CHAPTER 2

THE DIFFUSION MODEL

2.1. Description of the Diffusion Model

In this chapter the diffusion model is suggested to describe the behaviour of the mass transfer within an extraction column [8,9]. In this model the two phases flow inopposite directions, with uniform superficial velocities F_x and F_y with each phases undergoing longitudinal dispersion, where mass transfer occurs from one phase to another. For purposes of mathematical modelling back mixing is represented by a piston diffusion flow in both phases. This is represented schematically in Figure 2-1.

In many actual two phase flow operations, one phase remains dispersed in the form of liquid drops. The actual situation in a counter current liquid - liquid extraction column may be described by Figure 2-2. The term back - mixing refers to a situation in which each phase undergoes some sort of mixing caused by back flow and transverse flow.

It is found that the concentrations within the equipment and at the outlet are shown to depend upon four dimension less parameters. There are functions of the dispersion rates and velocities, the equilibrium partition coefficient, and the "true" over-all mass transfer coefficient.

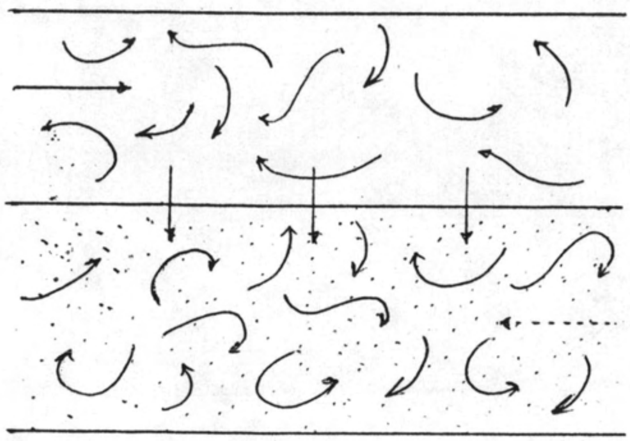


Figure 2-1

Schematic representation of piston diffusion flow in both phases.

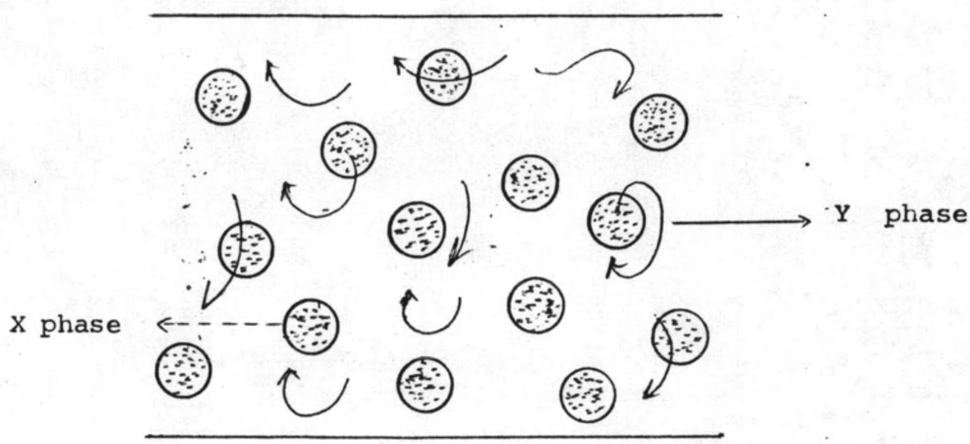


Figure 2-2

A description of back mixing in a countercurrent extraction column.

2.2. Derivation of the Miyauchi Model differential equations and boundary conditions.

The diffusion model to be derived in this section assumes piston flow in both phases with mass transfer between the two phases. The solute balance in differential element of column as shown in Figure 2-3 for the case z=0 at the bottom of column, Y phase enters at the bottom, X phase enters at the top, z=L ; mass transfer occurs from phase Y to Phase X.

For the Y phase:

$$u_y A_T (1-\emptyset) Y \Big|_{z} - u_y A_T (1-\emptyset) Y \Big|_{z+\Delta z} + J_y A_T \Big|_{z} - J_y A_T \Big|_{z+\Delta z}$$

$$-K_y . a' . A_T . \Delta z . (Y - Y^*) = 0 \qquad (2\text{-}1)$$

$$(1-\emptyset)\, u_y \frac{\left(-Y\Big|_{z+\Delta z} + Y\Big|_{z}\right)}{\Delta z} + \frac{\left(D_y \frac{dy}{dz}\Big|_{z+\Delta z} - D_y \frac{dy}{dz}\Big|_{z}\right)}{\Delta z}$$

$$-K_y . a' . (Y - Y^*) . = 0 \qquad (2\text{-}2)$$

taking the limit as $\Delta z \longrightarrow 0$

$$-(1-\emptyset)\, u_y \frac{dY}{dz} + D_y \frac{d^2Y}{dz^2} - K_y . a' . (Y - Y^*) = 0 \qquad (2\text{-}3)$$

The equation for the other phase (X phase) is derived as above and taking the limit as $\Delta z \longrightarrow 0$ one obtains

$$\emptyset u_x \frac{dX}{dz} + D_x \frac{d^2X}{dz^2} + K_y . a' . (Y - Y^*) = 0 \qquad (2\text{-}4)$$

Upon rearrangement the equations of the diffusion model may be presented as follows with P_x, P_y and R_y as independent parameters.

