ผลของสนามสเกลาร์แบบคามีเลียนต่อเส้นโค้งการหมุนและปรากฏการณ์เลนส์โน้มถ่วงของกาแลกซี่

นายศิรจักร์ พันธุ์พานิช

ศูนย์วิทยทรัพยากร
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วิทยานิพนธ์นี้เป็นส่วนหนึ่งของการศึกษาตามหลักสูตรปริญญาวิทยาศาสตรมหาบัณฑิต
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ลิขสิทธิ์ของจุฬาลงกรณ์มหาวิทยาลัย

EFFECTS OF CHAMELEON SCALAR FIELD ON ROTATION CURVES AND GRAVITATIONAL LENSING OF THE GALAXIES

Mr. Sirachak Panpanich

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ศิรจักร์ พันธุ์พานิช: ผลของสนามสเกลาร์แบบคามีเลียนต่อเส้นโค้งการหมุนและปรากฏการณ์เลนส์โน้มถ่วงของกาแลกซี่. (EFFECTS OF CHAMELEON SCALAR FIELD IN GRAVITATIONAL LENSING FROM DARK MATTER) อ. ที่ปรึกษาวิทยานิพนธ์หลัก: อ. ดร. ปิยบุตร บุรีคำ, 69 หน้า.

เราทำการหาผลของสนามสเกลาร์แบบคามีเลียนบนความหนาแน่นยังผลและความดันยังผลของสสารมืดทรงกลด เนื่องจากความหนาแน่นของสนามสเกลาร์น้อยกว่าความหนาแน่นของสสารมืดมาก ดังนั้นเราจึงไม่สามารถเห็นผลของสนามสเกลาร์แบบคามีเลียนบนมุมหักเหของแสงในปรากฏการณ์เลนส์โน้มถ่วงได้ อย่างไรก็ตามแรงที่ห้า (fifth force) ที่มาจากสนามสเกลาร์แบบคามีเลียนโดยสสารมืดสามารถปรับแต่งเส้นโค้งการหมุนของกาแลกซี่ได้ เราพบว่าแรงที่ห้าสามารถทำให้ความชันของเส้นโค้งการหมุนชันมากขึ้น ในบริเวณรอบ ๆ ศูนย์กลางกาแลกซี่ของทุก ๆ ข้อมูลเฉพาะตัวของสสารมืดตั้งแต่ข้อมูลเฉพาะตัวแบบนาวาร์โร - เฟรงค์-ไวท์ ไปยังแบบซูโดไอโซเทอร์มอล (pseudo-isothermal) ค่าคงตัวการคู่ควบระหว่างสสารกับคามีเลียนจากเส้นโค้งการหมุนของกาแลกซี่ที่มีความสว่างน้อยถูกจำกัดด้วยระดับความเชื่อมั่น 95% โดยที่การจำกัดที่มากที่สุดมาจากข้อมูลเฉพาะตัวแบบซูโดไอโซเทอร์มอล $\left(\approx 10^{-3}\right)$ นอกจากนี้เราแสดงให้เห็นว่าเงื่อนไขขอบเขตแบบทิกค์เชลล์ (thick-shell) ที่ไม่ทำให้เกิดจุดเอกฐาน นำไปสู่การจำกัดที่เข้มงวดของค่าคงตัวการคู่ควบระหว่างสสารกับคามีเลียน $\left(\lesssim 10^{-7}\right)$ เมื่อประยุกต์ใช้กับกาแลกซี่

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We investigate the effects of chameleon scalar field on the effective density and effective pressure of a dark matter halo. Since the density from the scalar field is far smaller than the mass density of dark matter, we cannot see the effects of the chameleon scalar field on the deflection angle of light in the galactic scale. However, the fifth force from the chameleon experienced by the dark matter could modify the rotation curve of galaxies. We found that the fifth force makes the slope of rotation curves steeper around the center of galaxy for any dark matter profiles starting from the Navarro-Frenk-White (NFW) to the pseudo-isothermal (ISO) profile. The coupling constants of matter-chameleon from the rotation curves of low surface brightness (LSB) galaxies are constrained by 95% C.L. where the strongest constraint comes from ISO profile ($\approx 10^{-3}$). Moreover, we demonstrate that the thick-shell non-singular boundary condition which forbids singular point leads to extremely stringent constraint on the matter-chameleon coupling ($\lesssim 10^{-7}$) when applied to galaxy.

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Chapter I

INTRODUCTION

At present, we know that our universe is expanding with acceleration [1, 2, 3], but we do not know the exact cause. We believe that the expansion of the universe is driven by some energy. The popular solution for the problem is called dark energy where we use the word "dark" because we do not know what kind of energy it is. The dark energy has many candidates such as the cosmological constant [4], the 3-form field [5] and the scalar field [6]. The famous models are scalar-tensor theories [7] because it clearly explains how the universe can expand with acceleration. It also explains the inflation [8] generated by the scalar field to drive the exponential expansion of the universe. However, these models have a crucial problem that if we have the scalar field filling space throughout the universe, then why we have never detected it. For example, Quintessence model [6, 9] uses scalar field to drive the universe by supposing that the scalar field is rolling down a potential in a runaway form. At late-time the kinetic energy of the scalar field can be neglected because the driving force from the potential is very small and the friction force from the Hubble parameter is large. We call this the slow-roll condition. This condition can cause the equation of state parameter equal to negative one. The required parameter in the equation of state for the accelerated expansion is less than $-1/3$. Moreover, from the observation data the equation of state parameter today is -1.1 ± 0.14 [3]. But the scalar field of the model is massless, namely interaction range of the scalar field is infinity. Thus, we should have detected the interaction of the scalar field, but we never have. Model which can solve this problem is the Chameleon dark energy model [10, 11, 12, 13, 14] which we will review in chapter 2. The model is one of the scalar-tensor theories as a Quintessence model, but the chameleon scalar field is not massless. The model is situated on the hypothesis that the scalar field has coupling with matter through the conformal coupling term, then mass of the scalar field depends on the local density. Therefore, we call the scalar field a chameleon. The model can address the vital problem of the scalar-tensor theories because in the high density region, the mass of the scalar field is very large, but it is tiny in the low density region. Thus, we cannot see the effects of the scalar field on the earth because the interaction range is very short due to large density with respect to outer space density. Experiments for finding interaction of the chameleon found constraints on the chameleon-photon coupling [15, 16] and the strongest constraints on the chameleon-matter coupling come from particle colliders [17, 18]. Namely, the coupling constant is not arbitrary and the exact value has not been found.

From observation of the rotation curve of galaxies, we found that the circular velocity of objects which orbit around the center of the galaxy is nearly constant, while the Newton's law predicts that the velocity should reduce at large distance. In order to keep the Newton's law, the galaxy must have more unobserved mass which we call dark matter. The dark matter does not interact with anything except through gravity. The dark matter should forms a halo that covers the entire galaxy (or galaxies). Moreover, from gravitational lensing data, it is found that we need more mass of the galaxy in the same order with the rotation curve's missing mass. Since we do not know the exact profile of dark matter in the halo, many theoretical profiles for dark matter halo are proposed which we review briefly in chapter 2.

In chapter 3, we calculate effects of the chameleon scalar field on the rotation curves by using fifth force. The fifth force comes from the coupling of matter with the scalar field in the Einstein frame. We calculate effects in the gravitational lensing by using the effective density and effective pressure of the dark matter halo. In addition, in this chapter we establish constraints of the chameleon-matter coupling constant on the galaxies. The results are shown in chapter 4, and our conclusions are available in chapter 5.

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Chapter II

THEORETICAL REVIEWS

2.1 Dark Energy from the Scalar-Tensor Theories

∗ Note : We use signature $(- + ++)$ and Roman indices (e.g. μ, ν) are $0, 1, 2, 3$ while English character indices (e.g. i, j) are $1, 2, 3$. ∗

Dark Energy and Equation of State Parameter

Our universe has two large scale properties, isotropy and homogeneity. Namely, the universe looks the same in all directions and everywhere. The two properties lead to Friedmann-Robertson-Walker (FRW) metric as the following [19, 20]

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1 - Kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right), \tag{2.1}$$

where $a(t)$ is the scale factor which represents an expansion factor for the distance in the universe and K is a constant which describes the geometry of the spatial section of spacetime. From the observational data [1, 2, 3], we can assume the FRW metric to be flat FRW metric ($K = 0$). Then, the flat FRW metric with cartesian coordinate is

$$ds^2 = -dt^2 + a^2(t)(dx^2 + dy^2 + dz^2). \tag{2.2}$$

At present, we knew that our universe is expanding with acceleration. We can calculate an acceleration of the universe as the following.

From the flat FRW metric, we obtain the metric tensor for expanding universe as the following

$$g_{\mu\nu} = \begin{pmatrix} -1 & & & \\ & a^2 & & \\ & & a^2 & \\ & & & a^2 \end{pmatrix}, \tag{2.3}$$

and

$$g^{\mu\nu} = \begin{pmatrix} -1 & & & \\ & a^{-2} & & \\ & & a^{-2} & \\ & & & a^{-2} \end{pmatrix}. \qquad (2.4)$$

We choose matter in the universe to be perfect fluid which rest in the comoving coordinates and have isotropic pressure for satisfying the isotropic property. Then, the energy-momentum tensor is given by

$$T_{\mu\nu} = (\rho + P)U_\mu U_\nu + P g_{\mu\nu}, \qquad (2.5)$$

where ρ is a matter density of the fluid which equal to energy density in the Natural unit, P is pressure of the fluid and U^μ is a four-velocity of the fluid. Since the perfect fluid is at rest in the comoving coordinates, the four-velocity becomes

$$U^\mu = (1, 0, 0, 0), \qquad (2.6)$$

Thus, the energy-momentum tensor for the (rest) perfect fluid becomes

$$T_{\mu\nu} = \begin{pmatrix} \rho & & & \\ & a^2 P & & \\ & & a^2 P & \\ & & & a^2 P \end{pmatrix}. \qquad (2.7)$$

From definition of the Christoffel symbol:

$$\Gamma^\rho_{\mu\nu} = \frac{1}{2} g^{\rho\lambda}(\partial_\mu g_{\nu\lambda} + \partial_\nu g_{\lambda\mu} - \partial_\lambda g_{\mu\nu}). \qquad (2.8)$$

The non-zero Christoffel symbols are

$$\begin{aligned}
\Gamma^0_{ij} &= \frac{1}{2} g^{0\lambda}(\partial_i g_{j\lambda} + \partial_j g_{\lambda i} - \partial_\lambda g_{ij}), \\
&= \frac{1}{2} g^{00}(\partial_i g_{j0} + \partial_j g_{0i} - \partial_0 g_{ij}), \\
&= \frac{1}{2} g^{00}(-\partial_0 g_{ij}), \\
&= \frac{1}{2}(-1)(-\partial_0 a^2 \delta_{ij}), \\
&= \frac{1}{2}(-1)(-2)\dot{a}a\delta_{ij}, \\
\therefore \Gamma^0_{ij} &= \dot{a}a\delta_{ij}. \qquad (2.9)
\end{aligned}$$

In the similar way, we obtain

$$\begin{aligned}
\Gamma^i_{j0} &= \frac{1}{2}g^{i\lambda}(\partial_j g_{0\lambda} + \partial_0 g_{\lambda j} - \partial_\lambda g_{j0}), \\
&= \frac{1}{2}g^{ii}(\partial_j g_{0i} + \partial_0 g_{ij} - \partial_i g_{j0}), \\
&= \frac{1}{2}g^{ii}(\partial_0 g_{ij}), \\
&= \frac{1}{2}a^{-2}(\partial_0 a^2 \delta^i_j), \\
&= \frac{1}{2}a^{-2}(2a\dot{a}\delta^i_j), \\
\therefore \Gamma^i_{j0} &= \frac{\dot{a}}{a}\delta^i_j,
\end{aligned} \tag{2.10}$$

where the other Christoffel symbols are zero. From the Einstein's field equation

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G T_{\mu\nu}, \tag{2.11}$$

where $R_{\mu\nu}$ is the Ricci tensor and R is the Ricci scalar. Take the metric tensor on the equation, we obtain

$$\begin{aligned}
g^{\mu\nu}(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R &= 8\pi G T_{\mu\nu}), \\
R - \frac{1}{2}(4)R &= 8\pi G T, \\
\therefore R &= -8\pi G T,
\end{aligned} \tag{2.12}$$

where T is a trace of the energy-momentum tensor. Then, the Einstein's field equation can be written as

$$\begin{aligned}
R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}(-8\pi G T) &= 8\pi G T_{\mu\nu}, \\
R_{\mu\nu} &= 8\pi G\left(T_{\mu\nu} - \frac{1}{2}T g_{\mu\nu}\right).
\end{aligned} \tag{2.13}$$

The trace of energy-momentum tensor of the perfect fluid is

$$\begin{aligned}
T &= T^\mu_\mu = g^{\mu\nu}T_{\mu\nu}, \\
&= \mathrm{Tr}\left(\begin{pmatrix} -1 & & & \\ & a^{-2} & & \\ & & a^{-2} & \\ & & & a^{-2} \end{pmatrix}\begin{pmatrix} \rho & & & \\ & a^2P & & \\ & & a^2P & \\ & & & a^2P \end{pmatrix}\right), \\
&= \mathrm{Tr}\begin{pmatrix} -\rho & & & \\ & P & & \\ & & P & \\ & & & P \end{pmatrix}, \\
&= -\rho + 3P.
\end{aligned} \tag{2.14}$$

For $\mu\nu = 00$, the Einstein's field equation becomes

$$R_{00} \;=\; 8\pi G\left(T_{00} - \frac{1}{2}Tg_{00}\right).$$

From a definition of the Ricci tensor

$$R_{\sigma\nu} = R^{\lambda}_{\sigma\lambda\mu} = \partial_\lambda \Gamma^{\lambda}_{\sigma\nu} - \partial_\nu \Gamma^{\lambda}_{\sigma\lambda} + \Gamma^{\lambda}_{\lambda\omega}\Gamma^{\omega}_{\sigma\nu} - \Gamma^{\lambda}_{\mu\omega}\Gamma^{\omega}_{\sigma\lambda}, \tag{2.15}$$

the Ricci tensor component 00 is

$$\begin{aligned}
R_{00} &= \partial_\lambda \Gamma^{\lambda}_{00} - \partial_0 \Gamma^{\lambda}_{0\lambda} + \Gamma^{\lambda}_{\lambda\omega}\Gamma^{\omega}_{00} - \Gamma^{\lambda}_{0\omega}\Gamma^{\omega}_{0\lambda}, \\
&= 0 - \partial_0\left(3\frac{\dot a}{a}\right) + 0 - \left(\frac{\dot a}{a}\right)^2 \delta^i_j \delta^j_i, \\
&= -3\partial_0\left(\frac{\dot a}{a}\right) - 3\left(\frac{\dot a}{a}\right)^2, \\
&= -3\left(\frac{\ddot a}{a} - \left(\frac{\dot a}{a}\right)^2\right) - 3\left(\frac{\dot a}{a}\right)^2, \\
&= -3\frac{\ddot a}{a}.
\end{aligned} \tag{2.16}$$

Thus

$$\begin{aligned}
-3\frac{\ddot a}{a} &= 8\pi G\left(\rho - \frac{1}{2}(-\rho + 3P)(-1)\right), \\
-3\frac{\ddot a}{a} &= 4\pi G\left(\rho + 3P\right), \\
\therefore \frac{\ddot a}{a} &= -\frac{4\pi G}{3}\left(\rho + 3P\right).
\end{aligned} \tag{2.17}$$

The above equation is called the acceleration equation of the universe. If we take the covariance derivative on the Einstein's field equations, we obtain

$$\nabla^\mu\left(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\right) = 8\pi G \nabla^\mu T_{\mu\nu}. \tag{2.18}$$

The left hand side (LHS) is equal to zero because of the Bianchi identity, $\nabla_{[\lambda}R_{\rho\sigma]\mu\nu} = 0$. Therefore, the right hand side (RHS) is equal to zero too. We obtain

$$\nabla^\mu T_{\mu\nu} = 0. \tag{2.19}$$

The above equation means the energy-momentum tensor is conserved. Since μ are dummy indices and we can contract with the metric tensor, then

$$\nabla^\mu T_{\mu\nu} = \nabla_\mu T^\mu_\nu = g_{\alpha\nu}(\nabla_\mu T^{\mu\alpha}). \tag{2.20}$$

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The best way to obtain density and pressure from the energy-momentum tensor is to obtain from T^{μ}_{ν} because it does not depend on any coordinates (invariance). The T^{μ}_{ν} can be written as

$$\begin{aligned}
T^{\mu}_{\nu} &= T_{\mu\sigma}g^{\sigma\nu}, \\
&= \begin{pmatrix} \rho & & & \\ & a^2P & & \\ & & a^2P & \\ & & & a^2P \end{pmatrix} \begin{pmatrix} -1 & & & \\ & a^{-2} & & \\ & & a^{-2} & \\ & & & a^{-2} \end{pmatrix}, \\
&= \begin{pmatrix} -\rho & & & \\ & P & & \\ & & P & \\ & & & P \end{pmatrix}.
\end{aligned} \tag{2.21}$$

For $\nu = 0$, we obtain

$$\begin{aligned}
0 &= \nabla_{\mu}T^{\mu}_{0}, \\
&= \partial_{\mu}T^{\mu}_{0} + \Gamma^{\mu}_{\mu\lambda}T^{\lambda}_{0} - \Gamma^{\lambda}_{\mu 0}T^{\mu}_{\lambda}, \\
&= \partial_0 T^0_0 + \sum_{i=1}^{3}\Gamma^{i}_{i0}T^{0}_{0} - \sum_{i=1}^{3}\Gamma^{i}_{i0}T^{i}_{i}, \quad \because \Gamma^{0}_{00} = 0, \\
&= -\dot{\rho} - 3\left(\frac{\dot{a}}{a}\right)\rho - 3\left(\frac{\dot{a}}{a}\right)P, \quad \because \Gamma^{i}_{i0} = \frac{\dot{a}}{a}, \\
\therefore 0 &= \dot{\rho} + 3\frac{\dot{a}}{a}(\rho + P).
\end{aligned} \tag{2.22}$$

The above equation is called the continuity equation which is very important in cosmology. The equation of state, relationship between ρ and P, is

$$P = w\rho. \tag{2.23}$$

Substitute the equation of state into the continuity equation. Then

$$\begin{aligned}
0 &= \dot{\rho} + 3\frac{\dot{a}}{a}(\rho + w\rho), \\
0 &= \frac{\dot{\rho}}{\rho} + 3\frac{\dot{a}}{a}(1 + w), \\
\therefore \frac{\dot{\rho}}{\rho} &= -3(1 + w)\frac{\dot{a}}{a}.
\end{aligned} \tag{2.24}$$

If w is a constant, the equation can be solved,

$$\begin{aligned}
\frac{1}{\rho}\frac{d\rho}{dt} &= -3(1 + w)\frac{1}{a}\frac{da}{dt}, \\
\frac{d}{dt}(\ln\rho) &= -3(1 + w)\frac{d}{dt}(\ln a), \\
\int d(\ln\rho) &= -3(1 + w)\int d(\ln a), \\
\ln\rho &= -3(1 + w)(\ln a) + \text{Const.},
\end{aligned}$$

At present, we define $t = t_0$, $a = a_0$ and $\rho = \rho_0$. For simplicity, we define a_0 equal to 1. Then

$$\begin{aligned} \ln\rho_0 &= -3(1+w)(\ln 1) + \text{Const.}, \\ \therefore \ln\rho_0 &= \text{Const.} \end{aligned} \tag{2.25}$$

The solution becomes

$$\begin{aligned} \ln\rho &= -3(1+w)(\ln a) + \ln\rho_0, \\ \ln\left(\frac{\rho}{\rho_0}\right) &= \ln a^{-3(1+w)}, \\ \therefore \rho &= \rho_0 a^{-3(1+w)}. \end{aligned} \tag{2.26}$$

The pressure from matter which acts on the universe can be approximated to be zero because we take the matter to be non-relativistic particles. We call the matter that is a dust form. From $P = w\rho$ and the density ρ is not equal to zero. Thus

$$w_{matter} = 0, \tag{2.27}$$

and we obtain

$$\rho_m \propto a^{-3}. \tag{2.28}$$

Then, the energy density of matter decreases as the universe expands. For radiation that includes photon and relativistic matters, the Lagrangian density is

$$\mathcal{L}_{EM} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu}, \tag{2.29}$$

where $F_{\mu\nu}$ is the field strength tensor. From a definition of the energy-momentum tensor

$$T^{\mu\nu} \equiv \frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}\mathcal{L})}{\delta g_{\mu\nu}}. \tag{2.30}$$

Thus, the energy-momentum tensor of radiation is

$$T^{\mu\nu} = \frac{2}{\sqrt{-g}}\left(\frac{\delta\sqrt{-g}}{\delta g_{\mu\nu}}\mathcal{L}_{EM} + \sqrt{-g}\frac{\delta\mathcal{L}_{EM}}{\delta g_{\mu\nu}}\right),$$

where

$$\begin{aligned} \delta\sqrt{-g} &= \frac{\sqrt{-g}}{2}g^{\mu\nu}\delta g_{\mu\nu}, \\ \frac{\delta\mathcal{L}_{EM}}{\delta g_{\mu\nu}} &= -\frac{1}{4}\left(\frac{\delta F^{\lambda\sigma}}{\delta g_{\mu\nu}}F_{\lambda\sigma} + F^{\lambda\sigma}\frac{\delta F_{\lambda\sigma}}{\delta g_{\mu\nu}}\right), \end{aligned}$$

$$\begin{aligned}
\frac{\delta\mathcal{L}_{EM}}{\delta g_{\mu\nu}} &= -\frac{1}{4}\left(2F^{\lambda\sigma}\frac{\delta F_{\lambda\sigma}}{\delta g_{\mu\nu}}\right), \\
&= -\frac{1}{2}F^{\lambda\sigma}\frac{\delta F_{\lambda\sigma}}{\delta g_{\mu\nu}}, \\
&= -\frac{1}{2}F^{\lambda\sigma}\frac{\delta(g_{\sigma\nu}F^{\nu}_{\lambda})}{\delta g_{\mu\nu}}, \\
&= -\frac{1}{2}\delta^{\mu}_{\sigma}F^{\lambda\sigma}F^{\nu}_{\lambda}, \\
&= -\frac{1}{2}F^{\lambda\mu}F^{\nu}_{\lambda}, \\
&= \frac{1}{2}F^{\mu\lambda}F^{\nu}_{\lambda}, \qquad \because F^{\mu\nu} = -F^{\nu\mu}. \qquad (2.31)
\end{aligned}$$

$$\begin{aligned}
\therefore T^{\mu\nu} &= \frac{2}{\sqrt{-g}}\left(\frac{\sqrt{-g}}{2}g^{\mu\nu}\mathcal{L}_{EM} + \sqrt{-g}\frac{\delta\mathcal{L}_{EM}}{\delta g_{\mu\nu}}\right), \\
&= g^{\mu\nu}\mathcal{L}_{EM} + 2\frac{\delta\mathcal{L}_{EM}}{\delta g_{\mu\nu}}, \\
&= -\frac{1}{4}g^{\mu\nu}F^{\lambda\sigma}F_{\lambda\sigma} + 2\frac{\delta\mathcal{L}_{EM}}{\delta g_{\mu\nu}}. \qquad (2.32)
\end{aligned}$$

Therefore, the energy-momentum tensor of radiation is

$$T^{\mu\nu} = -\frac{1}{4}g^{\mu\nu}F^{\lambda\sigma}F_{\lambda\sigma} + F^{\mu\lambda}F^{\nu}_{\lambda}. \qquad (2.33)$$

Trace of the energy-momentum tensor of radiation is

$$\begin{aligned}
T = T^{\mu}_{\mu} &= g_{\mu\nu}T^{\mu\nu}, \\
&= g_{\mu\nu}\left(F^{\mu\lambda}F^{\nu}_{\lambda} - \frac{1}{4}g^{\mu\nu}F^{\lambda\sigma}F_{\lambda\sigma}\right), \\
&= F^{\mu\lambda}F_{\mu\lambda} - \frac{1}{4}(4)F^{\lambda\sigma}F_{\lambda\sigma}, \\
&= 0, \qquad (2.34)
\end{aligned}$$

Since the trace of energy-momentum tensor for the radiation equal to zero, thus

$$\begin{aligned}
-\rho_r + 3P_r &= 0, \\
-\rho_r + 3(w\rho_r) &= 0, \\
\therefore w_{radiation} &= \frac{1}{3}. \qquad (2.35)
\end{aligned}$$

Then, we obtain

$$\rho_r \propto a^{-4}. \qquad (2.36)$$

The energy density of radiation reduces as universe expands where the decreasing rate faster than the energy density of matter.

