CHAPTER III

THEORETICAL CONSIDERATION

The computer program for oil reserve calculation that will be developed in this study will use volumetric method to calculate oil reserve and can report the uncertainty of calculated oil reserve using Monte Carlo method. The knowledge of volumetric method for oil reserve calculation and Monte Carlo simulation for obtaining the uncertainty of output are necessary. In addition, statistical relationship between input variables and statistical relationship of values of input variables in space will be incorporated in the calculation. For the statistical relationship between input variables, procedure for incorporating this statistical relationship will be considered and used. The incorporation of statistical relationship of values of input variables in space called spatial correlation will be discussed and will be accomplished by using turning bands method. This method is a numerical method for generating random variables following desired statistical properties in space of input variables.

Topics involving development of the computer program for oil reserve calculation using Monte Carlo simulation with incorporation of statistical relationship between input variables and spatial correlation are as follows:

1. Oil reserve estimation by volumetric method,

2. Monte Carlo simulation and oil reserve calculation using Monte Carlo simulation,

3. Improvement of oil reserve estimation by considering statistical relationship between two input variables,

4. Improvement of oil reserve estimation by considering spatial correlation of an input variable.

Oil Reserve Estimation by Volumetric Method.

Oil reserve estimation by volumetric method is explained in several petroleum engineering text books, e.g., Amyx, Bass, and Writing (1960), Nind (1964), Walstrom et al. (1967), Dickey (1981), and so on. In this method, the volume of oil existing in a given reservoir called oil initially in place can be calculated by

$$N = 7758 \ A \ h_g \ R_1 \ R_2 \ \phi \ (1 - S_w)/B_{oi}, \qquad (3.1)$$

where

7758 = conversion constant

N = oil initially in place, stock tank barrel or barrel at standard condition

A = area of reservoir, acre

h_g = gross thickness, foot

R_1 = net to gross thickness ratio

R_2 = oil sand to net sand ratio

ϕ = porosity, fraction of voids in total rock matrix

S_w = water saturation, volume fraction of the voids filled with interstitial water.

B_{oi} = initial oil formation volume factor, reservoir barrels divided by stock tank barrels.

All variables on the right hand side of the Equation (3.1) will become available after the discovery well has been drilled.

Oil reserve is a quantity of crude oil that can be recovered from a reservoir. The amount of oil reserve is affected by the following factors:

1. reservoir fluid density and viscosity,

2. gas solubility,

3. shape of the relative permeability curve,

4. reservoir pressure,

5. presence of a connate water phase,

6. amount of gas cap and its method of expansion,

7. rates of withdrawal and pressure drawdown,

8. type of geological structure.

The oil reserve can be obtained by modifying Equation (3.1) as

$$N_R = 7758 \ A \ h_g \ R_1 \ R_2 \ \phi \ (1 - S_w) \ E_R \ / \ B_{oi}, \qquad (3.2)$$

where

N_R = oil reserve (recoverable oil), stock tank barrels,

E_R = recovery factor, fraction of the oil in place that will

probably be recovered.

Both reservoir fluid and rock properties are involved in the

determination of the recoverable fraction of the oil initially in place

in a reservoir.

In oil reserve estimation, the geological data e.g., net volume

of reservoir rock, porosity, water saturation, and so on, obtained from

any interested field are used in Equation (3.2). These geological data

will be sampled from various points in a reservoir. The important

problem is that data for oil reserve calculation obtained from the interested field will have uncertainty in their values. The uncertainty of these data is possibly due to two causes. Firstly, the deposition of reservoir rock occured naturally. Therefore, the geological characteristics in each location in the reservoir is likely to be different from one another causing the values of rock and fluid properties obtained from various points to be different. Secondly, measuring process may cause uncertainty in values of these data. Values of most variables cannot be determined accurately, e.g., some variables cannot be measured directly and values of some variables are obtained by using an averaging process. The uncertainty in value of input variables causes the uncertainty in the calculated oil reserve.

It is likely that the values of thickness in each position within a reservoir are not same due to geologic structure of an reservoir and there is also uncertainty in values of measured thickness.

Uncertainty in value of porosity and water saturation is due to nature of these variables and the measuring process.

Uncertainty in the recovery factor is attributed to the nature of the reservoir recovery mechanism and inability to precisely and accurately specify the recovery factor value. The value of recovery factor may be

determined from past experience in the area.

The oil formation volume factor was obtained from PVT analysis and laboratory measurements of recombined fluid samples from the reservoir. The uncertainty of this variable is due to difference in composition of oil under pressure at different position and errors introduced during measurement.

Because of uncertainty in values of all input variables, the resulting value of reserve, N_R, will also have uncertainty. Therefore, statistical methods should be incorporated in calculation of the reserve. This will help in assessing uncertainty of the calculated reserve value. The statistical method used in this study to assess uncertainty is Monte Carlo simulation.

Monte Carlo Simulation and Oil Reserve Calculation Using Monte Carlo Simulation.

There are two alternative methods that can be used to solve a probabilistic problem:

1. Analytical approach, combining of distributions by algebraic means using stochastic parameters such as the mean, standard deviation, skewness and other basic shape factors (or moments);

2. Numerical approach, combining of variables by random

selection of values from distributions.

In the analytical approach, the moments of the final distribution

are directly derived from those of the constituent data distributions via

addition and other algebraic operations in the traditional manner. The

advantage of this method lies in the fact that computational time is low,

the main disadvantage being its limitation to treat complex problems.

The numerical method, including Monte Carlo method will use a random

number generator and input-data distributions which many have any form.

Normally, several thousand passes (similar calculations) are carried out,

the actual number depending on the desired accuracy and skewness of basic

data distributions.

In this study, Monte Carlo simulation is used to solve the

reserve calculation problem by repetitively solving for a set of reserve

values, each of which is an equally probable representation of the volume

of oil to be recovered from the oilfield.

Monte Carlo method is a statistical method used to access the

uncertainty of dependent variables caused by uncertainty in values of

input variables. This method uses random numbers as an important tool.

The input variables having uncertainty in their values are known as

"random variable" and the problem that involving the random variables is known as "probabilistic problem". At present, the Monte Carlo method is a well-known method. In addition to solving the probabilistic problem, it can also be used to solve a deterministic problem, e.g., calculation of π , integration, and so on.

