# CHAPTER V

# EVALUATION PROCUDURES OF TRAFFIC FLOW MODELS

## Background

May and Keller[14] showed that microscopic and macroscopic theories of traffic flow can be reduced to the equation of the general car-following model, formulated by Gazis, Herman, and Rothery.[13] The equation is as follows

$$\ddot{x}_{n+1}(t+T) = \frac{\alpha\left[\dot{x}_{n+1}(t+T)\right]^{m}}{\left[x_n(t) - x_{n+1}(t)\right]^{l}} \left[\dot{x}_n(t) - \dot{x}_{n+1}(t)\right] \quad \text{- - - (25)}$$

In the above equation $\ddot{x} = d\dot{x}/dt$, $\dot{x} = dx/dt$, and x is the position of the $n^{th}$ or $(n+1)^{th}$ vehicle at time t or (t+T), respectively. T, $\alpha$, m and l are constant.

## Matrix Development

The steady-state flow formulation of this equation can be obtained by integrating the above equation and is given by Gazis et al to be

$$f_m(u) = c' + cf_l(s) \quad \text{- - - - - - - - - (26)}$$

where u = steady-state speed of a traffic stream,

s = constant average spacing, and

c and c' = some appropriate constants, consistent with physical restrictions.

The integration constant c' is related to a free speed, $u_f$, or a jam spacing, $s_j$, depending on the values of m and l. The jam spacing, $s_j$, can be transformed to jam density, $k_j$, by $s_j = 1/k_j$. By selecting proper combinations of the exponents m and l in Eq. 25 and 26, known microscopic and macroscopic traffic flow models can be obtained.

Equation 26 has six variables, two of which (speed u and spacing s) can be observed directly. While c' and c $\left[c'=f(c)\right]$ can be determined by regression analysis for a set of observations of u and s $\left[s = f(k), \text{ where } k \text{ is density}\right]$, the exponent m and l remain unknown.

By using this general solution of Gazis et al, a maxtrix of steady-flow equation for different m and l values was developed. The general expressions are shown in Table 3. An inspection of these two matrices reveals that all the previously reported microscopic and macroscopic models and several other possible models can be located in terms of m and l combinations.

| l \ m | $m<1$ | $m = 1$ | $m>1$ |
|---|---|---|---|
| $l<1$ | $u^{1-m}=ck_j^{l-1}+ck^{l-1}$ | Boundary conditions not satisfied | $u^{1-m}=u_f^{1-m}+ck^{l-1}$ |
| $l=1$ | $u^{1-m}=c\ln(1/k_j)+c\ln(1/k)$ | $\ln u=c\ln(1/k_j)+c\ln(1/k)$ | $u^{1-m}= c\ln(1/k_j)+c\ln(1/k)$ |
| $l>1$ | $u^{1-m}=ck_j^{l-1}+ck^{l-1}$ | $\ln u=\ln u_f+ck^{l-1}$ | $u_f^{1-m}=u_f^{1-m}+ck^{l-1}$ |

Table 3 Matrix of steady-state flow equations for different m,l values in $f_m(u) = c' + cf_l(s)$

By choosing particular m and l combinations, a wide variety of shaped curves for the speed-density relation can be selected (Figure 21). One also can recognize certain trends in the shape of the curves by keeping one of the expenents, m or l, constant. It should be noted that non-integer m and l values can be utilized, and consequently an expression can be determined which more closely represents actual speed-density relation. The curve is shown in Figure 22.

The matrix of m and l values has permitted the development of analytical techniques for evaluating deterministic traffic-flow models using speed-density measurements.

<u>Analytical Procedure for Evaluating Deterministic Integer and Non-integer Traffic-flow Models</u>

The speed-density relation rather than the flow-density or speed-flow relation was selected as the relationship for evaluation. Once this equation is evaluated, the other relationships can be obtained by using the steady-state equation $q = uk$. The speed-density has the advantage of being easier to handle mathematically.

Variety of different models can be obtained through the selection of different m, l values. To justify or to conclude on a certain m, l combination, it proves to be very valuable to use m and l as coordinates of a plane and to project on this plane some certain parameters. The study of this behavior on the m, l matrix permits very constraining conclusions as to m, l combinations that best represent speed-density data sets. To define a specific equation, the 4 parameters m, l, c and c' must be determined.