From the acceleration equation of the universe, we found that the universe can expand with acceleration when $w < -\frac{1}{3}$.

$$\begin{aligned} \frac{\ddot{a}}{a} &= -\frac{4\pi G}{3}\left(\rho + 3w\rho\right), \\ \frac{\ddot{a}}{a} &= -\frac{4\pi G}{3}\left(1 + 3w\right)\rho, \\ \therefore \ddot{a} > 0 \quad &\text{when} \quad w < -\frac{1}{3}. \end{aligned} \tag{2.37}$$

Thus, matter and radiation are not the cause of accelerated expansion, and dark energy must have equation of state parameter as

$$w_{\text{dark energy}} < -\frac{1}{3}. \tag{2.38}$$

This is a condition for all of the dark energy candidates.

For example, the cosmological constant, Λ, is one of the dark energy candidates proposed by Einstein. Original model was proposed to obtain a static universe, namely Einstein believed that our universe does not expand or shrink. Then, energy density of this model is constant ($\rho_\Lambda = Const.$). From the continuity equation, we obtain

$$\begin{aligned} \dot{\rho}_\Lambda + 3\frac{\dot{a}}{a}(\rho_\Lambda + P) &= 0, \\ 0 + 3\frac{\dot{a}}{a}(\rho_\Lambda + P) &= 0, \\ P &= -\rho_\Lambda, \qquad \because a, \dot{a} \neq 0 \\ \therefore w_\Lambda &= -1. \end{aligned} \tag{2.39}$$

Then, the cosmological constant can be the dark energy. However, it is not definitive because from the observational data the equation of state parameter is not exactly equal to minus one. It has uncertainties around the minus one value (-1.1 ± 0.14 [3]).

Therefore, the equation of state parameter (w) can tell us whether something is or is not a dark energy candidate. The equation of state parameter of dark energy must be less than minus one-third ($w_{\text{DE}} < -\frac{1}{3}$) in order to accelerate the expansion of the universe.

Scalar-Tensor Theories

The theory is situated on a hypothesis that we have scalar field in the space throughout the universe, and it is rolling down a scalar potential. Then, at the late-time, the scalar field becomes dark energy because of the slow-roll condition [19, 20]. We can illustrate this mechanism as in Figure 2.1.

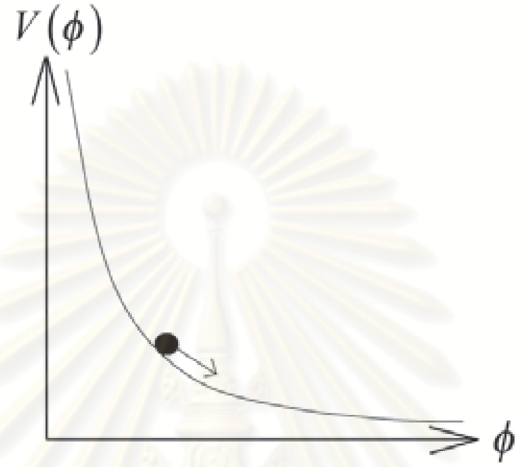

Figure 2.1: The scalar field rolls down along the scalar potential by driving force.

Consider the following action

$$S = \int d^4x\sqrt{-g}\left(-\frac{1}{2}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi - V\left(\phi\right)\right), \qquad (2.40)$$

where ϕ is the scalar field and $V(\phi)$ is the scalar potential. The energy-momentum tensor for the scalar field is (we will prove in section 2.2.1)

$$T^{(\phi)}_{\mu\nu} = \nabla_\mu\phi\nabla_\nu\phi - g_{\mu\nu}\left(\frac{1}{2}g^{\alpha\beta}\nabla_\alpha\phi\nabla_\beta\phi + V(\phi)\right). \qquad (2.41)$$

Since ϕ is a scalar quantity, then $\nabla\phi = \partial\phi$. We obtain

$$T^{\mu(\phi)}_{\nu} = \partial^\mu\phi\partial_\nu\phi - \delta^\mu_\nu\left(\frac{1}{2}g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi + V(\phi)\right). \qquad (2.42)$$

The energy density of scalar field is (the energy-momentum tensor component $\mu\nu = 00$)

$$-\rho = g^{00}\partial_0\phi\partial_0\phi - (1)\left(\frac{1}{2}g^{00}\partial_0\phi\partial_0\phi + \frac{1}{2}g^{ii}\partial_i\phi\partial_i\phi + V(\phi)\right),$$

We assume that the scalar field is homogeneous throughout the space ($\partial_i\phi = 0$).

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Then, we obtain

$$
\begin{aligned}
\rho &= \partial_0\phi\partial_0\phi + \left(\frac{1}{2}g^{00}\partial_0\phi\partial_0\phi + V(\phi)\right), \\
&= \dot{\phi}^2 + \left(-\frac{1}{2}\dot{\phi}^2 + V(\phi)\right), \\
&= \frac{1}{2}\dot{\phi}^2 + V(\phi).
\end{aligned} \tag{2.43}
$$

And the pressure of the scalar field is (the energy-momentum tensor component $\mu\nu = ii$)

$$
\begin{aligned}
P &= g^{ii}\partial_i\phi\partial_i\phi - (1)\left(\frac{1}{2}g^{00}\partial_0\phi\partial_0\phi + \frac{1}{2}g^{ii}\partial_i\phi\partial_i\phi + V(\phi)\right), \\
&= -\left(\frac{1}{2}(-1)\partial_0\phi\partial_0\phi + V(\phi)\right), \\
&= \frac{1}{2}\dot{\phi}^2 - V(\phi).
\end{aligned} \tag{2.44}
$$

Therefore, the equation of state parameter of the scalar field is

$$
w_\phi = \frac{P}{\rho} = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)}. \tag{2.45}
$$

The w_ϕ depends on kinetic energy and scalar potential of the scalar field. Equation of motion of the scalar field can be obtained after varying the action with respect to the field as the following

$$
\begin{aligned}
\delta S &= \int d^4x\sqrt{-g}\left(-\delta\left(\frac{1}{2}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi\right) - \delta V(\phi)\right) \\
&\quad + \int d^4x\delta\sqrt{-g}\left(-\frac{1}{2}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi - V(\phi)\right),
\end{aligned} \tag{2.46}
$$

where

$$
\begin{aligned}
\delta\left(\frac{1}{2}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi\right) &= \frac{1}{2}\left(\delta\nabla_\mu\phi\nabla^\mu\phi + \nabla_\mu\phi\delta\nabla^\mu\phi\right), \\
&= \nabla_\mu\phi\delta\nabla^\mu\phi, \\
&= \nabla_\mu\phi\nabla^\mu\delta\phi,
\end{aligned} \tag{2.47}
$$

$$
\delta V(\phi) = \frac{\partial V}{\partial\phi}\,\delta\phi, \tag{2.48}
$$

$$
\delta\sqrt{-g} = -\frac{\sqrt{-g}}{2}g_{\mu\nu}\delta g^{\mu\nu}. \tag{2.49}
$$

Then

$$
\begin{aligned}
\delta S &= \int d^4x\sqrt{-g}\left(-\nabla_\mu\phi\nabla^\mu\delta\phi - \frac{\partial V}{\partial\phi}\,\delta\phi\right) \\
&\quad - \int d^4x\frac{\sqrt{-g}}{2}g_{\mu\nu}\delta g^{\mu\nu}\left(-\frac{1}{2}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi - V(\phi)\right).
\end{aligned} \tag{2.50}
$$

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Integrate by parts on the first term of RHS

$$\begin{aligned}\int d^4x\sqrt{-g}\,(-\nabla_\mu\phi\nabla^\mu\delta\phi) &= -\sqrt{-g}\nabla_\mu\phi\delta\phi|_{\text{boundary}} + \int d^4x\delta\phi\nabla^\mu(\nabla_\mu\phi\sqrt{-g}), \\ &= \int d^4x\sqrt{-g}(\nabla^\mu\nabla_\mu\phi)\delta\phi, \qquad \because \text{at boundary}\,\delta\phi = 0, \\ &= \int d^4x\sqrt{-g}(\nabla^2\phi)\delta\phi. \qquad (2.51)\end{aligned}$$

So, we obtain

$$\frac{\delta S}{\delta\phi} = \sqrt{-g}\left(\nabla^2\phi - \frac{\partial V}{\partial\phi}\right). \qquad (2.52)$$

Since $\frac{\delta S}{\delta\phi} = 0$, the equation of motion of the scalar field is

$$\nabla^2\phi - \frac{\partial V}{\partial\phi} = 0. \qquad (2.53)$$

For flat spacetime (Minkowski spacetime), $\nabla^2 = \partial_\mu\partial^\mu \equiv \Box$, the equation is called the Klein-Gordon equation. From the assumption that the scalar field is homogeneous throughout the space,

$$\begin{aligned}\nabla^2\phi &= \nabla_\mu(\nabla^\mu\phi) = \nabla_\mu(\partial^\mu\phi), \\ &= \partial_\mu(\partial^\mu\phi) + \Gamma^\mu_{\mu\lambda}(\partial^\lambda\phi), \\ &= g^{00}\partial_0(\partial_0\phi) + \Gamma^\mu_{\mu 0}(\partial^0\phi), \quad \because \partial_i\phi = 0, \\ &= -\ddot{\phi} + 3\frac{\dot{a}}{a}g^{00}\partial_0\phi, \\ &= -\ddot{\phi} - 3\frac{\dot{a}}{a}\dot{\phi}. \qquad (2.54)\end{aligned}$$

From a definition of the Hubble parameter, H, which is a ratio of the relative velocity and distance to the earth of a galaxy $H \equiv \frac{v}{d} = \frac{\dot{a}}{a}$. The equation of motion becomes

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0, \qquad (2.55)$$

where the Hubble parameter acts as a friction term and $\frac{dV}{d\phi}$ acts as a driving term. Thus, if the scalar potential is in the runaway form such as a power-law potential in the Quintessence model, $V(\phi) = \frac{M^{4+n}}{\phi^n}$, the field will roll down the potential. And the field slow rolls when the driving force is small and the friction is large where H is large at late-time. Then, the kinetic term is much smaller than the potential term, and it can be neglected. The equation of state parameter can be approximated to be

$$\begin{aligned}w_\phi &\approx \frac{-V(\phi)}{V(\phi)}, \\ \therefore w_\phi &\approx -1. \qquad (2.56)\end{aligned}$$

So, the scalar field can be the dark energy and also flexible value than the cosmological constant because if we keep the kinetic term, the equation of state parameter can change from minus one a little.

2.2 Chameleon Dark Energy Model

The chameleon dark energy model is one of the scalar-tensor theories, which can explain why we have never found an interaction of the scalar field on the earth. However, the scalar-tensor theories do not have only one problem. The other problem is the scalar-tensor theories assume that our universe is dominated by the scalar field. One way to solve this problem is coupling the scalar field with matter. This coupling also solves the coincidence problem [21, 22]. The coincidence problem is the question of why in the present day we have the matter density in roughly the same order of magnitude as the dark energy density (the density parameter of matter is about 0.276 and of dark energy about 0.72 [3]) while the density parameter of radiation is nearly zero (about 8.24×10^{-5} [3]). If scalar field couples with matter, it will increase the matter density and reduce the dark energy density to the same order of magnitude. In the chameleon model, the scalar field couple with matter through the conformal coupling term which come from conformal transformation. The conformal transformation is a transformation between general action of the scalar-tensor theories in the Jordan frame and action in the Einstein frame which has the Einstein-Hilbert action term. We choose the Einstein frame to be our frame because the Einstein-Hilbert action term leads to the Einstein's field equation. Moreover, the matter and the scalar field couple together in this frame.

2.2.1 Chameleon Equation of Motion

We consider a general action with a single scalar field:

$$S = \int d^4x\sqrt{-g}\left(\frac{M_{Pl}^2}{2}R - \frac{(\partial\phi)^2}{2} - V(\phi)\right) - \int d^4x\mathcal{L}_m\left(\psi_m, \tilde{g}_{\mu\nu}\right), \qquad (2.57)$$

where ϕ is the chameleon scalar field, M_{Pl} is the reduced Planck's mass ($M_{Pl} = 2.43 \times 10^{18}$GeV) and ψ_m is the fermion field. The key point of this model is the conformal coupling of the scalar field and fermion field where the fermion field follows the geodesics of a metric $\tilde{g}_{\mu\nu}$. The metric $\tilde{g}_{\mu\nu}$ relates to the Einstein frame metric $g_{\mu\nu}$ as the following

$$\tilde{g}_{\mu\nu} = A^2(\phi) g_{\mu\nu}, \tag{2.58}$$

where $A(\phi)$ is the conformal coupling term. We now vary this action in order to obtain the equation of motion of chameleon scalar field:

$$\begin{aligned} \delta S &= \int d^4x \sqrt{-g} \left(\frac{M_{Pl}^2}{2} \delta R - \delta \frac{(\partial\phi)^2}{2} - \delta V(\phi) \right) - \int d^4x \delta \mathcal{L}_m (\psi_m, \tilde{g}_{\mu\nu}) \\ &+ \int d^4x \delta \sqrt{-g} \left(\frac{M_{Pl}^2}{2} R - \frac{(\partial\phi)^2}{2} - V(\phi) \right), \end{aligned} \tag{2.59}$$

where

$$\begin{aligned} \delta R &= \delta R_{\mu\nu} g^{\mu\nu} + R_{\mu\nu} \delta g^{\mu\nu}, \qquad (2.60) \\ \delta V(\phi) &= \frac{\partial V}{\partial \phi} \delta\phi, \\ \delta\sqrt{-g} &= -\frac{\sqrt{-g}}{2} g_{\mu\nu} \delta g^{\mu\nu}, \end{aligned}$$

and

$$\begin{aligned} \delta \frac{(\partial\phi)^2}{2} &= \delta \frac{(\nabla\phi)^2}{2}, \\ &= \frac{1}{2} \left(\delta \nabla_\mu \phi \nabla^\mu \phi + \nabla_\mu \phi \delta \nabla^\mu \phi \right), \\ &= \nabla_\mu \phi \delta \nabla^\mu \phi, \\ &= \nabla_\mu \phi \nabla^\mu \delta\phi, \qquad (2.61) \\ \delta \mathcal{L}_m &= \frac{\partial \mathcal{L}_m}{\partial \tilde{g}_{\mu\nu}} \delta \tilde{g}_{\mu\nu} + \frac{\partial \mathcal{L}_m}{\partial \psi_m} \delta \psi_m. \qquad (2.62) \end{aligned}$$

Then, we obtain

$$\begin{aligned} \delta S &= \int d^4x \sqrt{-g} \left(\frac{M_{Pl}^2}{2} \left(\delta R_{\mu\nu} g^{\mu\nu} + R_{\mu\nu} \delta g^{\mu\nu} \right) - \nabla_\mu \phi \nabla^\mu \delta\phi - \frac{\partial V}{\partial \phi} \delta\phi \right) \\ &- \int d^4x \left(\frac{\partial \mathcal{L}_m}{\partial \tilde{g}_{\mu\nu}} \delta \tilde{g}_{\mu\nu} + \frac{\partial \mathcal{L}_m}{\partial \psi_m} \delta \psi_m \right) \\ &- \int d^4x \frac{\sqrt{-g}}{2} g_{\mu\nu} \delta g^{\mu\nu} \left(\frac{M_{Pl}^2}{2} R - \frac{(\partial\phi)^2}{2} - V(\phi) \right). \end{aligned} \tag{2.63}$$

Integrate by parts the second term of the RHS:

$$\begin{aligned} \int d^4x \sqrt{-g} \left(-\nabla_\mu \phi \nabla^\mu \delta\phi \right) &= -\sqrt{-g} \nabla_\mu \phi \delta\phi |_{\text{boundary}} + \int d^4x \delta\phi \nabla^\mu (\nabla_\mu \phi \sqrt{-g}), \\ &= \int d^4x \sqrt{-g} (\nabla^\mu \nabla_\mu \phi) \delta\phi, \qquad \because \text{at boundary } \delta\phi = 0, \\ &= \int d^4x \sqrt{-g} (\nabla^2 \phi) \delta\phi. \end{aligned} \tag{2.64}$$

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Thus

$$\begin{aligned}\delta S &= \int d^4x\sqrt{-g}\left(\frac{M_{Pl}^2}{2}\left(\delta R_{\mu\nu}g^{\mu\nu}+R_{\mu\nu}\delta g^{\mu\nu}\right)+(\nabla^2\phi)\delta\phi-\frac{\partial V}{\partial\phi}\,\delta\phi\right)\\ &\quad -\int d^4x\left(\frac{\partial\mathcal{L}_m}{\partial\tilde{g}_{\mu\nu}}\,\delta\tilde{g}_{\mu\nu}+\frac{\partial\mathcal{L}_m}{\partial\psi_m}\,\delta\psi_m\right)\\ &\quad -\int d^4x\frac{\sqrt{-g}}{2}g_{\mu\nu}\delta g^{\mu\nu}\left(\frac{M_{Pl}^2}{2}R-\frac{(\partial\phi)^2}{2}-V\left(\phi\right)\right).\end{aligned} \tag{2.65}$$

After variation with respect to ϕ, we obtain

$$\begin{aligned}\sqrt{-g}\left(\nabla^2\phi-\frac{\partial V}{\partial\phi}\right)-\frac{\partial\mathcal{L}_m}{\partial\tilde{g}_{\mu\nu}}\frac{\partial\tilde{g}_{\mu\nu}}{\partial\phi} &= 0, \qquad \because\frac{\delta S}{\delta\phi}=0,\\ \nabla^2\phi-\frac{\partial V}{\partial\phi}-\frac{1}{\sqrt{-g}}\frac{\partial\mathcal{L}_m}{\partial\tilde{g}_{\mu\nu}}\frac{\partial\tilde{g}_{\mu\nu}}{\partial\phi} &= 0,\end{aligned}$$

where

$$\frac{\partial\tilde{g}_{\mu\nu}}{\partial\phi}=2A\frac{\partial A}{\partial\phi}g_{\mu\nu}. \tag{2.66}$$

Then, we obtain

$$\begin{aligned}\nabla^2\phi-\frac{\partial V}{\partial\phi}-\frac{2}{\sqrt{-g}}\frac{\partial\mathcal{L}_m}{\partial\tilde{g}_{\mu\nu}}A\frac{\partial A}{\partial\phi}g_{\mu\nu} &= 0,\\ \nabla^2\phi-\frac{\partial V}{\partial\phi}-\frac{2}{\sqrt{-g}}\frac{\partial\mathcal{L}_m}{\partial\tilde{g}_{\mu\nu}}\left(\frac{1}{A}\frac{\partial A}{\partial\phi}\right)A^2g_{\mu\nu} &= 0,\\ \nabla^2\phi-\frac{\partial V}{\partial\phi}-\frac{2}{\sqrt{-g}}\frac{\partial\mathcal{L}_m}{\partial g_{\mu\nu}}\left(\frac{\partial g_{\mu\nu}}{\partial\tilde{g}_{\mu\nu}}\right)\left(\frac{1}{A}\frac{\partial A}{\partial\phi}\right)A^2g_{\mu\nu} &= 0.\end{aligned}$$

From a definition of the energy-momentum tensor of the matter:

$$T^{\mu\nu}\equiv\frac{2}{\sqrt{-g}}\frac{\delta S_m}{\delta g_{\mu\nu}}. \tag{2.67}$$

From action (2.57), the matter Lagrangian is $S_m=-\int d^4x\mathcal{L}_m\left(\psi_m,A^2\left(\phi\right)g_{\mu\nu}\right)$. Thus, the energy-momentum tensor becomes

$$T^{\mu\nu}=-\frac{2}{\sqrt{-g}}\frac{\partial\mathcal{L}_m}{\partial g_{\mu\nu}}. \tag{2.68}$$

