$

$$\Leftrightarrow \int_t^T r + \mu(s) - \frac{\sigma^2(s)}{2} ds + \sqrt{\int_t^T \sigma^2(s)ds}\; Z > \ln(\frac{K}{S(t)})$$

$$\Leftrightarrow Z > \frac{\ln(\frac{K}{S(t)}) - \int_t^T r + \mu(s) - \frac{\sigma^2(s)}{2} ds}{\sqrt{\int_t^T \sigma^2(s)ds}} =: -d_1$$

□

Lemma 5.4. $E_{\mathbb{Q}}[I|\mathcal{F}_t] = \Phi(d_1)$

where

Φ *is a cumulative distribution function of the standard normal distribution.*

$$d_1 := \frac{\int_t^T r + \mu(s) - \frac{\sigma^2(s)}{2} ds - \ln(\frac{K}{S(t)})}{\sqrt{\int_t^T \sigma^2(s) ds}}$$

and

$$I = \begin{cases} 1 & if\ S(T) > K \\ 0 & if\ S(T) \leq K \end{cases}$$

Proof.

$$\begin{aligned}
E_{\mathbb{Q}}[I|\mathcal{F}_t] &= P(S(T) > K) \\
&= P(Z > \frac{\ln(\frac{K}{S(t)}) - \int_t^T r + \mu(s) - \frac{\sigma^2(s)}{2} ds}{\sqrt{\int_t^T \sigma^2(s) ds}}) \qquad \text{(Lemma 5.3)} \\
&= P(Z < \frac{\int_t^T r + \mu(s) - \frac{\sigma^2(s)}{2} ds - \ln(\frac{K}{S(t)})}{\sqrt{\int_t^T \sigma^2(s) ds}}) \\
&= \Phi(\frac{\int_t^T r + \mu(s) - \frac{\sigma^2(s)}{2} ds - \ln(\frac{K}{S(t)})}{\sqrt{\int_t^T \sigma^2(s) ds}}) \\
&= \Phi(d_1) \qquad \text{where } d_1 := \frac{\int_t^T r + \mu(s) - \frac{\sigma^2(s)}{2} ds - \ln(\frac{K}{S(t)})}{\sqrt{\int_t^T \sigma^2(s) ds}}
\end{aligned}$$

$\square$

Lemma 5.5. $E_{\mathbb{Q}}[S(T)I|\mathcal{F}_t] = S(t)e^{\int_t^T \mu(s)+r\ ds}\Phi(d_2)$

where

$$d_2 := d_1 + \sqrt{\int_t^T \sigma^2(s) ds}$$

$$d_1 := \frac{\int_t^T r + \mu(s) - \frac{\sigma^2(s)}{2} ds - \ln(\frac{K}{S(t)})}{\sqrt{\int_t^T \sigma^2(s) ds}}$$

and I is defined as in previous lemma which is

$$I = \begin{cases} 1 & if\ S(T) > K \\ 0 & if\ S(T) \le K \end{cases}$$

Proof.