In the Monte Carlo method, random numbers, the numbers distributing evenly between 0 and 1, are used to be a tool for selecting value of input variables from their cumulative distribution function to obtain set of values of input variables for calculating an outcome (in this case it is oil reserve). A set of values of input variables obtained from selection by random numbers and the calculated output are called "a realization". This process will be performed repeatedly to construct probability density function of oil reserve.

Consider the Monte Carlo simulation of the Equation (3.2), where A, $\emptyset$, h_g, R_1, R_2, S_w, E_R, and B_{oi} are input random variables. From their respective distributions, values of the eight variables would be selected at random expressed graphically in Figure 3.1.
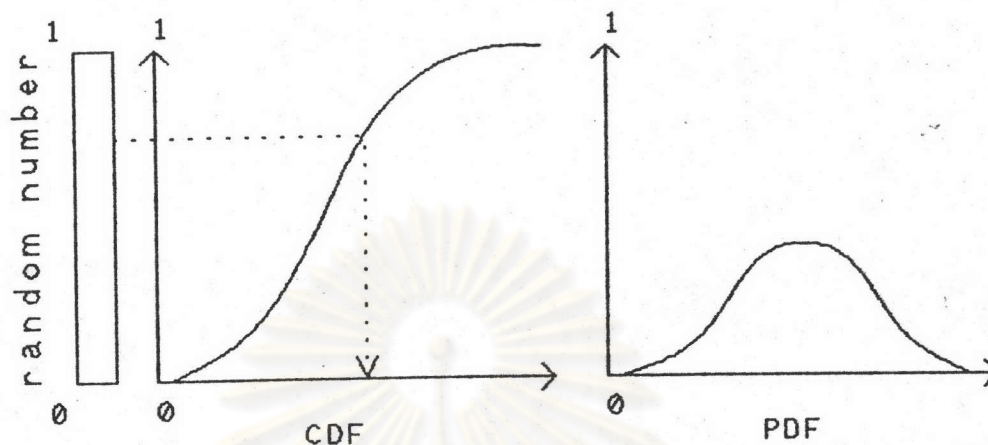
Figure 3.1. Cumulative distribution function (CDF) and probability

density function (PDF) and use of random number as a tool for obtaining

a realization.

Substituting these values into Equation (3.2) yields a value of

N_R. Subsequent values of variable N_R are obtained by repeating the

simulation process with additional sets of randomly sampled values of

the eight input variables. Approximate distribution approaches true

distribution as the number of computations or realizations incerases.

The process is repeated as many times as necessary to build up the

desired confidence level for the oil reserve distribution. The shape of

probability distribution for the calculated variable will reflect its

uncertainty. The narrowwer the distribution, the less uncertainty of the reserve values. On the other hand, a flat distribution with considerable spread between the upper and lower limits indicates a greater uncertainty in the reserve values. The flowchart of Monte Carlo method for oil reserve calculation is shown in Figure 3.2.

Generally, use of Monte Carlo simulation for oil reserve calculation in each simulation process, the oil reservoir will be assumed to have uniform rock and fluid properties, i.e. the whole oil reservoir has the same value of each variable on the right hand side of Equation (3.2). This is rather crude because it is generally accepted that in reality there is no such thing as a homogeneous oil reservoir. To improve on this matter, the oil reservoir will be divided into a finite number of blocks in this study. Each block is assumed to have uniform rock and fluid properties. A set of realizations on the right hand side of Equation (3.2) will be assigned to the blocks and the total oil reserve is the summation of oil volume in each block.

```
┌─────────────────────────────┐
│    Estimate distributions    │
│                              │
│      of input variables      │
└─────────────────────────────┘
              │
              ▼
┌─────────────────────────────┐
│     select a value for each  │
│                              │
│  variable using random number│
│                              │
│  and respective distribution │
└─────────────────────────────┘
              │
              ▼
┌─────────────────────────────┐
│     calculate oil reserve    │
│                              │
│   from the Equation (3.2)    │
└─────────────────────────────┘
              │
              ▼
┌─────────────────────────────┐
│    repeat steps 2 and 3      │
│                              │
│    using another set of      │
│                              │
│       values selected        │
└─────────────────────────────┘
              │
              ▼
┌─────────────────────────────┐
│  after obtaining determined  │
│                              │
│    number of repetitions,    │
│                              │
│    construct probablity for  │
│                              │
│         oil reserve          │
└─────────────────────────────┘
              │
              ▼
┌─────────────────────────────┐
│      compute mean and        │
│                              │
│     standard deviation       │
└─────────────────────────────┘
```
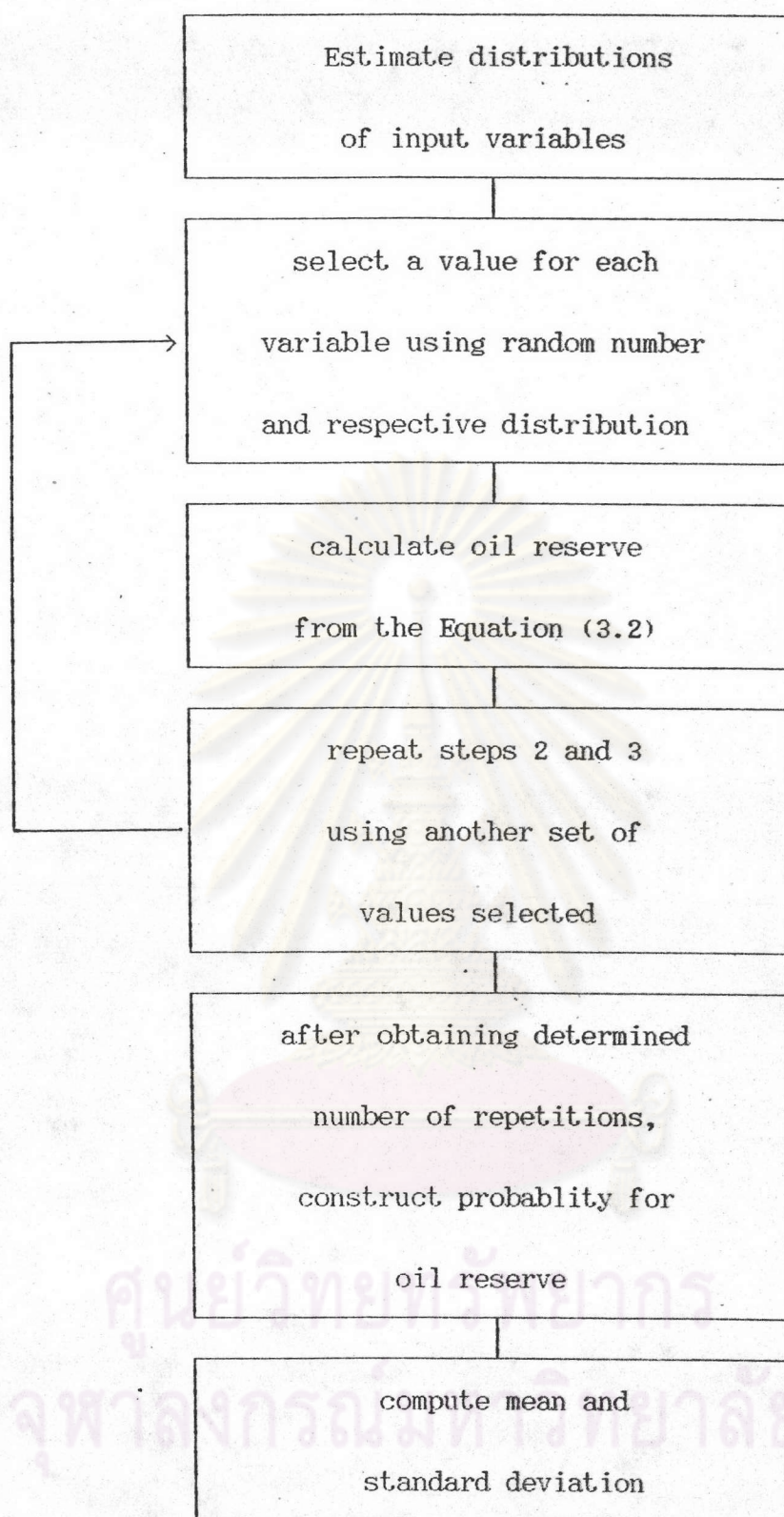
Figure 3.2. Procedure for oil reserve estimation using Monte Carlo

