CHAPTER 4

ANALYSIS

4.1 The Mathematical Model

The open loop transfer function from equation 2.9 can be rewritten as

$$Go(s) = Ksf \frac{KA}{n} Gho(s)Gm(s) \qquad (4.1)$$

Substitute for

$Ksf = 0.000146$ sec/rad

$KA = 21739.0$ volts/sec

$n = 76.56$

$Km = 35.5$ rad/sec/volt

$Tm = 0.0149$ sec

$$Gho(s) = \frac{1-e^{-sT}}{s}$$

$$Gm(s) = \frac{Km}{s(Tm\ s + 1)}$$

Then equation (4.1) becomes

$$Go(s) = 1.472 \cdot \frac{1 - e^{-sT}}{s^2(Tm\ s + 1\ )} \qquad (4.2)$$

$$= 1.472\ (1-e^{-sT})\left(\frac{1}{s^2} - \frac{Tm}{s} + \frac{Tm}{s+1/Tm} \right) \qquad (4.3)$$

Then

$$Go(z) = 1.472\ (1-z^{-1})\left(\frac{zT}{(z-1)^2} - \frac{zTm}{z-1} + \frac{zTm}{z-e^{-T/Tm}} \right) \qquad (4.4)$$

Let $T' = T/Tm$ (4.5)

$$Go(z) = 0.02193\ \frac{(T'+e^{-T'}-1)z + (1-T'e^{-T'}-e^{-T'})}{(z-1)(z-e^{-T'})} \qquad (4.6)$$

4.2 The Critical Sampling Period

For dead-zone $d = 1\% \times 0.92\text{x}10^{-3} = 0.92\text{x}10^{-5}$,

then from equation (2.25), we have

$$E\left(\frac{1}{N(z)}\text{min.}\right) = \sqrt{2}\text{x}0.92\text{x}10^{-5} = 1.3\text{x}10^{-5} \text{ sec} \qquad (4.7)$$

and from section (3.7) implies that

$$V = \text{Tem} = 0.92\text{x}10^{-3} \text{ sec} \qquad (4.8)$$

Then from equation (2.24), for E=V, we have

$$-\frac{1}{N(z)}\text{min} = -\frac{\pi}{4} \times 2 \times \frac{d}{E}$$

$$= -\frac{\pi}{4} \times 2 \times \frac{1}{100}$$

$$= -1.57 \times 10^{-2} \text{ sec} \qquad (4.9)$$

From equation (4.6), we can calculate the critical sampling period to be used by equating

$$-\frac{1}{N(z)} = Go(z) \qquad (4.10)$$

then

$$-1.57 \times 10^{-2} = 0.02193 \frac{(T'+e^{-T'}-1)z + (1-T'e^{-T'}-e^{-T'})}{(z-1)(z-e^{-T'})} \qquad (4.11)$$

by substitution, we have the approximate critical sampling period as

$$Tcr = 30 \text{ ms.}$$

4.3 The Nyquist Diagram

From equation (4.6), we make a Nyquist plot for Go(z) at verious sampling period T.

For T = 2 ms

$$Go(z) = \frac{0.0001864z+0.0001821}{(z-1)(z-0.8743)} \quad (4.12)$$

For T = 20 ms

$$Go(z) = \frac{0.01322z+0.00852}{(z-1)(z-0.2611)} \quad (4.13)$$

For T = 50 ms

$$Go(z) = \frac{0.0524z+ 0.01859}{(z-1)(z-0.0349)} \quad (4.14)$$

The values of Go(z) at various sampling periods and angles are tabulated as follows :

Z	T = 2 ms	T = 20 ms	T = 50 ms
$1\angle 45^\circ$	$5.720\times10^{-4}\angle 167^\circ$	$0.03150\angle -142.6^\circ$	$0.119\angle 168.2^\circ$
$1\angle 90^\circ$	$1.387\times10^{-4}\angle 139.5^\circ$	$0.01070\angle 177.6^\circ$	$0.0390\angle -154.1^\circ$
$1\angle 135^\circ$	$0.439\times10^{-4}\angle 114.2^\circ$	$0.00423\angle 148.7^\circ$	$0.0118\angle -160.7^\circ$
$1\angle 180^\circ$	$1.147\times10^{-6}\angle 180^\circ$	$0.00186\angle 180^\circ$	$0.0163\angle 180^\circ$
$1\angle 270^\circ$	$1.387\times10^{-4}\angle -139.5^\circ$	$0.01070\angle -177.6^\circ$	$0.0390\angle 154.1^\circ$
$1\angle 360^\circ$	∞	∞	∞

4.4. Sammary

The Nyquist plots for the open loop system at various sampling periods are shown in Figure 4.1, 4.2 and 4.3. The system is found to be stable for 2 ms and 20 ms sampling intervals. For the sampling interval of 50 ms the system is not stable, since the Go(z) locus encircles the $-\frac{1}{N(z)}$ locus.