$$\begin{aligned}
&E_{\mathbb{Q}}[S(T)I|\mathcal{F}_t] \\
&= \int_{-\infty}^{\infty} S(T)If(y)dy \\
&= \int_{-d_1}^{\infty} S(t)e^{\int_t^T \mu(s)+r-\frac{\sigma^2(s)}{2}ds+\sqrt{\int_t^T \sigma^2(s)ds}\ y}\frac{1}{\sqrt{2\pi}}e^{-\frac{y^2}{2}}dy \quad (I=0 \text{ if } S(T)\le K \text{ and lemma 5.3.}) \\
&= \frac{S(t)e^{\int_t^T \mu(s)+r-\frac{\sigma^2(s)}{2}ds}}{\sqrt{2\pi}}\int_{-d_1}^{\infty} e^{\sqrt{\int_t^T \sigma^2(s)ds}\ y-\frac{y^2}{2}}dy \\
&= \frac{S(t)e^{\int_t^T \mu(s)+r-\frac{\sigma^2(s)}{2}ds}}{\sqrt{2\pi}}\int_{-d_1}^{\infty} e^{-\frac{1}{2}\left(y^2-2\sqrt{\int_t^T \sigma^2(s)ds}\ y+\int_t^T \sigma^2(s)ds\right)+\frac{1}{2}\int_t^T \sigma^2(s)ds}\ dy \\
&= \frac{S(t)e^{\int_t^T \mu(s)+r-\frac{\sigma^2(s)}{2}ds+\frac{1}{2}\int_t^T \sigma^2(s)ds}}{\sqrt{2\pi}}\int_{-d_1}^{\infty} e^{-\frac{1}{2}(y-\sqrt{\int_t^T \sigma^2(s)ds})^2}dy \quad \left(\text{Let } x = y-\sqrt{\int_t^T \sigma^2(s)ds.}\right) \\
&= \frac{S(t)e^{\int_t^T \mu(s)+r-\frac{\sigma^2(s)}{2}ds+\frac{1}{2}\int_t^T \sigma^2(s)ds}}{\sqrt{2\pi}}\int_{-d_2}^{\infty} e^{-\frac{x^2}{2}}dx \quad \left(\text{Let } d_2 = d_1+\sqrt{\int_t^T \sigma^2(s)ds.}\right) \\
&= S(t)e^{\int_t^T \mu(s)+r-\frac{\sigma^2(s)}{2}ds+\frac{1}{2}\int_t^T \sigma^2(s)ds}\frac{1}{\sqrt{2\pi}}\int_{-d_2}^{\infty} e^{-\frac{x^2}{2}}dx \\
&= S(t)e^{\int_t^T \mu(s)+r-\frac{\sigma^2(s)}{2}ds+\frac{1}{2}\int_t^T \sigma^2(s)ds}P(Z > -d_2) \\
&= S(t)e^{\int_t^T \mu(s)+r-\frac{\sigma^2(s)}{2}ds+\frac{1}{2}\int_t^T \sigma^2(s)ds}P(Z < d_2) \\
&= S(t)e^{\int_t^T \mu(s)+rds}\Phi(d_2)
\end{aligned}$$

□

Theorem 5.6. *The no-arbitrage European SET50 Index call options prices with strike price K is*

$$C(T,t,S(t),K) = S(t)e^{\int_t^T \mu(s)ds}\Phi(d_2) - Ke^{-r(T-t)}\Phi(d_1)$$

where

$$d_1 := \frac{\int_t^T r+\mu(s)-\frac{\sigma^2(s)}{2}ds - \ln(\frac{K}{S(t)})}{\sqrt{\int_t^T \sigma^2(s)ds}}.$$

and

$$d_2 := d_1 + \sqrt{\int_t^T \sigma^2(s)ds}$$

Proof. We omit writing the risk neutral measure $\mathbb{Q}$ and the filtration $\mathcal{F}_t$ in this proof to avoid confusion about notations. From the equation (5.1), we obtain

$$\begin{aligned} C(T,t,S(t),K) &= e^{-r(T-t)}E\left[max(0,S(T)-K)\right] \\ &= e^{-r(T-t)}E\left[(S(T)-K)^+\right] \\ &= e^{-r(T-t)}E\left[I(S(T)-K)\right] \\ &= e^{-r(T-t)}E\left[IS(T)\right] - Ke^{-r(T-t)}E\left[I\right] \end{aligned} \tag{5.2}$$

where I is the indicator random variable as previously define that is

$$I = \begin{cases} 1 & if\ S(T) > K \\ 0 & if\ S(T) \leq K \end{cases}$$

From the equation (5.2), we apply lemma 5.4 and 5.5 to find $E\left[I(S(T)\right]$ and $E\left[I\right]$ in this equation. We then complete the proof. □

Next we will consider SET50 Index put options which are European put options.