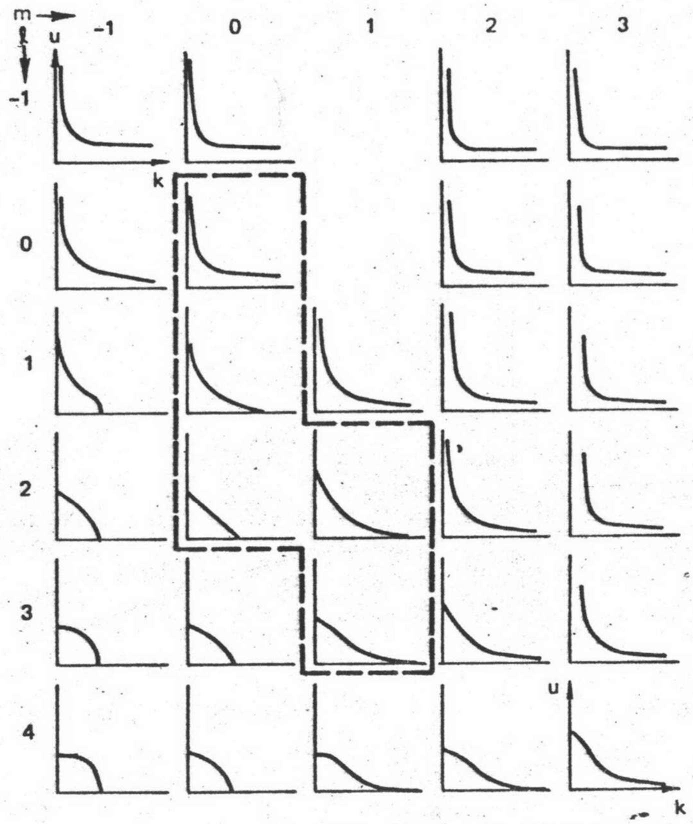


Figure 21 Matrix of speed-density relationships for various *m*, *l* combinations of the general car-following equation. (Dashed lines enclose limiting values of *l* and *m* used in Table 8 )

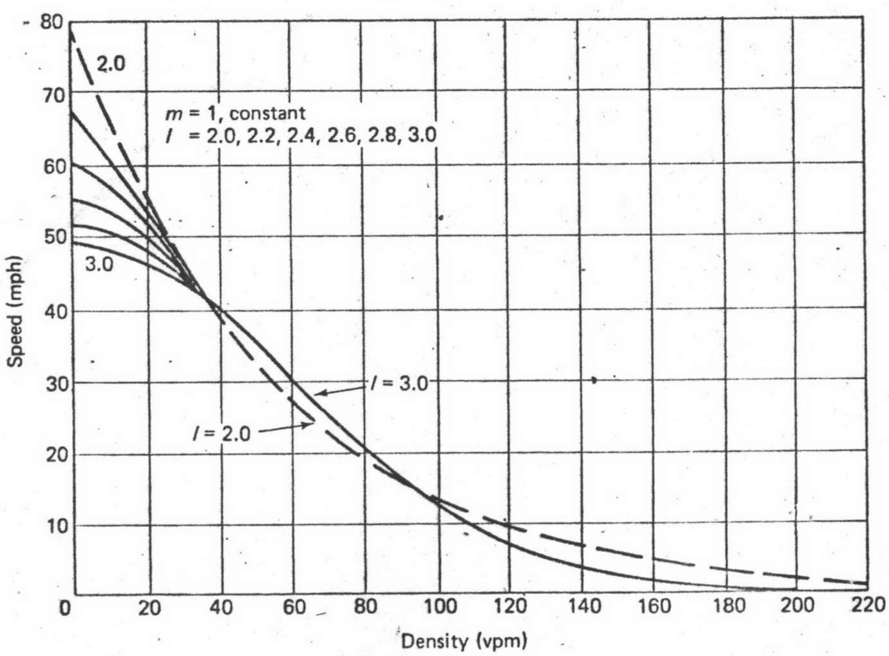


Figure 22 Influence of the use of non-integer exponents on the speed-density relation. (Source: May and Keller, "Non-Integer Car-Following Models," p. 24.)

Two types of parameters were used to design a procedure for the evaluation of different traffic flow models.

1) The statistical parameter used was the mean deviation,s. It is defined as the square root of the sum of squqred deviation of observations from the model considered, divided by the number of observation:

$$s = \sqrt{\frac{\sum(u-u_{estimated})^2}{n}} \quad \text{- - - - - - -} \quad (27)$$

and taking this as a measurement of the goodness of fit.