We obtain

$$\nabla^2\phi-\frac{\partial V}{\partial\phi}+T^{\mu\nu}\left(\frac{\partial g_{\mu\nu}}{\partial\tilde{g}_{\mu\nu}}\right)\left(\frac{1}{A}\frac{\partial A}{\partial\phi}\right)A^2g_{\mu\nu} = 0,$$

where

$$\frac{1}{A}\frac{\partial A}{\partial\phi} = \frac{\partial\ln A}{\partial\phi}\equiv\alpha_\phi, \tag{2.69}$$

$$\frac{\partial\tilde{g}_{\mu\nu}}{\partial g_{\mu\nu}} = A^2(\phi)\Rightarrow\frac{\partial g_{\mu\nu}}{\partial\tilde{g}_{\mu\nu}}=\frac{1}{A^2(\phi)}, \tag{2.70}$$

$$\frac{\partial V}{\partial\phi} \equiv V_{,\phi} \tag{2.71}$$

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Thus

$$\nabla^2\phi - V_{,\phi} + \alpha_\phi T^{\mu\nu} g_{\mu\nu} = 0. \tag{2.72}$$

The trace of energy-momentum tensor can be written as

$$T^{\mu\nu} g_{\mu\nu} = T_{\mu\nu} g^{\mu\nu} = T^\mu_\mu = T. \tag{2.73}$$

Therefore, the equation of motion of the chameleon scalar field in the Einstein frame is

$$\nabla^2\phi = V_{,\phi} - \alpha_\phi T^\mu_\mu. \tag{2.74}$$

We approximate that matter in the universe is a perfect fluid which is pressureless (non-relativistic). Therefore, the trace of energy-momentum tensor of matter is $-\rho_m$. But the energy-momentum tensor in the Einstein frame does not conserve because the matter couples with the scalar field. Thus, the conserved energy-momentum tensor becomes the total energy-momentum tensor.

$$\nabla^\mu T^{(total)}_{\mu\nu} = \nabla^\mu T^{(m)}_{\mu\nu} + \nabla^\mu T^{(\phi)}_{\mu\nu} = 0. \tag{2.75}$$

In order to get $T^{(\phi)}_{\mu\nu}$ we consider action for scalar field from the action (2.57):

$$S_\phi = \int d^4x\sqrt{-g}\left(-\frac{(\partial\phi)^2}{2} - V(\phi)\right). \tag{2.76}$$

We now vary the above action

$$\delta S_\phi = \int d^4x\sqrt{-g}\left(-\delta\frac{(\partial\phi)^2}{2} - \delta V(\phi)\right) + \int d^4x\delta\sqrt{-g}\left(-\frac{(\partial\phi)^2}{2} - V(\phi)\right), \tag{2.77}$$

where

$$\begin{aligned} \delta\sqrt{-g} &= -\frac{\sqrt{-g}}{2} g_{\mu\nu}\delta g^{\mu\nu}, \\ \frac{(\partial\phi)^2}{2} &= \frac{1}{2} g^{\alpha\beta}\nabla_\alpha\phi\nabla_\beta\phi, \qquad (2.78) \\ \delta\frac{(\partial\phi)^2}{2} &= \frac{1}{2}\delta g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi + \frac{1}{2} g^{\mu\nu}\delta\nabla_\mu\phi\nabla_\nu\phi + \frac{1}{2} g^{\mu\nu}\nabla_\mu\phi\delta\nabla_\nu\phi, \qquad (2.79) \\ \delta V(\phi) &= \frac{\partial V}{\partial\phi}\,\delta\phi. \end{aligned}$$

Then

$$\begin{aligned} \delta S_\phi = & \int d^4x\sqrt{-g}\left(-\frac{1}{2}\delta g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi - \frac{1}{2} g^{\mu\nu}\delta\nabla_\mu\phi\nabla_\nu\phi - \frac{1}{2} g^{\mu\nu}\nabla_\mu\phi\delta\nabla_\nu\phi - \frac{\partial V}{\partial\phi}\,\delta\phi\right) \\ & + \int d^4x\left(-\frac{\sqrt{-g}}{2} g_{\mu\nu}\delta g^{\mu\nu}\right)\left(-\frac{(\nabla\phi)^2}{2} - V(\phi)\right). \end{aligned} \tag{2.80}$$

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After variation with respect to $g^{\mu\nu}$, we obtain

$$\begin{aligned}\frac{\delta S_\phi}{\delta g^{\mu\nu}} &= \sqrt{-g}\left(-\frac{1}{2}\nabla_\mu\phi\nabla_\nu\phi\right) + \sqrt{-g}\left(-\frac{g_{\mu\nu}}{2}\right)\left(-\frac{1}{2}g^{\alpha\beta}\nabla_\alpha\phi\nabla_\beta\phi - V(\phi)\right), \\ &= -\frac{\sqrt{-g}}{2}\left(\nabla_\mu\phi\nabla_\nu\phi - g_{\mu\nu}\left(\frac{1}{2}g^{\alpha\beta}\nabla_\alpha\phi\nabla_\beta\phi + V(\phi)\right)\right). \qquad (2.81)\end{aligned}$$

From a definition of the energy-momentum tensor of the scalar field:

$$T^{(\phi)}_{\mu\nu} \equiv -\frac{2}{\sqrt{-g}}\frac{\delta S_\phi}{\delta g^{\mu\nu}}. \qquad (2.82)$$

Therefore

$$T^{(\phi)}_{\mu\nu} = \nabla_\mu\phi\nabla_\nu\phi - g_{\mu\nu}\left(\frac{1}{2}g^{\alpha\beta}\nabla_\alpha\phi\nabla_\beta\phi + V(\phi)\right). \qquad (2.83)$$

Since the covariant derivative of the scalar field corresponds to the partial derivative, thus

$$T^{(\phi)}_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}\left(\frac{1}{2}g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi + V(\phi)\right). \qquad (2.84)$$

The covariant derivative of the energy-momentum tensor of scalar field is

$$\begin{aligned}\nabla^\mu T^{(\phi)}_{\mu\nu} &= (\nabla^\mu\partial_\mu\phi)\partial_\nu\phi + \partial_\mu\phi(\nabla^\mu\partial_\nu\phi) - (\nabla^\mu g_{\mu\nu})\left(\frac{1}{2}g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi + V(\phi)\right) \\ &\quad -g_{\mu\nu}\nabla^\mu\left(\frac{1}{2}g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi + V(\phi)\right). \qquad (2.85)\end{aligned}$$

From the metric compatibility ($\nabla^\mu g_{\mu\nu} = 0$), we obtain

$$\begin{aligned}\nabla^\mu T^{(\phi)}_{\mu\nu} &= (\nabla^\mu\partial_\mu\phi)\partial_\nu\phi + \partial_\mu\phi(\nabla^\mu\partial_\nu\phi) - g_{\mu\nu}\nabla^\mu\left(\frac{1}{2}g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi + V(\phi)\right), \\ &= (\nabla^\mu\nabla_\mu\phi)\partial_\nu\phi + \partial_\mu\phi(\nabla^\mu\partial_\nu\phi) - g_{\mu\nu}(\nabla^\mu(\frac{1}{2}\partial_\alpha\phi\partial^\alpha\phi) + \nabla^\mu V(\phi)).\end{aligned}$$

We use chain rule at the last term ($\nabla^\mu V(\phi) = \partial^\mu V(\phi) = V_{,\phi}\,\partial^\mu\phi$). Then

$$\begin{aligned}\nabla^\mu T^{(\phi)}_{\mu\nu} &= (\nabla^2\phi)\partial_\nu\phi + \partial_\mu\phi(\nabla^\mu\partial_\nu\phi) - g_{\mu\nu}\partial^\alpha\phi\nabla^\mu\partial_\alpha\phi - g_{\mu\nu}V_{,\phi}\,\partial^\mu\phi, \\ &= (\nabla^2\phi)\partial_\nu\phi + \partial_\mu\phi(\nabla^\mu\partial_\nu\phi) - \partial^\alpha\phi\nabla_\nu\partial_\alpha\phi - V_{,\phi}\,\partial_\nu\phi, \\ &= (\nabla^2\phi - V_{,\phi})\partial_\nu\phi + \partial_\mu\phi(\nabla^\mu\partial_\nu\phi) - \partial^\alpha\phi\nabla_\nu\partial_\alpha\phi,\end{aligned}$$

where

$$\nabla_\nu\partial_\alpha\phi = \partial_\nu\partial_\alpha\phi - \Gamma^\lambda_{\nu\alpha}\partial_\lambda\phi, \qquad (2.86)$$

$$\begin{aligned}\nabla^\mu\partial_\nu\phi &= g^{\mu\alpha}\partial_\alpha\partial_\nu\phi - g^{\mu\alpha}\Gamma^\lambda_{\alpha\nu}\partial_\lambda\phi, \\ &= \partial^\mu\partial_\nu\phi - g^{\mu\alpha}\Gamma^\lambda_{\alpha\nu}\partial_\lambda\phi. \qquad (2.87)\end{aligned}$$

Then

$$\begin{aligned}\partial_\mu\phi(\nabla^\mu\partial_\nu\phi) - \partial^\alpha\phi\nabla_\nu\partial_\alpha\phi &= \partial_\mu\phi(\partial^\mu\partial_\nu\phi - g^{\mu\alpha}\Gamma^\lambda_{\alpha\nu}\partial_\lambda\phi) - \partial^\alpha\phi(\partial_\nu\partial_\alpha\phi - \Gamma^\lambda_{\nu\alpha}\partial_\lambda\phi), \\ &= -\partial^\alpha\phi\Gamma^\lambda_{\alpha\nu}\partial_\lambda\phi + \partial^\alpha\phi\Gamma^\lambda_{\nu\alpha}\partial_\lambda\phi, \\ &= 0. \qquad (2.88)\end{aligned}$$

From the equation of motion, we obtain

$$\begin{aligned}\nabla^{\mu}T^{(\phi)}_{\mu\nu} &= (\nabla^2\phi - V_{,\phi})\partial_{\nu}\phi, \\ &= -\alpha_{\phi}T^{(m)}\partial_{\nu}\phi,\end{aligned} \tag{2.89}$$

and

$$\nabla^{\mu}T^{(m)}_{\mu\nu} = \alpha_{\phi}T^{(m)}\partial_{\nu}\phi. \tag{2.90}$$

We now see that the energy-momentum tensor of matter in Einstein frame is not conserved. Nevertheless, the density in the Einstein frame must obey the continuity equation ($\rho \propto a^{-3(1+w)}$). Then, we have to define a new density which is conserved in Einstein frame.

From $\nabla^{\mu}T^{(m)}_{\mu\nu} = \nabla_{\mu}T^{\mu(m)}_{\nu}$ and we know that for $\nu = 0$ the covariant derivative of energy-momentum tensor is the continuity equation. Therefore, we obtain

$$-\dot{\rho} - 3\left(\frac{\dot{a}}{a}\right)\rho - 3\left(\frac{\dot{a}}{a}\right)P = \alpha_{\phi}T^{(m)}\partial_0\phi, \tag{2.91}$$

or

$$\dot{\rho} + 3H(\rho + P) = -\alpha_{\phi}\dot{\phi}(-\rho + 3P). \tag{2.92}$$

Since we assume that the matter in the universe is pressureless ($P = 0$). Then

$$\dot{\rho} + 3H\rho = \alpha_{\phi}\dot{\phi}\rho. \tag{2.93}$$

We define a new density which conserve in the Einstein frame as

$$\rho \equiv \rho^{(c)}A(\phi), \tag{2.94}$$

where $\rho^{(c)}$ is the conserved density and ρ is the non-conserved density from the energy-momentum tensor. From the continuity equation, we obtain

$$\begin{aligned}\rho^{(c)}\dot{A} + \dot{\rho}^{(c)}A + 3H\rho^{(c)}A &= \alpha_{\phi}\dot{\phi}\rho^{(c)}A, \\ \rho^{(c)}\frac{\partial A}{\partial\phi}\dot{\phi} + \dot{\rho}^{(c)}A + 3H\rho^{(c)}A &= \alpha_{\phi}\dot{\phi}\rho^{(c)}A, \\ \rho^{(c)}\left(\frac{1}{A}\frac{\partial A}{\partial\phi}\right)\dot{\phi} + \dot{\rho}^{(c)} + 3H\rho^{(c)} &= \alpha_{\phi}\dot{\phi}\rho^{(c)}, \\ \alpha_{\phi}\dot{\phi}\rho^{(c)} + \dot{\rho}^{(c)} + 3H\rho^{(c)} &= \alpha_{\phi}\dot{\phi}\rho^{(c)}, \\ \therefore \dot{\rho}^{(c)} + 3H\rho^{(c)} &= 0.\end{aligned} \tag{2.95}$$

Finally, the equation of motion of the chameleon scalar field which depends on conserved density is

$$\nabla^2\phi = V_{,\phi} + \alpha_{\phi}\rho^{(c)}_{m}A(\phi). \tag{2.96}$$

However, the definition of conserved density in the most of original chameleon models is $\rho^{(c)} \equiv \tilde{\rho} A^3(\phi)$. Because, there define with respect to $\tilde{T}^{\mu}_{\mu}$ while the previous definition defines with respect to T^{μ}_{μ}. We will show a consistency of the two definitions as the following.

Since $\det(CA_{n\times n}) = C^n \det(A_{n\times n})$, we obtain

$$\begin{aligned} \det(\tilde{g}_{\mu\nu}) &= \det(A^2(\phi) g_{\mu\nu}), \\ &= A^8(\phi) \det(g_{\mu\nu}), \\ \therefore \sqrt{-\tilde{g}} &= A^4(\phi)\sqrt{-g}. \end{aligned} \tag{2.97}$$

Therefore, the trace of energy-momentum tensor can transform to be

$$\begin{aligned} T^{\mu\nu} g_{\mu\nu} &= -\frac{2}{\sqrt{-g}} \frac{\partial \mathcal{L}_m}{\partial g_{\mu\nu}} g_{\mu\nu}, \\ &= -\frac{2A^4}{\sqrt{-\tilde{g}}} \frac{\partial \mathcal{L}_m}{\partial g_{\mu\nu}} g_{\mu\nu}, \\ &= -\frac{2A^4}{\sqrt{-\tilde{g}}} \frac{\partial \mathcal{L}_m}{\partial \tilde{g}_{\mu\nu}} \frac{\tilde{g}_{\mu\nu}}{g_{\mu\nu}} g_{\mu\nu}, \\ &= A^4 \left(-\frac{2}{\sqrt{-\tilde{g}}} \frac{\partial \mathcal{L}_m}{\partial \tilde{g}_{\mu\nu}} \right) A^2 g_{\mu\nu}, \\ \therefore T^{\mu\nu} g_{\mu\nu} &= A^4(\phi) \tilde{T}^{\mu\nu} \tilde{g}_{\mu\nu}. \end{aligned} \tag{2.98}$$

Thus

$$\rho = \tilde{\rho} A^4(\phi), \tag{2.99}$$

$$P = \tilde{P} A^4(\phi). \tag{2.100}$$

From $\rho \equiv \rho^{(c)} A(\phi)$, we obtain

$$\rho^{(c)} \equiv \tilde{\rho} A^3(\phi). \tag{2.101}$$

Therefore, the two definitions are consistent with each other.

2.2.2 Chameleon Mechanism

ϕ_{min} and Mass of the Chameleon Scalar Field

Since we define the scalar field to be dark energy, the scalar potential $V(\phi)$ must be a runaway form because it can satisfy slow-roll condition in the late-time universe.

We thus choose the scalar potential in a power-law potential as in quintessence models.

$$V(\phi) = \frac{M^{4+n}}{\phi^n}, \tag{2.102}$$

where M is a constant in unit of mass. From the Klein-Gordon equation, $\nabla^2\phi - V_{,\phi} = 0$, we found that the equation of motion of chameleon had effective potential:

$$\begin{aligned}
\nabla^2\phi &= V_{,\phi} + \alpha_\phi \rho_m A(\phi), \\
&= V_{,\phi} + \left(\frac{1}{A}\frac{\partial A}{\partial\phi}\right)\rho_m A, \\
&= V_{,\phi} + \left(\frac{\partial A}{\partial\phi}\right)\rho_m, \\
&= \frac{\partial}{\partial\phi}\left(V(\phi) + \rho_m A(\phi)\right), \\
\therefore V_{eff}(\phi) &= V(\phi) + \rho_m A(\phi).
\end{aligned} \tag{2.103}$$

We choose the conformal coupling is the exponential form $A(\phi) = e^{\beta\phi/M_{Pl}}$ where β is coupling constant between the matter and chameleon scalar field. We can illustrate the effective potential of the chameleon scalar field as in Figure 2.2.

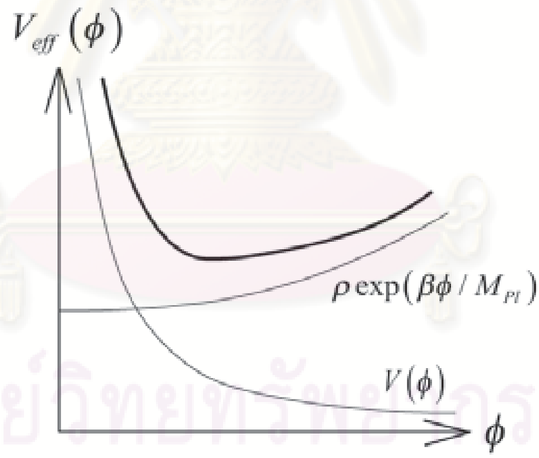

Figure 2.2: The chameleon effective potential is sum of the two lines: one from the scalar potential $V(\phi)$, and the other from its coupling to matter density ρ.

We now see that the potential has minimum when β is positive. Then, value of the scalar field which minimize the effective potential (ϕ_{min}) is

$$\begin{aligned}
\frac{\partial V_{eff}}{\partial\phi}|_{\phi_{min}} = 0 &= \frac{\partial}{\partial\phi}\left(V(\phi) + \rho_m A(\phi)\right)|_{\phi_{min}}, \\
0 &= -n\frac{M^{4+n}}{\phi_{min}^{n+1}} + \frac{\beta}{M_{Pl}}\rho_m e^{\beta\phi_{min}/M_{Pl}},
\end{aligned}$$

$$\begin{aligned} n\frac{M^{4+n}}{\phi_{min}^{n+1}} &= \frac{\beta}{M_{Pl}}\rho_m e^{\beta\phi_{min}/M_{Pl}}, \\ \phi_{min}^{n+1} &= \frac{n}{\beta}\frac{M_{Pl}}{\rho_m}M^{4+n}\frac{1}{e^{\beta\phi_{min}/M_{Pl}}}, \\ \phi_{min}^{n+1} &= \frac{n}{\beta}\frac{M_{Pl}^{5+n}}{\rho_m}\frac{M^{4+n}}{M_{Pl}^{4+n}}\frac{1}{e^{\beta\phi_{min}/M_{Pl}}}. \end{aligned}$$

Since reduced Planck's mass is a very large value ($M_{Pl} = 2.43 \times 10^{18}$GeV), we assume that $e^{\beta\phi_{min}/M_{Pl}} \simeq 1$. Thus

$$\phi_{min} = M_{Pl}\left(\frac{n}{\beta}\frac{M_{Pl}^4}{\rho_m}\left(\frac{M}{M_{Pl}}\right)^{4+n}\right)^{1/(n+1)}. \tag{2.104}$$

And then, mass of small fluctuation about minimum is

$$\begin{aligned} m^2 &= \frac{\partial^2 V_{eff}}{\partial\phi^2}|_{\phi_{min}}, \qquad (2.105) \\ &= \frac{\partial}{\partial\phi}\left(-n\frac{M^{4+n}}{\phi^{n+1}} + \frac{\beta}{M_{Pl}}\rho_m e^{\beta\phi/M_{Pl}}\right)|_{\phi_{min}}, \\ &= n(n+1)\frac{M^{4+n}}{\phi_{min}^{n+2}} + \frac{\beta^2}{M_{Pl}^2}\rho_m e^{\beta\phi/M_{Pl}}, \end{aligned}$$

and again we assume $e^{\beta\phi_{min}/M_{Pl}} \simeq 1$, thus

$$m = \left(n(n+1)\frac{M^{4+n}}{\phi_{min}^{n+2}} + \frac{\beta^2}{M_{Pl}^2}\rho_m\right)^{1/2}. \tag{2.106}$$

We now see that mass and ϕ_{min} depend on local density where at the higher density region such as Earth, the mass is higher than the mass in empty space. Namely, interaction range of the mediated scalar field on the Earth is very short. We then have never detected its effects. So, this scalar field is called a chameleon. The mechanism of ϕ_{min} and mass of chameleon are illustrated as in Figure 2.3.

Moreover, when the scalar field stays at the minimum (ϕ_{min}) we can neglect the kinetic energy term. This leads to the slow-roll condition and late-time acceleration of the universe. Therefore, it is called that chameleon dark energy model.

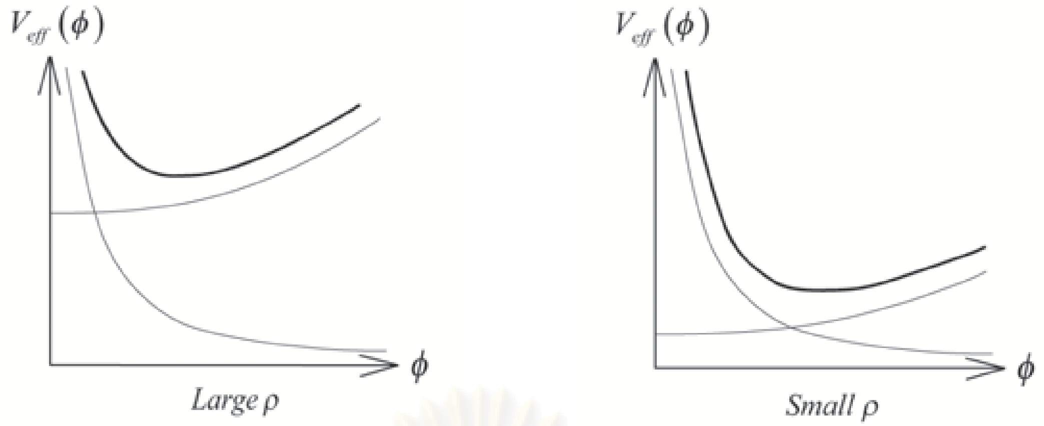

Figure 2.3: Left picture represents the effective potential for large matter density and right picture represents the effective potential for small matter density.