Theorem 5.7 (S.Rujivan [5])**.** *Under the no-arbitrage assumptions in a futures market, the put option price must equal to the present value of the expected payoff of the put option under the risk neutral measure* $\mathbb{Q}$*. The no-arbitrage options price on day* t *with maturity date* T *and excercise price* K*, denoted by* $P(T,t,S(t),K)$*, must satisfy*

$$P(T,t,S(t),K) = e^{-r(T-t)}E_{\mathbb{Q}}\left[max(0,K-S(T))|\mathcal{F}_t\right] \tag{5.3}$$

Next we will use the similar steps as we use to find European call options price that are rewriting the equation (5.3) and finding $E[I]$ and $E[I(S(T))]$. Then we will substitute these terms into the equation that we derive from the equation (5.3). However, the random variable I will be differently defined.

Lemma 5.8. $S(T) < K \Leftrightarrow Z < -d_1$

where

$$d_1 := \frac{\int_t^T r + \mu(s) - \frac{\sigma^2(s)}{2} ds - \ln(\frac{K}{S(t)})}{\sqrt{\int_t^T \sigma^2(s) ds}}$$

K is a real constant and Z is a standard normal random variable.

Proof. Let Z be a standard normal variable.

$$\begin{aligned}
S(T) < K &\Leftrightarrow S(t)e^{\int_t^T r+\mu(s)-\frac{\sigma^2(s)}{2}ds + \int_t^T \sigma(s)dW(s)} > K \quad \text{(Use the equation (3.6).)} \\
&\Leftrightarrow \int_t^T r + \mu(s) - \frac{\sigma^2(s)}{2} ds + \int_t^T \sigma(s) dW(s) > \ln(\frac{K}{S(t)}) \\
&\Leftrightarrow \int_t^T r + \mu(s) - \frac{\sigma^2(s)}{2} ds + \sqrt{\int_t^T \sigma^2(s) ds}\ Z > \ln(\frac{K}{S(t)}) \\
&\Leftrightarrow Z < \frac{\ln(\frac{K}{S(t)}) - \int_t^T r + \mu(s) - \frac{\sigma^2(s)}{2} ds}{\sqrt{\int_t^T \sigma^2(s) ds}} =: -d_1
\end{aligned}$$

□

d_1 here is the same as it is previously defined in the lemma 5.3.

Lemma 5.9. $E_{\mathbb{Q}}[I|\mathcal{F}_t] = \Phi(-d_1)$

where

Φ is a cumulative distribution function of the standard normal distribution.

$$d_1 := \frac{\int_t^T r + \mu(s) - \frac{\sigma^2(s)}{2} ds - \ln(\frac{K}{S(t)})}{\sqrt{\int_t^T \sigma^2(s) ds}}$$

and

$$I = \begin{cases} 1 & if\ S(T) < K \\ 0 & if\ S(T) \geq K \end{cases}$$

Proof.

$$\begin{aligned}
E_{\mathbb{Q}}[I|\mathcal{F}_t] &= P(S(T) < K) \\
&= P(Z < -d_1) \qquad \text{(Lemma 5.8)} \\
&= \Phi(-d_1)
\end{aligned}$$

□

Lemma 5.10. $E_{\mathbb{Q}}[S(T)I|\mathcal{F}_t] = S(t)e^{\int_t^T \mu(s)+rds}\Phi(-d_2)$

where

$$d_2 := d_1 + \sqrt{\int_t^T \sigma^2(s)ds}$$

$$d_1 := \frac{\int_t^T r + \mu(s) - \frac{\sigma^2(s)}{2}ds - \ln(\frac{K}{S(t)})}{\sqrt{\int_t^T \sigma^2(s)ds}}$$

and I is defined as in the previous lemma (the lemma 5.9) which is

$$I = \begin{cases} 1 & if\ S(T) < K \\ 0 & if\ S(T) \geq K \end{cases}$$

Proof.

