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Modus Tollens: Interaction between the Humanities and the Sciences

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ABSTRACT

This paper describes how the disciplines of the Humanities (especially philosophy) and the Sciences--both "formal" (mathematics) and "empirical" (the natural and social sciences), are brought together through the task of theory testing in the natural and social sciences. The fundamental tool in this amazing task is "Modus Tollens"-a logical form discovered by the group of ancient Greek philosophers known as the Stoics. The relevance of this logical form in theory testing is demonstrated through some hypothetical examples in everyday life, through some examples in the natural sciences such as Torricelli's theory of the structure of earth's atmosphere, and through some examples in the social sciences such as Zajonc's confluence theory of intelligence. Finally, a testing of a recent theory in the field of education--on the relationship between class size and student academic achievement, is suggested.

Modus Tollens: Interaction between the Humanities and the Sciences

Categories of Academic Disciplines

All of the disciplines offered or taught in colleges or universities can be divided into two major categories-the humanities and the sciences. This division is a classical one (see, for example, Ruediger, 1910).

1. Humanities. The humanities include fields of studies such as painting, music, sculpture, dancing, theater, literature and philosophy. The study of "logic" is part of philosophy.

A form of logic known as **"Modus Tollens"** discovered by the Stoics around 300-129 B.C. (see Sedley, 1998), is of particular interest in the present paper. This logical form could be stated as followed: "If the first (statement is true), the second (statement is true); but not the second (statement is true); therefore not the first (statement is true)." Its structure could be represented as shown below:

$$\begin{array}{ccc} P & \supset & Q \\ & \sim & Q \\ \hline & & \sim P \end{array} \quad (1)$$

where P stands for a statement and Q stands for another statement that would make sense. The symbol ⊃ stands for "implies" and the symbol ~ stands for "not (or, it is 'not' the case that)." We could read the above structure (Structure 1) as "P implies Q (or If P then Q); not Q; therefore, not P."

As an example, let P be the statement "it rained on the street (assuming it was not too long ago)" and Q be the statement "the street is wet." The entire Modus Tollens structure shown above would then read "if it rained on the street (assuming it was not too long ago) then the street is wet; it is not the case that the street is wet; therefore, it is not the case that it rained on the street." - which makes perfect sense. Note that the first line in the three-line logical structure ($P \supset Q$) tends to be an armchair thinking while the second line ($\sim Q$) tends to be an actual observation. The third line ($\sim P$) is the conclusion that results from the previous two lines. The readers of this paper might want to try other examples as an exercise.

In the above example, the street is observed "not wet" (~ Q). What conclusion (line 3) could be drawn if the street is observed "wet" (Q)? Could we conclude that "It rained on the street" (P)? A little reflection will make one realize that we could not make such conclusion with certainty for the street could have been wet by other means such as a water truck has just watered the street. The more correct conclusion would be "maybe it rained on the street" (maybe P). The structure of reasoning in this latter case is shown below:

$$\begin{array}{ccc} P & \supset & Q \\ & & Q \\ \hline & & \text{maybe } P \end{array} \qquad (2)$$

where "maybe P" indicates that the conclusion is not certain. This is contrary to Structure 1 in which the conclusion of "~P" is quite certain. It is important to note here that the logical form Modus Tollens refers to Structure 1, not Structure 2. No official name has been given to Structure 2 since it is not a "valid" (conclusion with certainty) deductive form. The author would like to refer to Structure 2 simply as **"Modus Maybe."** It will be seen later in this paper that these two complementary structures are used in testing theories in the natural and social sciences.

2. Sciences. The sciences are subdivided into the FORMAL sciences and the EMPIRICAL sciences. The formal sciences include fields of studies such as algebra, geometry, trigonometry, calculus and topology. The empirical sciences are further divided into the NATURAL and SOCIAL sciences. The natural sciences include fields of studies such as physics, chemistry and biology. The social sciences include fields of studies such as sociology, economics, political sciences, anthropology, and psychology.

While the formal sciences rely on "deductive reasoning" (the main branch of logic) to prove the truth of their statements or theorems, the empirical sciences rely on "observation" to prove the truth or credibility of the statements claimed. Empirical knowledge is then simply a set of statements proven true or credible by the method of observation. Three types of these statements could be identified-the particulars, laws, and theories.

