CHAPTER 2

THEORY

2.1 General

A pulse-width modulation control system is shown in Figure 2.1. The desired angular position is set, there by controlling the sampling time of the sampler. In effect, the sampler generates a pulse of width Tr. called commanding pulse, which is proportional to the angular position being set. The feedback pulse, Tf, is also generated in the same manner. The two pulses are compared and an error pulse of width Te = Tr - Tf is generated. The error pulse is fed to a data reconstruction circuit and followed by an on-off switcher which switches on the amplifier to drive the motor. The motor is coupled to a load and a sampler by which the feedback pulse is generated.

2.2 The System

The pulse-width modulation control system shown in Figure 2.1 can be rearranged and shown in Figure 2.2. The signal flow graph may be represented as shown in Figure 2.3. Then we have[2]

$$E^*(S) = Ksi\ R^*(S) - Ksf\ C^*(S) \qquad (2.1)$$

$$C(S) = \frac{KA}{n} N^*(S) Gho(S) Gm(S)\ E^*(S) \qquad (2.2)$$

Equation 2.2 can be writen as

$$C^*(S) = \frac{KA}{n} N^*(S)\ \overline{Gho\ Gm}^*(S)\ E^*(S) \qquad (2.3)$$

Substitute $E^*(S)$ in equation 2.2 we have

$$C^*(S) = \frac{KA}{n} N^*(S)\ \overline{Gho\ Gm}^*(S) \left[Ksi\ R^*(S) - Ksf\ C^*(S)\right] \qquad (2.4)$$

and

$$\frac{C^*(S)}{R^*(S)} = Ksi \frac{KA}{n} \frac{N^*(S)\ \overline{Gho\ Gm}^*(S)}{1 + \frac{Ksf\ KA}{n} N^*(S)\ \overline{Gho\ Gm}^*(S)} \qquad (2.5)$$

Hence

$$\frac{C(Z)}{R(Z)} = Ksi \frac{KA}{n} \frac{N(Z)\ Gho\ Gm(Z)}{1 + Ksf \frac{KA}{n} N(Z)\ Gho\ Gm(Z)} \qquad (2.6)$$