$$
\begin{aligned}
&E_{\mathbb{Q}}[S(T)I|\mathcal{F}_t] \\
&\quad = \int_{-\infty}^{\infty} S(T)If(y)dy \\
&\quad = \int_{-\infty}^{-d_1} S(t)e^{\int_t^T \mu(s)+r-\frac{\sigma^2(s)}{2}ds+\sqrt{\int_t^T \sigma^2(s)ds}\ y}\frac{1}{\sqrt{2\pi}}e^{-\frac{y^2}{2}}dy \quad (I = 0 \text{ if } S(T) \geq K \text{ and lemma 5.8.)} \\
&\quad = \frac{S(t)e^{\int_t^T \mu(s)+r-\frac{\sigma^2(s)}{2}ds}}{\sqrt{2\pi}}\int_{-\infty}^{-d_1} e^{\sqrt{\int_t^T \sigma^2(s)ds}\ y-\frac{y^2}{2}}dy \\
&\quad = \frac{S(t)e^{\int_t^T \mu(s)+r-\frac{\sigma^2(s)}{2}ds}}{\sqrt{2\pi}}\int_{-\infty}^{-d_1} e^{-\frac{1}{2}\left(y^2-2\sqrt{\int_t^T \sigma^2(s)ds}\ y+\int_t^T \sigma^2(s)ds\right)+\frac{1}{2}\int_t^T \sigma^2(s)ds}dy \\
&\quad = \frac{S(t)e^{\int_t^T \mu(s)+r-\frac{\sigma^2(s)}{2}ds+\frac{1}{2}\int_t^T \sigma^2(s)ds}}{\sqrt{2\pi}}\int_{-\infty}^{-d_1} e^{-\frac{1}{2}(y-\sqrt{\int_t^T \sigma^2(s)ds})^2}dy \quad \left(\text{Let } x = y - \sqrt{\int_t^T \sigma^2(s)ds.}\right) \\
&\quad = \frac{S(t)e^{\int_t^T \mu(s)+r-\frac{\sigma^2(s)}{2}ds+\frac{1}{2}\int_t^T \sigma^2(s)ds}}{\sqrt{2\pi}}\int_{-\infty}^{-d_2} e^{-\frac{x^2}{2}}dx \quad \left(\text{Let } d_2 = d_1 + \sqrt{\int_t^T \sigma^2(s)ds.}\right) \\
&\quad = S(t)e^{\int_t^T \mu(s)+r-\frac{\sigma^2(s)}{2}ds+\frac{1}{2}\int_t^T \sigma^2(s)ds}\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{-d_2} e^{-\frac{x^2}{2}}dx \\
&\quad = S(t)e^{\int_t^T \mu(s)+r-\frac{\sigma^2(s)}{2}ds+\frac{1}{2}\int_t^T \sigma^2(s)ds}P(Z < -d_2) \\
&\quad = S(t)e^{\int_t^T \mu(s)+rds}\Phi(-d_2)
\end{aligned}
$$

$\square$

Theorem 5.11. *The no-arbitrage European SET50 Index put options prices with strike price K is*

$$P(T,t,S(t),K) = -S(t)e^{\int_t^T \mu(s)ds}\Phi(-d_2) + Ke^{-r(T-t)}\Phi(-d_1)$$

where

$$d_1 := \frac{\int_t^T r + \mu(s) - \frac{\sigma^2(s)}{2}ds - \ln(\frac{K}{S(t)})}{\sqrt{\int_t^T \sigma^2(s)ds}}.$$

and

$$d_2 := d_1 + \sqrt{\int_t^T \sigma^2(s)ds}$$

Proof. We omit writing the risk neutral measure $\mathbb{Q}$ and the filtration $\mathcal{F}_t$ in this proof to avoid confusion about notations. From the equation (5.3), we obtain

$$\begin{aligned} P(T,t,S(t),K) &= e^{-r(T-t)}E\left[max(0, K - S(T))\right] \\ &= e^{-r(T-t)}E\left[(K - S(T))^+\right] \\ &= e^{-r(T-t)}E\left[I(K - S(T))\right] \\ &= Ke^{-r(T-t)}E\left[I\right] - e^{-r(T-t)}E\left[IS(T)\right] \end{aligned} \tag{5.4}$$

where I is the indicator random variable as previously define that is

$$I = \begin{cases} 1 & if\ S(T) < K \\ 0 & if\ S(T) \geq K \end{cases}$$

From the equation (5.4), we apply lemma 5.9 and 5.10 to find $E\left[I(S(T)\right]$ and $E\left[I\right]$ in this equation. We then complete the proof. $\square$