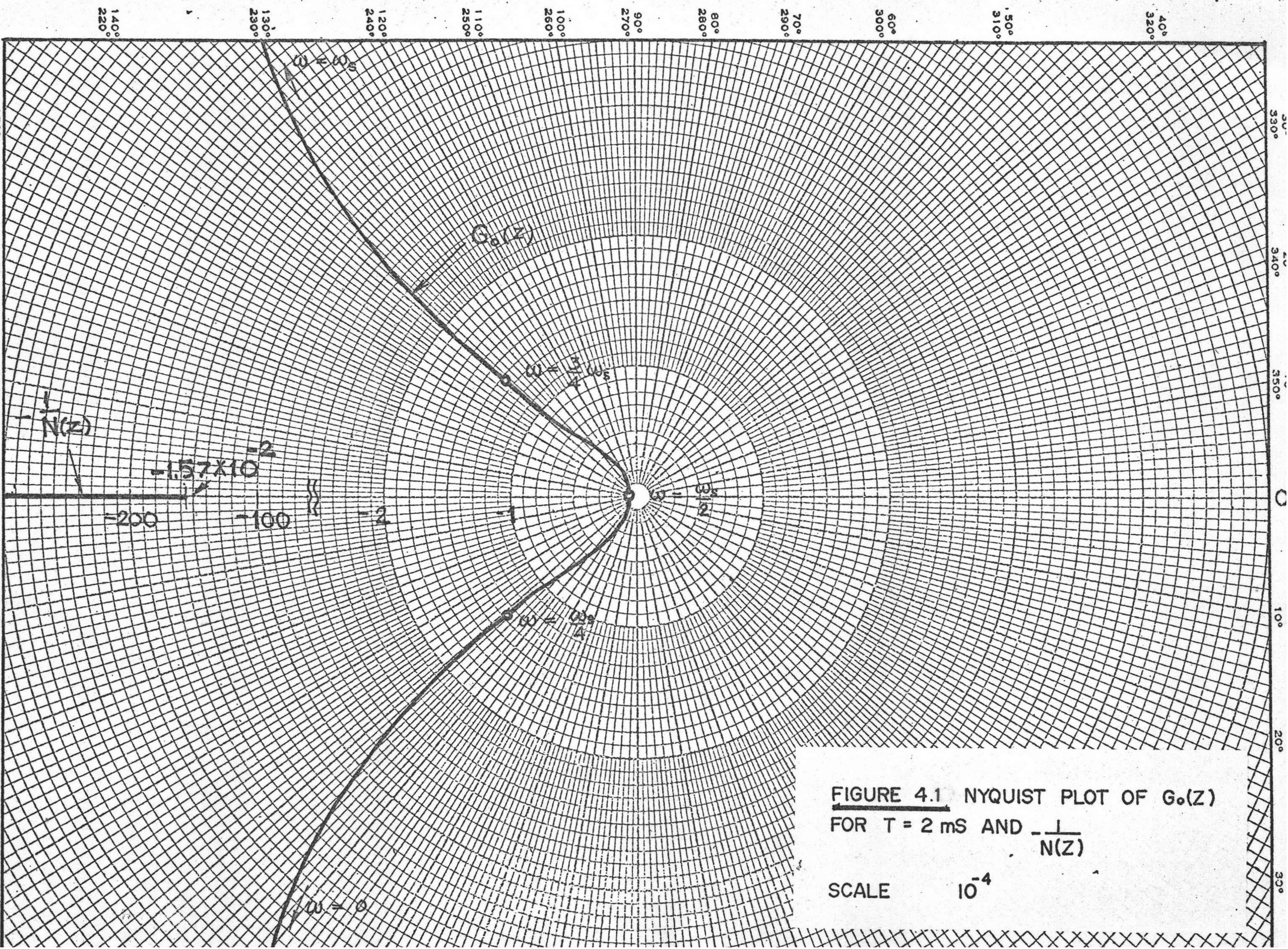

FIGURE 4.1 NYQUIST PLOT OF $G_o(Z)$ FOR T = 2 mS AND $-\frac{1}{N(Z)}$

SCALE 10^{-4}