Thin-shell and Thick-shell Regimes

The dynamics of the chameleon scalar field on a massive object can be separated into two regimes, the thin-shell and the thick-shell regime. We consider a massive object has spherical symmetry with homogeneous density, ρ_c, and radius, R_c. From

$$\begin{aligned}\nabla^2\phi &= \nabla_\mu(\nabla^\mu\phi) = \nabla_\mu(\partial^\mu\phi),\\ &= \partial_\mu(\partial^\mu\phi) + \Gamma^\mu_{\mu\lambda}\partial^\lambda\phi.\end{aligned}$$

We assume that the chameleon scalar field is static (does not depend on time) and has spherical symmetry for consistency with the object. Thus

$$\begin{aligned}\nabla^2\phi &= \partial_r(\partial^r\phi) + \Gamma^\mu_{\mu r}\partial^r\phi,\\ &= \partial_r(g^{rr}\partial_r\phi) + \Gamma^\mu_{\mu r}(g^{rr}\partial_r\phi).\end{aligned}$$

The metric tensor of the flat spherical coordinate is

$$g^{\mu\nu} = \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & \frac{1}{r^2} & \\ & & & \frac{1}{r^2\sin^2\theta}\end{pmatrix}. \tag{2.107}$$

We use the flat spherical coordinate because we neglect effect of the spacetime curvature on the chameleon scalar field. Then, we obtain

$$\begin{aligned}\nabla^2\phi &= \partial_r\partial_r\phi + \Gamma^\theta_{\theta r}\partial_r\phi + \Gamma^\phi_{\phi r}\partial_r\phi, \quad \because \Gamma^t_{tr} = \Gamma^r_{rr} = 0,\\ &= \partial_r^2\phi + \frac{1}{r}\partial_r\phi + \frac{1}{r}\partial_r\phi, \quad \because \Gamma^\theta_{\theta r} = \Gamma^\phi_{\phi r} = \frac{1}{r},\\ &= \partial_r^2\phi + \frac{2}{r}\partial_r\phi. \end{aligned} \tag{2.108}$$

Then, the equation of motion of the chameleon is reduced to

$$\frac{d^2\phi}{dr^2} + \frac{2}{r}\frac{d\phi}{dr} = V_{,\phi} + \alpha_\phi \rho(r) A(\phi). \tag{2.109}$$

where $\rho(r) = \rho_c$ for $r < R_c$ and $\rho(r) = \rho_\infty$ (density outside the object) for $r > R_c$. We now see that solution or profile of chameleon on massive object depends on the boundary conditions. If we demand the solution to be non-singular at the origin and the field value outside the object depends on the outside density. Then, the boundary conditions are

$$\frac{d\phi}{dr} = 0 \qquad \text{at} \qquad r = 0, \tag{2.110}$$

$$\phi \to \phi_\infty \qquad \text{as} \qquad r \to \infty, \tag{2.111}$$

where ϕ_∞ is the field value which minimizes the effective potential at the density ρ_∞ (outside the object) according to Eqn. (2.104) and for inside the object (density ρ_c) we denote by ϕ_c. The ϕ_{min} will change into the new minimum when local density changes because of the driving force, $\frac{dV_{eff}}{d\phi}$. During the change, the field is not at rest at the minimum; instead it is rolling on effective potential. Then, ϕ and $\frac{d\phi}{dr}$ must be continuous.

The field begins at rest at the center of the object according to the boundary condition $d\phi/dr = 0$ at $r = 0$, from some initial value (does not necessarily be ϕ_{min}) which we define as the following

$$\phi_i \equiv \phi(r = 0). \tag{2.112}$$

Thin-shell regime: $(\phi_i - \phi_c) \ll \phi_c$. The driving term is negligible and the dynamics are dominated by the friction term, $\frac{1}{r}\frac{d\phi}{dr}$. So, the field value stays at the initial value $\phi_i \approx \phi_c$ until the friction force becomes small (when r becomes large). Namely

$$\phi(r) \approx \phi_c \qquad \text{for} \qquad 0 < r < R_{roll}, \tag{2.113}$$

$$\frac{d\phi}{dr} = 0 \qquad \text{at} \qquad r = R_{roll}, \tag{2.114}$$

where R_{roll} is a radius which the field begins to roll ($R_{roll} < R_c$). Then, in this regime, we have the short range that the chameleon scalar field rolls out from the minimum point of the effective potential between the two difference density (ρ_c and ρ_∞).

Thick-shell regimes: The friction term is not dominating in this case, then the field is displaced from ϕ_c and begin to rolls on the effective potential as soon as it is released at origin (or $R_{roll} \to 0$). Then

$$\phi_i \gtrsim \phi_c \qquad \text{at} \quad r = 0, \tag{2.115}$$

Therefore, in this regime, the range that the chameleon scalar field rolls out from the minimum point of the effective potential is a long range.

2.3 Gravitational lensing

The gravitational lensing is a phenomenon of bending of light trajectory by a massive object. Then, star or galaxy that we see in the universe near a galaxy (or galaxies) is an only image. We can see the effect of gravitational lensing from a galaxy better than a star because it has more mass. The trajectory of light around a galaxy can be illustrated as in Figure 2.4.

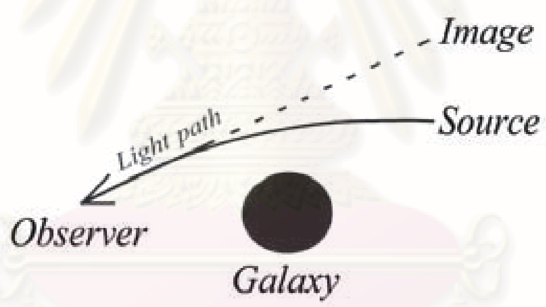

Figure 2.4: A galaxy can bend the light path which cause image of star or galaxy in the background universe.

First, we consider the metric which has a spherical symmetry:

$$ds^2 = -A(r)\,c^2 dt^2 + B(r)\,dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2. \tag{2.116}$$

Divide the metric by proper time, $d\tau$, we obtain a Lagrangian.

$$\begin{aligned} L &= -A(r)\,c^2 \frac{dt^2}{d\tau^2} + B(r)\,\frac{dr^2}{d\tau^2} + r^2 \frac{d\theta^2}{d\tau^2} + r^2 \sin^2\theta \frac{d\phi^2}{d\tau^2}, \\ &= -A(r)\,c^2 \dot{t}^2 + B(r)\,\dot{r}^2 + r^2 \dot{\theta}^2 + r^2 \sin^2\theta \dot{\phi}^2. \end{aligned} \tag{2.117}$$

For simplicity, we suppose that light travels on $\theta = \frac{\pi}{2}$ plane. Then

$$L = -A(r)\,c^2 \dot{t}^2 + B(r)\,\dot{r}^2 + r^2 \dot{\phi}^2. \tag{2.118}$$

We now take this Lagrangian into the Euler-Lagrange equation:

$$\frac{\partial L}{\partial q_i} = \partial_\tau \left(\frac{\partial L}{\partial \dot{q}_i}\right). \tag{2.119}$$

For component t:

$$\begin{aligned} \frac{\partial L}{\partial t} &= \partial_\tau \left(\frac{\partial L}{\partial \dot{t}}\right), \\ 0 &= \partial_\tau(-2Ac^2\dot{t}), \\ \therefore Ac^2\dot{t} &= \text{constant.} \end{aligned} \tag{2.120}$$

For component ϕ:

$$\begin{aligned} \frac{\partial L}{\partial \phi} &= \partial_\tau \left(\frac{\partial L}{\partial \dot{\phi}}\right) \\ 0 &= \partial_\tau(2r^2\dot{\phi}), \\ \therefore r^2\dot{\phi} &= \text{constant.} \end{aligned} \tag{2.121}$$

We define two constants of motion as

$$E \equiv Ac\dot{t}, \tag{2.122}$$

$$L \equiv r^2\dot{\phi}, \tag{2.123}$$

where L is the angular momentum of light. Therefore, the Lagrangian becomes

$$\begin{aligned} L &= -Ac^2\left(\frac{E}{Ac}\right)^2 + B\dot{r}^2 + r^2\left(\frac{L}{r^2}\right)^2, \\ &= -\frac{E^2}{A} + B\dot{r}^2 + \frac{L^2}{r^2}. \end{aligned} \tag{2.124}$$

Since the trajectory of light is a null path, ds^2 of light equal to zero. Then, $L = \dfrac{ds^2}{d\tau^2} = 0$. We obtain

$$\begin{aligned} 0 &= -\frac{E^2}{A} + B\dot{r}^2 + \frac{L^2}{r^2}, \\ B\dot{r}^2 &= \frac{E^2}{A} - \frac{L^2}{r^2}, \\ \dot{r}^2 &= \frac{E^2}{AB} - \frac{L^2}{Br^2}. \end{aligned} \tag{2.125}$$

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For simplicity, we define $u \equiv 1/r$. Then

$$\begin{aligned} \dot{r}^2 &= \left(\frac{dr}{d\tau}\right)^2, \\ &= \left(-r^2\frac{du}{d\tau}\right)^2, \qquad \because du = -\frac{1}{r^2}dr, \\ &= \left(-r^2\frac{du}{d\phi}\frac{d\phi}{d\tau}\right)^2, \\ &= \left(-r^2\dot{\phi}\frac{du}{d\phi}\right)^2, \\ &= L^2\left(\frac{du}{d\phi}\right)^2. \end{aligned} \tag{2.126}$$

Therefore, we obtain

$$\begin{aligned} L^2\left(\frac{du}{d\phi}\right)^2 &= \frac{E^2}{AB} - \frac{L^2u^2}{B}, \\ \left(\frac{du}{d\phi}\right)^2 &= \frac{1}{AB}\frac{E^2}{L^2} - \frac{u^2}{B}, \\ \therefore \frac{du}{d\phi} &= \sqrt{\frac{1}{AB}\frac{E^2}{L^2} - \frac{u^2}{B}}. \end{aligned} \tag{2.127}$$

From the above equation, we can calculate the deflection angle of light when we know the metric component $A(r)$ and $B(r)$. The deflection angle of light defines as in Figure 2.5.

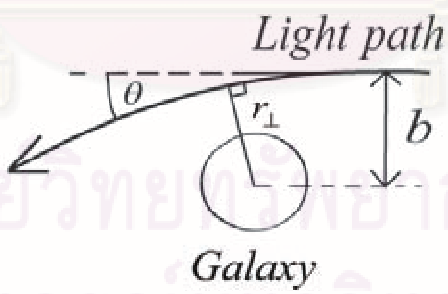

Figure 2.5: θ is the deflection angle of light, b is the impact parameter and $r_\perp$ is a shortest distance between the trajectory of light and center of the lensing galaxy.

In addition, we can fix the constants E and L by the boundary conditions at infinity [23, 24].

$$\frac{dt}{d\tau} = 1 \qquad \text{as} \quad r \to \infty, \tag{2.128}$$

and $A(\text{outside object}) = A(\text{Schwarzschild}) = 1 - \frac{2GM}{rc^2}$. Then

$$A = 1 \qquad \text{as} \quad r \to \infty. \tag{2.129}$$

The constant E at infinity becomes

$$E = Ac\dot{t} = c. \tag{2.130}$$

Since E is constant, it equals to E inside the spherical symmetry metric. Moreover, the constant L is the angular momentum of light. Then

$$L = r^2\dot{\phi} = r_{\perp}c \approx bc. \tag{2.131}$$

Because if the deflection angle is very small, we can assume $r_{\perp} \approx b$ (in fact, the deflection angle of the galaxy or galaxy cluster has value in unit of arcsec, then this approximation is justified). Therefore, we obtain the simple equation to calculate the deflection angle of light as [23, 24]

$$\frac{du}{d\phi} = \sqrt{\frac{1}{AB}\frac{1}{b^2} - \frac{u^2}{B}}. \tag{2.132}$$

2.4 Spherically Symmetric Metric Inside a Star

From a spherically symmetric metric in natural unit $(c = 1, \hbar = 1)$

$$ds^2 = -A(r)\,dt^2 + B(r)\,dr^2 + r^2 d\theta^2 + r^2\sin^2\theta d\phi^2. \tag{2.133}$$

The components of Ricci tensor can be written as [25]

$$\begin{aligned} R^t_t &= R^{tr}_{tr} + R^{t\theta}_{t\theta} + R^{t\phi}_{t\phi}, \\ &= -\frac{A''}{2AB} + \frac{A'B'}{4AB^2} + \frac{A'^2}{4A^2B} - \frac{A'}{rAB}, \end{aligned} \tag{2.134}$$

$$\begin{aligned} R^r_r &= R^{rt}_{rt} + R^{r\theta}_{r\theta} + R^{r\phi}_{r\phi}, \\ &= -\frac{A''}{2AB} + \frac{A'B'}{4AB^2} + \frac{A'^2}{4A^2B} + \frac{B'}{rB^2}, \end{aligned} \tag{2.135}$$

$$\begin{aligned} R^\theta_\theta &= R^{\theta t}_{\theta t} + R^{\theta r}_{\theta r} + R^{\theta\phi}_{\theta\phi}, \\ &= -\frac{A'}{2rAB} + \frac{B'}{2rB^2} + \frac{(B-1)}{Br^2}, \end{aligned} \tag{2.136}$$

$$R^\phi_\phi = R^\theta_\theta, \qquad \because \text{spherical symmetry},$$

where $A' \equiv \dfrac{\partial A}{\partial r}$, $B' \equiv \dfrac{\partial B}{\partial r}$ and $A'' \equiv \dfrac{\partial^2 A}{\partial r^2}$ respectively. From the Einstein's field equation:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G T_{\mu\nu},$$

or

$$R^{\mu}_{\nu} - \frac{1}{2}\delta^{\mu}_{\nu} R \equiv G^{\mu}_{\nu}, \tag{2.137}$$

where G^{μ}_{ν} is the Einstein's tensor ($G^{\mu}_{\nu} = 8\pi G T^{\mu}_{\nu}$). For component t and r, we obtain

$$\begin{aligned}
G^{t}_{t} &= R^{t}_{t} - \frac{1}{2}R, \\
&= R^{t}_{t} - \frac{1}{2}\left(R^{t}_{t} + R^{r}_{r} + R^{\theta}_{\theta} + R^{\phi}_{\phi}\right), \\
&= -\frac{(B-1)}{Br^2} - \frac{B'}{B^2 r}, \qquad (2.138) \\
G^{r}_{r} &= R^{r}_{r} - \frac{1}{2}R, \\
&= R^{r}_{r} - \frac{1}{2}\left(R^{t}_{t} + R^{r}_{r} + R^{\theta}_{\theta} + R^{\phi}_{\phi}\right), \\
&= -\frac{(B-1)}{Br^2} + \frac{A'}{rAB}. \qquad (2.139)
\end{aligned}$$

We assume that matter inside a star is perfect fluid. Thus, the energy-momentum tensor is

$$T^{\mu}_{\nu} = \begin{pmatrix} -\rho & & & \\ & P & & \\ & & P & \\ & & & P \end{pmatrix}. \tag{2.140}$$

Then, Eqn. (2.138) and (2.139) become

$$\begin{aligned}
G^{t}_{t} = -\frac{(B-1)}{Br^2} - \frac{B'}{B^2 r} &= 8\pi G T^{t}_{t} = -8\pi G\rho, \\
\therefore \frac{(B-1)}{Br^2} + \frac{B'}{B^2 r} &= 8\pi G\rho, \qquad (2.141) \\
G^{r}_{r} = -\frac{(B-1)}{Br^2} + \frac{A'}{rAB} &= 8\pi G T^{r}_{r} = 8\pi G P, \\
\therefore \frac{(B-1)}{Br^2} - \frac{A'}{rAB} &= -8\pi G P. \qquad (2.142)
\end{aligned}$$

Therefore, we can calculate the metric $A(r)$ and $B(r)$ inside the star when we know density and pressure or vise verse.

2.5 Rotation Curve and Dark Matter Halo

From observation of circular velocity of star orbiting around the center of galaxies, we found that the velocity is higher than predicted. The predicted value comes

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from the Newton's law through gravity. Then, the Newton's law for an object circularly orbiting around the galactic center can be written as

$$\frac{GM(r)m}{r^2} = \frac{mv_c^2}{r}, \tag{2.143}$$

where m is a mass of an object, v_c is a circular velocity and $M(r)$ is a mass of a galaxy at radius r. Thus, the circular velocity is

$$v_c = \sqrt{\frac{GM(r)}{r}}. \tag{2.144}$$

Now, we see that at large distance the velocity must be decreasing because far from the galactic center the $M(r)$ is a constant. Plot of circular velocity versus distance is called a rotation curve. From observational data the average velocity is quite constant and higher than the predicted value in the far region. According to the Newton's law, we must have more mass in the galaxy than observed, however we had never detected it. We should also assume that those invisible masses do not have any other interactions except gravity. We call the invisible mass is a dark matter. For the dark matter covering a galaxy called a dark matter halo. Then, the rotation curve is a crucial evidence for the existence of dark matter. After calculating the whole mass of the galaxy from observation velocity, we found that dark matter is much more than the visible mass. Furthermore, in the gravitational lensing phenomenon we found that the mass of the galaxy which causes the deflection of light is more than the observed mass. It has the same order of magnitude as those implied from the rotation curves. If we assert that the dark matter does not exist, it means the theory of gravitation such as Newton's law or General Relativity is incomplete and need certain modifications.

Although, we know the amount of masses of the dark matter halo, but we do not know the exact profile of dark matter because it had never been detected directly. Several profiles of dark matter were proposed. We will show some profiles that we are using in this work.

2.5.1 NFW Profile

The Navarro-Frenk-White (NFW) profile [26], which comes from the N-body simulation of the cold dark matter (CDM) is often used to describe the mass distribution of dark matter halo because the density is proportional to $1/r^3$. However, the profile has a singularity at the origin of the halo. The mathematical expression is

$$\rho_{NFW}(r) = \frac{\rho_0}{\frac{r}{a}\left(1+\frac{r}{a}\right)^2}, \tag{2.145}$$

where a is the scale radius, ρ_0 is the characteristic density which depends on critical density of the universe determined by the Hubble parameter. The two parameters are used to fit data of the rotation curves. Mass at arbitrary distance can be obtained from

$$M(r) = \int_0^r 4\pi\rho(r)r^2 dr. \tag{2.146}$$

Since the density diverges at the origin, the physical mass of a galaxy is then given by

$$\begin{aligned} M(r) &= \int_0^r 4\pi\rho(r)r^2 dr, \\ &= \int_0^r 4\pi \frac{\rho_0}{\frac{r}{a}\left(1+\frac{r}{a}\right)^2} r^2 dr, \\ &= 4\pi a^3 \rho_0 \left(\frac{a}{a+r} - \frac{a}{a} + \ln(a+r) - \ln(a) \right), \\ &= 4\pi a^3 \rho_0 \left(\ln\left(\frac{a+r}{a}\right) - \frac{r}{a+r} \right). \end{aligned} \tag{2.147}$$

As $r \to \infty$ the mass became divergent. We then take the edge of the halo to be the virial radius, r_{200}, where the average kinetic energy is equal to 1/2 of average potential energy of the system. It has relation as the following

$$r_{200} = ca, \tag{2.148}$$

where c is the concentration parameter (dimensionless). Then, the accumulated mass is

$$\begin{aligned} M(r_{200}) &= 4\pi a^3 \rho_0 \left(\ln\left(\frac{a+r_{200}}{a}\right) - \frac{r_{200}}{a+r_{200}} \right), \\ &= 4\pi a^3 \rho_0 \left(\ln\left(\frac{a+ca}{a}\right) - \frac{ca}{a+ca} \right), \\ &= 4\pi a^3 \rho_0 \left(\ln(1+c) - \frac{c}{1+c} \right). \end{aligned} \tag{2.149}$$

Since the bracket term in RHS is a constant, then it can be written as mean density within the virial radius, r_{200}. Therefore

$$\begin{aligned} M(r_{200}) &= 4\pi a^3 \rho_0 \left(\ln(1+c) - \frac{c}{1+c} \right), \\ &= 4\pi \frac{r_{200}^3}{c^3} \rho_0 \left(\ln(1+c) - \frac{c}{1+c} \right), \\ &= \frac{4\pi r_{200}^3}{3} \rho_0 \frac{3}{c^3} \left(\ln(1+c) - \frac{c}{1+c} \right), \\ &= \frac{4\pi r_{200}^3}{3} \rho_{\text{mean}}, \end{aligned} \tag{2.150}$$

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where $\rho_{\text{mean}} = \rho_0 \frac{3}{c^3}\left(\ln(1+c) - \frac{c}{1+c}\right)$. In addition, we know that the characteristic density of the halo and the critical density of the universe are related by

$$\rho_0 = \delta_c \rho_{crit}, \tag{2.151}$$

where δ_c is a dimensionless constant which is called the characteristic overdensity of the halo and $\rho_{crit} = \dfrac{3H^2}{8\pi G}$ (where H is the Hubble parameter). Then

$$\rho_{\text{mean}} = \delta_c \rho_{crit} \frac{3}{c^3}\left(\ln(1+c) - \frac{c}{1+c}\right), \tag{2.152}$$

and the mean density within r_{200} is $200 \times \rho_{crit}$. Thus, we obtain

$$\begin{aligned} 200 \times \rho_{crit} &= \delta_c \rho_{crit} \frac{3}{c^3}\left(\ln(1+c) - \frac{c}{1+c}\right), & (2.153)\\ \therefore \delta_c &= \frac{200}{3}\frac{c^3}{(\ln(1+c) - c/(1+c))}. & (2.154)\end{aligned}$$

Therefore, the accumulated mass is

$$M(r_{200}) = 200\rho_{crit}\left(\frac{4\pi r_{200}^3}{3}\right). \tag{2.155}$$

Now, we will find the circular velocity of the star in the NFW profile. From Eqn. (2.147), we obtain

$$\begin{aligned} M(r) &= 4\pi a^3 \rho_0 \left(\ln\left(1+\frac{r}{a}\right) - \frac{r/a}{1+r/a}\right),\\ &= \frac{4\pi r_{200}^3 \rho_0}{c^3}\left(\ln\left(1+\frac{rc}{r_{200}}\right) - \frac{rc/r_{200}}{1+rc/r_{200}}\right),\\ &= \frac{4\pi r_{200}^3 \rho_0}{c^3}\left(\ln(1+cx) - \frac{cx}{1+cx}\right), \qquad (2.156)\end{aligned}$$

where $x = r/r_{200}$. We obtain the circular velocity at virial radius:

$$V_{200} = \sqrt{\frac{GM(r_{200})}{r_{200}}}. \tag{2.157}$$

Then

$$\begin{aligned} \frac{v_c^2(r)}{V_{200}^2} &= \frac{GM(r)}{r}\frac{r_{200}}{GM(r_{200})},\\ &= \frac{M(r)}{M(r_{200})}\frac{r_{200}}{r},\\ &= \frac{M(r)}{M(r_{200})}\frac{1}{x},\\ &= \frac{4\pi r_{200}^3\rho_0/c^3\left(\ln(1+cx) - \frac{cx}{1+cx}\right)}{4\pi r_{200}^3\rho_0/c^3\left(\ln(1+c) - \frac{c}{1+c}\right)}\frac{1}{x},\end{aligned}$$

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because $x = 1$ at $r = r_{200}$.

$$\therefore \frac{v_c^2(r)}{V_{200}^2} = \frac{\left(\ln(1+cx) - \frac{cx}{1+cx}\right)}{\left(\ln(1+c) - \frac{c}{1+c}\right)} \frac{1}{x}.$$

Therefore, the circular velocity of the NFW profile is

$$v_c(r) = V_{200}\sqrt{\frac{1}{x}\frac{\ln(1+cx) - cx/(1+cx)}{\ln(1+c) - c/(1+c)}}. \tag{2.158}$$

Although, the NFW profile is supported by simulation from the ΛCDM model (model of the universe which contains only cold dark matter and the cosmological constant), but results from simulation and observational data have contrast. From observations on low surface brightness (LSB) galaxies and gas-rich dwarf galaxies, the rotation curves around the center of galaxies quite are a constant slope while the simulations indicate a steep slope which called the cusp. This is known as the core-cusp problem [27] where the word "core" means the density around the center of LSB galaxies is approximately constant.