Corollary 5.12. *From both the theorem 5.6 and 5.11, we have the relation in the following form*

$$P(T,t,S(t),K) - C(T,t,S(t),K) = Ke^{-r(T-t)} - S(t)e^{\int_t^T \mu(s)ds}$$

5.2.1 Simulation

We end this chapter by showing the evolution of the options prices obtained from the closed-form solution of call option prices in the theorem 5.6 and put option prices in the theorem 5.11 with the parameters $K = 850, T = 1.475, r = 0.01, \epsilon = 0.005, \mu_1 = 0.15, \mu_2 = 0.2, \mu_3 = 0.1, \sigma_1 = 0.4, \sigma_2 = 0.5, \sigma_3 = 0.55$. Then

$$\mu(t) = \begin{cases} 0.15 & 0 \leq t \leq 0.475 \\ t - 0.325 & 0.475 \leq t \leq 0.525 \\ 0.2 & 0.525 \leq t \leq 0.975 \\ -2t + 2.15 & 0.975 \leq t \leq 1.025 \\ 0.1 & 1.025 \leq t \leq 1.475 \end{cases}$$

and

$$\sigma(t) = \begin{cases} 0.4 & 0 \leq t \leq 0.475 \\ 2t - 0.55 & 0.475 \leq t \leq 0.525 \\ 0.5 & 0.525 \leq t \leq 0.975 \\ t - 0.475 & 0.975 \leq t \leq 1.025 \\ 0.55 & 1.025 \leq t \leq 1.475 \end{cases}$$

From the figure 5.6, at the fixed time, the call option prices are higher when the spot prices are higher. This is consistent with our intuition since the call option contracts buyer have rights to buy the asset at the agreed price (850 points in this case). If we expect the performance to make pofit/loss $(\mu(t), \sigma(t))$ of these different price assets to be the same in the future (in other words during the time $[t, T]$), then for him the higher the price of the asset at time t is, the more valuable the contract is; we all like to buy the valuable asset at the low price. On the contrary, the opposite is true for put option case since people like to sell the low price asset at a high price; then it is better when the spot price is lower. See the figure 5.7.

The other point we want to make here is that at the maturity date the options prices can be found by looking at its payoff functions; notice both the figure 5.6