2.1 Particulars. A particular is a true-by-observation statement referring to a specific event that happens at a particular place and time. Some philosophers of sciences also referred to this type of statements as "singulars" (e.g., Ayer, 1972, p. 134). An example of a particular or singular is statement such as:

"(Look!) The level of water is rising."

as uttered by an elementary school teacher while performing an experiment in which she poured some water into a glass tube and moved the tube over a lamp.

2.2 Laws. A law is a true-by-observation statement referring to a more general event that could happen in many places and over a period of time. Hosper (1967, p. 232) referred to this type of statements as "universal" empirical propositions. An example of a law is:

"Water in general, when heated, expands."

This assertion is true of any water that happens in any place and at any time. (The author may need to add here that this truth is limited by the range of temperature from 4 to100 degrees Celsius at the sea level.)

A law could also be viewed as describing a "connection", or relationship, between "concepts" or variables (see Brodbeck, 1968, p. 7). A "variable" is a concept. In the above example, "heat" or *temperature* is a variable, and "expansion" or *volume* is also a variable. To say that "water expands when heated," is equivalent to saying that "higher-temperature water has larger volume and lower-temperature water has smaller volume."

2.3 Theories. A theory is a "credible (by observation)" statement or set of statements referring to objects or events that are NOT open to direct observation. The events referred to tend to be those that happen behind the scene, or happen beneath the surface (an underlying process). Thus, a theory tends to have an aura of mystery. This meaning of the term "theory," as used in the empirical sciences, coincides with the meaning of the term as used in everyday language. In our daily life--say, if our house was burglarized last night and we did not see who did it but we have a hunch, we might say "I have a theory of who did it." We only have a "theory" of who did it because we "did not actually see or observe" who did it! A theory is about an event that is NOT, or NO LONGER is, open to direct observation. An example of a theory in the empirical sciences is the **molecular theory of liquid** in general, or water in particular. Virtually all of us learned this theory earlier in schools (elementary or secondary). The theory is somewhat like the following:

> "Water consists of tiny tiny molecules. Each molecule consists of two hydrogen atoms and one oxygen atom. These molecules vibrate all the time except at the absolute zero temperature. The vibration of these molecules goes hand-in- hand with the temperature-the more the vibration the higher the temperature or vice versa."

Has anyone really seen the molecules or atoms described above? Has anyone seen them vibrate? If no one has, that is simply because it is a theory. Remember? Theories usually describe events that are not open to direct observation. This sets theories apart from either laws or particulars. A particular or a law MUST be about what we can OBSERVE more or less directly. Thus, in the example of a "particular" given above, we can directly observe the "level of water" and how it "rises." In the example of a "law" given above, we can directly observe the heating, the rising of temperature as indicated by the rising mercury, and the expansion of the volume or water.

Examples of Particulars, Laws and Theories in the Social Sciences

The examples of particulars, laws, and theories given above are from the area of natural sciences (or physical sciences, to be more exact). There are, of course, parallels in the social sciences. The following examples are from the area of psychology.

Particular: "John Doe Jr. is very smart with an IQ of 150."

This statement could be, for example, confirmed from the school psychologist's file at school.

Law: "An earlier born child tends to have higher IQ than a later born child in a Family." This statement has some data supporting it (see, for example, Zajonc and Markus, 1975, p. 75; Zajonc, 1976, p. 228). Note also that the statement describes a relationship between the "birth order" variable and the "IQ" variable. As mentioned earlier, a scientific law often refers to relationship between variables.

Theory: An example of a theory in the social science is the **"Confluence Theory of Intelligence."** The full form including the mathematics of it can be found in Zajonc and Markus (1975). The following is an abbreviated and simplified version:

> "Within each family, the intellectual growth of every member is affected by the family's intellectual environment. The family's intellectual environment is the average of all the members' intellectual units."

Note that the above set of statements refers to an event that is not open to direct observation. How could any person witness the effect of family intellectual environment on a child's intelligence the same way one could witness the effect of burning fire on a piece of wood or plastic? Similarly, a family member's intellectual units could be conceived of as "invisible" bundles of intelligence in the head. The more of these invisible bundles a person possesses, the more intelligent the person becomes.