2.5.2 ISO Profile

Pseudo-isothermal (ISO) profile [28] supposes that dark matter stay in the isothermal sphere because we want the constant circular velocity at large distance, then the kinetic energy of each dark matter particle should be equal. The expression of the pseudo-isothermal profile is

$$\rho_{ISO}(r) = \frac{\rho_0}{1 + \left(\frac{r}{R_c}\right)^2}, \tag{2.159}$$

where ρ_0 and R_c are the central density and core radius of the dark matter halo respectively. At large distance, the profile becomes the singular isothermal sphere (SIS) profile (the SIS profile depends on $1/r^2$) which can produce the constant circular velocity. The pseudo-isothermal profile can produce a constant slope of the rotation curves around the center of the galaxy. The mass of ISO halo can be written as

$$\begin{aligned} M(r) &= \int_0^r 4\pi\rho(r)r^2 dr, \\ &= \int_0^r 4\pi\frac{\rho_0}{1 + \left(\frac{r}{R_c}\right)^2} r^2 dr, \\ &= 4\pi\rho_0 R_c^2\left(r - R_c \arctan\left(\frac{r}{R_c}\right)\right). \end{aligned}$$

The above equation is equal to zero when $r = 0$, then $M(0) = 0$.

$$\therefore M(r) \;=\; 4\pi\rho_0 R_c^2\left(r - R_c \arctan\left(\frac{r}{R_c}\right)\right). \tag{2.160}$$

Then the circular speed becomes

$$\begin{aligned} v_c &= \sqrt{\frac{GM(r)}{r}}, \\ \therefore v_c &= \sqrt{4\pi G\rho_0 R_c^2\left(1 - \frac{R_c}{r}\arctan\left(\frac{r}{R_c}\right)\right)}. \end{aligned} \tag{2.161}$$

The circular velocity of the ISO halo depends on two fitting parameter (ρ_0 and R_c).

2.6 Equation of Motion of the Perfect Fluid

From the energy-momentum tensor of perfect fluid in SI unit:

$$T^{\mu\nu}_{\text{perfect fluid}} = (\rho + \frac{P}{c^2})U^\mu U^\nu + Pg^{\mu\nu}, \tag{2.162}$$

and $U^\mu = (c, \vec{v})$. Then, in the cartesian coordinate, we obtain

$$T^{\mu\nu} = \begin{pmatrix} \rho c^2 & \rho c v_x + P\frac{v_x}{c} & \rho c v_y + P\frac{v_y}{c} & \rho c v_z + P\frac{v_z}{c} \\ \rho c v_x + P\frac{v_x}{c} & \rho v_x^2 + P\frac{v_x^2}{c^2} + P & \rho v_x v_y + P\frac{v_x v_y}{c^2} & \rho v_x v_z + P\frac{v_x v_z}{c^2} \\ \rho c v_y + P\frac{v_y}{c} & \rho v_y v_x + P\frac{v_y v_x}{c^2} & \rho v_y^2 + P\frac{v_y^2}{c^2} + P & \rho v_y v_z + P\frac{v_y v_z}{c^2} \\ \rho c v_z + P\frac{v_z}{c} & \rho v_z v_x + P\frac{v_z v_x}{c^2} & \rho v_z v_y + P\frac{v_z v_y}{c^2} & \rho v_z^2 + P\frac{v_z^2}{c^2} + P \end{pmatrix}, \tag{2.163}$$

where $\dfrac{v}{c} = \beta$. For the non-relativistic fluid, $\beta \ll 1$, we will neglect the terms that contain β for the non-relativistic fluid. Thus

$$T^{\mu\nu} = \begin{pmatrix} \rho c^2 & \rho c v_x & \rho c v_y & \rho c v_z \\ \rho c v_x & \rho v_x^2 + P & \rho v_x v_y & \rho v_x v_z \\ \rho c v_y & \rho v_y v_x & \rho v_y^2 + P & \rho v_y v_z \\ \rho c v_z & \rho v_z v_x & \rho v_z v_y & \rho v_z^2 + P \end{pmatrix}. \tag{2.164}$$

For approximately flat cartesian spacetime, we obtain

$$\nabla_\nu T^{\mu\nu} = \partial_\nu T^{\mu\nu} = 0.$$

For $\mu = 0$

$$\begin{aligned} \partial_0 T^{00} + \partial_i T^{0i} &= 0, \\ \frac{1}{c}\partial_t(\rho c^2) + \partial_i(\rho c v^i) &= 0, \\ \therefore \partial_t \rho + \vec{\nabla}\cdot(\rho\vec{v}) &= 0, \end{aligned} \tag{2.165}$$

which is the continuity equation.

For $\mu = i$

$$\begin{aligned}
\partial_0 T^{i0} + \partial_j T^{ij} &= 0, \\
\frac{1}{c}\partial_t(\rho c v^i) + \partial_i(\rho v^i v^i + P) + \partial_j(\rho v^i v^j) &= 0, \\
\partial_t(\rho v^i) + \partial_i(\rho v^i v^i) + \partial_j(\rho v^i v^j) &= -\partial_i P, \\
\rho(\partial_t v^i + v^j \partial_j v^i + v^i \partial_j v^j + 2v^i \partial_i v^i) &= -\partial_i P.
\end{aligned}$$

If the fluid does not have source for the flow, we can neglect the divergence term $(\partial_i v^i)$. Then

$$\rho(\partial_t v^i + \vec{v} \cdot \vec{\nabla} v^i) = -\partial_i P.$$

Therefore, we obtain

$$\rho(\partial_t \vec{v} + \vec{v} \cdot \vec{\nabla} \vec{v}) = -\vec{\nabla} P. \tag{2.166}$$

The above equation is the equation of motion or the Euler equation of fluid.

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Chapter III

EFFECTS OF CHAMELEON SCALAR FIELD

In order to find effects of the chameleon scalar field on the rotation curve and gravitational lensing, the essential components are pressure and density of the scalar field. Since the chameleon scalar field has coupling with matter, energy-momentum tensor of matter is not conserved. The energy-momentum tensor of the scalar field can have visible effects on the dark matter halo.

3.1 Effective Pressure and Effective Density

We suppose the dark matter is perfect fluid and has spherically symmetry. Then, from spherically symmetric metric inside a star, the Einstein's field equation can be written as

$$\frac{(B-1)}{Br^2} + \frac{B'}{B^2 r} = 8\pi G\rho, \tag{3.1}$$

$$\frac{(B-1)}{Br^2} - \frac{A'}{rAB} = -8\pi GP. \tag{3.2}$$

From Bianchi's identity the energy-momentum tensor must be conserved, but the energy-momentum tensor of matter is not served because of the coupling with the scalar field. Then, conserved quantity in Einstein's field equations becomes the total energy-momentum tensor.

$$\nabla_\mu T^{\mu\nu}_{(total)} = 0, \tag{3.3}$$

where

$$\nabla_\mu T^{\mu\nu}_{(total)} = \nabla_\mu T^{\mu\nu}_{(matter)} + \nabla_\mu T^{\mu\nu}_{(\phi)}. \tag{3.4}$$

The energy-momentum tensor of matter (perfect fluid) and scalar field are

$$T_\nu^{\mu(matter)} = \begin{pmatrix} -\rho & & & \\ & P & & \\ & & P & \\ & & & P \end{pmatrix}, \tag{3.5}$$

$$T_\nu^{\mu(\phi)} = \partial^\mu\phi\partial_\nu\phi - \delta^\mu_\nu\left(\frac{1}{2}g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi + V(\phi)\right). \tag{3.6}$$

Since we suppose that the halo has spherical symmetry, for consistency, we suppose the chameleon has spherical symmetry and static too. Then

$$\begin{aligned} T_t^{t(\phi)} &= \partial^t\phi\partial_t\phi - \delta^t_t\left(\frac{1}{2}(g^{tt}\partial_t\phi\partial_t\phi + g^{rr}\partial_r\phi\partial_r\phi + g^{\theta\theta}\partial_\theta\phi\partial_\theta\phi + g^{\phi\phi}\partial_\phi\phi\partial_\phi\phi) + V(\phi)\right), \\ &= g^{tt}\partial_t\phi\partial_t\phi - \left(\frac{1}{2}(g^{tt}\partial_t\phi\partial_t\phi + g^{rr}\partial_r\phi\partial_r\phi) + V(\phi)\right), \\ &= -\frac{1}{2}g^{rr}\partial_r\phi\partial_r\phi - V(\phi), \end{aligned} \tag{3.7}$$

and

$$\begin{aligned} T_r^{r(\phi)} &= \partial^r\phi\partial_r\phi - \delta^r_r\left(\frac{1}{2}(g^{tt}\partial_t\phi\partial_t\phi + g^{rr}\partial_r\phi\partial_r\phi + g^{\theta\theta}\partial_\theta\phi\partial_\theta\phi + g^{\phi\phi}\partial_\phi\phi\partial_\phi\phi) + V(\phi)\right), \\ &= g^{rr}\partial_r\phi\partial_r\phi - \left(\frac{1}{2}(g^{tt}\partial_t\phi\partial_t\phi + g^{rr}\partial_r\phi\partial_r\phi) + V(\phi)\right), \\ &= \frac{1}{2}g^{rr}\partial_r\phi\partial_r\phi - V(\phi). \end{aligned} \tag{3.8}$$

From spherical symmetry metric, the inverse metric tensor is

$$g^{\mu\nu} = \begin{pmatrix} -\frac{1}{A(r)} & & & \\ & \frac{1}{B(r)} & & \\ & & \frac{1}{r^2} & \\ & & & \frac{1}{r^2\sin^2\theta} \end{pmatrix}. \tag{3.9}$$

Therefore, the density and pressure of the chameleon scalar field are [29]

$$T_t^{t(\phi)} = -\rho_{(\phi)} \quad\Rightarrow\quad \rho_{(\phi)} = \frac{\phi'^2}{2B} + V(\phi), \tag{3.10}$$

$$T_r^{r(\phi)} = P^r_{(\phi)} \quad\Rightarrow\quad P^r_{(\phi)} = \frac{\phi'^2}{2B} - V(\phi). \tag{3.11}$$

Then, the Einstein's field equations with total energy-momentum tensor can be written as

$$\frac{(B-1)}{Br^2} + \frac{B'}{B^2r} = 8\pi G(\rho_m + \frac{\phi'^2}{2B} + V(\phi)), \tag{3.12}$$

$$\frac{(B-1)}{Br^2} - \frac{A'}{rAB} = 8\pi G(-P_m - \frac{\phi'^2}{2B} + V(\phi)). \tag{3.13}$$

We will set the pressure of matter to zero by assuming that the dark matter is the non-relativistic fluid. We now define the effective density and effective pressure as

$$\rho_{eff} \equiv \rho_m + \frac{\phi'^2}{2B} + V(\phi), \tag{3.14}$$

$$P_{eff} \equiv \frac{\phi'^2}{2B} - V(\phi). \tag{3.15}$$

There are useful tools to find the effect on gravitational lensing, but we must know a profile of the scalar field in the dark matter halo. We will calculate the profile in the next section.

3.2 Chameleon Profile for Dark Matter Halo

First, we consider the equation of motion of the chameleon scalar field

$$\nabla^2\phi = V_{,\phi} + \alpha_\phi \rho_m A(\phi). \tag{3.16}$$

We assume that the scalar field is static and spherical symmetric. The effect of the spacetime curvature for the halo on the scalar field is negligible. Then

$$\frac{d^2\phi}{dr^2} + \frac{2}{r}\frac{d\phi}{dr} = V_{,\phi} + \alpha_\phi \rho(r) A(\phi), \tag{3.17}$$

where $\rho(r)$ depends on profile of dark matter halo such as Navarro-Frenk-White (NFW) or pseudo-isothermal (ISO). We choose a self-interacting potential (power-law potential), $V(\phi) = \frac{M^{4+n}}{\phi^n}$, and conformal coupling in the exponential form $A(\phi) = e^{\beta\phi/M_{Pl}}$ as the original chameleon dark energy model. Therefore

$$\frac{d^2\phi}{dr^2} + \frac{2}{r}\frac{d\phi}{dr} = -n\frac{M^{4+n}}{\phi^{n+1}} + \frac{\beta}{M_{Pl}}\rho(r)e^{\beta\phi/M_{Pl}}. \tag{3.18}$$

We can obtain the profile of the chameleon by a numerical method because this is a non-linear differential equation. We set the dynamics of chameleon in the thick-shell regime [30]. Namely, we assume that the value of the scalar field which minimizes the effective potential (ϕ_{min}) only stay at the exterior of dark matter halo. The value of the scalar field in the interior of halo is not ϕ_{min} but $\phi(r)$ that is determined by the matter density of the halo.

Then, we will solve Eqn. (3.18) from the edge to the center of dark matter halo. In this work, we choose a constant in a self-interacting potential $M = 10^{-3}$eV because we want to refer the constraint of M from profile of the chameleon scalar field on the Earth [10, 11, 12, 13, 14].

For an analytic solution, we approximate the equation to be a linear differential equation by neglect the potential term and set $e^{\beta\phi/M_{Pl}} \simeq 1$ since M_{Pl} is very large and M is small. We obtain

$$\begin{aligned} \frac{d^2\phi}{dr^2} + \frac{2}{r}\frac{d\phi}{dr} &= \frac{\beta}{M_{Pl}}\rho(r), \\ \frac{1}{r^2}\frac{d}{dr}\left(\phi' r^2\right) &= \frac{\beta}{M_{Pl}}\rho(r). \end{aligned} \tag{3.19}$$

We integrate the above equation from $r = 0$ to r:

$$\begin{aligned} \int_{\phi' r^2|_{r=0}}^{\phi' r^2} d(\phi' r^2) &= \frac{\beta}{M_{Pl}}\int_0^r \rho(r) r^2 dr, \\ \phi' r^2 - (\phi' r^2)|_{r=0} &= \frac{\beta}{4\pi M_{Pl}}\int_0^r 4\pi\rho(r) r^2 dr. \end{aligned}$$

The integral on the RHS becomes the mass. Thus

$$\phi' r^2 - (\phi' r^2)|_{r=0} = \frac{\beta}{4\pi M_{Pl}}(M(r) - M(0)).$$

We define $M(r) - M(0) \equiv M(r)$. Then

$$\begin{aligned} \phi' r^2 - (\phi' r^2)|_{r=0} &= \frac{\beta}{4\pi M_{Pl}} M(r), \\ \phi'(r) &= \frac{\beta}{4\pi M_{Pl}}\frac{M(r)}{r^2} + \frac{1}{r^2}(\phi' r^2)|_{r=0}. \end{aligned} \tag{3.20}$$

Since $(\phi' r^2)|_{r=0}$ is a constant, we define to be a constant C.

$$\phi'(r) = \frac{\beta}{4\pi M_{Pl}}\frac{M(r)}{r^2} + \frac{C}{r^2}. \tag{3.21}$$

The $\phi'(r)$ at r_{max} is not necessarily equal to zero, we suppose $\phi'(r_{max}) = \alpha$. The r_{max} is the distance at the density of dark matter equal to the density of the background universe ($\approx 10^{-26}\text{kg/m}^3$). Then

$$\begin{aligned} \alpha &= \frac{\beta}{4\pi M_{Pl}}\frac{M(r_{max})}{r_{max}^2} + \frac{C}{r_{max}^2}, \\ \therefore C &= \alpha r_{max}^2 - \frac{\beta}{4\pi M_{Pl}} M(r_{max}). \end{aligned}$$

Substitute C into Eqn. (3.21), we obtain

$$\begin{aligned} \phi'(r) &= \frac{\beta}{4\pi M_{Pl}}\frac{M(r)}{r^2} + \frac{1}{r^2}\left(\alpha r_{max}^2 - \frac{\beta}{4\pi M_{Pl}} M(r_{max})\right), \\ &= \frac{\beta}{4\pi M_{Pl} r^2}(M(r) - M(r_{max})) + \frac{\alpha r_{max}^2}{r^2}. \end{aligned}$$

We define

$$\alpha r_{max}^2 \equiv \frac{\beta\alpha'}{4\pi M_{Pl}}. \tag{3.22}$$

Thus

$$\phi'(r) \;=\; \frac{\beta}{4\pi M_{Pl} r^2}(M(r) - M(r_{max}) + \alpha'), \tag{3.23}$$

where $M(r_{max})$ is an accumulated mass of the halo and α' relates to changing of field value at the edge of halo. This is a chameleon profile for the dark matter halo when we approximate the equation of motion to be a linear differential equation.

3 Solutions of Chameleon Profile

From the above equation beside the dark matter profiles, the chameleon profile depends on α' (or α) so that we can separate into 4 cases as the following;

Case I : $\alpha' = 0$. We obtain

$$\phi'(r) \;=\; \frac{\beta}{4\pi M_{Pl} r^2}(M(r) - M(r_{max})). \tag{3.24}$$

For $r < r_{max}$, $M(r) - M(r_{max})$ is always negative. So, the chameleon field will increase from the edge to the center of halo and become to singular as $r \to 0$.

Case II : $\alpha' < M(r_{max})$.
For $r \to r_{max}$, the mass of halo can assume to be $M(r) \simeq M(r_{max})$. Thus

$$\phi'(r_{max}) \;=\; \frac{\beta}{4\pi M_{Pl} r^2}(\alpha'). \tag{3.25}$$

The field decrease from the edge to the center of halo. But, for $r \to 0$ the mass of halo tends to zero $M(r) \to 0$. Then

$$\phi'(r \to 0) \;=\; \frac{\beta}{4\pi M_{Pl}(r \to 0)^2}(-M(r_{max}) + \alpha'). \tag{3.26}$$

The field dramatically increases and becomes singular. Therefore, the profile looks like a valley.

Case III : $\alpha' = M(r_{max})$. Then

$$\phi'(r) \;=\; \frac{\beta}{4\pi M_{Pl} r^2}(M(r)). \tag{3.27}$$

The field always decreases from the edge to the center of halo, and for $r = 0$ we obtain the non-singular boundary condition

$$\frac{d\phi}{dr}(r = 0) = 0. \tag{3.28}$$

Then, in this case, the field does not become singular at the origin which it is a physical boundary condition and always used in all the original chameleon dark energy models.

Case IV : $\alpha' > M(r_{max})$. Therefore, as $r \to 0$

$$\phi'(r \to 0) \quad = \quad \frac{\beta}{4\pi M_{Pl}(r \to 0)^2}(-M(r_{max}) + \alpha'). \tag{3.29}$$

The field always decreases similar to case III, but it truncates at finite r (before $r \to 0$) because $\phi'(r)$ is very large. This case is unphysical.