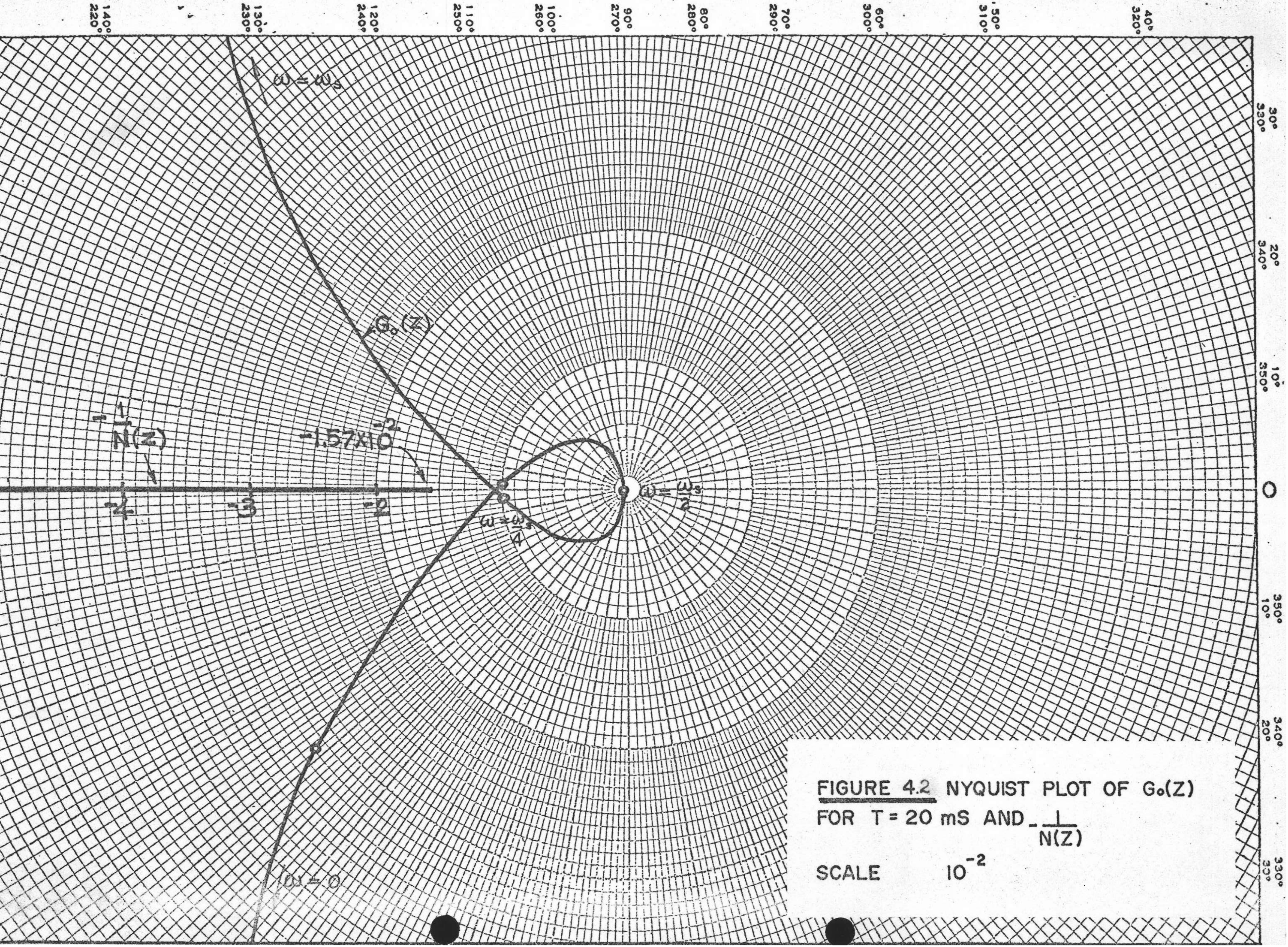

FIGURE 4.2 NYQUIST PLOT OF $G_0(Z)$ FOR T = 20 mS AND $-\frac{1}{N(Z)}$

SCALE 10^{-2}