simulation.

Improvement of Oil Reserve Estimation by Considering Statistical

Relationship Between Two Input Variables.

In the previous section it has been shown that Monte Carlo

simulation can report the uncertainty of calculated oil reserve. In the

Monte Carlo simulation random numbers, being the very important tool,

will be used to select the values of variables from cumulative

distribution function of input variables. So, the distribution of value

of a variable will depend only on its uncertainty. In some case, values

of an input variable tend to have statistical relationship with one

another. At the same sampling point the value of one variable may not

distribute randomly but depending on the value of another variable.

The considerations of the statistical relationship between two

input variables for oil reserve calculation using Monte Carlo simulation

are illustrated by Walstrom et al. (1967), and McCray (1975).

To incorporate the statistical relationship between two input

variable into the calculation, the curve fitting is necessarily performed.

The equation obtained by this curve fitting will represent the

deterministic relationship. However, the uncertainty of the statistical

relationship should be considered. The random part should be assigned

into the relationship representing this uncertainty. Therefore, the

statistical relationship (for example, between porosity and water
saturation) can be expressed as follows:

$$S_w = A_1 \phi^2 + A_2 \phi + A_3 + \delta , \qquad (3.3)$$

where A_1, A_2, and A_3 are constants and δ is a random part.

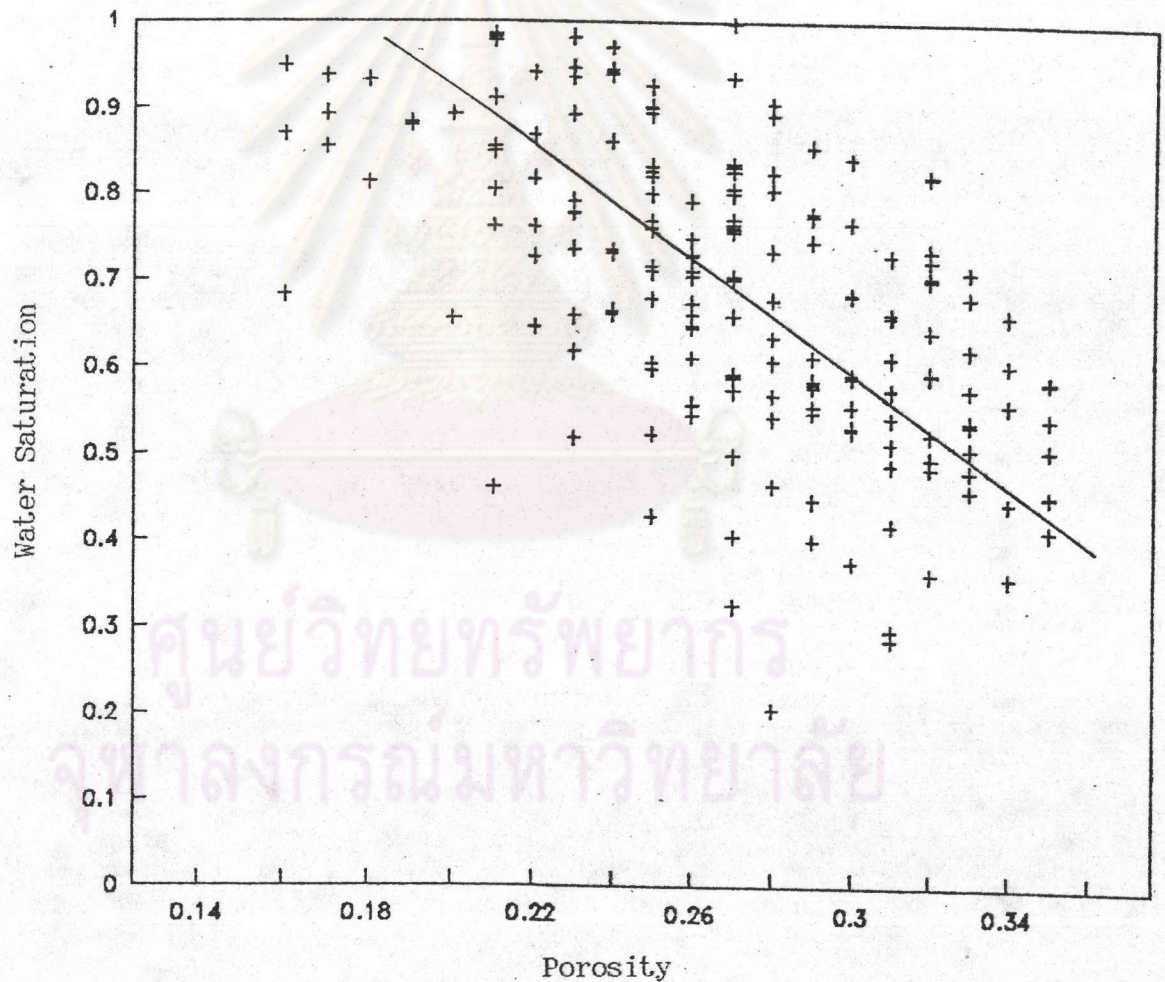Figure 3.3 shows the relationship between porosity and water
saturation.