Readers who are interested in a more "formal" description of a theory are referred to Hempel (1970).

How do we determine that a Particular, a Law, or a Theory is True or Credible-By-Observation?

How do we know if a particular, a law, or a theory is true or credible-by observation? For a particular or a law, this is simple. Because a particular or a law describes an event that is open to direct observation, we simply observe the event referred to by the "particular" or the "law" statement. If there is a correspondence between the event observed and the statement claimed, then the statement is true-by-observation. If we indeed observe that the level of water is rising, we say that the statement: "the level of water is rising," is true. If we indeed observe the level of water rising when heated at various places and times, we say that the statement "water in general, when heated, expands" is true.

For a theory, the situation is not as simple. A theory describes an event that is not directly observable. We cannot simply observe the event referred to by the theory, for we will see nothing.

The structure of Theory Testing

What a scientist or researcher does in theory testing is to use the theory (which is about the unobservable) to LOGICALLY derive a so-called **"observable consequence."** This observable consequence-also called **"test implication"** by Hempel (1966), of course, is represented by another statement. However, this "observable consequence" statement now refers to an event that is open to direct observation. The scientist or the researcher then observes the event referred to by this "observable consequence" statement. If there is a correspondence between the event observed and the "observable consequence" statement claimed, then the "observable consequence" statement is "directly" true, and the theory becomes "indirectly" credible-by-observation.

A simple example will clarify the above method of testing a theory. Let's assume that the author proposes a **theory of "The Invisible Rabbit"** which is in the following form:

"There is an invisible rabbit sitting in the corner of this room." .... (1)

This statement (statement #1) qualifies as a theory because it refers to an unobservable event. To logically derive an "observable" consequence, we could think of the fact that all rabbits, invisible or not, need to eat. What do rabbits like to eat? The answer may be "carrots." How long can a rabbit go without eating--maximum? The answer may be "perhaps, 24 hours." We, therefore, can derive an "observable consequence" as follows:

"A carrot left in the corner of the room is eaten (will be eaten) within 24 hours." ....(2)

Note that this statement (statement #2), unlike statement #1, refers to an event that is completely open to direct observation. Now, if we observe the corner of the room and find the carrot disappear (eaten) little by little in bits and pieces before the day is over, statement #2 will be directly confirmed. At the same time, statement #1 (the Theory of Invisible Rabbit) will, indirectly, become "credible." Note that I use the term "credible" here rather than "true" because it may be an invisible jerboa (or another invisible animal of a different species), not an invisible rabbit, that eats the carrot. However, the notion that the animal is an invisible rabbit has now become a possibility. On the other hand, if we have observed the corner of the room for an entire day and the carrot is still safe and sound, statement #2 (the observable consequence) will be directly rejected and statement #1 (the Theory of Invisible Rabbit) will be indirectly rejected. Note that the rejection of the theory is quite forceful because if the carrot is safe and sound, it obviously is NOT the case that there is any invisible animal (including the invisible rabbit) there to eat it!

The theory testing process described above can be represented in a diagram as follows.

Case 1:

Step 1: IF statement #1(unobs) is true	THEN	statement #2(obs) is true
Step 2:		statement #2 is true
--		
Step 3: THEREFORE	MAYBE	statement #1 is true

Case 2:

Step 1: IF statement #1(unobs) is true	THEN	statement #2(obs) is true
Step 2:		statement #2 is false
--		
Step 3: THEREFORE		statement #1 is false

Again, note that the conclusion(Step 3) in Case 2 is more forceful than in Case 1 as explained earlier.

Note also that in the above two cases, **Case 1** corresponds precisely to the **"Modus Maybe"** structure while **Case 2** corresponds precisely to the **"Modus Tollens"** structure described at the beginning of this paper. The role of the logical form Modus Tollens in theory testing is now obvious.

