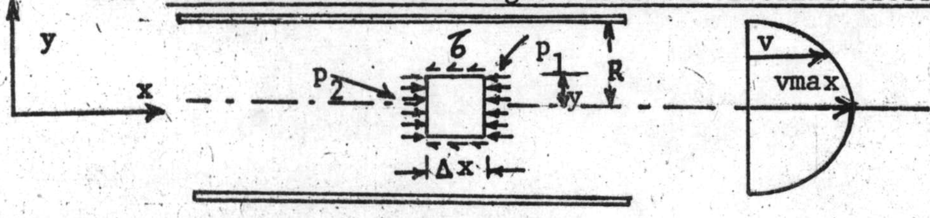
CHAPTER III

THEORETICAL CONSIDERATION

3.1 Viscous Flow in Straight Tubes of Circular Cross Section



Consider the cylindrical fluid element of length dx and radius y as shown above.

Newton's law;

$$\tau \propto \frac{-dv_x}{dy}$$

$$\tau = -\eta \frac{dv_x}{dy} \qquad \text{———————} \qquad (1)$$

η = Coefficient of absolute viscosity

ΣF_H = o;

$$\pi y^2 (P_2 - P_1) - 2\pi y \Delta x \tau = 0$$

letting

Δhr = $(P_2 - P_1)$ and substituting $-\eta \frac{dv_x}{dy}$ for τ

$$\pi y^2 r \Delta h = -2\pi y \Delta x \; \eta \frac{dv_x}{dy}$$

$$\frac{dv_x}{dy} = \frac{-yr\Delta h}{2\eta \Delta x}$$

Integrating

$$v_x = \frac{-r}{2\eta}\frac{\Delta h}{\Delta x}\cdot\frac{y^2}{2} + C$$

Boundary Condition;

$$v_x = 0, \quad y = R.$$

therefore, $$C = \frac{r\Delta h}{2\eta\Delta x}\frac{R^2}{2}$$

$$v_x = \frac{r\Delta h}{4\eta\Delta x}(R^2 - y^2)$$

As $\Delta x \longrightarrow 0$; $\frac{\Delta h}{\Delta x} \longrightarrow \frac{dh}{dx} = s$ (hydraulic gradient)

$$v_x = \frac{r}{4\eta} s (R^2 - y^2)$$

If $y = 0$; $v_x = v_{max.}$

$$v_{max.} = \frac{r}{4\eta} s R^2$$

$$v_{av} = \frac{v_{max}}{2}$$ ($\because$ Velocity distribution is a paraboloid of revolution)

$$q = \int v_x da$$

$$= v_{av} a$$

$$= \frac{1}{2}\left(\frac{r}{4\eta} sR^2\right) a$$

$$= \frac{1}{2}\frac{rs}{\eta} R_h^2 \, a$$

R_h = hydraulic radius of tube = $\frac{R}{2}$

3.2 Viscous Flow between Wide Parallel Plates

Let z = width of plates

$$\Sigma F_h = 2yz\,(P_2 - P_1) - 2z\Delta x\tau = 0$$

$$2yz\, r\Delta h + 2z\Delta x\eta \frac{dv_x}{dy} = 0$$

$$\frac{dv_x}{dy} = \frac{-r\Delta h}{\eta\Delta x} y$$

$$\frac{dv_x}{dy} = -\frac{r}{\eta} sy$$

$$v_x = -\frac{rs}{\eta} \frac{y^2}{2} + c$$

B.C.

$$y = D, \quad V_x = 0 ;$$

$$c = \frac{rs}{\eta} \frac{D^2}{2}$$

D = one-half distance between plates

$$v_x = \frac{rs}{2\eta} (D^2 - y^2)$$

$$v_{av} = \frac{2}{3} v_{max}$$ (The velocity distribution is parabolic)

$$q = 2/3 \left(\frac{rs}{2\eta} D^2\right) a$$

$$= 1/3 \frac{rs}{\eta} D^2 . a$$

$$= 1/3 \frac{rs}{\eta} R_h^2 . a$$

($R_h = \frac{2DZ}{2Z} = D =$ hydraulic radius of plates)

3.3 Viscous Flow through Any Cross-section Shape

$$q = c_s \frac{rs}{\eta} R_h^2 . a \quad \text{———} \quad (2)$$

This equation 2 is known as Hagen - Poiseuille equation

where c_s = shape factor

Kozeny's Equation

$$R_h = \frac{\text{Cross - sectional of flow}}{\text{Wetted perimeter}} \left(\frac{L}{L}\right)$$

$$= \frac{\text{Volume of voids}}{\text{Void (or particle) surface area}}$$

$$= \frac{V_s}{S} \left(\frac{E}{1-E}\right)$$

$$= \frac{1}{S_v} \left(\frac{E}{1-E}\right)$$

Substituting R_h in equation (2)

$$\frac{q}{a} = \frac{c_s rs}{\eta} \frac{1}{S_v^2} \frac{E^2}{(1-E)^2}$$

$$u_e = c_s \frac{r}{\eta} \frac{\Delta h}{Le} \left(\frac{L}{L}\right) \frac{1}{S_v^2} \frac{E^2}{(1-E)^2}$$

$$u/E = \frac{c_s}{\eta} \frac{\Delta P}{L_e} \left(\frac{L}{L}\right) \frac{1}{S_v^2} \frac{E^2}{(1-E)^2}$$

$$u = \frac{c_s}{\eta} \left(\frac{L}{L_e}\right) \frac{\Delta P}{L} \frac{1}{S_v^2} \frac{E^3}{(1-E)^2} \qquad (3)$$

This equation was derived by Kozeny.