Figure 3.3. Relationship between porosity and water saturation.

The statistical relationship may consist of an exact equation (solid curve) and random part. In this study the random part will be assumed to have a normal distribution with zero mean and a specified (depending on data) standard deviation. A simulated value of water saturation can be obtained as follows:

1. Obtaining a value of porosity by using its probability density function and a random number,

2. Calculating the exact part of water saturation using the exact part of the relationship, Equation (3.3),

3. Calculating the random part of water saturation using a random number and a specified normal distribution of the random part,

4. Obtaining the simulated value of water saturation by addition of the exact and random part of the water saturation.

In this study, the oil reserve calculation using Monte Carlo simulation will be modified by incorporating this interrelation between two input variables, porosity and water saturation, and the study of effects of this relationship on the calculated oil reserve will be investigated.

Improvement of Oil Reserve Estimation by Considering Spatial Correlation
of an Input Variable.

Observations of geological variables sometimes show significant

statistical relationship in space which may be called spatial

correlation. The values of geologic informations at different position

in a reservoir usually have different magnitude, e.g., the value of

porosity of porous medium in different positions is usually different.

Though a variable has different values at different position, its values

may have some statistical relationship. For example, at two adjacent

points if the value at one point is high, the value at the other point

will tend to be high, and vice versa. On the other hand, for two points

that are far away from each other, the value at one point would have less

influence on the value at the other point.

Simulation of spatial correlation of an input variable will be

accomplished by using turning bands method. The parameter representing

spatial correlation is a covariance function. The turning bands method

for the generation of synthetic random variables has become quite popular

in the past few years in mining and hydrology. The algorithm involves

the generation of a series of one-dimensional random processes along

lines radiating from a coordinate origin and their subsequent projection

and combination at arbitrary points in space, yielding discrete values
or realizations of the random variable. An important advantage of the
method lies in its computational efficiency.

In this study, the spatial correlation will be incorporated in
oil reserve calculation program and the study of effect of this spatial
correlaiton of an input variable on the calculated oil reserve will be
investigated.

A concept of spatial correlation and the technique that will be
used to generate values of a variable which has spatial correlation will
be discussed as follows:

1. Spatial correlation,

2. The turning bands method,

3. Spectral Representation of a two-dimensional process.

1. Spatial Correlation.

The theory about spatial correlation can be explained as
follows (Journel and Huijbregts, 1978; Horn, 1988).

Let $x = (X_1, X_2, \ldots, X_n)$ represent a point in n-dimensional
space R^n, $Z(x)$ be a random variable corresponding to point x, and
a random function is definded as the set $\{[x, Z(x)] | x \in R^n\}$. Note that,

a random function may also be called a stochastic process.

Spatial correlation of a variable can be characterized by covariance function. The covariance function is a statistical property of a variable that imply the distribution of values of variable in space. The covariance function is defined as

$$C(x_1, x_2) = E[\{Z(x_1) - m(x_1)\}\{Z(x_2) - m(x_2)\}]$$

$$= E[Z(x_1)Z(x_2)] - m(x_1)\,m(x_2), \qquad (3.4)$$

where $E[\ ]$ is an expectation operator and $m(x)$ is mean of a random variable which

$$m(x) = E[Z(x)]. \qquad (3.5)$$

A stochastic process is called a second order stationary process if the following conditions are satisfied:

1) The mean is constant at any point in space R^n:

$$E[Z(x)] = m(x) = m, \qquad \forall x \in R^n \qquad (3.6)$$

2) The covariance function depends only on the vector difference $(x_1 - x_2)$ and not on each particular vector x_1, x_2:

$$C(x_1, x_2) = C(x_1 - x_2) = C(h). \qquad (3.7)$$

The second order stationary process is usually used for spatial correlation simulation due to the simplicity of the calculation.

A second-order stationary precess is isotropic if the covariance function does not depend on the direction $h = x_1 - x_2$ of the distance vector, but only on the vector length $|h|$. Then we can write

$$C(h) = C(r), \qquad (3.8)$$

where $r = |h|$.

To obtain a covariance function for any input variable, the process of fitting a theoretical covariance function or model to an observed covariance function must be performed.

There are many model usually used in the hydrology and geological engineering such as spherical model, Gaussian model, and exponential model. Selection of these models depends on the correspondence properties of the data.

The model used in this study is the exponential model. The equation of the exponential model is as follows:

$$C(r) = \sigma^2 \exp(-br), \qquad (3.9)$$

where

      r = distance

      b = (correlation length)$^{-1}$

      σ^2 = variance of the process.