2) Parameters based on flow characteristics included free flow speed, jam density, and maximum flow.

For evaluation, the results (mean deviation, free-flow speed, jam density, and maximum flow) are plotted in an m, l plane and developed curves of equal mean deviation, free flow speed, jam density, and maximum flow. The trends of these curves show how specific m, l combinations or models fit the requirements.

The mean deviation contour lines in the m, l matrix for Sukhumvit Road, as an example,are shown in Figure F-1.That portion of the m, l matrix which meets the statistical criteria is shaded.

The free flow speed contour lines in the m, l matrix for Sukhumvit Road, as an example, are shown in Figure F-2. The shaded area of the figures indicate that portion of the m, l matrix which meets the free flow speed criteria. A finite value for the free flow velocity exists only for traffic flow model with $l > 1$.

The jam density contour lines in the m, l matrix for Sukhumvit Road, as an example, are shown in Figure F-3. The shaded area of the figures indicate that portion of the m, l matrix which meets the jam density criteria. A finite value for the jam density exists only for traffic flow models with $m<1$.

The maximum flow contour lines in the m, l matrix for Sukhumvit Road, as an example, are shown in Figure F-4. The shaded area of the figures indicate that portion of the m, l matrix which meets the maximum flow criteria.

After assigning constraints to the parameters, preselected statistical and traffic flow criteria, the previous individual m, l matrices were overlayed and the superposition of these matrices give the area or point on the m, l matrix where all or most criteria are satisfied, as shown in Figure F-5 for Sukhumvit Road, as an example. This area or point in the m, l plane is considered to be an indication of the best m, l combination.

This procedure was applied to single-regime traffic flow models for 17 data sets by using computers. Flow chart and computer porgram for this evaluation are shown in Appendix E. Sample of results of analytical procedure are shown in Appendix F.

For the single-regime model, only models with an x-intercept (jam density) and a y-intercept (free-flow speed) were considered. This condition limited the investigation of the m, l matrix to the region where $m<1$ and $l>1$. Further, it was required that in Eq.25 the speed function and the spacing function of the sensitivity component remain in the numerator and denominator, respectively. This condition limited the investigation of the m, l matrix to the region where $m\geqslant 0$ and $l\geqslant 0$. The combination of these two requirements restricted the investigation of the m, l matrix to the region where $0\leqslant m\leqslant 1$ and $l>1$.

Data Selection

Since the analysis was aimed at evaluating a number of functional relations over the full range of operating characteristics, it was necessary to give equal consideration to all flow conditions which might occur. Sampling procedure was designed so as to efficiently collect a relevant set of unbiased data. The feature of the observed original data set was the points seemed to occur in bunches. One solution to the problem would have been to arrange the set of one-minute observation in the order of increasing density and divide into ranges of approximately 5 vehicles per kilometer. The number of observations falling in the most sparse 5 vehicles per kilometer was determined and a like number of data points was randomly sampled from each of the other ranges. This statistical procedure resulted in a considerable degree of uniformity of sample density throughout the range of the independent variable, selected sample of observations, as shown in Figure D-1 to D-51, particularly when compared with observed data in Figure C-1 to C-17.

The 17 data sets of selected sample of speed-density observations, input data, were analyzed by this evaluation process. Some characteristics of each data sets are shown in Table 4. As can be seen from the plot of the data as shown in Figure D-1 to D-51.

Table 4 Some characteristics of the data sets.