3.3 Constraints on Matter-Chameleon Coupling Constant

Normally, we require there is no singularity of the field anywhere in the entire space. Then, the boundary conditions are

$$\begin{aligned} \frac{d\phi}{dr} = 0 \qquad & \text{at} \qquad r = 0, \\ \phi \to \phi_\infty \qquad & \text{as} \qquad r \to \infty. \end{aligned}$$

The solution which matches with these boundary conditions is case III. Thus, the profile is

$$\phi'(r) = \frac{\beta}{4\pi M_{Pl} r^2}(M(r)). \tag{3.30}$$

Integrate the above equation from $r = 0$ to $r = r_{max}$. Then

$$\begin{aligned} \int_{\phi(0)}^{\phi_\infty} d\phi &= \frac{\beta}{4\pi M_{Pl}} \int_0^{r_{max}} \frac{M(r)}{r^2}\, dr, \\ \phi_\infty - \phi(0) &= \frac{\beta}{4\pi M_{Pl}} \int_0^{r_{max}} \frac{M(r)}{r^2}\, dr, \\ \beta &= (\phi_\infty - \phi(0)) \frac{4\pi M_{Pl}}{\int_0^{r_{max}} \frac{M(r)}{r^2}\, dr}, \end{aligned} \tag{3.31}$$

where $\phi(0)$ is the field value at the origin ($r = 0$). Now, we see that the coupling constant will be the maximum when $\phi(0) = 0$. Moreover, the maximum value will decrease when the halo becomes more massive. Therefore

$$\beta_{max} = \frac{4\pi M_{Pl} \phi_\infty}{\int_0^{r_{max}} \frac{M(r)}{r^2}\, dr}. \tag{3.32}$$

Since we have

$$\phi_\infty = M_{Pl}\left(\frac{n}{\beta}\frac{M_{Pl}^4}{\rho_\infty}\left(\frac{M}{M_{Pl}}\right)^{4+n}\right)^{1/(n+1)}, \tag{3.33}$$

where ρ_∞ is the background density of universe ($\approx 10^{-26}\text{kg/m}^3$), then

$$\begin{aligned}
\beta_{max} &= \frac{4\pi M_{Pl}}{\int_0^{r_{max}} \frac{M(r)}{r^2}\,dr} M_{Pl}\left(\frac{n}{\beta_{max}}\frac{M_{Pl}^4}{\rho_\infty}\left(\frac{M}{M_{Pl}}\right)^{4+n}\right)^{1/(n+1)}, \\
&= \frac{4\pi M_{Pl}}{\int_0^{r_{max}} \frac{M(r)}{r^2}\,dr} M_{Pl}\left(n\frac{M_{Pl}^4}{\rho_\infty}\left(\frac{M}{M_{Pl}}\right)^{4+n}\right)^{1/(n+1)}\left(\frac{1}{\beta_{max}}\right)^{1/n+1},
\end{aligned}$$

$$\begin{aligned}
\beta_{max}^{1+\frac{1}{n+1}} &= \frac{4\pi M_{Pl}^2}{\int_0^{r_{max}} \frac{M(r)}{r^2}\,dr}\left(n\frac{M_{Pl}^4}{\rho_\infty}\left(\frac{M}{M_{Pl}}\right)^{4+n}\right)^{1/(n+1)}, \\
\beta_{max}^{\frac{n+2}{n+1}} &= \frac{4\pi M_{Pl}^2}{\int_0^{r_{max}} \frac{M(r)}{r^2}\,dr}\left(n\frac{M_{Pl}^4}{\rho_\infty}\left(\frac{M}{M_{Pl}}\right)^{4+n}\right)^{1/(n+1)}, \\
\therefore \beta_{max} &= \left(\frac{4\pi M_{Pl}^2}{\int_0^{r_{max}} \frac{M(r)}{r^2}\,dr}\right)^{\frac{n+1}{n+2}}\left(n\frac{M_{Pl}^4}{\rho_\infty}\left(\frac{M}{M_{Pl}}\right)^{4+n}\right)^{1/(n+2)}. \qquad (3.34)
\end{aligned}$$

Surprisingly, we found that the coupling constant has maximum value which depends on profile of the dark matter. But, in the case I and case II we do not have the constraint because $\phi(0)$ becomes ∞, then the β becomes $-\infty$ (no constraint) accordingly.

3.4 Rotation Curves

3.4.1 The Fifth Force

Since the chameleon scalar field couples with matter, the energy-momentum tensor of matter is not conserved. From

$$\nabla_\mu T^{\mu\nu}_{(total)} = \nabla_\mu T^{\mu\nu}_{(matter)} + \nabla_\mu T^{\mu\nu}_{(\phi)} = 0,$$

and

$$\begin{aligned}
\nabla_\mu T^{\mu\nu}_{(\phi)} &= -\alpha_\phi T_{(m)}\partial^\nu\phi, \\
\therefore \nabla_\mu T^{\mu\nu}_{(matter)} &= \alpha_\phi T_{(m)}\partial^\nu\phi.
\end{aligned}$$

From the approximation that the dark matter is the non-relativistic fluid (pressureless), we obtain

$$\nabla_\mu T^{\mu\nu}_{(matter)} = -\alpha_\phi\rho\partial^\nu\phi. \tag{3.35}$$

If $\nu = i$, the LHS will equal to the equation of motion of the fluid. Thus

$$\rho(\partial_t v^i + \vec{v} \cdot \vec{\nabla} v^i) = -\alpha_\phi \rho \partial_i \phi, \quad \because P = 0. \tag{3.36}$$

In this case the fluid has only rotation while the gradient has only the radial direction due to spherical symmetry, then $\vec{v} \cdot \vec{\nabla} = 0$. We obtain

$$\rho(\partial_t v^i) = -\alpha_\phi \rho \partial_i \phi, \tag{3.37}$$

$$\therefore \vec{a} = -\alpha_\phi \vec{\nabla} \phi. \tag{3.38}$$

This is the acceleration of the fifth force [13] which acts on matter in the Einstein frame ($g_{\mu\nu}$). The fifth force does not occur in the $\tilde{g}_{\mu\nu}$ frame because we set the fermions follow the geodesics of the metric $\tilde{g}_{\mu\nu}$.

3.4.2 Effects on Rotation Curves

Since we choose the Einstein frame to be our frame, the object which orbits around the center of a galaxy has gravitational force and the fifth force acts on it (we ignore pressure of dark matter from annihilation in Ref. [31]). Then, the Newton's laws can be written as

$$\vec{F}_{Gravity} + \vec{F}_{5th} = m\vec{a}.$$

As we suppose that the dark matter halo has spherical symmetry, then the fifth force has only direction in the radial direction.

$$\vec{F}_{5th} = -m\alpha_\phi \frac{d\phi}{dr}\hat{r}. \tag{3.39}$$

Moreover, the gravitational force has only radial direction, then the object has a centripetal acceleration (a_c).

$$\left(-\frac{GM(r)m}{r^2}\right)\hat{r} + \left(-m\alpha_\phi \frac{d\phi}{dr}\right)\hat{r} = \left(-\frac{mv_c^2}{r}\right)\hat{r}, \quad (\because \vec{a}_c = -\frac{v_c^2}{r}\hat{r})$$

$$\frac{GM(r)m}{r^2} + m\alpha_\phi \frac{d\phi}{dr} = \frac{mv_c^2}{r}.$$

Therefore, the circular velocity which includes the fifth force is

$$v_c(r) = \sqrt{\frac{GM(r)}{r} + \alpha_\phi r \frac{d\phi}{dr}}. \tag{3.40}$$

In this work, we use the exponential form for the coupling term ($A(\phi) = e^{\beta\phi/M_{Pl}}$), then

$$\therefore v_c(r) = \sqrt{\frac{GM(r)}{r} + \frac{\beta r}{M_{Pl}} \frac{d\phi}{dr}}. \tag{3.41}$$

Thus, the circular velocity depends on the profile of the dark matter halo and the profile of the chameleon scalar field. Furthermore, from Eqn. (3.23) the fifth force depends on β^2. Then, if the chameleon scalar field does not couple with matter (or $\beta = 0$), the effect from the fifth force will not occur.

NFW Halo

From the NFW profile, $\rho_{NFW}(r) = \dfrac{\rho_0}{\frac{r}{a}\left(1+\frac{r}{a}\right)^2}$, the mass of NFW halo is

$$M_{NFW}(r) = 4\pi a^3 \rho_0 \left(\ln\left(\frac{a+r}{a}\right) - \frac{r}{a+r} \right). \tag{3.42}$$

It depends on the characteristic density, ρ_0, and the scale radius, a. In this work, we use data from Ref. [32]. The paper tells us only the concentration parameter, c, and circular velocity at virial radius, V_{200}. Then, we have to find the relationship between the parameters in Eqn. (3.42) and the given parameters in Ref. [32]. From the accumulated mass at the virial radius, r_{200},

$$M(r_{200}) = 200\rho_{crit}\left(\frac{4\pi r_{200}^3}{3}\right),$$

and circular velocity at virial radius is

$$V_{200} = \sqrt{\frac{GM(r_{200})}{r_{200}}}. \tag{3.43}$$

We obtain

$$\begin{aligned} V_{200} &= \sqrt{\frac{G}{r_{200}}(200\rho_{crit})\left(\frac{4\pi r_{200}^3}{3}\right)}, \\ &= \sqrt{\frac{G}{r_{200}}(200\frac{3H^2}{8\pi G})\left(\frac{4\pi r_{200}^3}{3}\right)}, \\ &= \sqrt{100H^2 r_{200^2}}, \\ \therefore V_{200} &= 10Hr_{200}. \end{aligned} \tag{3.44}$$

And we have $r_{200} = ca$. Therefore, we obtain the scale radius which depends on V_{200} and c (where $H = 72\text{km s}^{-1}\text{Mpc}^{-1}$ [3]).

$$\therefore a = \frac{V_{200}}{10Hc}. \tag{3.45}$$

For the characteristic density, we start at the circular velocity of the NFW profile

$$v_c(r) = V_{200}\sqrt{\frac{1}{x}\frac{\ln(1+cx) - cx/(1+cx)}{\ln(1+c) - c/(1+c)}}.$$

Then

$$\begin{aligned}
\sqrt{\frac{GM(r)}{r}} &= V_{200}\sqrt{\frac{1}{x}\,\frac{\ln{(1+cx)}-cx/(1+cx)}{\ln{(1+c)}-c/(1+c)}}, \\
\frac{GM(r)}{r} &= V_{200}^2\frac{r_{200}}{r}\,\frac{\ln{(1+cx)}-cx/(1+cx)}{\ln{(1+c)}-c/(1+c)}, \\
\therefore M(r) &= \frac{V_{200}^2 r_{200}}{G}\left(\frac{\ln{(1+cx)}-cx/(1+cx)}{\ln{(1+c)}-c/(1+c)}\right), && (3.46) \\
&= \frac{V_{200}^2 r_{200}}{G}\left(\frac{\ln{(1+cr/r_{200})}-cr/r_{200}/(1+cr/r_{200})}{\ln{(1+c)}-c/(1+c)}\right). && (3.47)
\end{aligned}$$

And since

$$\rho(r) = \frac{dM(r)}{dV} = \frac{1}{4\pi r^2}\frac{dM(r)}{dr}, \tag{3.48}$$

then

$$\begin{aligned}
\frac{dM(r)}{dr} &= \frac{V_{200}^2 r_{200}}{G}\left(\frac{1}{\ln{(1+c)}-c/(1+c)}\right)\left(\frac{1}{1+cr/r_{200}}(c/r_{200})\right. \\
&\quad \left.-\frac{(1+cr/r_{200})c/r_{200}-(cr/r_{200})(c/r_{200})}{(1+cr/r_{200})^2}\right), \\
&= \frac{V_{200}^2 r_{200}}{G}\left(\frac{1}{\ln{(1+c)}-c/(1+c)}\right)\left(\frac{c^2 r/r_{200}^2}{(1+cr/r_{200})^2}\right). && (3.49)
\end{aligned}$$

Thus

$$\rho(r) = \frac{V_{200}^2 c}{4\pi r^2 G}\left(\frac{1}{\ln{(1+c)}-c/(1+c)}\right)\left(\frac{cr/r_{200}}{(1+cr/r_{200})^2}\right). \tag{3.50}$$

From the NFW profile, the characteristic density is

$$\begin{aligned}
\frac{\rho_0}{\frac{r}{a}\left(1+\frac{r}{a}\right)^2} &= \frac{V_{200}^2 c}{4\pi r^2 G}\left(\frac{1}{\ln{(1+c)}-c/(1+c)}\right)\left(\frac{cr/r_{200}}{(1+cr/r_{200})^2}\right), \\
\frac{\rho_0}{cr/r_{200}\left(1+cr/r_{200}\right)^2} &= \frac{V_{200}^2 c}{4\pi r^2 G}\left(\frac{1}{\ln{(1+c)}-c/(1+c)}\right)\left(\frac{cr/r_{200}}{(1+cr/r_{200})^2}\right), \\
\rho_0 &= \frac{V_{200}^2 c}{4\pi r^2 G}\left(\frac{c^2 r^2/r_{200}^2}{\ln{(1+c)}-c/(1+c)}\right), \\
&= \frac{V_{200}^2 c}{4\pi G}\left(\frac{c^2/r_{200}^2}{\ln{(1+c)}-c/(1+c)}\right), \\
\therefore \rho_0 &= \frac{V_{200}^2}{4\pi G a^2}\left(\frac{c}{\ln{(1+c)}-c/(1+c)}\right). && (3.51)
\end{aligned}$$

The above equation uses only the concentration parameter (c) and the circular velocity at virial radius (V_{200}) where we can calculate the scale radius (a) from Eqn. (3.45). Therefore, from these relations, we can find the mass of the NFW halo by the data from Ref. [32].

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ISO Halo

From ISO profile, $\rho_{ISO}(r) = \dfrac{\rho_0}{1+\left(\frac{r}{R_c}\right)^2}$, the mass of the ISO halo is

$$M_{ISO}(r) = 4\pi\rho_0 R_c^2\left(r - R_c \arctan\left(\frac{r}{R_c}\right)\right).$$

Then, the mass depends on the central density, ρ_0, and the core radius, R_c. Fortunately, the two parameters are available in [32], then it is easy to calculate.

The Parametrized Model

General form of the NFW profile can be written as

$$\rho_{PM}(r) = \frac{\rho_0}{\left(\frac{r}{r_s}\right)^\alpha \left(1+\frac{r}{r_s}\right)^{3-\alpha}}, \tag{3.52}$$

where α is a parameter, ρ_0 and r_s are fitting parameters. If $\alpha = 1$, it becomes the NFW profile. Mass of the model can be written as

$$\begin{aligned} M(r) &= \int_0^r 4\pi\rho(r)r^2 dr, \\ &= \int_0^r 4\pi \frac{\rho_0}{\left(\frac{r}{r_s}\right)^\alpha \left(1+\frac{r}{r_s}\right)^{3-\alpha}} r^2 dr, \\ &= 4\pi\rho_0 r^{3-\alpha} r_s^\alpha \frac{{}_2F_1(3-\alpha, 3-\alpha, 4-\alpha, -r/r_s)}{3-\alpha}, \end{aligned} \tag{3.53}$$

where ${}_2F_1$ is a hypergeometric function. We obtain

$$M(r) = \frac{4\pi\rho_0 r^3}{3-\alpha}\left(\frac{r}{r_s}\right)^{-\alpha} {}_2F_1(3-\alpha, 3-\alpha, 4-\alpha, -r/r_s). \tag{3.54}$$

Then, the circular velocity of parametrized model is

$$v_c(r) = \sqrt{\frac{4\pi G\rho_0 r^2}{3-\alpha}\left(\frac{r}{r_s}\right)^{-\alpha} {}_2F_1(3-\alpha, 3-\alpha, 4-\alpha, -r/r_s)}, \tag{3.55}$$

where we fit the parameters α, r_s and ρ_0 with the observational data.

3.5 Gravitational Lensing

We suppose that light comes from infinity (outside the dark matter halo), and then travels through the dark matter halo as in Figure 3.1.

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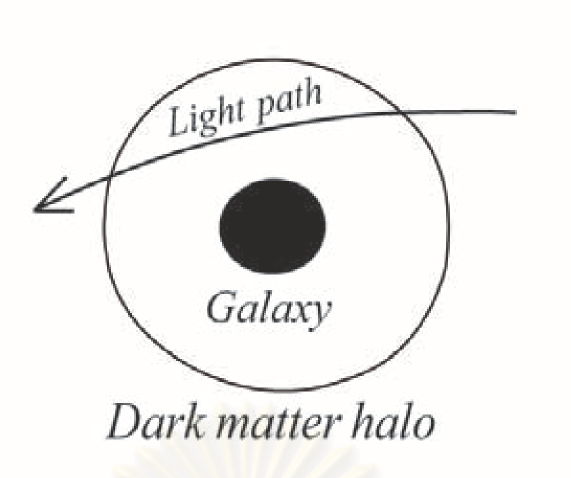

Figure 3.1: Light travels through the dark matter halo.

The metric inside and outside the halo then are not unique, but they still have continuity conditions together. The metric outside the halo is the Schwarzschild metric:

$$ds^2 = -\left(1 - \frac{2GM}{rc^2}\right)c^2dt^2 + \frac{dr^2}{(1 - \frac{2GM}{rc^2})} + r^2d\theta^2 + r^2\sin^2\theta d\phi^2. \tag{3.56}$$

While we do not know the exact metric inside the halo, we use a general form of spherically symmetry metric inside a star for the metric inside the halo:

$$ds^2 = -A(r)c^2dt^2 + B(r)dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2. \tag{3.57}$$

We can calculate the deflection angle of light by Eqn. (2.132) as

$$\frac{du}{d\phi} = \sqrt{\frac{1}{A(r)B(r)}\frac{1}{b^2} - \frac{u^2}{B(r)}}.$$

For the Schwarzschild metric we know the metric components $A(r)$ and $B(r)$, then we can find the deflection angle easily. But, for the metric inside the halo we must find the way to obtain those parameters.

3.5.1 Deflection Angle from the Schwarzschild Metric

For Schwarzschild metric used outside the halo, the light comes from infinity and then approaches the dark matter halo. Thus, the metric components are

$$A_{SC}(r) = 1 - \frac{2GM}{rc^2}, \tag{3.58}$$

$$B_{SC}(r) = \frac{1}{1 - \frac{2GM}{rc^2}}, \tag{3.59}$$

where M is a total mass of the halo. We then obtain

$$\begin{aligned}\frac{du}{d\phi} &= \sqrt{\frac{1}{b^2} - u^2\left(1 - \frac{2GM}{rc^2}\right)},\\ &= \sqrt{\frac{1}{b^2} - u^2\left(1 - \frac{2GMu}{c^2}\right)}.\end{aligned} \tag{3.60}$$

The limit of integration is from $u = 0$ (or $r = \infty$) to $u = \frac{1}{R_{halo}}$ because the metric is the Schwarzschild metric until light reaches the edge of halo. Thus

$$\Delta\phi = \int_{u=0}^{u=1/R_{halo}} \frac{1}{\sqrt{\frac{1}{b^2} - u^2\left(1 - \frac{2GMu}{c^2}\right)}}\, du, \tag{3.61}$$

where $\Delta\phi$ is the angle between $u = 0$ and $u = \frac{1}{R_{halo}}$ as in Figure 3.2.

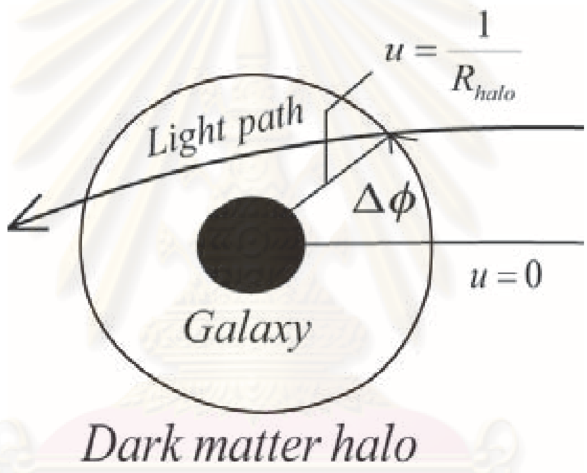

Figure 3.2: The $\Delta\phi$ is a sweeping angle in the Schwarzschild metric.

3.5.2 Deflection Angle from Metric Inside the Halo

From the spherically symmetry metric inside a star, we obtain the Einstein's field equations as Eqn. (3.1) and (3.2). In order to find effects of the chameleon scalar field on a deflection angle in the dark matter halo, we also add the energy-momentum tensor of the scalar field in these equations. Since we assume the pressure of dark matter in the halo to be zero, then the Einstein's field equations inside the halo with effective pressure and effective density are

$$\frac{(B-1)}{Br^2} + \frac{B'}{B^2 r} = 8\pi G\left(\rho_m + \frac{\phi'^2}{2B} + \frac{M^{4+n}}{\phi^n}\right), \tag{3.62}$$

$$\frac{(B-1)}{Br^2} - \frac{A'}{rAB} = 8\pi G\left(-\frac{\phi'^2}{2B} + \frac{M^{4+n}}{\phi^n}\right). \tag{3.63}$$

In this work, we use a numerical method to find the functions $A(r)$ and $B(r)$ where we solve $B(r)$ by the upper equation and then solve $A(r)$ by the lower equation. We substitute $A(r)$ and $B(r)$ from the numerical method into the following equation

$$\frac{du}{d\phi} = \sqrt{\frac{1}{AB}\frac{1}{b^2} - \frac{u^2}{B}},$$

where the limit of integration is from $u = \frac{1}{R_{halo}}$ to u_{max} ($r_{min} = r_{\perp}$). We cannot integrate from $u = 0$ (incoming light at infinity) to $u = 0$ (outgoing light at infinity) because the result of the integral becomes zero.

We define the angle from the Schwarzschild metric between $u = 0$ and $u = \frac{1}{R_{halo}}$ to be $D1$ and the angle from metric inside the halo between $u = \frac{1}{R_{halo}}$ and u_{max} to be $D2$. Thus, the deflection angle of light, δ, is

$$\delta = 2\left(D1 + D2 - \frac{\pi}{2}\right). \tag{3.64}$$

We can illustrate the definitions of each angle as Figure 3.3.

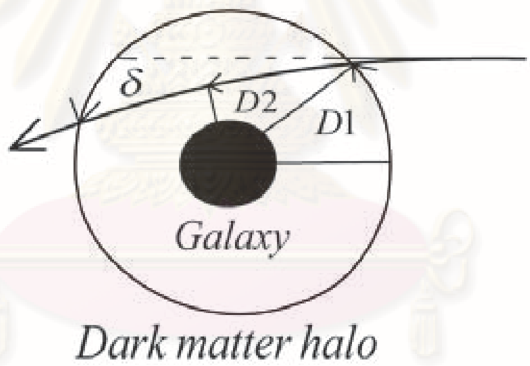

Figure 3.3: The essential angles for calculating the deflection angle.

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Chapter IV

RESULTS AND DISCUSSIONS

In this work, we use low surface brightness (LSB) galaxies for the investigation of effects from the chameleon scalar field because the LSB galaxies are dominated by dark matter. For LSB galaxies DDO47, U4325 and U3371 the NFW profile cannot fit data without unrealistic values for the rotation velocities. Therefore, we calculate only the ISO profile.

4.1 Constraints on Matter-Chameleon Coupling Constant from the Non-Singular Solution

The constraints on matter-chameleon coupling constant for the non-singular solution can be obtained from Eqn. (3.34) where we choose $n = 1$ and $M = 10^{-3}$eV ($M_{Pl} = 2.43 \times 10^{18}$GeV and $\rho_\infty = 10^{-26}$kg/m^3).