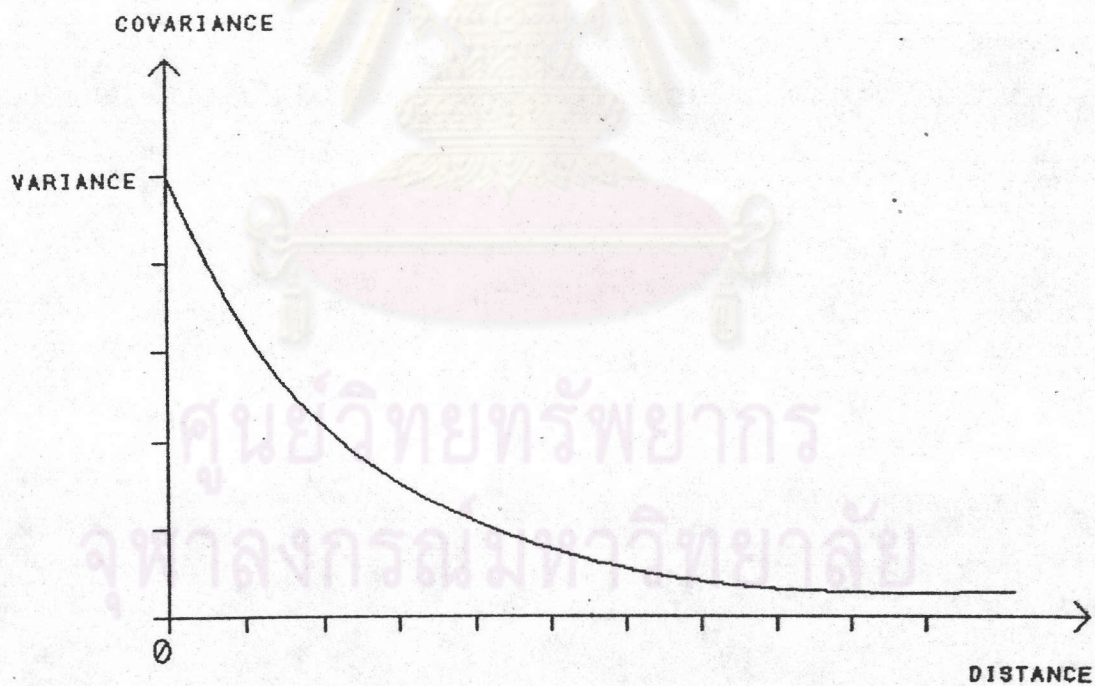Shape of the exponential model is shown in Figure 3.4.

Figure 3.4. An example of an exponential covariance function model.

Realizations of a random variable having spatial correlation must have their distribution in space corresponding to covariance function and having uncertainty of its values corresponding to its probability density function.

There is a method that is popular in geology and hydrology that can determine realization in each block with less time consumption called turning bands method. The theory of turning bands method for determining the process (random variable) in multi-dimensional space will be described in the next section. In this study the turning bands method will be use for determining realizations in a two-dimensional space with desired spatial correlation.

2. The Turning Bands Method.

The use of turning bands method to generate random field (or variable) in two-dimension will be described here. In this study it is assumed that the field to be simulated is second-order stationary and isotropic. It is also assumed that the covariance $C(r)$ of the field to be simulated is known.

Let P represent the two-dimensional field to be simulated by generating values at discrete points as shown in Figure 3.5. Choose

an arbitrary origin O in R^n and generate lines corresponding to direction

vectors u which are uniformly distributed on the unit circle. The angle

θ_i is uniformly distributed between O and 2π.

Along each line i, generate a second-order stationary

uni-dimensional discrete process having zero mean and covariance

function $C_i(\zeta)$, where ζ is the coordinate on line i. Onto line i,

orthogonally project those points of the field where we want to generate

values, and assign to them the corresponding values of the

uni-dimensional discrete process. If N is a point of the region having

location vector x_N, then the assigned value from line i will be $z_i(\zeta_{Ni})$

where $\zeta_{Ni} = x_N \cdot u_i$ is the projection of the vector x_N onto line i, u_i the

unit vector on line i, and $x_N \cdot u_i$ represents the inner product of the

vectors x_N and u_i. Take L lines such as i. For each line generate a

uni-dimensional realization using $C_i(\zeta)$ as the covariance function.

Then at every point N of the region, there are L assigned values

$z_i(\zeta_{Ni}) = z_i(x_N \cdot u_i)$, where $i = 1, \ldots, L$, from the uni-dimensional

realizations. Finally, assign to the point N the value $z_s(x_N)$ given by

$$z_s(x_N) = L^{-1/2} \sum_{i=1}^{L} z_i(x_N \cdot u_i). \qquad (3.10)$$
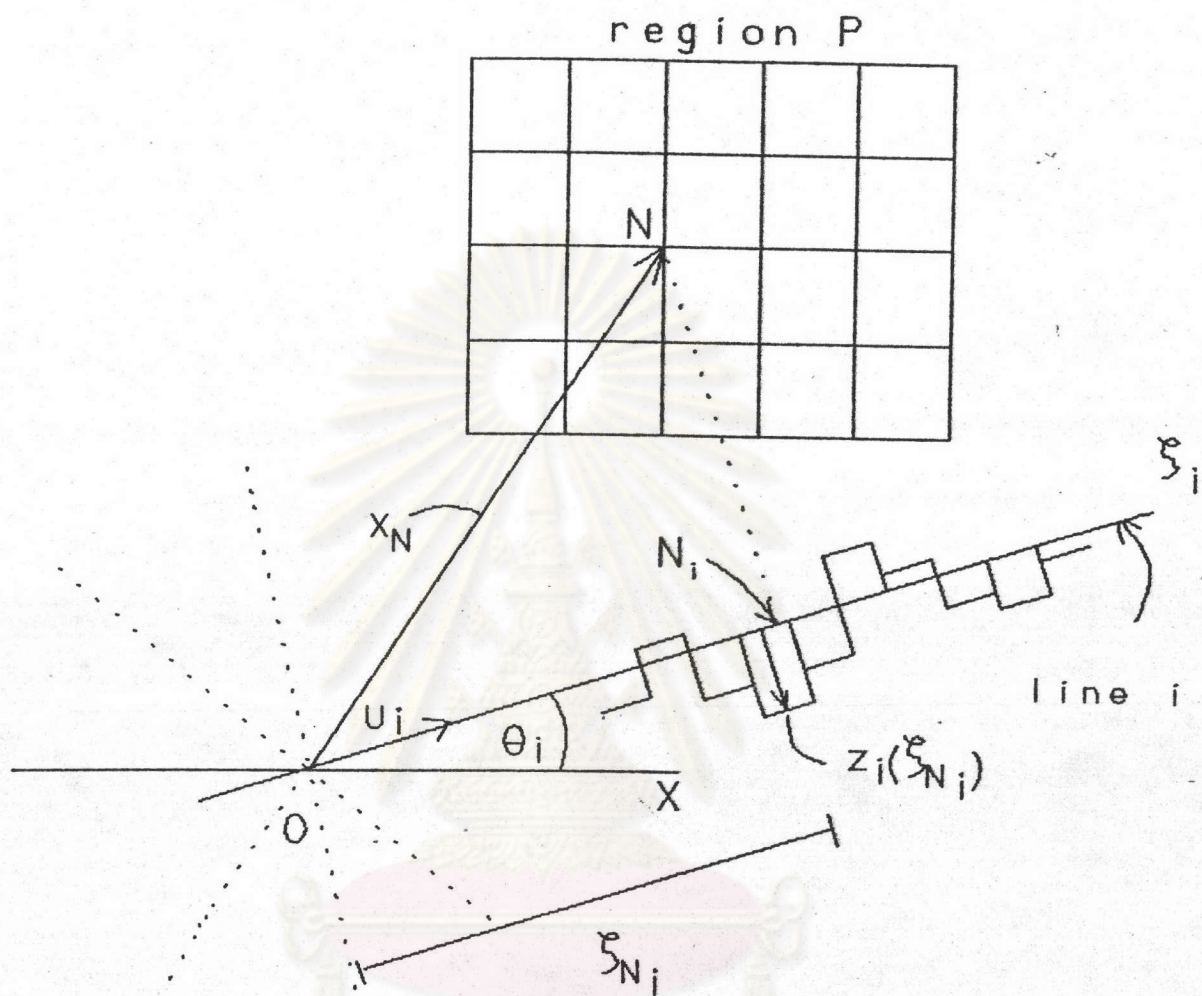
Figure 3.5. Schematic representation of the two-dimensional field P and the turning bands lines (after Mantoglou and Wilson, 1982).