| Streets | number of observations | speed characteristics | | | density characteristics | | |
|---|---|---|---|---|---|---|---|
| | | lowest speed | highest speed | mean speed | lowest density | highest density | mean density |
| 1. Rama I Road | 59 | 2.1 | 65.6 | 19.3 | 12.8 | 520.3 | 199.1 |
| 2. Rama IV Road | 40 | 14.0 | 76.4 | 39.0 | 13.4 | 300.0 | 123.5 |
| 3. Yaowaraj Road | 68 | 3.5 | 45.4 | 12.5 | 18.5 | 535.7 | 295.0 |
| 4. Ratchadamnoen Khang Road | 35 | 12.5 | 48.4 | 31.5 | 24.8 | 260.0 | 121.7 |
| 5. Phaholyothin Road | 41 | 5.9 | 56.4 | 23.1 | 11.7 | 303.5 | 130.7 |
| 6. Sukhumvit Road | 33 | 10.0 | 56.0 | 31.9 | 8.1 | 206.5 | 92.5 |
| 7. New Petchbury Road | 37 | 5.9 | 72.4 | 29.9 | 18.2 | 294.5 | 125.6 |
| 8. Raj Prarop Road | 48 | 3.3 | 43.3 | 16.4 | 18.0 | 305.3 | 138.8 |
| 9. Charoen Krung Road | 24 | 8.0 | 36.8 | 23.0 | 31.0 | 226.1 | 106.7 |
| 10. Raj Vithee Road | 31 | 5.1 | 66.7 | 26.1 | 10.8 | 214.0 | 98.6 |
| 11. Lat Phrao Road | 25 | 13.0 | 67.3 | 38.6 | 9.8 | 208.5 | 82.8 |
| 12. Phrachao Taksin Road | 26 | 12.5 | 52.1 | 31.1 | 27.5 | 172.5 | 93.5 |
| 13. Phran Nok Road | 26 | 8.0 | 58.3 | 27.7 | 9.9 | 202.5 | 83.6 |
| 14. Ramkhamhaeng Road | 26 | 11.5 | 91.2 | 35.2 | 12.5 | 214.5 | 91.2 |
| 15. Soi Sena Nikhom 1 | 28 | 11.9 | 51.9 | 32.0 | 2.8 | 95.5 | 39.7 |
| 16. Soi Aree | 20 | 7.1 | 35.5 | 19.0 | 2.8 | 71.5 | 29.3 |
| 17. Chula Soi 12 | 26 | 5.5 | 40.9 | 26.0 | 2.9 | 77.7 | 34.7 |

Criteria for Selection of the Best Models

For the single-regime model, four criteria were used to select the best model: mean deviation, jam density, free-flow speed, and maximum flow.

The criteria preselected for the statistical parameter was that those traffic flow models (m, l combinations) which exhibited a mean deviation (s) within 10% of minimum mean deviation would meet this statistical criteria. The maximum acceptable mean deviation therefore were shown in Table 5.

Traffic flow characteristic levels were also preselected in order to evaluate and select the most representative traffic flow model or models. According to the situation at each location, Table 5 indicates the range in free flow speed, jam density, and maximum flow which would be acceptable: traffic flow models which exhibited free flow speed, jam density, and maximum flow values within the indicated ranges met these criteria. The selection of these acceptable ranges were based on knowledge of the locations and other flow studies as well as examination of the speed-density data point plots.

Table 5 The criteria preselected for statistical and traffic flow characteristic parameters.

| streets | free flow speed(kph) | jam density (vpk) | maximum flow(vph) | mean deviation (kph) |
|---|---|---|---|---|
| 1. Rama I Road | 50-65 | 375-600 | 2200-3500 | 4.7 |
| 2. Rama IV Road | 60-70 | 375-600 | 5000-5500 | 5.2 |
| 3. Yaowaraj Road | 40-50 | 375-600 | 2600-3500 | 2.0 |
| 4. Ratchadamnoen Khang Road | 50-65 | 500-800 | 3800-4200 | 5.6 |
| 5. Phaholyothin Road | 60-70 | 250-400 | 2200-2800 | 3.4 |
| 6. Sukhumvit Road | 55-65 | 250-400 | 2600-3100 | 3.3 |
| 7. New Petchbury Road | 60-75 | 250-400 | 3000-3900 | 9.9 |
| 8. Raj Prarop Road | 35-55 | 250-400 | 1800-2500 | 3.7 |
| 9. Charoen Krung Road | 40-50 | 250-400 | 2200-2600 | 2.2 |
| 10. Raj Vithee Road | 60-75 | 250-400 | 1800-2600 | 4.4 |
| 11. Lat Phrao Road | 60-70 | 250-400 | 2800-3400 | 3.3 |
| 12. Phrachao Taksin Road | 55-70 | 250-400 | 2600-3200 | 3.3 |
| 13. Phran Nok Road | 45-60 | 250-400 | 2000-2500 | 2.8 |
| 14. Ramkhamhaeng Road | 60-70 | 250-400 | 2600-3000 | 5.1 |
| 15. Soi Sena Nikhom 1 | 50-60 | 125-200 | 1300-1600 | 3.6 |
| 16. Soi Aree | 30-40 | 125-200 | 460-600 | 3.6 |
| 17. Chula Soi 12 | 35-45 | 125-200 | 700-1000 | 3.4 |