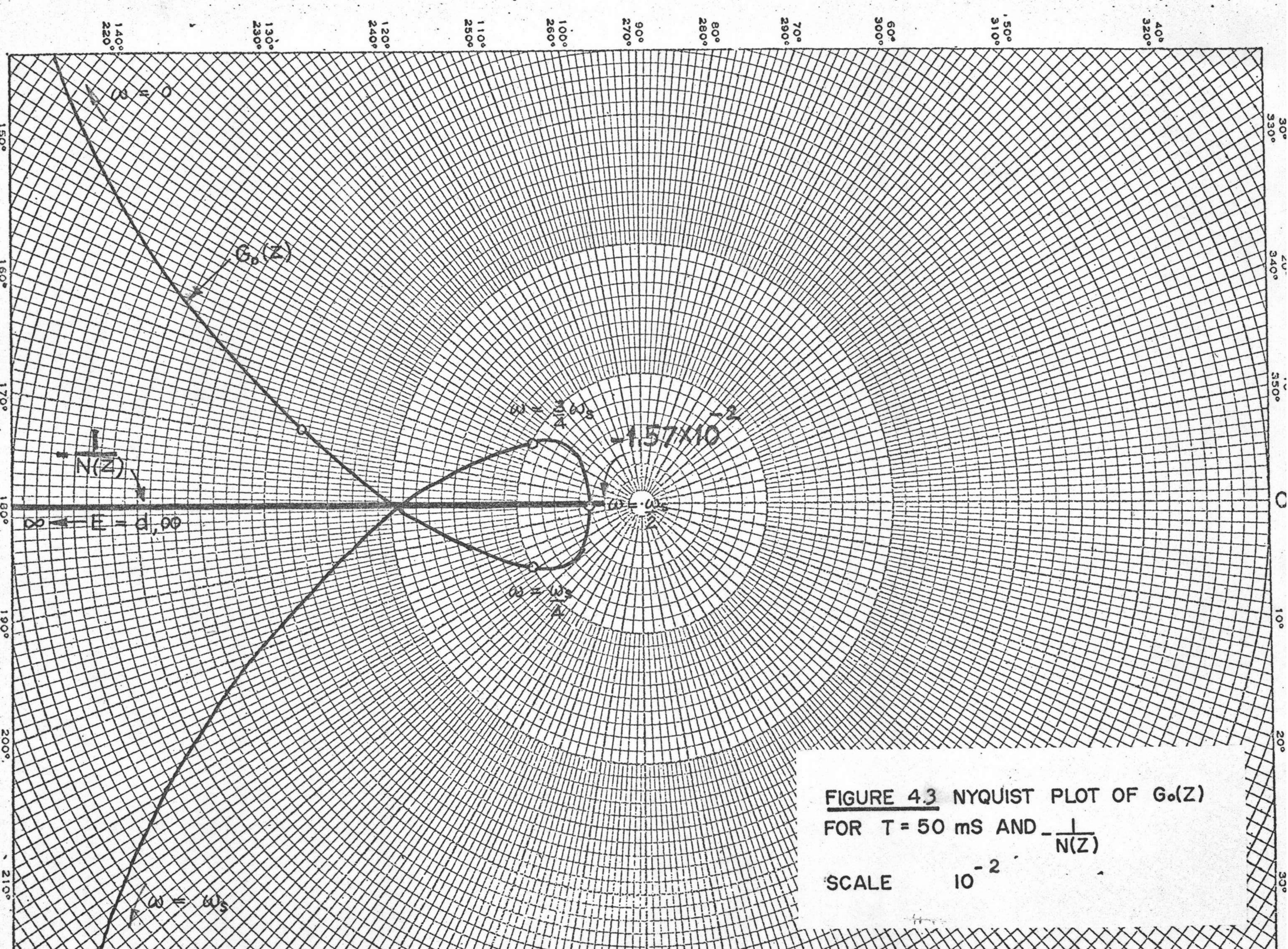

FIGURE 4.3 NYQUIST PLOT OF $G_o(Z)$ FOR $T = 50$ mS AND $-\frac{1}{N(Z)}$

SCALE 10^{-2}