The assigned lines are assumed to be uniformed distributed in two-dimensional space, as taken from a uniform distribution on the unit circle. In three dimensions, Journel and Huijbregts (1978) suggested that a group of 15 lines, joining the midpoints of the opposite edges of a regular icosahedron, is adequate for typical applications. For two-dimensional fields, Mantoglou and Wilson (1982) suggested that 4-16 lines should be sufficient, depending on the accuracy desired.

The uni-dimensional covariance function $C_1(\zeta)$ obtained has to be corresponding to the two-dimensional covariance function. In obtaining $C_1(\zeta)$, the followings are considered. Take two points of the field having location vectors x_1, x_2, respectively. The simulated values corresponding to these points are given by Equation (3.10) and the covariance function of the simulated field is (Mantoglou and Wilson, 1982)

$$C_s(r) = (1/2\pi) \int_{\text{unit cycle}} C_1(h.u) \, du, \qquad (3.11)$$

where $r = |h|$ and $h = x_1 - x_2$.

Orthogonal axes (x,y) is defined in the plane of the field, with origin at point x_1 and the y axis in the direction of the vector

$h = x_2 - x_1$ as shown in Figure 3.6.
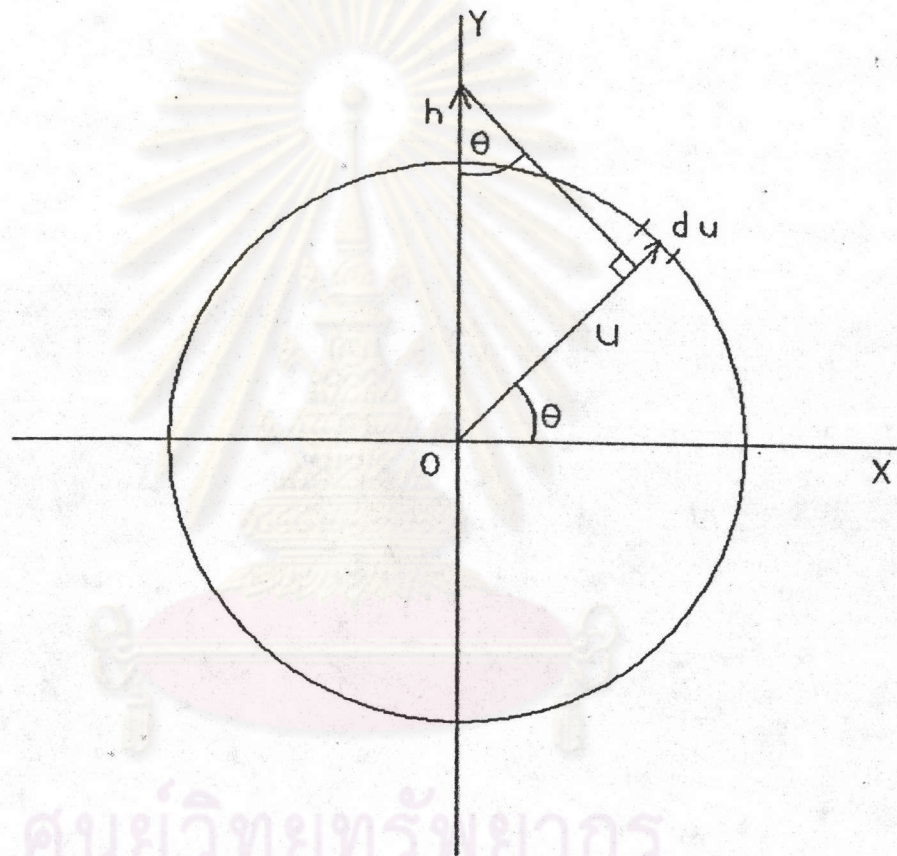
Figure 3.6. Definition sketch for the two-dimensional case, showing unit

circle (after Mantoglou and Wilson, 1982).

In polar coordinates we can write $\mathbf{h}.\mathbf{u} = r \sin\theta$ and $du = d\theta$.

Equation (3.11) then becomes

$$C_s(r) = (1/2\pi) \int_0^{2\pi} C_1(r\sin\theta)\, d\theta$$

$$= (2/\pi) \int_0^r \frac{C_1(\zeta)}{(r^2 - \zeta^2)^{1/2}}\, d\zeta , \qquad (3.12)$$

where

$$\zeta = r\sin\theta.$$

Substituting $C_s(r) = C(r)$, in order to preserve the known

covariance, we get

$$\int_0^r \frac{C_1(\zeta)}{(r^2 - \zeta^2)^{1/2}}\, d\zeta = (\pi/2) C(r). \qquad (3.13)$$

This equation relates the two-dimensional covariance function

$C(r)$ to the corresponding uni-dimensional $C_1(\zeta)$ along the turning bands

lines. It is an integral equation in which $C_1(\zeta)$ cannot be directly

expressed as a function of $C(r)$. To circumvent this difficulty, an

expression for the spectral density function of the uni-dimensional

process as a function of the radial spectral density function of the

two-dimensional process is used. The line process can then be generated

easily using a spectral method.