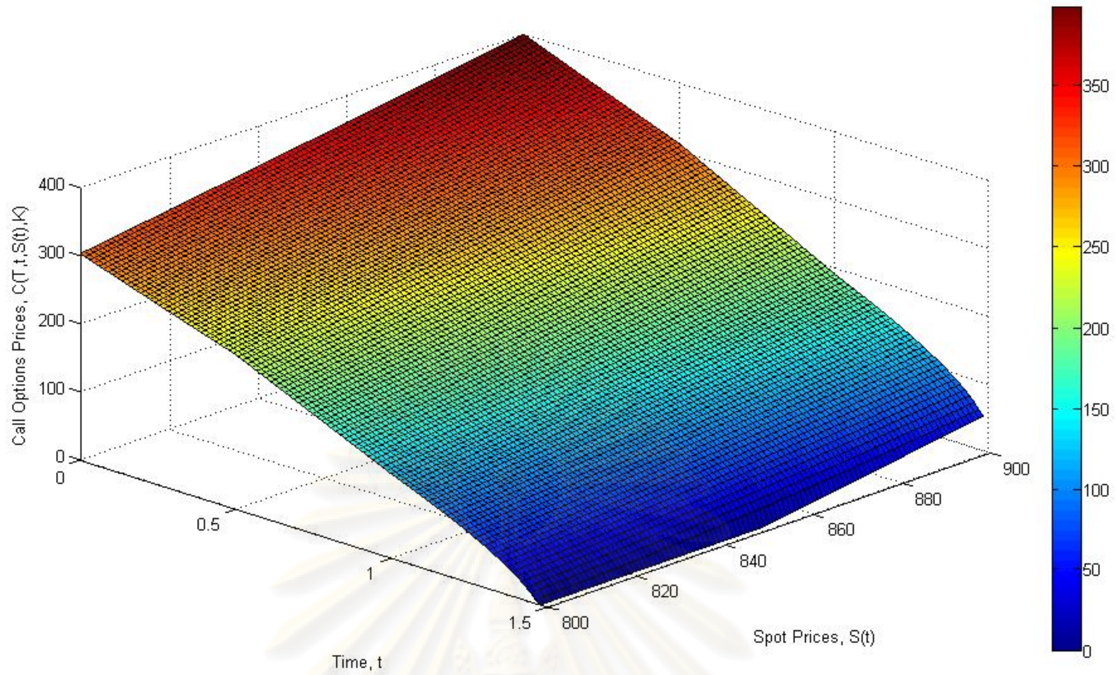

Figure 5.6: Evolution of the call option prices

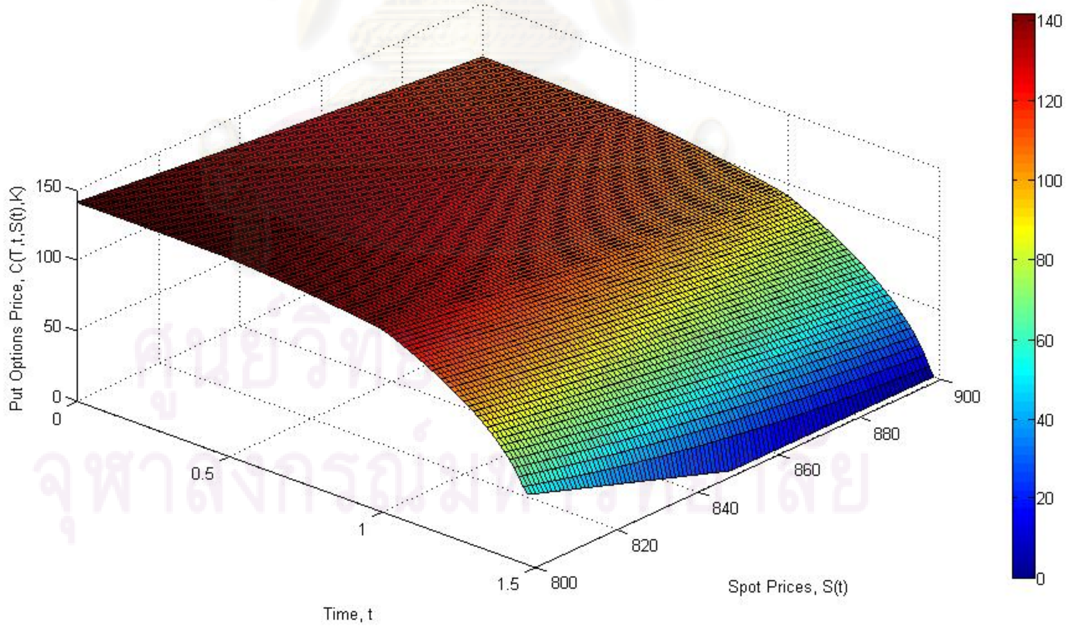

Figure 5.7: Evolution of the call option prices

and 5.7 when the time $t = 1.475$. Comparing to (a) and (c) in the figure 5.5, we can see that in the options prices on maturity date behave like payoff functions. We can see it more clearly if we change the view we look at these two figures from the figure 5.6 and 5.7 to their front view (the figure 5.8 and 5.9).