Name	Profiles	β_{max}
U5750	NFW	1.53092×10^{-7}
	ISO	2.06357×10^{-7}
U5005	NFW	1.76533×10^{-7}
	ISO	1.69763×10^{-7}
DDO189	NFW	3.55203×10^{-7}
	ISO	2.79221×10^{-7}
DDO47	ISO	1.75644×10^{-7}
U4325	ISO	7.30803×10^{-8}
U3371	ISO	1.67904×10^{-7}

Table 4.1: Constraints on the matter-chameleon coupling constant from the non singular solution.

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The maximum values of matter-chameleon coupling constant are about 10^{-7}. Then, the chameleon effects on the rotation curves from the non-singular solution of case III are hardly detected.

4.2 Chameleon Profiles

We use the numerical method with Eqn. (3.18) because we need actually profile where $n = 1$, $M = 10^{-3}$eV. While β, we choose from Table 4.1 because we want to see the chameleon profile in all cases of solutions. In order to avoid the singularity in the numerical method, we will solve Eqn. (3.18) from the edge of dark matter halo to $r_{min} = 0.001$ kpc. The chameleon profiles from numerical method are represented in the black lines.

Chameleon Profiles of the Case $\alpha' = 0$

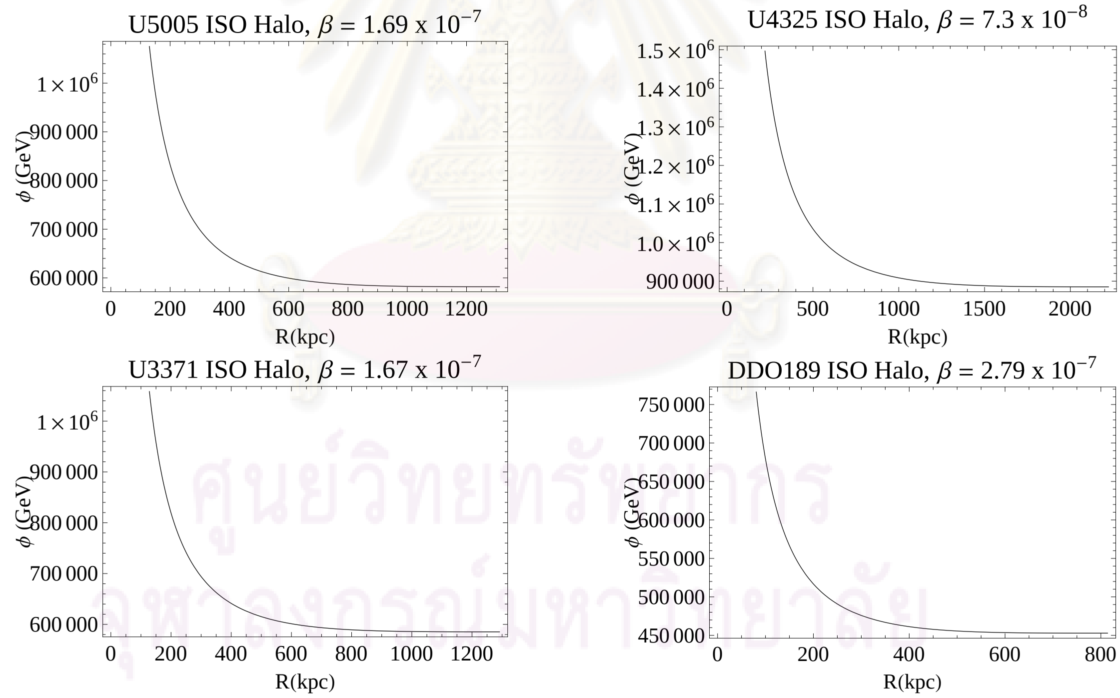

Figure 4.1: The chameleon profiles in the ISO halo of U5005, U4325, U3371 and DDO189 galaxy with $\alpha' = 0$.

The profiles of all galaxies for the case $\alpha' = 0$ are singular at the origin as Eqn. (3.24).

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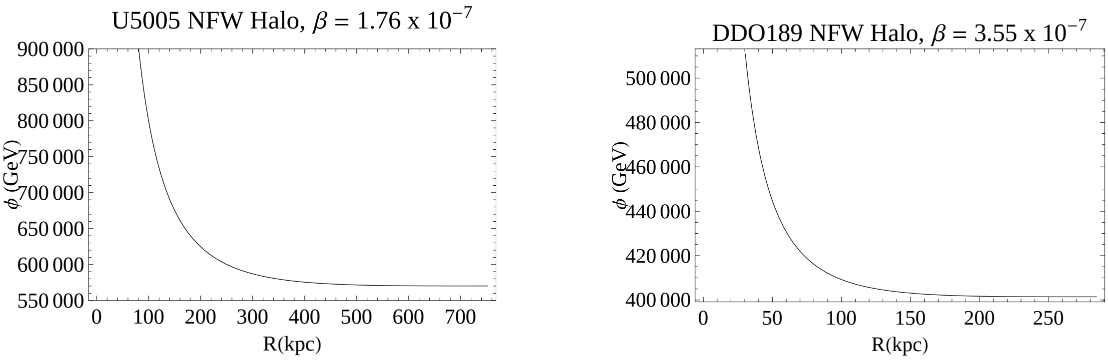

Figure 4.2: The chameleon profiles in the NFW halo of U5005 and DDO189 galaxy with $\alpha' = 0$.

Chameleon Profiles of the Case $\alpha' > 0$

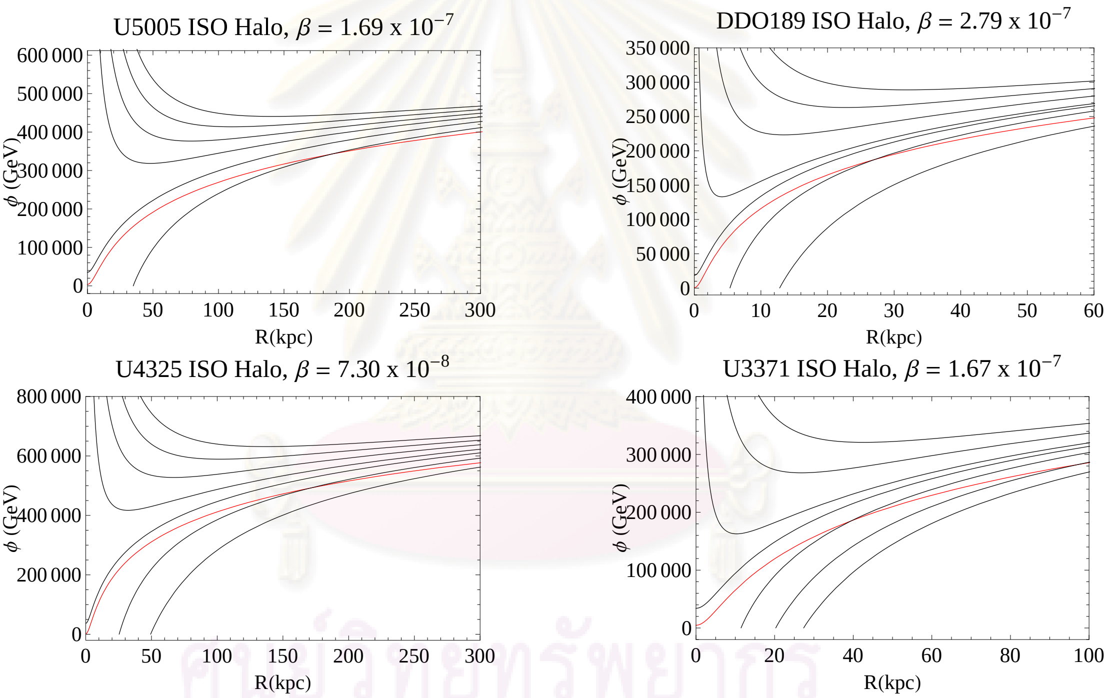

Figure 4.3: The chameleon profiles of U5005, DDO189, U4325 and U3271 galaxy with increasing α' from top to bottom where the red lines represent the analytic approximation of case III.

The profiles of the case $\alpha' > 0$ obey Eqn. (3.25), (3.27) and (3.29). Since the potential term is negligible, the approximate solutions can adequately explain behavior of the chameleon scalar field in the dark matter halo.

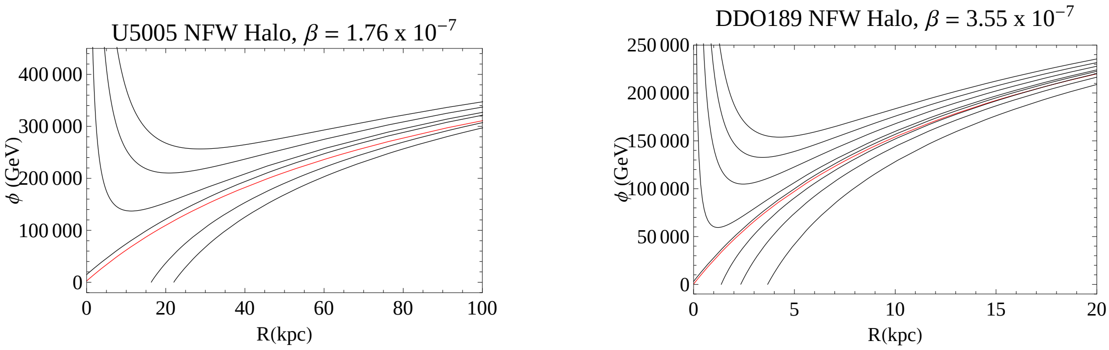

Figure 4.4: The chameleon profiles of U5005 and DDO189 galaxy with increasing α' from top to bottom where the red lines represent the analytic approximation of case III.

4.3 Acceleration of the Fifth Force

Since the coupling constant for case III has constrained value, which is very small ($\approx 10^{-7}$), then we cannot see the effects of the chameleon scalar field from the profiles in case III. Additionally, the case II at larger coupling constant has quite the same shape with case I. Thus, in this work, we use only the case I ($\alpha' = 0$) to find the effects of the chameleon scalar field on the rotation curves and gravitational lensing.

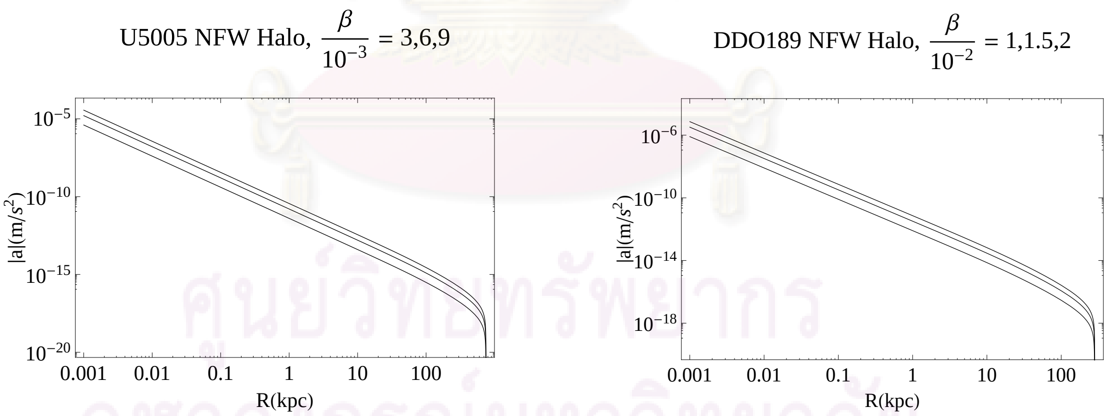

Figure 4.5: The acceleration of the fifth force in the NFW halo of U5005 and DDO189 galaxy increases with the coupling β.

The acceleration of the fifth force at the origin of all galaxies and dark matter profiles are considerably larger than the outside region because the main contribution of the acceleration comes from derivative of the field. We use the chameleon profile of case I, which the profile becomes singular at the origin. Thus, the gradient of the field becomes very high accordingly. Therefore, the fifth force pushes all objects which orbit around the center of the galaxy radially outward.

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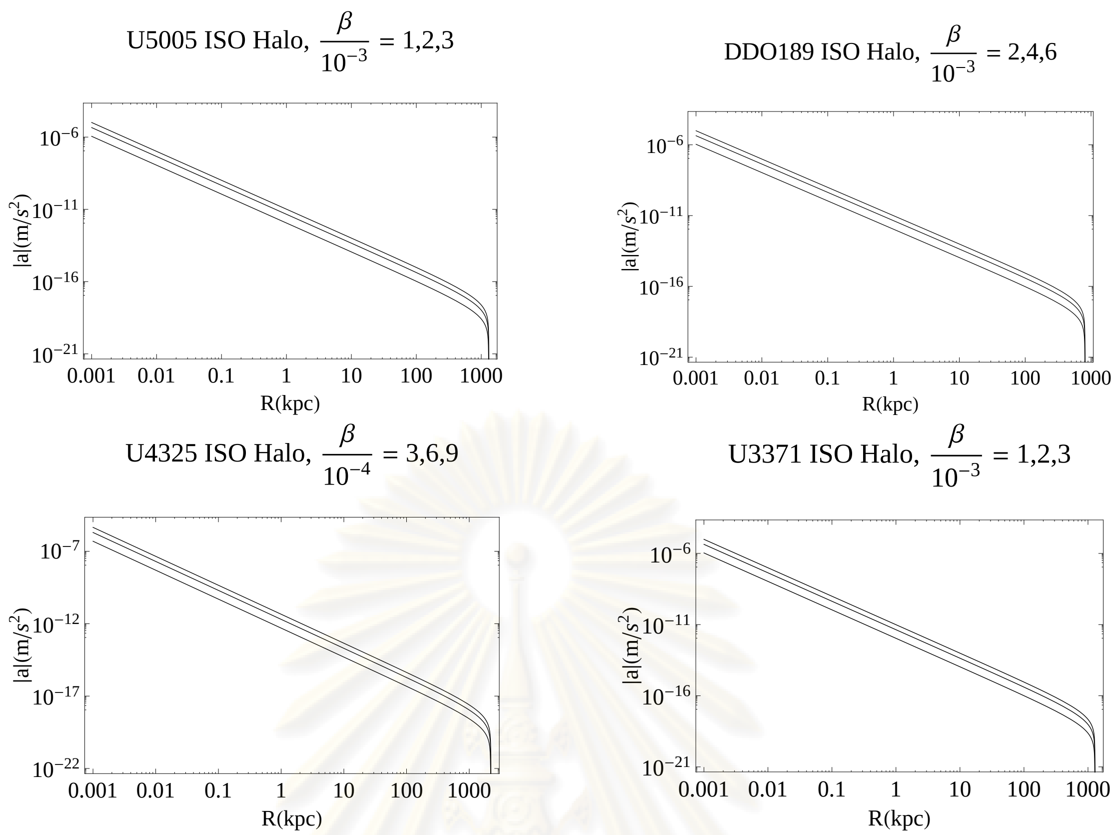

Figure 4.6: The acceleration of the fifth force in the ISO halo of U5005, U4325, U3371 and DDO189 galaxy increases with the coupling β.

4.4 Equation of State Parameter

Normally, the equation of state parameter of scalar field is equal to -1 because of the slow-roll condition. The chameleon scalar field becomes dark energy only outside the halo because it stays at the minimum only outside the halo. We can neglect the kinetic energy term of the scalar field due to the slow-roll condition. Since we choose the dynamics of the chameleon scalar field is the thick-shell regime. Inside the halo, the scalar field becomes singular and the derivative of the scalar field is very large, then the kinetic term dominates and the equation of state parameter of chameleon scalar field is equal to 1 accordingly.

In these results we use a very low coupling constant because we want to show the changing of equation of state parameter of the chameleon scalar field. If we use the coupling constant in the same order as the rotation curves, we will not see the difference of each line.

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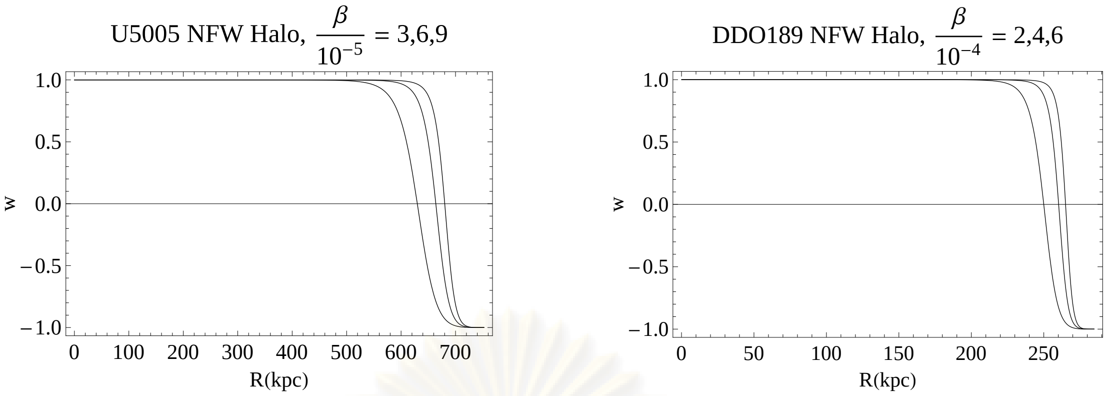

Figure 4.7: The equation of state parameter in the NFW halo of U5005 and DDO189 galaxy.

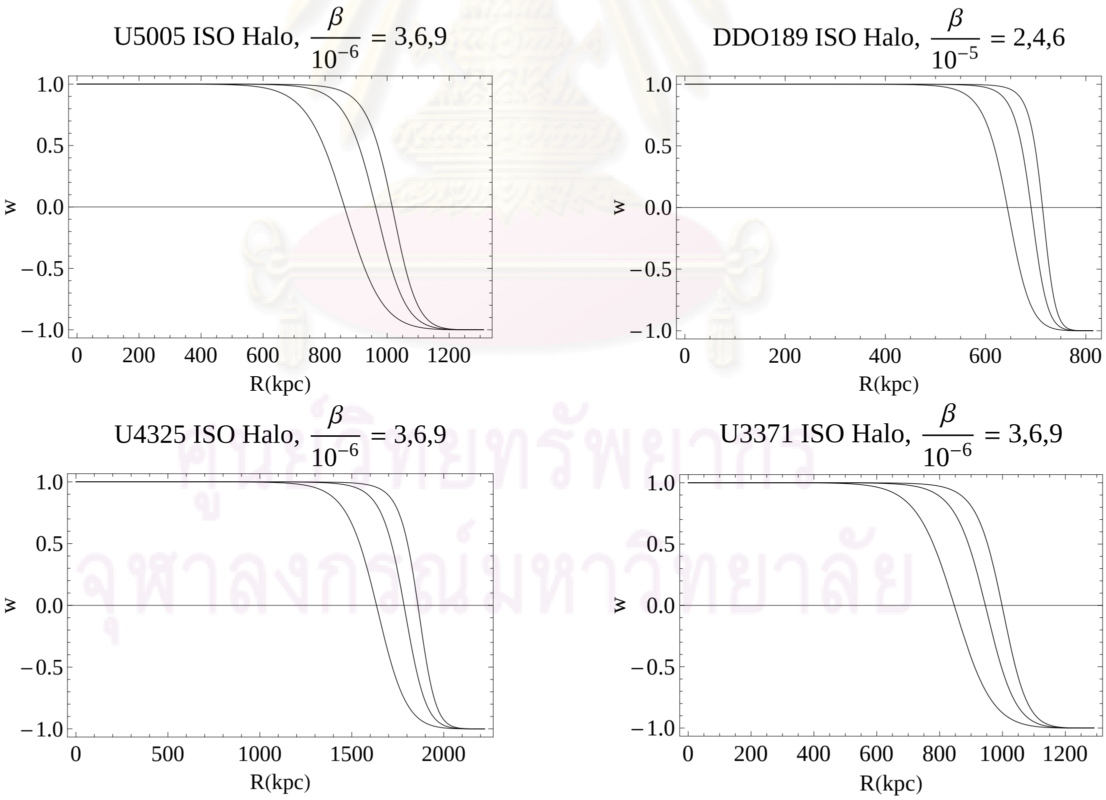

Figure 4.8: The equation of state parameter in the ISO halo of U5005, U4325, U3371 and DDO189 galaxy.

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4.5 Rotation Curves

We found that the fifth force from the chameleon scalar field can cause the slope of rotation curves steeper (cuspier) around the central region in any dark matter profile of any galaxies. Since the direction of the fifth force is outward, then the circular velocity is reduced. Furthermore, the rotation curves more cusps when we increase the coupling constant.

4.5.1 ISO Halo

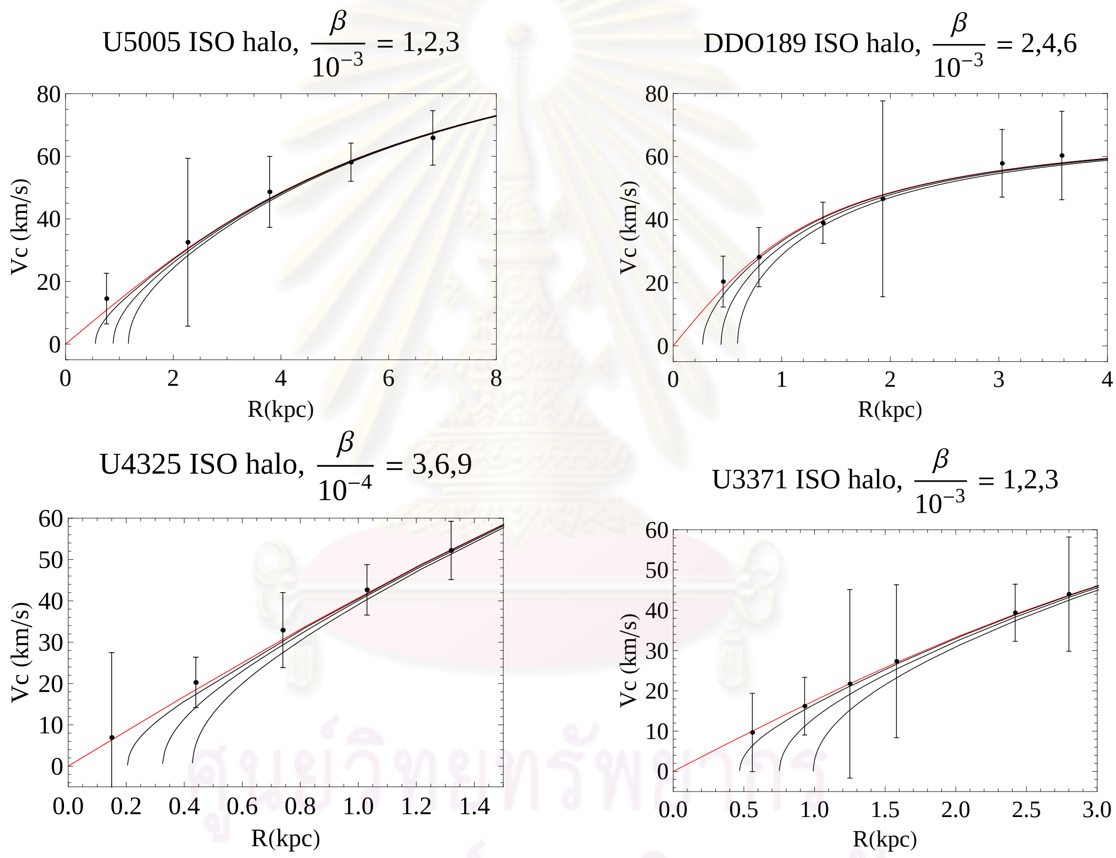

Figure 4.9: Rotation curves of U5005, DDO189, U4325 and U3371 ISO galaxy around the core region for varying β. The red lines represent rotation curves of the galaxy without the fifth force.