3. Spectral Representation of a Two-Dimensional Process.

If the covariance function $C(h)$ of the two-dimensional

process is continuous and tends to zero fast enough as $|h| \longrightarrow \infty$, it can

be expressed as a Fourier integral given by (Mantoglou and Wilson, 1982)

$$C(h) = \int_{R^2} e^{ih.w} S(w) \, dw, \qquad (3.14)$$

where i now represents $\sqrt{-1}$, $w = [w_1, w_2]$ is a vector of frequencies

(wave numbers), $dw = dw_1 dw_2$, and $h.w$ is the inner product of the vectors

h and w. The non-negative function $S(w) = S(w_1, w_2)$ is the spectral

density function of the two-dimensional process and is given by the

Fourier transform of $C(h)$:

$$S(w) = 1/(2\pi)^2 \int_{R^2} e^{-ih.w} C(h) \, dh. \qquad (3.15)$$

If the field is isotropic, then $S(w) = S(w)$, in which $w = |w|$.

The Fourier transform Equations (3.14) and (3.15) then becomes (Mantoglou

and Wilson, 1982)

$$C(r) \quad = \quad \sigma^2 \int_0^\infty f(w) J_o(wr) \, dw, \qquad (3.16)$$

$$f(w) \quad = \quad (w/\sigma^2) \int_0^\infty C(r) J_o(wr) r \, dr, \qquad (3.17)$$

where $r = |h|$, $J_o(\ )$ is a Bessel function of the first kind of order zero, and $f(w)$ is the radial spectral density function of the two-dimensional isotropic process defined by

$$f(w) \quad = \quad (1/\sigma^2) \int_{c_w} S(w) \, dw$$

$$= \quad (1/\sigma^2) \, S(w) \int_{c_w} dw$$

$$= \quad 2\pi \, w \, S(w)/\sigma^2, \qquad (3.18)$$

where c_w is a circle of radius w and dw is the differential length on circle c_w. If the two-dimensional isotropic covariance function $C(r)$ is known, we can use Equation (3.17) to calculate the corresponding radial spectral density function. The exponential scheme two-dimensional

covariance functions $C(r)$ and its radial spectral density function, $f(w)$

are shown in Equations (3.9) and (3.19), respectively, which this $f(w)$

is related to the spectral density function $S(w)$ of the two-dimensional

process through Equation (3.18) (Mantoglou and Wilson, 1982):

$$f(w) \quad = \quad (w/b^2)/[1 + (w/b)^2]^{3/2}, \qquad (3.19)$$

where

$$b \quad = \quad (\text{correlation length})^{-1}.$$

From Mantoglou and Wilson (1982) spectral representation of

the uni-dimensional process, $S_1(w)$ is

$$S_1(w) \quad = \quad (\sigma^2/2) \ f(w). \qquad (3.20)$$

The essential feature of Shinozuka and Jan (1972) is that a

random process can be simulated by a series of cosine functions with

random frequency (or wave number), and the uni-dimensional process on

line i can be generated by

$$z_i(\zeta) = 2 \sum_{k=1}^{M} [S_1(w_k) \ \Delta w]^{1/2} \cos(w'_k \zeta + \phi_k), \qquad (3.21)$$

where ϕ_k are independent random angles uniformly distributed between 0

and 2π, $w_k = (k-1/2) \Delta w$, and $w'_k = w_k + \delta w$, for $k = 1,\ldots,M$. It has

been assumed that the spectral density function $S_1(w)$ is insignificant

outside the region $[-\Omega, \Omega]$. The discretization frequency Δw is defined as $\Delta w = \Omega/M$, where M is the number of harmonics used. The frequency δw is a small random frequency added in order to avoid periodicities and is uniformly distributed between $-\Delta w'/2$ and $+\Delta w'/2$, where $\Delta w'$ is a small frequency, $\Delta w' << \Delta w$.

The accuracy of the uni-dimensional spectral method depends on the maximum frequency (Ω) at which the spectrum is truncated, and the number of harmonics (M). In Figure 3.7 one-dimensional spectral density for the exponential covariance function along some line i is shown. The frequency increment Δw must be kept small enough to ensure that a sufficient degree of accuracy is maintained, while the number of harmonics M must be kept large enough to properly account for the contributions of the spectral tail.

From a computational point of view, the line process $Z_i(\zeta)$ must be evaluated in a discrete fashion along line i. Assuming that a constant spacing is used on a given line it is easy to see qualitatively that choosing $\Delta\zeta$ larger than typical field grid spacing will reduce the effect of projecting the random information from the line to the field grid. It is quite possible that the same discrete line value will be projected to two neighboring points in the real field when, in fact, two

different values are theoretically called for. This problem can persist

in lines at an angle to the field grid and can never be fully eliminated

with a discrete line process. A pragmatic rule of thumb (Mantoglou and

Wilson, 1982) is to choose $\Delta \zeta < \min(\Delta x_1, \Delta x_2, \Delta x_3)$ for three-dimensional