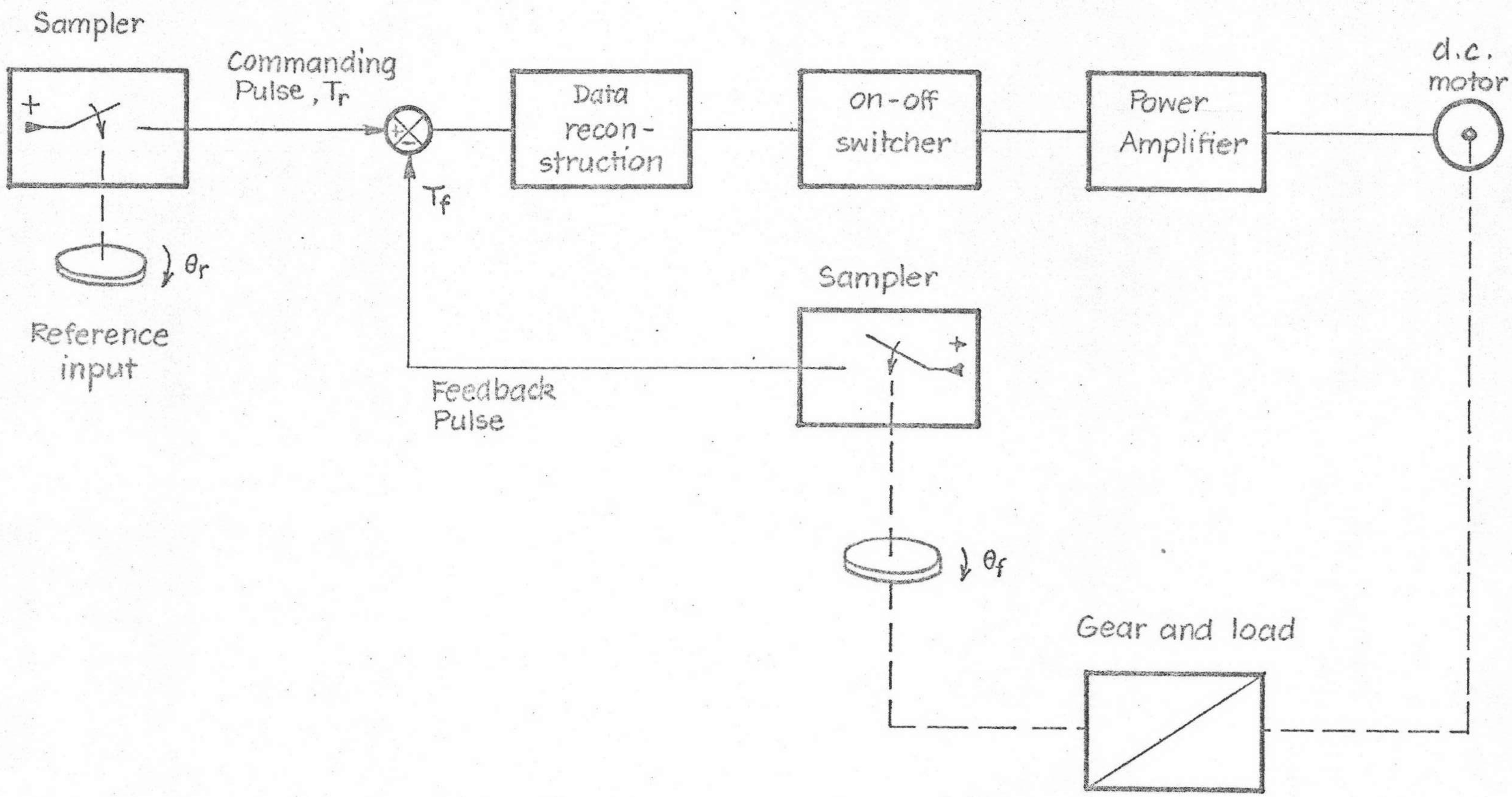

Figure 2.1 A pulse-width-modulation control system

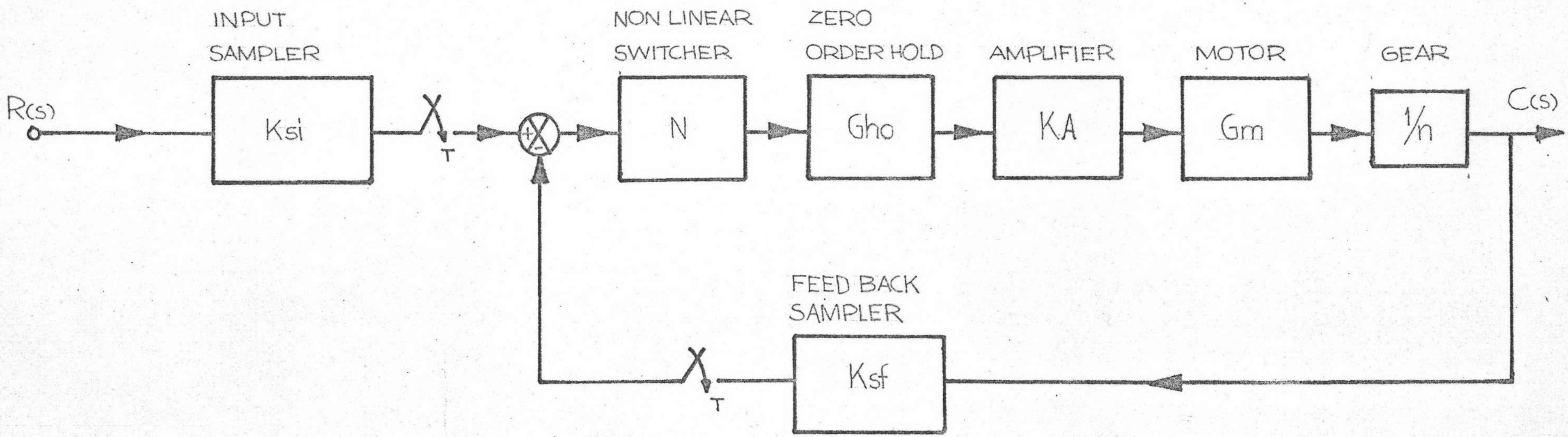

FIGURE 2.2 THE BLOCK DIAGRAM OF THE PULSE WIDTH MODULATION CONTROL SYSTEM

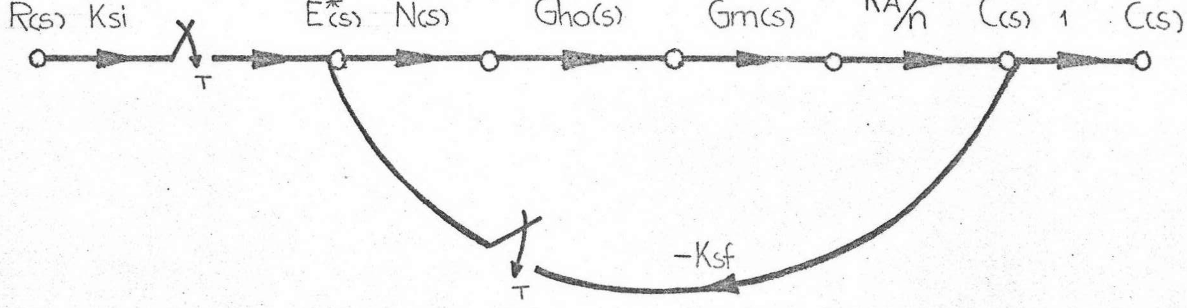

FIGURE 2.3 THE SIGNAL FLOW GRAPH OF THE SYSTEM
SHOWN IN FIGURE 2.2

The Characteristic equation is

$$1 + Ksf\ \frac{KA}{n}\ N(Z)\ Gho\ Gm\ (Z) = 0 \qquad (2.7)$$

then

$$Ksf\ \frac{KA}{n}\ Gho\ Gm\ (Z) = -\frac{1}{N(Z)} \qquad (2.8)$$

The open loop transfer function of the system without nonlinear element is

$$Go(Z) = Ksf\ \frac{KA}{n}\ Gho\ Gm\ (Z) \qquad (2.9)$$

If equation 2.8 is satisfied, then the system is unstable [2] and exhibit a limit cycle. This situation corresponds to the case where the KA $\frac{Ksf}{n}$ Gho Gm(Z) locus passes through the critical point.

In conventional frequency response analysis of linear control system. the critical point is the (-1, +j0) point.

In the describing function analysis, $-\frac{1}{N(Z)}$ locus becomes a locus of critical points.

To determine the stability of the system the $-\frac{1}{N(Z)}$ locus and the open loop transfer function are plotted the criterion for stability is [2,6]

1. The open - loop system is stable if the Nyquist plot of the open loop transfer function does not enclose the (- 1, + j0) point to ensure that the closed loops system is also stable.

2. The plot of the open loop transfer function does not encircle the $-\frac{1}{N(Z)}$ locus.

2.3 The Discrete Describing Function N(Z)

In the conventional nonlinear continuous-data control system, the describing function of a nonlinear element is defined to be the complex ratio of the fundamental harmonic component of the output to the sinusoidal input

For nonlinear sampled-data system, the discrete describing function is used which based on the assumption that the input signal to the nonlinear element is a sinusoidally modulated impulse train. The descrete describing function[2] is defined as the ratio of the Z - transform of the output $v^*(t)$ to the Z - transform of the input $e^*(t)$. That is

$$N(Z) = V(Z) / E(Z) \qquad (2.10)$$

Figure 2.4 shows the input and output characteristic curve for an on off nonlinearity, where as their waveforms are shown in Figure 2.5.

Consider Figure 2.5 (a)