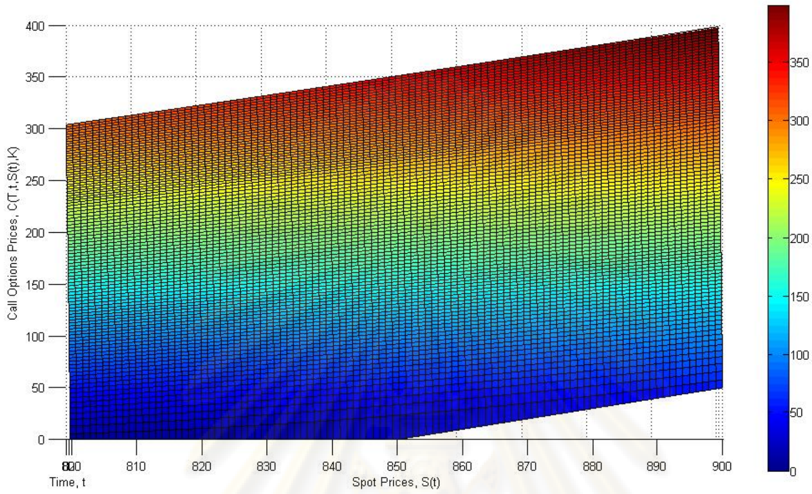

Figure 5.8: Call Option Prices VS Spot Prices

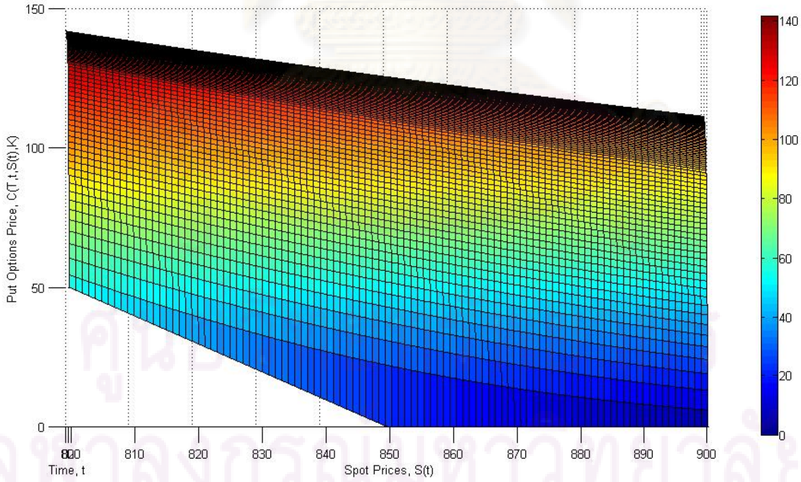

Figure 5.9: Put Option Prices VS Spot Prices

The reason that we can find options prices from payoff functions comes naturally. The payoff functions of option contracts buyer exhibit the financial benefits we get from holding the option contracts in each circumstance (when spot price of the underlying asset on the maturity date varies). Also,when we buy goods, it will be fair if its price is equal to its value. The concept can be applied to our situation here since the payoff function is the financial value of the option contract at the maturity date, so its price on this day is just the payoff function.

CHAPTER VI

Conclusions

By the characteristic of SET50 Index resulting from the procedure called *stock revision* or *index reconstitution*, we have developed a model for SET50 Index from the geometric Brownian motion. Moreover, we derive closed-form solutions for no-arbitrage prices of SET50 Index futures and SET50 Index options under the no-arbitrage assumptions. In addition, both the SET50 Index futures prices and the SET50 Index options prices depend on parameters such as spot prices, $S(t)$, maturity date, T, expected rate of return, $\mu(t)$, volatility $\sigma(t)$, and risk free rate of interest r. Also, these prices are consistent with our intuition. Moreover, one can use our model to predict SET50 Index in the future if the model parameters are estimated using historical data of SET50 Index. Then, the next work that could be completed is to estimate these parameters.

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