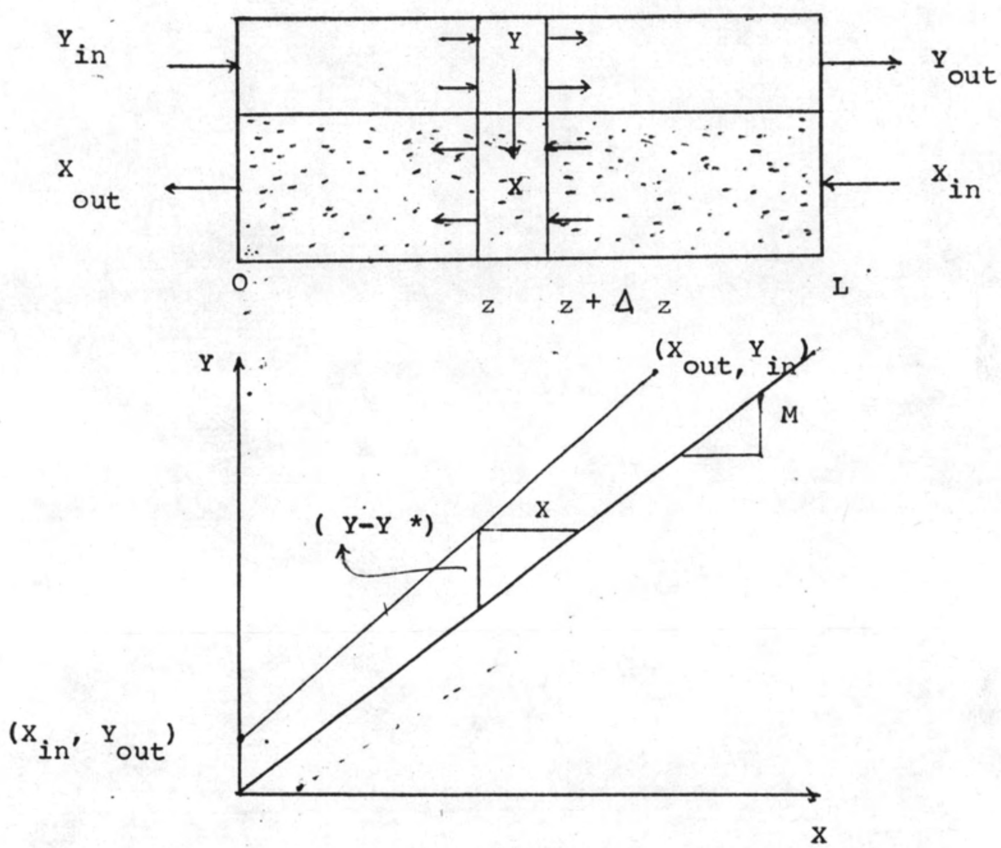


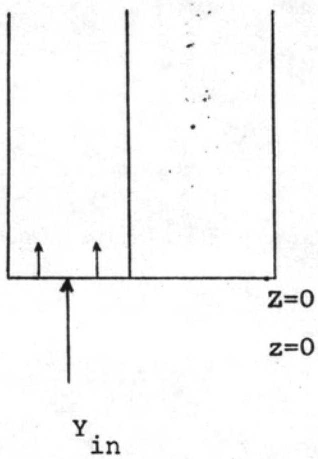
Figure 2-3

Schematic diagram of column and equilibrium diagram.

$$\frac{1}{P_y}\frac{d^2Y}{dZ^2} - \frac{dY}{dZ} - R_y\,(Y-Y^*) = 0 \qquad (2\text{-}5)$$

$$\frac{1}{P_x}\frac{d^2X}{dZ^2} + \frac{dX}{dZ} + R_x\,(Y-Y^*) = 0 \qquad (2\text{-}6)$$

A solute balance at the boundaries for the case of mass transfer from $Y \longrightarrow X$, gives the following boundary conditions.



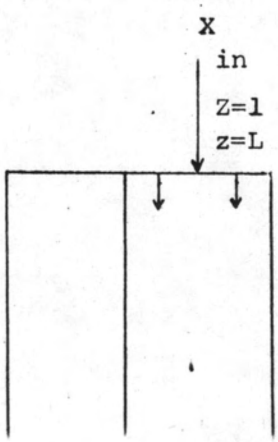
At $Z = 0$

$$\frac{dX(0)}{dZ} = 0 \qquad (2\text{-}7a)$$

$$G_y\,Y_{in} = A_T(1-\emptyset)u_y\,Y(0^+) + J_yA_T\Big|_{z=0}$$

$$F_y\,Y_{in} = F_yY(0^+) - D_y\frac{dY(0^+)}{dz}\Big|_{z=0}$$

$$Y_{in} = Y(0^+) - \frac{1}{P_y}\frac{dY(0^+)}{dZ} \qquad (2\text{-}7b)$$



At $Z = 1$

$$\frac{dY(1)}{dZ} = 0 \qquad (2\text{-}8a)$$

$$G_x\,X_{in} = A_T\emptyset u_x X(1^-) + J_xA_T\Big|_{z=L}$$

$$F_x\,X_{in} = F_xX(1^-) + D_x\frac{dX(1^-)}{dz}\Big|_{z=L}$$

$$X_{in} = X(1^-) + \frac{1}{P_x}\frac{dX}{dZ}(1^-) \qquad (2\text{-}8b)$$