$$e(t) = E \sin wt \qquad (2.11)$$

Taking Z - transformation[2], we have

$$E(Z) = \frac{E(Z \sin wT)}{Z^2 - 2Z \cos wT + 1} \qquad (2.12)$$

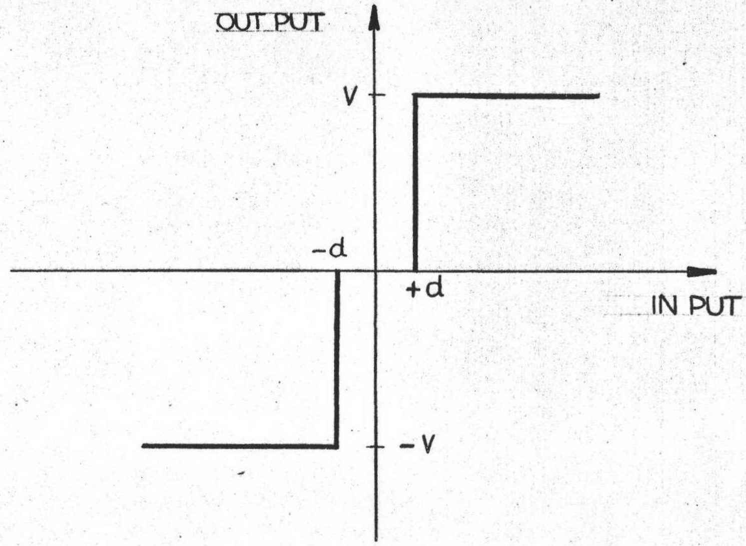

FIGURE 2.4 INPUT - OUTPUT CHARACTERISTIC CURVE FOR AN ON - OFF NONLINEARITY WITH DEAD ZONE

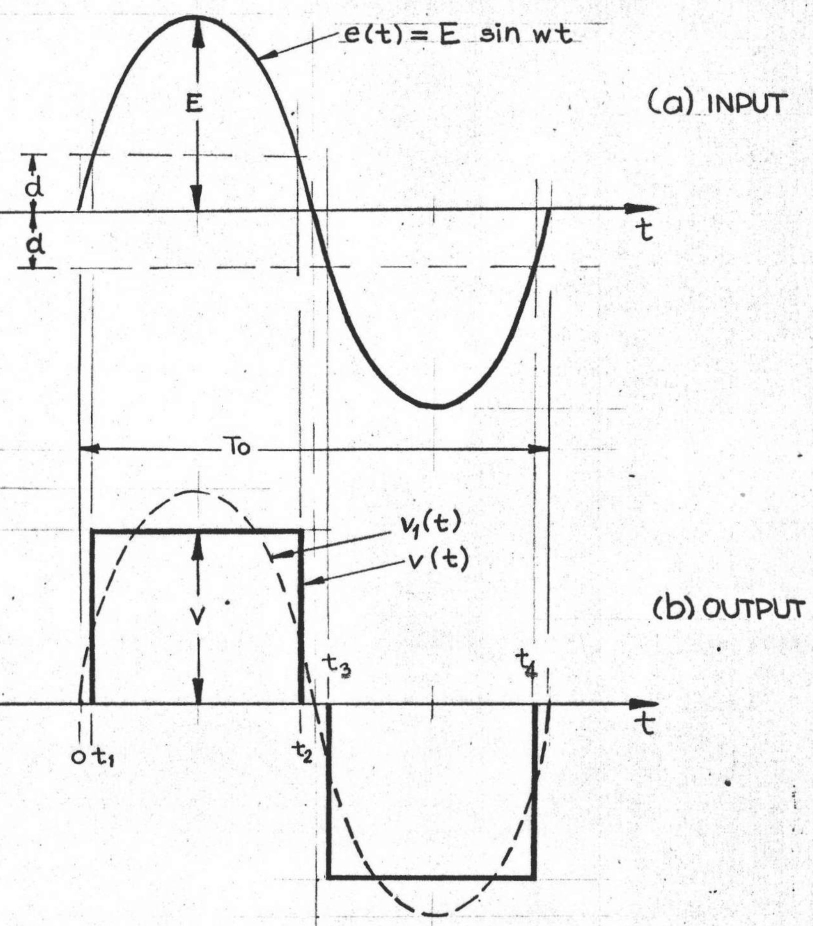

FIGURE 2.5 INPUT - OUTPUT WAVEFORMS FOR THE ON-OFF NONLINEARITY WITH DEAD ZONE

If the system is unstable, it will start oscillating. Since the period of self sustained oscillation[2] is an integral multiple of sampling period T, Let

$$T_o = \text{Period of oscillation}$$

$$m = \text{integer}$$

Then $$T_o = mT \qquad (2.13)$$

Consider Figure 2.5 (b), we may write

$$\frac{v(t)}{V} = U(t-t_1)-U(t-t_2)+U(t-t_3)+U(t-t_4)+\cdots \qquad (2.14)$$

The output may be expressed as a Fourier series as follows:[6]

$$v(t) = A_o + \sum_{n=1}^{\infty} (A_n \cos nwt + B_n \sin nwt) \qquad (2.15)$$

It is very complicated to derive the Z-transformation for equation (2.14) and (2.15). For approximation we assume that only the fundamental harmonic component of the output is significant. Since the output is an odd function, hence the fundamental harmonic component is

$$v_1(t) = B_1 \sin wt \qquad (2.16)$$

Where $$B_1 = \frac{1}{\pi}\int_o^{2\pi} v(t) \sin wt \, d(wt) \qquad (2.17)$$

$$= \frac{4V}{\pi} \cos wt_1 \qquad (2.18)$$

Since $$\sin wt_1 = d/E \qquad (2.19)$$

therefore $$B_1 = \frac{4V}{\pi}\sqrt{1-\left(\frac{d}{E}\right)^2} \qquad (2.20)$$

Then we have

$$v_1(t) = \frac{4V}{\pi} \cdot \sqrt{1-\left(\frac{d}{E}\right)^2} \quad \sin wt \tag{2.21}$$

Taking Z-transformation, we have

$$V(Z) = \frac{4V}{\pi} \sqrt{1-\left(\frac{d}{E}\right)^2} \; \frac{(Z \sin wT)}{Z^2-2Z\cos wT+1} \tag{2.22}$$

From equation (2.10), we have

$$-\frac{1}{N(Z)} = -\frac{E(Z)}{V(Z)} \tag{2.23}$$

Substitute for E(Z) and V(Z), equation (2.23) becomes

$$-\frac{1}{N(Z)} = -\frac{\pi E^2}{4V} \quad (E^2 - d^2)^{-\frac{1}{2}} \tag{2.24}$$

We can find that the $\frac{1}{N(Z)}$ term will have a minimum absolute value at

$$E\left(\frac{1}{N(Z)} \text{ min.}\right) = \sqrt{2}\, d \tag{2.25}$$

and

$$E\left(\frac{1}{N(Z)} = \infty\right) = d, \infty \tag{2.26}$$

Then we have , **for V = E = maximum error,**

$$\frac{1}{N(Z)} \text{ min.} = \frac{\pi}{2} \times \frac{d}{E}$$

$$= \frac{\pi}{2} \times (\text{ \% dead-zone}) \tag{2.27}$$