It was pointed out by P.C. Caman[5] that the pore velocity parallel to the direction of flow is $\frac{u}{E}$ if the fractional free area is E; but, as the actual path pursued by an element of fluid is tortuous, the true pore velocity must be higher. The time taken for such an element to pass over a tortuous track of length L_e, at a velocity $\left(\frac{u}{E}\right)\left(\frac{L_e}{L}\right)$, corresponds to the time taken to pass over a distance L at velocity $\frac{u}{E}$. Thus, we must replace $\frac{u}{E}$ by $\left(\frac{u}{E}\right)\left(\frac{L_e}{L}\right)$

$$\left(\frac{u}{E}\right)\left(\frac{L_e}{L}\right) = \frac{c_s}{\eta}\left(\frac{L}{L_e}\right)\left(\frac{\Delta P}{L}\right)\left(\frac{1}{S_v^2}\right)\frac{E^2}{(1-E)^2}$$

$$u = \frac{c_s}{\eta}\left(\frac{L}{L_e}\right)^2\left(\frac{\Delta P}{L}\right)\left(\frac{1}{S_v^2}\right)\frac{E^3}{(1-E)^2}$$

$$\frac{v}{At} = \frac{c_s}{\eta}\left(\frac{L}{L_e}\right)^2\left(\frac{\Delta P}{L}\right)\left(\frac{1}{S_v^2}\right)\frac{E^3}{(1-E)^2}$$

$$S_v^2 = c_s\left(\frac{L}{L_e}\right)^2\frac{At}{v\eta}\left(\frac{\Delta P}{L}\right)\frac{E^3}{(1-E)^2}$$

$$S_w^2 = \frac{c_s}{\rho^2}\left(\frac{L}{L_e}\right)^2\frac{At}{v\eta}\left(\frac{\Delta P}{L}\right)\frac{E^3}{(1-E)^2} \quad\text{———} \quad (4)$$

This equation is known as Kozeny-carman equation.

The range of variation in c_s is not large, and the use of a value of 1/2 would seldom result in errors above 25 percent. The determination of $\left(\frac{L}{L_e}\right)^2$ is still a matter of considerable controversy. Carman suggested that the product of $\frac{1}{c_s}\left(\frac{L_e}{L}\right)^2$ should be taken as 5 for unconsolidated porous media.

then,

$$S_w^2 = \frac{1}{5}\left(\frac{At}{\rho^2 v\eta}\right)\left(\frac{\Delta P}{L}\right)\frac{E^3}{(1-E)^2} \quad\text{———}\quad (5)$$

in c.g.s. unit the above equation may be written as

$$S_w^2 = \frac{980}{5}\left(\frac{At}{\rho^2 v\eta}\right)\left(\frac{\Delta P'}{L}\right)\frac{E^3}{(1-E)^2}$$

where $\Delta p'$ = pressure difference in $\frac{gm}{c.m.^2}$

$$S_w^2 = \frac{196}{\rho^2}\left(\frac{At}{v\eta}\right)\left(\frac{\Delta P'}{L}\right)\frac{E^3}{(1-E)^2}$$

$$S_w = \frac{14}{\rho}\sqrt{\left(\frac{At}{v\eta}\right)\left(\frac{\Delta P'}{L}\right)\frac{E^3}{(1-E)^2}} \quad\text{———}\quad (6)$$

For Blain air permeability apparatus, the calibration of the apparatus shall be made using the standard, fineness sample. Calculation of specific surface values shall be made according to the following formulas:

$$S_w = \frac{S_s \rho_s (1-E_s) \sqrt{E^3} \sqrt{T}}{\rho (1-E) \sqrt{E_s^3} \sqrt{T_s}} \tag{7}$$

$$S_w = \frac{S_s \rho_s (1-E_s) \sqrt{\eta_s} \sqrt{E^3} \sqrt{T}}{\rho (1-E) \sqrt{\eta} \sqrt{E_s^3} \sqrt{T_s}} \tag{8}$$

Equation (7) shall be used when the temperature of test of the test sample is **within** $\pm$ 3°c of the temperature of calibration test of the standard fineness sample, and eq. (8) is used if the temperature of tests is outside of this range.

<u>Limitations of Kozeny-Carman equation</u>

1) It is assume that the range of pore shapes is such that C_s is reasonably constant.

2) The Tortuosity is not very susceptible to variations in pore geometry.

3) It is valid for unconsolidated media with a random pore structure, but not for flow along parallel-oriented fibres.

4) Uniformity of pore size is implied.