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4.5.2 Parametrized Model

The parametrized model with $\alpha = 0.7$ cannot fit U4325 and U3371 without making unrealistically circular velocity.

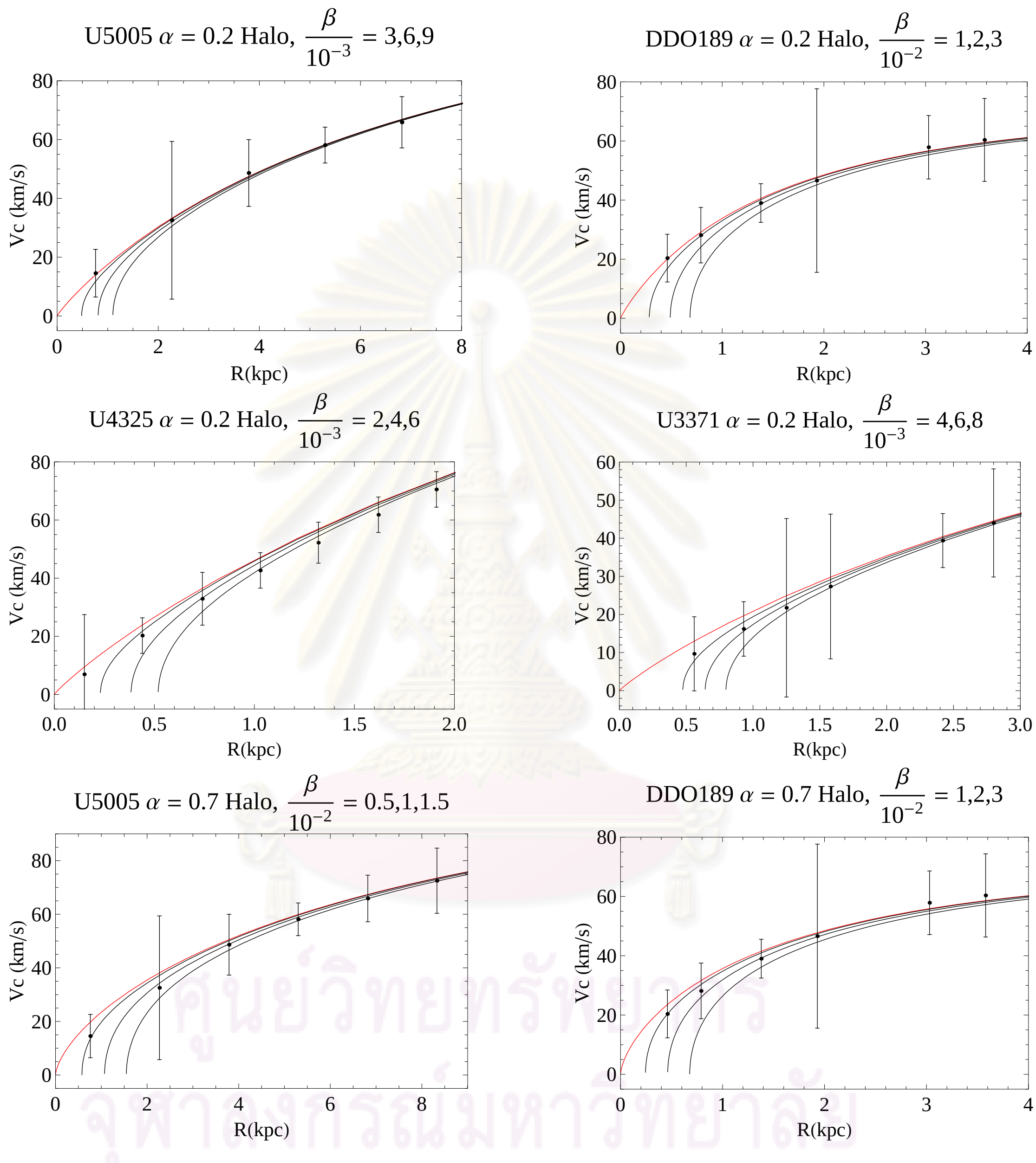

Figure 4.10: Rotation curves of U5005, DDO189, U4325 and U3371 parametrized model galaxy around the core region for varying β. The red lines represent the rotation curves of the galaxy without the fifth force.

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4.5.3 NFW Halo

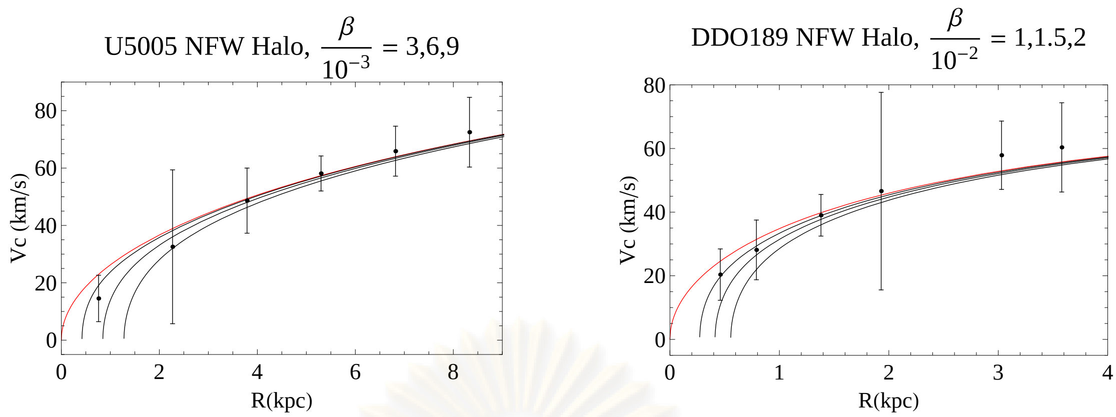

Figure 4.11: Rotation curves of U5005 and DDO189 NFW galaxy around the core region for varying β. The red lines represent the rotation curve of the galaxy without the fifth force.

4.5.4 Dependence on the Power of the Self-Interaction Potential

There are two parameters which can be varied, the power n in the scalar potential and the constant M in the scalar potential. From the chameleon dark energy model, parameter n can be an arbitrary value. If n is large, the scalar potential is very steep, then we often choose n in the order one, $O(1)$. For example, the simplest case is $n = 1$. For constant M, we still set $M = 10^{-3}$eV.

We found that the rotation curves change very little for various n and the equation of state parameter inside the halo approaches closer to -1 when the power n is reduced.

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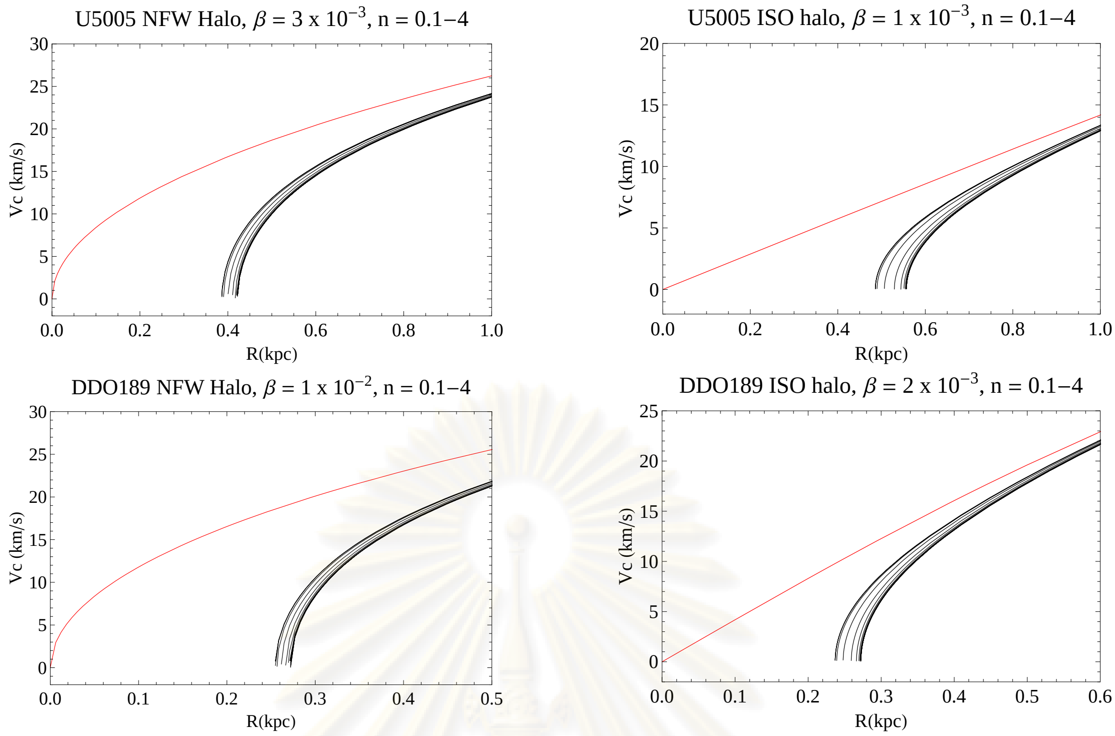

Figure 4.12: Rotation curves of galaxy U5005, DDO189 with $n = 0.1 - 4$ (left to right) using NFW, ISO profile. The red lines are the rotation curves without the chameleon.

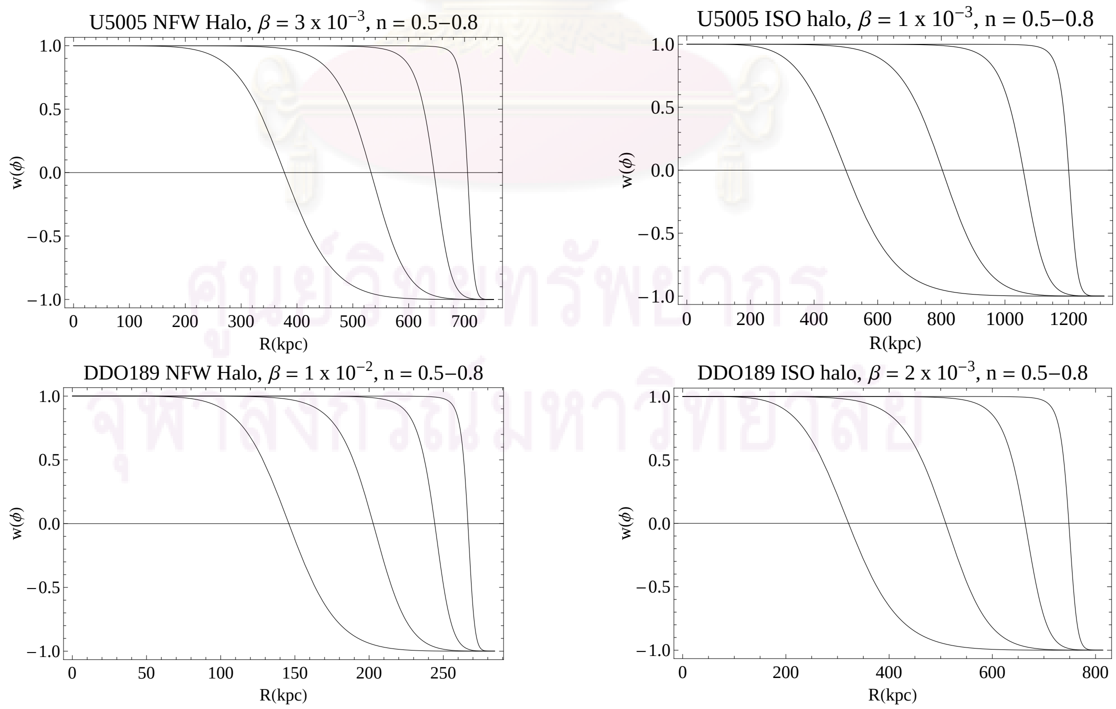

Figure 4.13: Equation of state parameter of galaxy U5005, DDO189 with $n = 0.5 - 0.8$ (left to right) using NFW, ISO profile.

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4.6 Constraints on Matter-Chameleon Coupling Constant from the Rotation Curves of LSB Galaxies

Since the chameleon scalar field can make the rotation curve more cusp, too large value of β will cause result in the rotation curves contradictory to the observation. So, we use the reduced chi-square method to find an upper bound of matter-chameleon coupling at 95% C.L. from rotation curves where degree of freedom of each galaxy is the following: U5005 d.o.f. $= 11 - 3 = 8$, DDO189 d.o.f. $= 11 - 3 = 8$, U4325 d.o.f. $= 16 - 3 = 13$ and U3371 $= 17 - 3 = 14$ respectively. The -3 come from three parameters which control the rotation curves: power n in self-interaction potential, two parameters for each LSB galaxy as Table A.1 and A.2.

LSB galaxy	upper bound on β at 95 % C.L.
U5005 (NFW)	6×10^{-3}
U5005 (ISO)	2×10^{-3}
U5005 (PM $\alpha = 0.2$)	6×10^{-3}
U5005 (PM $\alpha = 0.7$)	9×10^{-3}
DDO189 (NFW)	1.75×10^{-2}
DDO189 (ISO)	4.8×10^{-3}
DDO189 (PM $\alpha = 0.2$)	1.75×10^{-2}
DDO189 (PM $\alpha = 0.7$)	1.85×10^{-2}
U4325 (ISO)	1×10^{-3}
U4325 (PM $\alpha = 0.2$)	5.4×10^{-3}
U3371 (ISO)	2.7×10^{-3}
U3371 (PM $\alpha = 0.2$)	9.5×10^{-3}

Table 4.2: Constraints on the matter-chameleon coupling constant from the LSB galaxies.

According to Table 4.2, the ISO profile has the strongest constraint than other profiles which is about $1 \times 10^{-3} - 4.8 \times 10^{-3}$ where other profiles are about $1.75 \times 10^{-2} - 6 \times 10^{-3}$ for NFW profile, $1.75 \times 10^{-2} - 9.5 \times 10^{-3}$ for the parametrized model with $\alpha = 0.2$ and $1.85 \times 10^{-2} - 9 \times 10^{-3}$ for the parametrized model with $\alpha = 0.7$.

4.7 Deflection Angle of Light

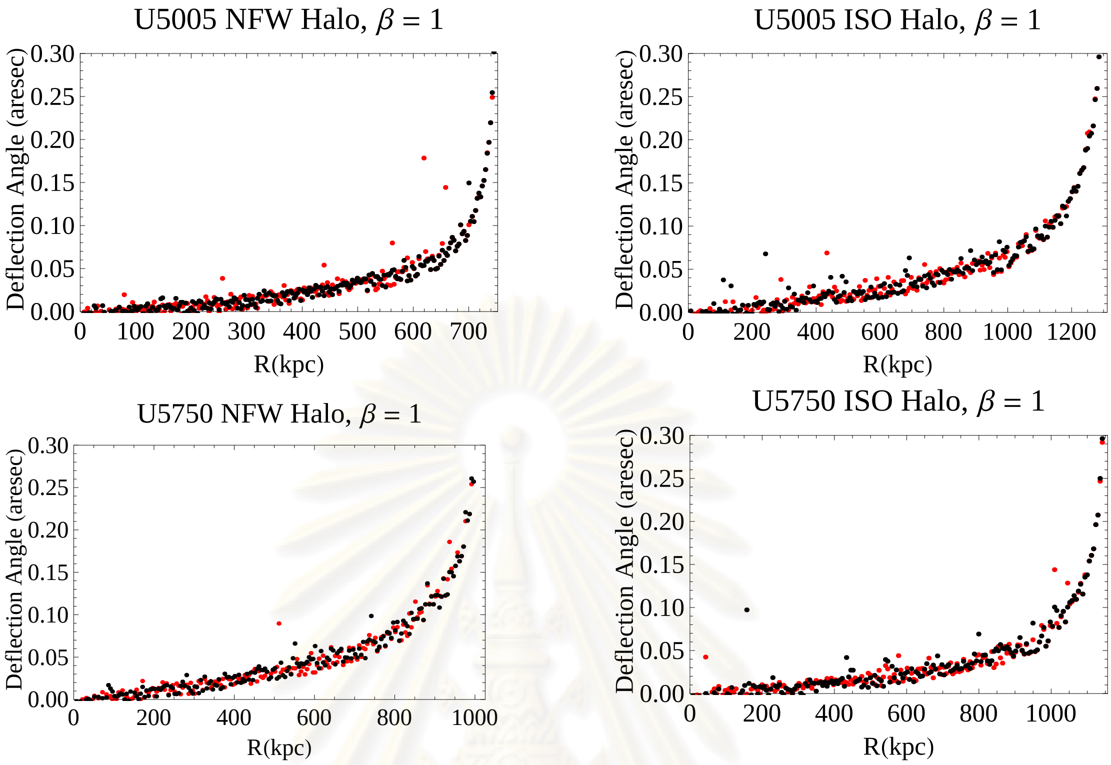

Figure 4.14: Deflection angle of galaxy U5005 and U5750 with $\beta = 1$.

The results are shown in multiple dots because we must set the impact parameter, b, and run the parameter as an integer number from the center ($b = 1$) to the edge ($b = r_{max}$) of dark matter halo. The red dots represent the deflection angle with chameleon while the black dots without chameleon. Moreover, the results are shown with $\beta = 1$ because the coupling of matter-chameleon is stronger than the low β ($\approx 10^{-3}$). Nevertheless, we found that there is no difference between the red dots and the black dots. Because the chameleon density is much smaller than the dark matter density and the effective pressure is very small which can be neglected. The effective density is dominated by the dark matter density. Therefore, the chameleon scalar field does not have any effects on the gravitational lensing.

Chapter V

CONCLUSIONS

The chameleon scalar field can modify the rotation curves of galaxies by the fifth force which comes from the derivative of the chameleon scalar field. The fifth force makes the rotation curve reduce (cuspier) around the center of the galaxies because the direction of the force is outward along the radial direction. Moreover, the effects on the rotation curves are controlled by the coupling constant because the fifth force is proportional to the matter-chameleon coupling β^2.

The non-singular boundary condition of the chameleon profile leads to stringent constraint on matter-chameleon coupling, which is a very small value ($\beta \lesssim 10^{-7}$). In such cases, we cannot see the effects of the chameleon scalar field. However, there are no physical reasons to prohibit the singular solution. We show that the central singularity is more likely to occur in general physical situation. Then, we investigate the effects of the chameleon from the profile with singularity at the origin.

For the singular chameleon solution, the upper bound of the matter-chameleon coupling from the rotation curves of LSB galaxies at 95% C.L. are $1.75\times10^{-2}-6\times10^{-3}$ for NFW profile, $1\times10^{-3}-4.8\times10^{-3}$ for ISO profile, $1.75\times10^{-2}-9.5\times10^{-3}$ for the parametrized model with $\alpha = 0.2$ and $1.85 \times 10^{-2} - 9 \times 10^{-3}$ for the parametrized model with $\alpha = 0.7$. Additionally, the change of parameter n in the scalar self-interacting potential has very small effects on the rotation curves.

Finally, we cannot see effect of the chameleon scalar field on the gravitational lensing because the density of the scalar field is much smaller than the matter density.

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APPENDICES

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Appendix A

DATA OF THE LSB GALAXIES

These tables are partial data of Ref. [32] where $M_{sun} = 1.98892 \times 10^{30}$Kg. and 1pc(parsec) $= 3.08568025 \times 10^{16}$m.

Name	c	V_{200}(km/s)
U5750	1.9	145.7
U5005	3.3	124.6
U4325	0.1	3331.6
U3371	0.1	875.6
DDO189	9.9	59.2408
DDO47	0.1	1332.5

Table A.1: Parameters for the NFW profile.

Name	R_c(kpc)	$\rho_0(M_{sun}/1000\text{pc}^3)$
U5750	5.0	7.9
U5005	4.7	11.5
U4325	2.7	100.1
U3371	3.7	18.0
DDO189	1.0	97.9
DDO47	2.1	47.5

Table A.2: Parameters for the ISO profile.

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Appendix B

UNIT TRANSLATIONAL TABLE

Natural unit	$\rightarrow$	SI unit
1 GeV	=	1.8×10^{-27}kg (mass)
1 GeV^{-1}	=	0.197×10^{-15} m (length)
1 GeV^{-1}	=	6.58×10^{-25} s (time)
SI unit	$\rightarrow$	**Natural unit**
1 m (length)	=	5.07614×10^{15} GeV^{-1}
1 kg (mass)	=	5.55556×10^{26} GeV
1 s (time)	=	1.51976×10^{24} GeV^{-1}

Table B.1: Unit translational table.

This table is more useful during calculations because most of the cosmological models use the natural unit ($c = 1$ and $\hbar = 1$) while the data from observation are in the SI units.

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VITAE

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Presentations

1. Effects of Chameleon Scalar Field on Rotation Curves of the Galaxies: The 19^{th} National Graduate Research Conference, Rajabhat Rajanagarindra University, Chachoengsao, Thailand (23-24 December 2010).

International Schools

1. 1^{st} CERN School Thailand 2010, Chulalongkorn University, Bangkok, Thailand (2-16 October 2010).

2. The 4^{th} Siam Symposium on General Relativity, High Energy Physics and Cosmology, Narasuan University, Phitsanulok, Thailand (26-28 July 2009).

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