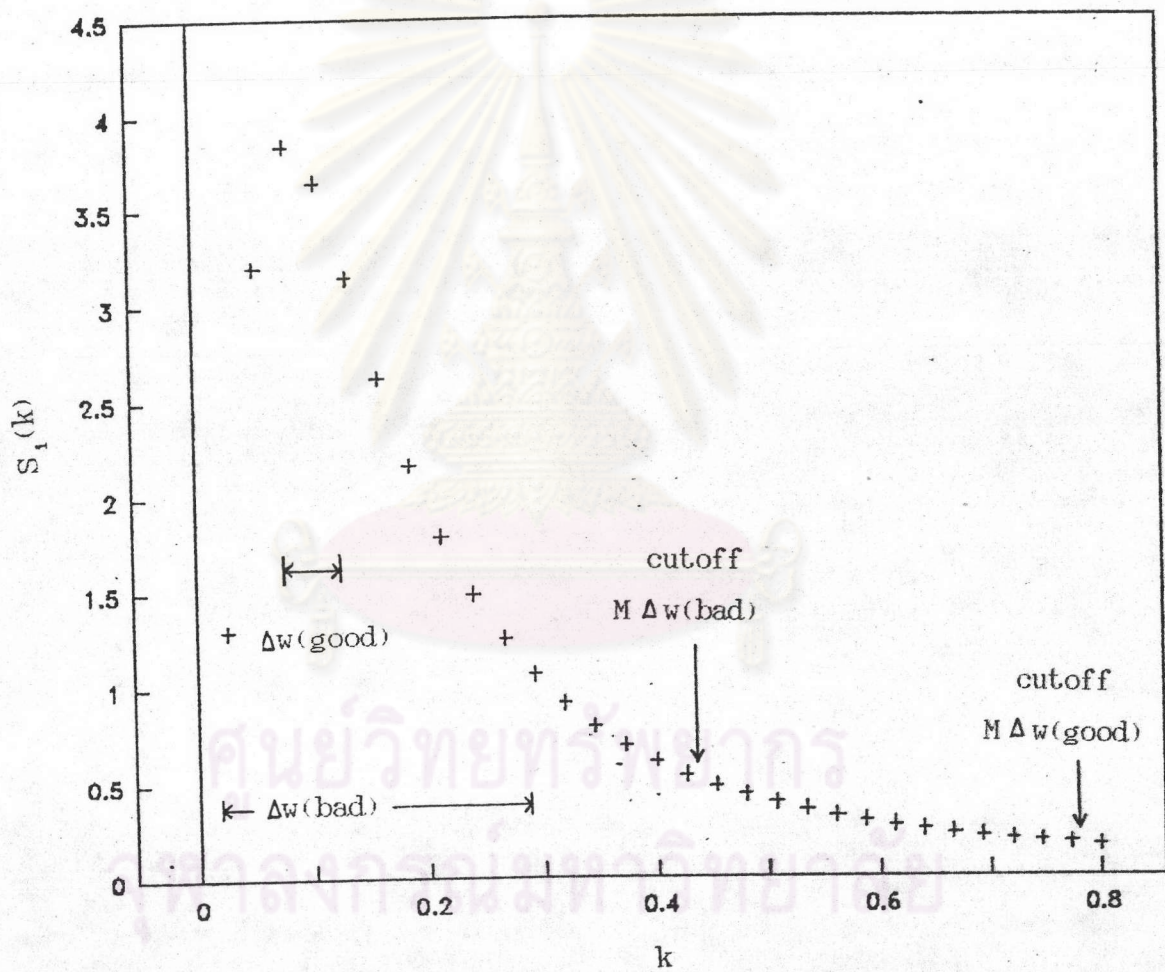space. This reduces the effect of gross errors the type shown in Figure

3.8.

Figure 3.7. General form of a line spectrum indicating choices of

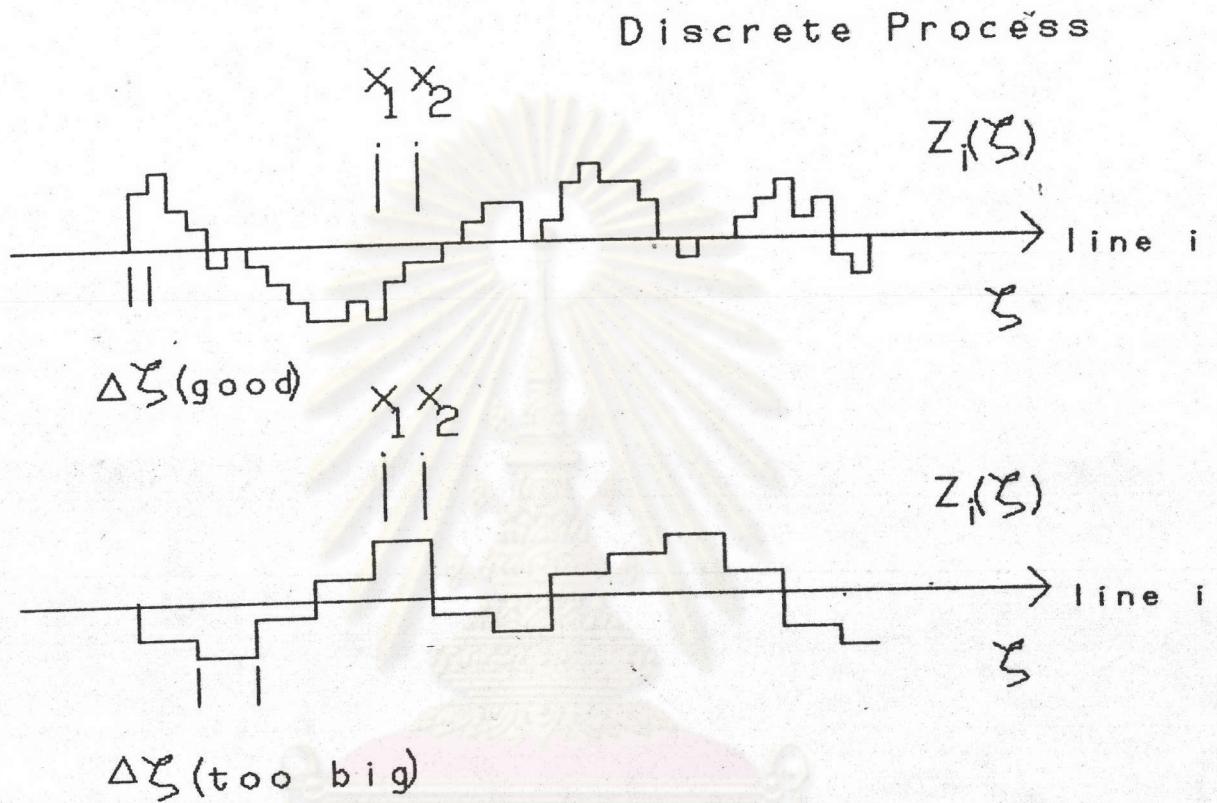harmonic number M and frequency range Ω (after Tompson et al., 1989).

Figure 3.8. Discrete implementation of line process (after Tompson et al., 1989).

In this study, the spatial correlation of an input variable is incorporated in oil reserve calculation to study effect of this correlation on calculated oil reserve.