It is important to also realize that there can be other possible observable consequences (or test implications) from the same theory. For example, another test implication from the invisible rabbit theory could be:

> "If we throw some sand into the corner where the invisible rabbit is and poke the corner with a stick, we will see rabbit footprints." .... (2.2)

Theory Testing in the Natural Sciences

Let's go back to **the molecular theory of water** repeated below for reading convenience:

> "Water consists of tiny tiny molecules. Each molecule consists of two hydrogen atoms and one oxygen atom. These molecules vibrate all the time except at the absolute zero temperature. The vibration of these molecules goes hand-in-hand with the temperature-the more the vibration the higher the temperature or vice versa."..(1)

This is the theory of invisible water molecules rather than invisible rabbit. What would be the "observable" consequence of this theory? We could perform the following logical thinking. If this theory is true, let's apply heat to the water and predict what would happen. The heat applied should increase the water temperature which should make the invisible water molecules vibrate more (as stated in the latter part of the theory shown above). As the molecules vibrate more, they would occupy more space. As a result, the volume of water would expand. Thus, the "observable" consequence of the theory of "invisible" water molecules is that:

> "Water in general, when heated, expands."..(2)

Since this observable consequence has long been observed and confirmed, we could conclude according to the Case 1 reasoning diagram shown above that the Molecular Theory of Water should be retained as a "maybe." This theory has become "credible" as a result of confirming its observable consequence. Could you now think of another possible observable consequence (or test implication)? Would the following statement work?

> "If we spin the water in a container with an egg beater, the water temperature will rise." .... (2.2)

Another example of theory testing in the natural sciences involves **Torricelli's theory of the structure of earth's atmosphere** (see Hempel, 1966). Simply put, the theory says:

> "The earth is surrounded by a sea of air. This sea of air exerts pressure on the surface below." .... (1)

Torricelli (1608-1647) had constructed a simple "mercury barometer" consisting of a glass tube sealed at one end, first filled with mercury and then inverted into a large container of mercury. At about sea level, the height of mercury in the inverted glass tube stood at about 30 inches. An interesting observable consequence (or, test implication) of Torricelli's theory was suggested by Pascal. Pascal reasoned that because there is more air pressing on any surface at sea level than at higher altitude, the level of mercury in Torricelli's inverted glass tube should decrease with increasing altitude. The observable consequence or test implication could be summarized in the following statement:

> "If Torricelli's mercury barometer is carried to increasing altitude (such as up a mountain), the mercury level will decrease." ....(2)

This test implication was carried out by Perier (Pascal's brother-in-law) who measured the mercury level as the barometer was being brought up the Puy-de-Dome mountain. Indeed, the level of mercury was observed to decrease and therefore, Torricelli's theory was retained as a "maybe."

Theory Testing in the Social Sciences

Let's now see how **the Confluence Theory of Intelligence** in psychology is tested. The theory is repeated below for convenience:

"Within each family, the intellectual growth of every member is affected by the family's intellectual environment. The family's intellectual environment is the average of all the members' intellectual units." ....(1)

Before attempting to draw an observable consequence from this theory, let us note that a person's intellectual growth does not increase forever. Actually, Piaget's theory of intellectual development proposes that a person's intellectual growth reaches the final stage of "formal operation" at about age 12 (see, for example, Piaget & Inhelder, 1969, p. 135; Hunt, 1961, p. 230). Other theories of intellectual development are more optimistic, showing a growth curve that levels off at about 25 years of age (see, for example, Hilgard, 1975, p. 415). However, different theories of intellectual growth, despite differences in the specifics, seem to agree on one general idea that after a while our intelligence does not grow any further. The most optimistic theory puts the leveling off point at about 25 years of age.

Now, what is the "observable" consequence of the Confluence Theory of Intelligence described above? To deduce an observable consequence from this theory, let's perform the following logical thinking. If in a family of three, the father who is 30 years old possesses 30 invisible bundles of intelligence, the mother who is 30 years old also possesses 30 invisible bundles of intelligence, and that of the new-born child possesses 0 bundle, the theory tells us that the "intellectual environment" of this family is (30+30+0)/3=20. Suppose four years later, another child is born. The intellectual environment according to the Confluence Theory will now be (30+30+4+0)/4=16. Note that the number of invisible bundles of intelligence for the father or mother remains 30 despite their advance in age to 34! This is in accordance with the theories of intellectual growth (described in the previous paragraph) which hold that intelligence should not grow further after age 25. However, the four-year-old child has now acquired 4 invisible bundles of intelligence and the second child, being new-born, has not acquired any bundle yet. It is now obvious from the calculation above that the second-born child was born into an intellectual environment of 16 units while the first-born was born into a higher intellectual environment of 20 units! Following this line of reasoning, if we calculate the intellectual environments that the third-born, fourth-born, ... would be born into, we would be forced to conclude that an earlier born child would have higher IQ than a later born child. To put the statement in a milder form, we would have:

> "An earlier born child tends to have higher IQ than a later born child in a Family." ....(2)

This above statement is an "observable" consequence of the Confluence Theory of Intelligence. We can observe from records who is earlier born and who is later born. We could observe from IQ test results who has what score. Many readers might have already noticed that this observable consequence is, in fact, an example of a "law" mentioned earlier! This is in fact the case in most situations in the empirical sciences. That is, a law is often the observable consequence of a theory.

Another example of theory testing in the social sciences is Archwamety's **Theory of Spatial Distance between Teacher and Student** (see Archwamety, 2000). The postulate of the theory is shown below:

$$A = Re^{-s(D-1)} + M \quad \text{..........} \quad (1)$$

where A = Academic achievement of a student.

R = Range (or difference) of achievement between the student who is seated nearest to the teacher and the student who is seated furthest.

e = exponential (approximate value = 2.7183)

s = steepness of the decaying exponential function.

D = Distance (physical) of the student from teacher.

M = Minimum achievement (of the student seated furthest from the teacher).

Simply put, a student achieves less the further the student is seated away from the teacher. This relationship is curvilinear as dictated by equation of the theory. The test implication of this theory is:

> "The academic achievement of students in a smaller class will be higher than that in a larger class." ....(2)

This test implication follows immediately from the theory considering the fact that the average distance between teacher and students tends to be longer as the class gets larger. This test implication was supported by Glass, Cahen, Smith, and Filby (1982) who found in their meta-analysis of research studies that as class size increases the students' achievement decreases.

Another test implication of the Spatial Distance between Teacher and Student theory is:

> "A rectangular classroom will do better with teacher located at the longer side of the rectangle than at the shorter side of the rectangle." ....(2.2)

This test implication could be checked by an interested educator in future research. Readers who are interested in the details of the "Spatial Distance between Teacher and Student" theory are referred to the full article by Archwamety (2000) listed in the References section of this paper.

References

Archwamety, T. (2000). A special distance explanation of the relationship between class size and achievement. **Journal of Research Methodology,** 13(2), 99-119.

Ayer, A .J. (1972). **Language, truth and logic.** Harmondsworth, England: Penguin Books.

Brodbeck, M. (1968). **Readings in the philosophy of the social sciences.** New York: Macmillan.

Glass, G. V., Cahen, L. S., Smith, M. L., & Filby, N. N. (1982). **School class size.** Beverly Hills, CA: Sage.

Hempel, C. G. (1966). **Philosophy of natural science**. Englewood Cliff, NJ: Prentice-Hall.

Hempel, C. G. (1970). On the "standard conception" of scientific theories. In M. Radner & S. Winokur (Eds.), **Analyses of theories and methods of physics and psychology** (pp.142-163). Minneapolis: University of Minnesota Press.

Hilgard, E. R., Atkinson, R. C., & Atkinson, R. L. (1975). **Introduction to psychology** (6th Ed.). New York: Harcourt Brace Jovanovich.

Hospers, J. (1967). **An introduction to philosophical analysis** (2nd Ed.). Englewoos Cliffs, NJ: Prentice Hall.

Hunt, J. McV. (1961). **Intellegence and experience**. New York: The Ronald Press Company.

Piaget, J., & Inhelder, B. (1969). **The psychology of the child.** New York: Basic Books.

Ruediger, W. C. (1910). **The principles of education.** Houghton Mifflin Company. (p. 177)

Sedley, D. (1998). Stoicism. In E. Craig (Ed.), **Routledge Encyclopedia of Philosophy** (pp. 9:141-161). London and New York: Routledge.

Zajonc, R. B., & Markus, G. B. (1975). Birth order and intellectual development. **Psychological Review,** 82(1), 74-88.

Zajonc, R. B. (1976). Family configuration and intelligence. **Science,** 192(4236), 227-236.